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Saltari, Enrico and Federici, Daniela

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Daniela Federici*– Enrico Saltari†

Abstract

The aim of this paper is to investigate the roots of the Italian total factor productivity slowdown. The analysis focusses on the specific pattern of technical progress in determining the TFP dynamics. This analysis can not be done with the Cobb-Douglas technology but requires the employ of a CES function which allows to distinguish between the direction and the bias of technical progress. We employ a CES specification embodying both labor- and capital-augmenting technical change, with a σ less than 1. We obtain three main results. 1) There seems to have been a structural break around the mid-nineties in the direction and bias of technological change; 2) The first half of the sample features a labor-augmenting technical change and a capital bias; 3) In the second part of the sample both these characteristics seem to disappear, and factor endowments evolution assumes a key role. This fact may be view as one of the potential causes of the Italian productivity stagnation.

JEL classification: C30; E 22; E23; O33.

Keywords: CES production function; Elasticity of substitution; Factor-augmenting technical progress and ICT technical change.

*University of Cassino and Southern Lazio. Email: d.federici@unicas.it.

†Sapienza, University of Rome. Email: enrico.saltari@uniroma1.it. Corresponding author.

1 Introduction

In this paper we build on the dynamic model of the Italian economy of Saltari et al. (2012, 2013). The main result of those papers is that the weakness of the Italian economy in the last two decades has been the total factor productivity (TFP) slowdown. The aim of this paper is to investigate the roots of this slowdown. The analysis focusses on the specific pattern of technical progress in determining the TFP dynamics. Of course, this analysis can not be done with the Cobb-Douglas technology, where technical progress is only Hicks neutral, but requires a CES production function which allows to distinguish between the direction and the bias of technical progress.

Differently from most of the literature, this investigation employs a CES specification with both labor- and capital-augmenting technical change. While for labor input we keep the traditional constant growth rate representation, for capital we impose a particular structure with ICT capital playing a key role. Moreover, in this exercise we do not calibrate the parameters of the CES production function but use our estimated values. It should be noted that the estimated elasticity of substitution is less than 1. Such as a value is by now well-grounded in the empirical literature (see for instance León-Ledesma 2010; for a critical discussion of the traditional methodology in estimating the elasticity of substitution, see Federici and Saltari 2014). On theoretical grounds, a $\sigma > 1$ implies that any amount of output can be produced with either zero amount of capital or zero amount of labor, which is clearly absurd (note that the Cobb Douglas almost shares this last property). The data on Italian economy refers to the period 1981:Q4–2005:Q2.[1]

We obtain three main results. 1) There seems to have been a structural break around the mid-nineties, i.e., at half of the sample, in the direction and bias of technological change; 2) The first half features a labor-augmenting technical change and a capital bias; 3) In the last part of the sample both these characteristics seem to disappear, and factor endowments evolution assumes a key role. The disappearance of technical progress contribution may be viewed as one of the potential causes of the Italian productivity stagnation.

The paper is organized as follows. The next section briefly recalls our production function and normalizes it. Section 3 compares the Cobb-Douglas and CES TFP computation; it also discusses the determinants of technological progress. Section 4 describes the evolution of the direction and factor bias. Section 5 concludes.

[1] The dataset is available from the authors upon request.

2 The technology

Our theoretical framework is one of dynamic disequilibrium with traditional and ICT investment functions, skilled and unskilled labour sectors, and price determination under imperfect competition (for details, see Saltari et al. 2012).

The production technology is given by the following CES aggregate production function

$$Y_t = \beta_3 \left[\left(C_t{}^{\gamma} K_t \right)^{-\beta_1} + \left(\beta_2 e^{\mu_K t} L \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}}. \qquad (1)$$

In equation (1), β_3 is a measure of the TFP and β_1 defines the elasticity of substitution through the relation $\sigma = \frac{1}{1+\beta_1}$. Moreover, we have two factor-augmenting technical progress. The efficiency of traditional capital is augmented by ICT capital, C, with a weighting factor equal to γ, a proxy of the relative share of the ICT in total capital. As for labor-augmenting technical progress, we follow the bulk of the literature in assuming that it grows at a constant rate $\mu = \lambda_K + \gamma \lambda_C$, where λ_K and λ_C are the rates of technical progress in the use of capital K and innovative (information and communication technology, ICT) capital, C, with β_2 as a scaling factor. That way, labor efficiency partly depends on the growth of ICT capital through $\gamma \lambda_C$. Thus, differently from most of the literature, labor efficiency is closely linked to capital efficiency. Finally, L denotes employment.

The model allows us to estimate, among others, the parameters of the production function for the sample period 1981:Q4-2005:Q2. For the reader's convenience, the production function parameters' estimates are reported in table 1.

Table 1 Estimated parameters

Parameters	β_1	$\sigma = \frac{1}{1+\beta_1}$	β_2	β_3	γ	λ_K	λ_C	$\mu = \lambda_K + \gamma \lambda_C$
Estimates	0.52	0.66	27.07	0.87	0.05	0.00134	0.0365	0.003

Notice that β_3, λ_K, λ_C and thus μ are all expressed on a quarterly basis. Thus, for instance the yearly growth rate of labor efficiency is about 1.2%.

2.1 Normalization

We normalize the production function so that the variables are independent of the unit of measure, i.e., are in index number form. Normalization is necessary for a number of aspects, such as securing the basic property of

CES production (the strictly positive relationship between the elasticity of substitution and the level of output; see Grandville 2009), and is useful to determine the direction and bias of technical progress (Acemoglu 2002).

We set the base period for the normalization at the middle of the sample, i.e., $t = 48$ corresponding to 1991:Q4, and denote it by the index 0. Normalization implies that all the variables are expressed in terms of their baseline values, i.e., K_0, L_0 and Y_0.

To normalize the production function, we start with our production function written as:

$$Y_t = \beta_3 \left[(KIT_t)^{-\beta_1} + \left(\beta_2 e^{\mu (t-t_0)} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \quad (2)$$

where t_0 is the base period used for normalization, and to simplify notation we set $KIT = C^{\gamma} K$.

Under imperfect competition, factor compensation is subject to a constant mark-up, denoted by β_{13}, so that in any period t the following relation holds:

$$(i_t KIT_t + w_t L_t)\, \beta_{13} = Y_t$$

where i_t is the real interest rate and w_t is the wage rate.

In the reference period capital compensation is:

$$i_0 = \frac{1}{\beta_{13}} \frac{\partial Y_0}{\partial KIT_0} = \frac{(\beta_3)^{-\beta_1}}{\beta_{13}} \left(\frac{Y_0}{KIT_0} \right)^{1+\beta_1}$$

so that total capital compensation over total factor income, or the capital share (π_0), in the base period is

$$\pi_0 = \frac{i_0 KIT_0}{Y_0} \beta_{13} = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{KIT_0} \right)^{\beta_1} \quad (3)$$

Proceeding in the same way for the labor share and substituting in (2), we get the normalized production function:

$$Y_t = \left[\pi_0 \, (KIT_t)^{-\beta_1} + (1 - \pi_0)\, LIT_t^{-\beta_1} \right]^{-\frac{1}{\beta_1}}, \quad (4)$$

where output, labor and capital are already expressed in index form, and $LIT = \left(e^{\mu (t-t_0)} L_t \right)^{-\beta_1}$. In the normalized production function the only crucial parameter is β_1.

Of course, in the Cobb-Douglas case (where $\beta_1 = 0$), the production function becomes:

$$Y_t^{CD} = (KIT_t)^{\pi_0} \, (LIT_t)^{1-\pi_0}.$$

3 Technical progress

Output growth rate is determined by the time log derivative of equation (4):

$$\begin{aligned}\frac{\dot{Y}_t}{Y_t} &= \varepsilon_{Y,KIT}\left(\frac{\dot{K}_t}{K_t}+\gamma\frac{\dot{C}_t}{C_t}\right)+\varepsilon_{Y,LIT}\left(\frac{\dot{L}_t}{L_t}+\mu\right) \\ &= \pi_0\left(\frac{Y_t}{KIT_t}\right)^{\beta_1}\left(\frac{\dot{K}_t}{K_t}+\gamma\frac{\dot{C}_t}{C_t}\right)+(1-\pi_0)\left(\frac{Y_t}{LIT_t}\right)^{\beta_1}\left(\frac{\dot{L}_t}{L_t}+\mu\right) \quad (5)\end{aligned}$$

where $\varepsilon_{Y,KIT} = \frac{\partial Y}{Y}/\frac{\partial KIT}{KIT}$ and $\varepsilon_{Y,LIT} = \frac{\partial Y}{Y}/\frac{\partial LIT}{LIT}$ are the elasticities of output with respect to inputs in efficiency units. In this framework, the capital-augmenting technical change is $\pi_0\left(\frac{Y_t}{KIT_t}\right)^{\beta_1}\gamma\frac{\dot{C}_t}{C_t}$, while the labor-augmenting factor is $(1-\pi_0)\left(\frac{Y_t}{LIT_t}\right)^{1+\beta_1}\mu$. Intuitively, each input-augmenting factor contribution to output growth rate can be split in two components: one is the pure technical progress ($\gamma\frac{\dot{C}_t}{C_t}$, μ); the other is the sensitivity of output with respect to the technical change ($\pi_0\left(\frac{Y_t}{KIT_t}\right)^{\beta_1}$, $(1-\pi_0)\left(\frac{Y_t}{LIT_t}\right)^{\beta_1}$). In the Cobb-Douglas case $\beta_1 = 0$, and the elasticities are simply the income shares.

It is worthwhile noticing that, differently from the traditional specification, capital-augmenting technical progress depends on the dynamics of ICT capital stock. This choice of capital-augmenting technical progress is motivated by the key role played by ICT on the productivity dynamics in industrialized countries at least since 90s. The ICT relevance is particularly important for Italy (although in a negative sense). However, by the impossibility theorem of Diamond et al. (1978), we cannot separately identify this role from that of the elasticity of substitution unless one imposes a specific structure to technical change. In defining this structure, we abandon the traditional specification of technical progress growing at a constant rate.

In particular, our model assumes that the efficiency of traditional fixed capital stock is augmented by ICT capital according to a weighting factor equal to γ. Since labour-augmenting is defined as $\mu = \lambda_K + \gamma\,\lambda_C$, the same factor also increases labour efficiency. That way, we are assuming that ICT investment also improves labour productivity. To our knowledge, this specification of technical progress was first introduced in Kaldor (1957) growth model.[2]

[2] Kaldor is explicit in affirming that one specific characteristic of his growth model is

4 The advantage of using a CES production function

The contribution of the technical progress to the output growth is generally computed using the Cobb-Douglas production function through the Solow residual. To see the relevance of the elasticity of substitution, let us compare the computation of TFP using the the Cobb-Douglas production function with that obtained with the CES. To this end, we calibrate equation (5) with our three key parameters' estimates reported in table 1 (σ, γ, μ):[3]

$$TFP_{CES} = \frac{\dot{Y}_t}{Y_t} - \left(\varepsilon_{Y,KIT} \frac{\dot{K}_t}{K_t} + \varepsilon_{Y,LIT} \frac{\dot{L}_t}{L_t} \right) \tag{6}$$

In the Cobb-Douglas case, the TFP_{CD} becomes:

$$TFP_{CD} = \frac{\dot{Y}_t^{CD}}{Y_t^{CD}} - \left(\pi_0 \frac{\dot{K}_t}{K_t} + (1-\pi_0) \frac{\dot{L}_t}{L_t} \right) \tag{7}$$

The result of these two growth accounting exercises is illustrated in figure 1. A notable feature of the graph is that in the first part of the sample period the TFP from the Cobb-Douglas lies above that of the CES,while in the second part they essentially overlap.

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that: "... it eschews any distinction between changes in techniques (and in productivity) which are induced by changes in the supply of capital relative to labour and those induced by technical invention or innovation — i.e., the introduction of new knowledge. The use of more capital per worker (whether measured in terms of the value of capital at constant prices, in terms of tons of weight of the equipment, mechanical power, etc.) inevitably entails the introduction of superior techniques" (p. 595).

[3] Employing observed data for capital, labour and output and our parameters estimates, the capital share for the Italian economy in the reference period, using equation (3), is:

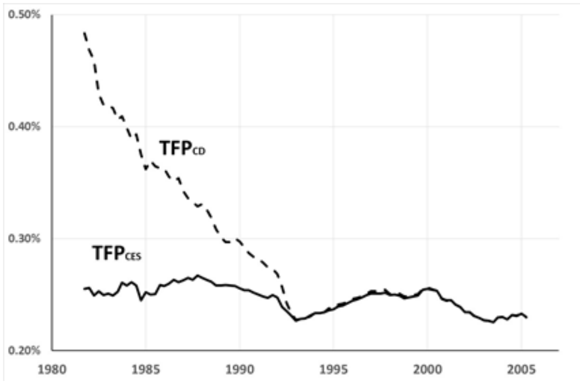
$$\pi_0 = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{KIT_0} \right)^{\beta_1} = 0.25$$

so that labour income share is

$$1 - \pi_0 = 0.75$$

Since these estimates are quite close at those present in several different databanks (such as OECD, EU KLEMS, AMECO), we decide to adopt these value of the income shares for the reference period.

Figure 1 The dynamics of TFP



A plausible interpretation is that our estimated σ is about two-thirds, while the Cobb-Douglas technology has a σ equal to 1. It follows that, from the property of general means (see Grandville 2009), the Cobb-Douglas output and correspondingly its growth rate is higher than in our CES case. In addition, there is a different weighting of the input growth rates in the two functions: the Cobb-Douglas uses fixed weights (equal to the income shares), while the CES uses the output-factor elasticities. As we will see, the gap between the two TFP, and its narrowing until it vanishes at about the middle of the sample period, can be explained by splitting the TFP into its components.

4.1 The decomposition of TFP

Whereas the Cobb-Douglas allows the computation of TFP only residually, a further advantage of the CES function is the possibility to decompose the TFP. This decomposition can best be done if we come back to our original framework. The tools are the output elasticities with respect to inputs, which represent a key feature of the CES production function. Indeed, they allow to split the contribution of each factor-augmenting technical change to the output growth rate. To appreciate the relevance of this property, we analyze the pattern of technical change of the Italian economy.

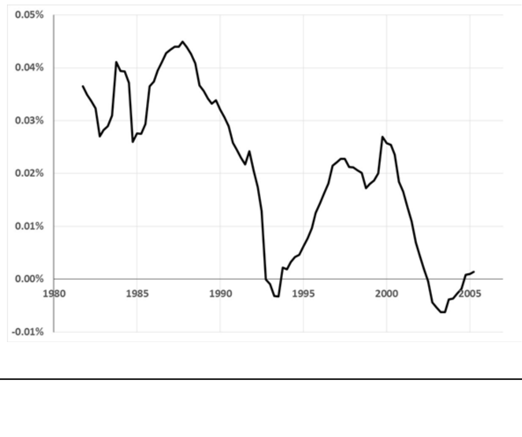
Let us start with the labor contribution to technical change, $\varepsilon_{Y,LIT} \cdot \mu$. Its dynamics is represented in figure 2.

Figure 2 The labor-augmenting technical change

It is straightforward to see that the labor contribution features two quite distinct patterns: in the first half of the sample period (1981:4, 1994:2) labor augmentation is steadily increasing. It is more troubling to detect a clear behavior in the second half. Indeed, it remains approximately constant. Hence, in the mid-90s seems to be present a structural break. The occurrence of such a break is confirmed by a simple Chow's breakpoint test. How sensitive is this result to changes in σ value? As a robustness check of the break timing, we tried values of σ closer or equal to 1 without finding any relevant differences.

A regime shift seems to be confirmed by the development of capital-augmentation, $\varepsilon_{Y,KIT} \cdot \gamma \frac{\dot{C}_t}{C_t}$. Its time evolution is quite volatile with a number of peaks; indeed, a test based on global information criteria indicate the existence of multiple breaks. However, a simple visual inspection of figure 3 shows that the relevant break occurs around the middle of 90s.

Figure 3 The capital-augmenting technical change



5 The factor bias

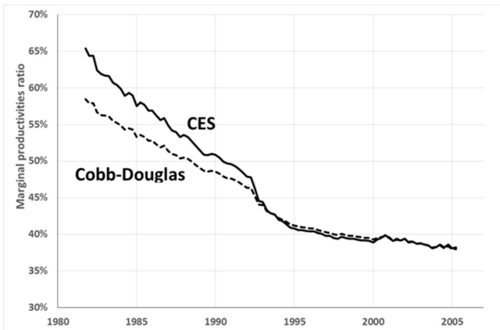
The CES production also sheds light on another aspect, the factor bias, which is defined by the ratio of marginal productivities of the inputs (not in efficiency units). From equation (4), we have:

$$\frac{\partial Y/\partial K}{\partial Y/\partial L} = \frac{\pi_0}{1-\pi_0}\left(\frac{e^{\mu\,(t-t_0)}}{C_t^{\gamma}}\right)^{\beta_1}\left(\frac{L_t}{K_t}\right)^{1+\beta_1}.$$

Technical progress is biased towards a factor if it increases its marginal product more than the other factor's. Following Acemoglu (2002), the bias can be divided in two parts. One is the traditional substitution effect, determined by the relative endowments of the two inputs, that favors the more scarce factor. The other component, that can be labelled the technical change effect, depends on the relative weight of factor-augmenting technical change. This second effect is absent in the Cobb-Douglas case.

The bias is clearly linked to the size of the elasticity of substitution. In our case, where $\beta_1 = 0.52$ ($\sigma = 0.66$), the dominance of labor-augmenting technical change in the first half of the sample implies that technical change is capital biased. Intuitively, the presence of capital bias means that technical change favors capital input.

Figure 4 The technological bias



In figure 4 the contribution of technical change to capital bias is given by the positive vertical distance separating the CES and the Cobb-Douglas (which includes only the substitution effect). Looking at the graph, it is worth noticing that, although present, the capital bias progressively reduces until it vanishes at the middle of 90s. To clarify this point, the vertical distance, a measure of technical progress contribution, is graphed in figure 5.

Figure 5 The contribution of technological progress

In fact, the graph clearly shows not only the disappearance of technical change but also verifies the occurrence of a structural break around the middle of 90s seen above. As in our technology representation (4), technical change is predominantly driven by ICT investment (see the definition of μ and of capital-augmenting factor), the disappearance of technical change contribution can be viewed as a failure to effectively employ innovative technologies in the Italian economy.

6 Conclusions

Most analyses of the current economic Italian stagnation focus on TFP slowdown without delving into its causes. In this paper we tried to make a step further looking at the determinants of TFP. To this end, we used our previous CES specification and estimated parameters. We find evidence of a structural break in the mid-nineties in the impact and nature of technical change. Labor augmentation and capital bias were found dominant in the first half of the sample period, while no evidence of technological progress of any type seems to be present in the second half. We believe that these results can be relevant not only for theoretical purposes but also for policy choices. This task is left for future research.

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