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Bell, Peter Newton

University of Victoria

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Choosing put option parameters based on quantiles from the distribution of portfolio value

Peter N. Bell

University of Victoria

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Contact: [pnbell@uvic.ca](mailto:pnbell@uvic.ca)

### Abstract

This paper explores how a put option changes the probability distribution of portfolio value. The paper extends the model introduced in Bell (2014) by allowing both the quantity and strike price to vary. I use the 5% quantile from the portfolio distribution to measure riskiness and compare different put options. I report a so-called 'quantile surface' that shows the quantile across different combinations of quantity and strike price. I find that it is possible to maximize the quantile by purchasing a put with quantity equal to one and strike deep in the money; however, the distribution with such a put option collapses to a single point because the option hedges all variation in stock price. This result is analogous to full-insurance in insurance economics, but has practical limitations. The quantile surface also shows that certain put options will decrease the quantile, which is equivalent to increasing the riskiness of the portfolio, and leads me to ask: what return will an investor receive in return for bearing that extra risk? I find that one such put option will cause the distribution to have an asymmetric shape with positive skewness, which is interesting to some speculators.

*Keywords:* Portfolio, put option, probability distribution, quantile, optimization, risk management, speculation

Use of quantiles from the distribution of portfolio value  
to choose put option parameters

This paper is concerned with the use of financial derivatives. I use a simple, toy model introduced in Bell (2014) where an agent holds one unit of put option and is allowed to trade some quantity of (at the money) put options. Here, I extend the model to consider choice of both quantity and strike price. Suppose that an agent holds some dollar value of stock, for example one unit of stock worth \$100. In my model, quantity refers to the notional amount of the put option in proportional to dollar value of stock: if quantity equals two ( $Q=2$ ), then the agent holds put options with notional value of \$200. The strike price refers to the level at which the put option begins to offset losses in the stock price. There is an analogy between the put option in my model and an insurance contract, but I focus on the context of financial derivatives in this short paper.

The paper uses simulation to generate insight into the model since it has no clear analytic solution. I use the same data generating process as in Bell (2014): the initial stock price is 100 and the price at option-expiry is governed by a log-normal distribution with mean zero and standard deviation 10%. This is a plain, vanilla model in financial economics that allows me to use the Black-Scholes option pricing formula for all options in the paper. I overlook concerns such as volatility smiles, where options deep out of the money have larger prices, or potential illiquidity of some deep in the money options.

One of striking results from Bell (2014) is associated with so-called quantile preferences. The 5% quantile from distribution of portfolio value is based on the Value at Risk measure used in market risk assessment: it represents an extremely low level for the portfolio, a value for which the portfolio will be larger with 95% confidence (in context of the model). Bell (2014) reported that the optimal quantity of put options based on quantile preferences is always precisely equal to one ( $Q=1.0$ ), which was striking because it is a knife-edge result that is robust to resampling. This result inspired me to investigate the quantile preferences further in this paper.

### The quantile surface

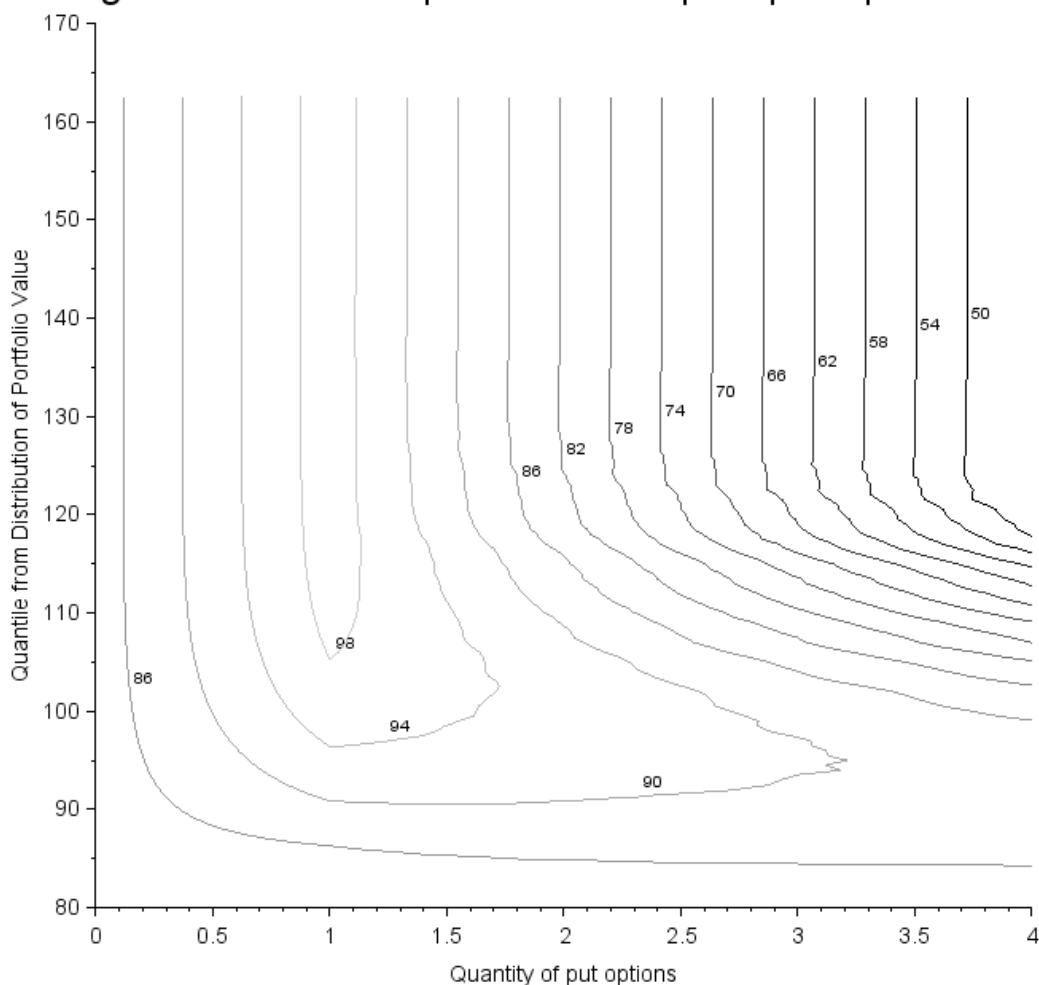
The quantile surface refers to a mathematical object that shows how the portfolio distribution changes with different types of put options. The put options are defined by two choice variables: quantity and strike price. I use a discrete approach to estimate the surface as follows. I specify a set of values for each choice variables and create a grid, where each point represents a put option. I estimate the probability distribution for the portfolio at each grid point and use the quantile from each distribution to determine the height of the surface at that grid point. I specify a generous range for each choice variable in order to explore the global behaviour of the quantile surface: I allow quantity to vary between 0 and 4, and strike price to vary between 80 and 160. I display the quantile surface as a level set diagram (topographical map) in Figure 1.

Figure 1 shows some level sets for the quantile surface. Each line has a corresponding numeric label, which specifies the value of the quantile for all combinations of quantity and strike price along the line. Higher values for the quantile are associated with less risk in the portfolio. The diagram shows several interesting features, which I discuss before characterizing the types of put options that are associated with the maximal value for the quantile.

One feature of interest in Figure 1 is the fact that the level sets become vertical lines for large values of the strike price (above 130 on vertical axis). This occurs because a put option with high strike will always expire in the money: the underlying normal probability model means that prices are almost surely less than 130. If the put option *always* expires in the money, then increasing the strike price further does not have any net effect within the model because the increased payoff is offset by increased price. The parallel level sets for high strikes suggest that changing strike price is inconsequential in this region.

The second feature of interest is the shape of the level set for lower strike prices. The negative slope for level sets suggests there is some tradeoff between quantity and strike, where an agent can achieve similar risk-management goals by increasing quantity and decreasing strike price, or *vice versa*. However, the tradeoff is non-constant based on the non-linear curvature of the level sets.

Figure 1 -- Portfolio quantile across put option parameters



In Bell (2014), I identify the quantity of put options that maximizes the quantile. It is possible to do so again with quantile surface, even though it may not be clear from Figure 1. To help clarify, I show the shape of the quantile surface around the optimum in Table 1.

**Table 1 -- Shape of quantile surface around optimum**

Quantity	Strike price					
	100	110	120	130	140	150
0.8	93.64	96.07	96.71	96.82	96.83	96.83
1	96.01	99.05	99.85	99.98	100.00	100.00
1.2	95.44	97.22	96.65	96.37	96.33	96.33

Table 1 shows that an agent can force the quantile to equal 100 if they buy an option with quantity equal to 1.0 and strike above 140. Any strike above 140 will ensure this result because the level sets are vertical lines, as discussed above. This implies that there is not one single put option that maximizes the quantile, instead there is a continuous set of ‘optimal’ put options that have quantity equal to one and strike deep in the money. This occurs because such put options hedge all variation in the stock price: the delta is approximately equal to negative one when they first buy the put option, and the option always expires in the money because the strike is so deep in the money.

Although Table 1 reveals that the optimal put option causes the quantile to equal 100, it does not provide any further information about the shape of distribution when using these options. Table 2 shows that the standard deviation of the optimal portfolio value approaches zero: the distribution collapses to a single point. Again, this occurs because such a put option hedges all variation in stock price – both upward and downward movement. It could be said that the optimal put option causes the portfolio to be ‘globally flat’, in contrast to the phrase ‘locally flat’ used by risk management professionals.

**Table 2 – Standard deviation of portfolio value around optimum**

Quantity	Strike price						
	100	110	120	130	140	150	
0.8	7.23	4.47	2.71	2.11	2.04	2.04	
1	6.69	3.51	1.39	0.33	0.00	0.00	
1.2	6.29	3.17	2.00	1.99	2.04	2.04	

The results in this section show that an agent can use a put options to control the quantile from the distribution of portfolio value. Furthermore, if they want to maximize this quantile then they will buy one unit of put options (proportional to stock position) with strike deep in the money and this option will eliminate all variation in portfolio value in context of the model. This is an extreme result, but it has some precedent in insurance economics where a risk averse agent chooses full insurance under risk neutral pricing, as discussed in Bell (2014).

### Speculative portfolio

Further inspection of the quantile surface in Figure 1 shows that there exists a region where the level sets become very close together (quantity from 2.0 to 4.0 and strike from 100 to 125). When the level sets are close together, the surface is very steep. This region raises concerns because it allows an agent to decrease the quantile, or increase the risk in their portfolio, which leads me to ask what sort of return they get in exchange for the increase in risk? To explore this region, I consider one portfolio with quantity equal to 3.0 and strike equal to 105.

Figure 2 -- Comparison of speculative and benchmark distributions

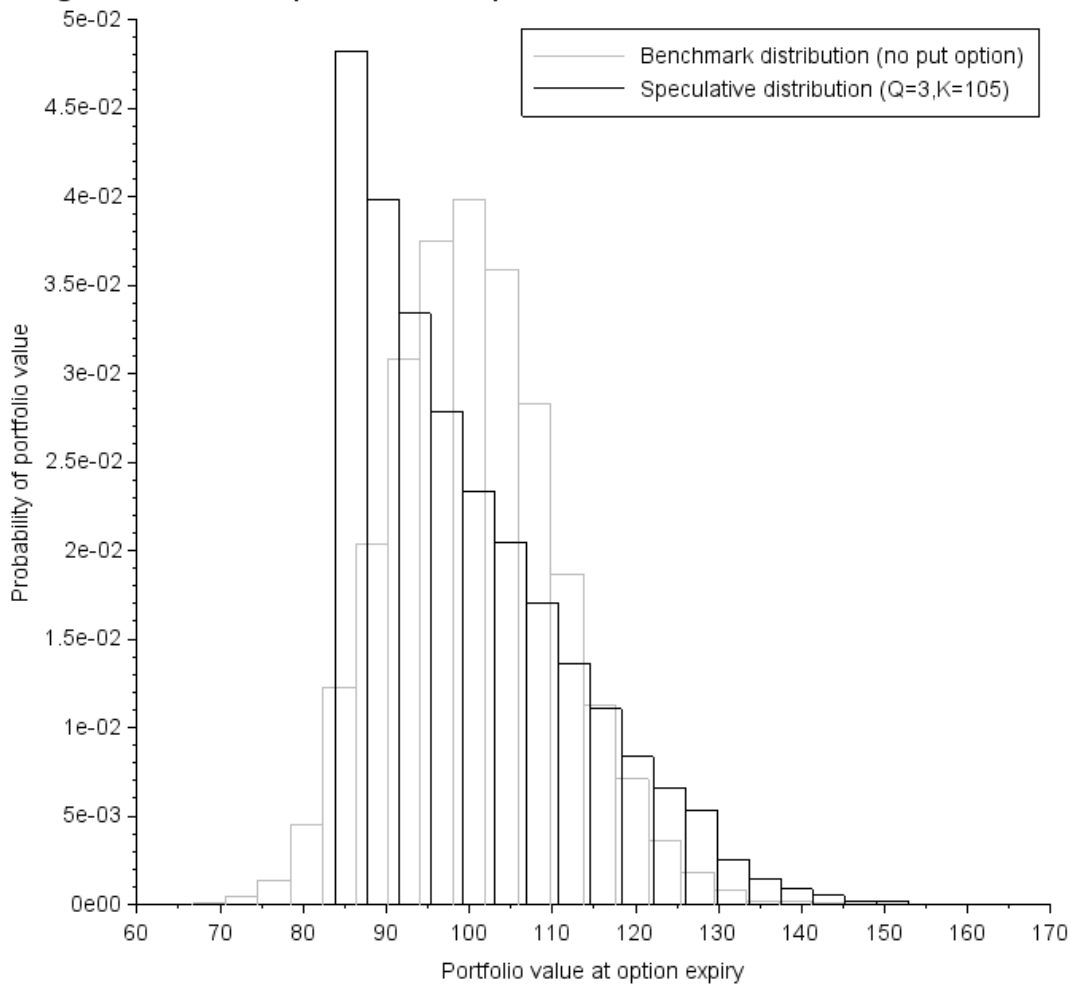




Figure 2 compares two different distributions of portfolio value. The dark line represents the probability distribution when the agent buys a put option from the region where the quantile surface is steep. I refer to this as a ‘Speculative distribution’ because it has a triangular, wedge shape. In contrast, the light gray line represents the distribution when the agent buys no options: by construction, the distribution has a normal, bell shape. There are several important differences between these two distributions.

Figure 2 shows that the speculative distribution has a larger, longer right tail than the benchmark distribution. This means that the speculator is more likely to get larger portfolio values. Figure 2 also shows that the speculative distribution has a smaller left tail, which is truncated at 85. This means that the speculator will not ever have as severely low values as the benchmark distribution. Together, this extended upside and reduced downside suggest that the speculative distribution reflects a particular combination of attributes that appeals to certain speculators: asymmetric returns with positive skewness. Mark Spitznagel (2013) describes how this concept is valuable inside finance, from futures trading in the pit to electronic trading in tail options, and outside of finance, as in ecology and ancient Chinese military philosophy.

**Table 3 -- Positive skewness for speculative portfolio**

Pearson's moment skewness coefficient	0.98
Pearson's median skewness coefficient	0.72

Table 3 shows that the speculative distribution does, indeed, have positive skewness. This result provides technical evidence to support the claim of asymmetric returns with positive skewness above. However, these results do not imply that the speculative distribution is necessarily superior to the benchmark. The quantiles for each distribution are very similar, approximately 82 for the speculative distribution and 84 for the benchmark distribution, but the shapes of the two distributions are very different. Notice that the peak of the speculative distribution is more pronounced and actually to the left of the benchmark distribution; this means the speculative portfolio will generally provide lower returns than the benchmark. In terms of decision theory, neither distribution has stochastic dominance over the other.

### Section 3: Conclusion

This short paper investigates a model where an agent holds one unit of stock and is allowed to buy a put option on the stock with some quantity and strike price. The agent uses the 5% quantile from the distribution of portfolio value to assess if the particular put option reduces risk in the portfolio or not. I report the quantile surface, which shows how the quantile varies across a grid of values for quantity and strike. The surface shows that there exists a set of put options with quantity one and strike deep in the money, that maximize the quantile and reduce all variation in the portfolio value. Such put options could be said to reduce all the risk in the portfolio, but they may be very illiquid in practice.

I also investigate a region of the quantile surface where the surface has the steepest gradient. I pick a put option from this region (quantity 3.0 and strike 105) that gives a similar value for the quantile as the benchmark case and compare the two portfolio distributions. I find that the distribution with the put option has properties that appeal to some speculators, asymmetric returns with positive skewness, which suggests that maximizing the quantile is not an appropriate objective for all types of agents.

In conclusion, I will identify two areas for further work. One is to replace the quantile measure used here with other types of utility functions or risk measures. The underlying distribution for portfolio values will be the same, but different measures may lead to different insights; for example, what is the tradeoff between strike price and quantity of put option for an agent with a risk averse utility function? The second direction of further work is to better develop the concept of time. The model used here only has two time periods, when the agent first buys the option and when the option expires. However, contemporary options markets have monthly term structures, which implies that there is another entire dimension for the put options: expiry time. It will be a challenge to incorporate multiple expiry times into the model as described here, but it could provide valuable insight into the dynamic use of financial derivatives.

## References

Bell, P.N. (2014). Optimal use of put options in a stock portfolio. Unpublished manuscript.

Available from <http://mpa.ub.uni-muenchen.de/54871/>

Spitznagel, M. (2013). The Dao of Capital: Austrian Investing in a Distorted World.

Hoboken, NJ: John Wiley & Sons.

## Appendix A

```

// Code Appendix for 'Choice of Put Option Parameters' paper.
// By Peter Bell, September 2014, written in SciLab 5.5.

// Section 1: Calculate level set for 5% quantile across strike, quantity
// Also creates Figure 1
clear
nSamp=1000; sigma=0.1; initPrice=100; rate=0;
diffRet=rand(nSamp,1,"normal");
priceExp=initPrice*exp(diffRet*sigma);
for quantCount = 1:100
    for strikeCount = 1:175
        quantLoop=quantCount/25;
        strikeLoop=75+strikeCount/2;
        d1=(1/sigma)*( log(initPrice/strikeLoop) + (rate+sigma^2/2) );
        d2=d1 - sigma;
        [p1,q1]=cdfnor("PQ",-d1,0,1);
        [p2,q2]=cdfnor("PQ",-d2,0,1);

        putPrice=strikeLoop*exp(-rate)*p2-initPrice*p1;
        // Using Black-Scholes price
        for i=1:nSamp
            portUnit(i)=priceExp(i)+quantLoop*(-putPrice+...
                max(strikeLoop-priceExp(i),0));
        end
        // Construct payoff as-if bought particular put option
        quantileVecTemp=gsort(portUnit);
        quantileMatrixResult(quantCount,strikeCount)=...
            quantileVecTemp(nSamp*0.95);
        // Key object is 'quantileMatrixResult' because
        // shows 5% quantile portfolio with each put option
    end
end

quantVec=(1:100)/25; strikeVec=75+(1:175)/2; quantLevel=50:4:110;
f=get("current_figure");
f.figure_size=[800,800] // Size of full window
f.color_map = graycolormap(16); scf(0);

contour2d(quantVec,strikeVec,quantileMatrixResult,quantLevel);

xlabel("Figure 1 -- Portfolio quantile across put option parameters", ...
    'Quantity of put options',...
    'Quantile from Distribution of Portfolio Value');
axesCurr=get("current_axes");

```

```

axesCurr.font_size=3;
title_label=axesCurr.title;
title_label.font_size=5;
x_label=axesCurr.x_label;
x_label.font_size=3;
y_label=axesCurr.y_label;
y_label.font_size=3;

xs2bmp(0,'Fig1.bmp'); // Creates Figure 1 from paper

// Section 2: Show behaviour of quantile around optimum parameter values
// Look at Q from 0.8 to 1.2 and K from 100 to 150.
for quantCount = 1:3
    for strikeCount = 1:6
        quantLoop=0.6+0.2*quantCount;
        strikeLoop=90+10*strikeCount;
        d1=(1/sigma)*( log(initPrice/strikeLoop) + (rate+sigma^2/2) );
        d2=d1 - sigma;
        [p1,q1]=cdfnrc("PQ",-d1,0,1);
        [p2,q2]=cdfnrc("PQ",-d2,0,1);

        putPrice=strikeLoop*exp(-rate)*p2-initPrice*p1;
        for i=1:nSamp
            portUnit(i)=priceExp(i)+quantLoop*(-putPrice+...
                max(strikeLoop-priceExp(i),0));
        end

        quantileVecTemp=gsort(portUnit);
        quantileTable(quantCount+1,1)=quantLoop;
        quantileTable(1,strikeCount+1)=strikeLoop;
        quantileTable(quantCount+1,strikeCount+1)=...
            quantileVecTemp(nSamp*0.95);
        // This is quantile around optimum
        devTable(quantCount+1,1)=quantLoop;
        devTable(1,strikeCount+1)=strikeLoop;
        devTable(quantCount+1,strikeCount+1)=...
            stdev(portUnit);
        // This is standard deviation of wealth around optimum
    end
end
csvWrite(quantileTable, 'Table1.csv');
csvWrite(devTable, 'Table2.csv');
// Tables 1 and 2 help characterize the character of the optimum

// Section 3: Show distribution for one interesting portfolio
// Pick quantity Q=3 and strike K=105 ad hoc, discussed in paper

```

```

clear
strike=105; quantPut=3;
nSamp=10000; sigma=0.1; initPrice=100; rate=0;
diffRet=rand(nSamp,1,"normal");
priceExp=initPrice*exp(diffRet*sigma);

d1=(1/sigma)*( log(initPrice/strike) + (rate+sigma^2/2) );
d2=d1 - sigma;
[p1,q1]=cdfnor("PQ",-d1,0,1);
[p2,q2]=cdfnor("PQ",-d2,0,1);
putPrice=strike*exp(-rate)*p2-initPrice*p1;
for i=1:nSamp
    portUnit(i)=priceExp(i)+quantPut*(...
        max(strike-priceExp(i),0)-putPrice);
end

skewVec = ( (portUnit-mean(portUnit))/stdev(portUnit) )^3;
skewStat = mean(skewVec);
skewPearson = 3*(mean(portUnit)-median(portUnit))/stdev(portUnit);
csvWrite([skewStat ; skewPearson] , 'Table3.csv');
// Table 3 shows the distribution has positive skewness

histplot(20,priceExp,style=[color("gray")])
histplot(20,portUnit)
hl=legend(['Benchmark distribution (no put option)'; ...
    'Speculative distribution (Q=3,K=105)']);
xtitle("Figure 2 -- Comparison of speculative and benchmark distributions", ...
    'Portfolio value at option expiry',...
    'Probability of portfolio value');
axesCurr=get("current_axes");
axesCurr.font_size=3;
title_label=axesCurr.title;
title_label.font_size=5;
x_label=axesCurr.x_label;
x_label.font_size=3;
y_label=axesCurr.y_label;
y_label.font_size=3;
l=axesCurr.children(1);
l.font_size = 3;
xs2bmp(0,'Fig2.bmp'); // Creates Figure 2 from paper

// End of file.
// By Peter Bell, on 8th September 2014, in Zurich, Switzerland.

```