The collusion incentive constraint

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Abstract: The collusion incentive constraint is an important economic measure of cartel stability. It weighs the profits of being in a cartel with those of cheating and punishment of the remaining cartel members. The constraint places no restrictions on firm cartel, cheating and punishment pricing, but is usually considered in a restricted competitive set up characterized by either Cournot or Bertrand competition. This paper examines the constraint under Bertrand competition and homogenous goods when assuming that cartel members have the same market power and then continues to examine if this is not so.

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1. Introduction

There are different ways to assess whether a strategy such as explicit collusion is a viable strategy for firms under different settings. One very handy tool is the collusion incentive constraint. The constraint makes use of the same way strategies are assessed in game theory.

2. Incentives in Cournot and Bertrand games

The collusion incentive constraint simply states that, if cartelization is a dominant strategy to all other strategies for a firm given all other firms' dominant choices, then it must be that, the individual profit from cartelization must be greater than or equal to the profit the firm receives by defecting from the cartel and the punishment the remaining cartel member can enforce on the cheater. Thus, the constraint is

$$\frac{\pi_i^c}{1-\delta} \geq \pi_i^d + \frac{\delta \pi_i^n}{1-\delta} \tag{1.1}$$

Where, π_i^c is firm i's individual profit from being a cartel member, δ is the common discount factor of the firm of the market, π_i^d is the individual deviation profit to the deviator and π_i^n is the game reversal profit, which is the profit firm i receives if it cheats in one period and the remaining cartel member punish or reward it forever for doing so. The reversal profit can be seen as the resulting profit for the cheater and the remaining cartel members when reacting on the cheater's defection. This is also called the cartel's trigger strategy.

A trigger strategy is a strategy specifying that if the remaining cartel members observe a deviation from the agreed cartel strategy, then they will do so and so to the cheater.

The collusion incentive constraint can either be written in quantity choices, assuming Cournot competition as the primary driver for competitive pricing, or written in price choices, assuming Bertrand as the main force of competition.

If written in quantity choices the collusion constraint is,

$$\frac{\pi_i^c(q_i^c, q_j^c)}{1-\delta} \geq \pi_i^d(q_i^d, q_j^c) + \frac{\delta \pi_i^n(q_i^n, q_j^n)}{1-\delta} \tag{1.2}$$

Where q^c is the agreed upon cartel quantity, q^d is the deviation quantity and q^n is the game reversal quantity.

And when written in price choices the constraint is,

$$\frac{\pi_i^c(p_i^c, p_j^c)}{1-\delta} \geq \pi_i^d(p_i^d, p_j^c) + \frac{\delta \pi_i^n(p_i^n, p_j^n)}{1-\delta} \tag{1.3}$$

3. The collusion constraint in a Bertrand game

An important thing to consider in connection with the collusion constraint is demand. Demand sets the restriction on the pricing decisions of the firm as well as the limits to the profit the firms are able to capture in the market.

In standard Bertrand games the usual way to describe demand is to assume we have a number of consumers each demanding one unit of the good at the lowest market price. Thus, the consumers' demand in the market can be described as the continuum $\left[0,1\right]$ and interpreted as the firm's market share.

The strategic choices of the consumers can then be written as the demand facing firm i as a function of its price, or as

$$D_i = \begin{cases} 0 & p_i > p_j \\ \frac{1}{2} \text{ if} & p_i = p_j \\ 1 & p_i < p_j \end{cases} \tag{1.4}$$

The firm's payoff or profit then depend on its pricing decision in relation to what other firms have set their price at.

4. The importance of symmetry for collusion

To examine how firms with the same market power views collusion we will assume that all the firms have the same marginal cost $(c_i = c_j), i \neq j$.

Using our knowledge about the demand of consumers and cartel prices we can immediately say the following about the prices in the collusion constraint.

a) Since firms in a cartel find a common price then $p_i^c = p_j^c$ and since the firms are symmetric then $\pi_i^c = \pi_j^c$. Thus, given symmetry firms have the same incentives to form a cartel.
b) We also know something about the deviation price of the firms. Since both firms are trying to maximize their profit having the same marginal cost, then $p_i^d = p_j^d$ and $\pi_i^d = \pi_j^d$. Thus, again the symmetry condition implies that firms have the same incentives to cheat from the cartel.
c) Following the demand conditions the dominant deviation price strategy implies setting it as close as possible to the cartel price, but always just low enough to capture the entire market or $p^d < p^c$.

Having the same market power implies that the remaining cartel members have essentially the same competitive strength as any cheater and as such any predatory pricing (p<MC) is a non-credible punishment and the only real option or the only credible strategy is for firms to revert to the

Bertrand equilibrium prices ($p = MC$) imply having profits of zero forever for both the remaining cartel members and the cheater.

Under these conditions we can now specify the collusion incentive constraint as,

$$\frac{\pi_i^c}{1-\delta} \geq \pi^d + \frac{\delta \pi^n}{1-\delta} \Rightarrow \frac{\left(p^c - c\right)\frac{1}{2}}{1-\delta} \geq \left(p^d - c\right) + \frac{\delta\left(c-c\right)\frac{1}{2}}{1-\delta}$$
$$\Updownarrow \qquad (1.5)$$
$$\frac{\left(p^c - c\right)\frac{1}{2}}{1-\delta} \geq \left(p^d - c\right)$$

As p^d is only a fraction of p^c we can write the inequality as,

$$\frac{\left(p^c - c\right)\frac{1}{2}}{\left(p^d - c\right)} \geq 1-\delta \Rightarrow \tfrac{1}{2} \geq 1-\delta \Leftrightarrow \delta \leq \tfrac{1}{2} \qquad (1.6)$$

The result indicates that collusion is the chosen strategy, if the discount factor is equal to or less than $\frac{1}{2}$. As the discount factor tell us something about the valuation of future profits and a discount factor $\delta = 1$ imply, that future profits is not valued at all and $\delta = 0$ imply valuing future profits as much as present profit. Thus, equation(1.6) tells us that whether or not collusion is chosen depend on the way the potential cartel members perceive profits, as well as the number of cartel members.

If firms only care about short run profits, then collusion is not a dominant strategy and firms will prefer to compete rather than cooperate. If they care about long run profits, then collusion is a dominant strategy and thus a real option for the firms.

The number of firms in a cartel is also important for cartel stability. If we look at equation(1.6), we know that the reason why the discount factor has to be less than $\frac{1}{2}$ is because we have two firms. If we assume that the market has n-firms then the equation become,

$$\frac{\left(p^c - c\right)\frac{1}{n}}{\left(p^d - c\right)} \geq 1-\delta \Rightarrow \tfrac{1}{n} \geq 1-\delta \Leftrightarrow \delta \leq 1-\tfrac{1}{n} \qquad (1.7)$$

Thus, the more cartel members we have the lower the discount factor must be for cooperation to be a dominant strategy.

Equation(1.7) implies that we can write the discount factor consistent with cooperation as,

$$0 < \delta \leq \tfrac{1}{2} \qquad (1.8)$$

5. The implication of asymmetry

The general point of view on asymmetry among cartel members is that asymmetry introduces greater instability than if the firms were symmetric, but what is asymmetry and when does the level of asymmetry matter for cartel instability?

Following the presentation of symmetry in the previous section then asymmetry implies $c_i \neq c_j$. To make things simpler we assume that we have two firms in the market L and H and assume that firm L have more market power than H or $c_H > c_L$.

We will start the analysis by considering the pre-cartel market conditions for the two firms. Since the asymmetry provides incentives for the low cost firm to set a price below the competitor's lowest possible price, we have to assume that such behavior is not considered by the firm.

The optimal price for the firms assuming Bertrand competition is $p_H = p_L = c_H$ and the pay-off to the two firms are,

$$\begin{aligned} &\pi_H = \left(c_H - c_H\right)\tfrac{1}{2} = 0 \\ &\pi_L = \left(c_H - c_L\right)\tfrac{1}{2} > 0,\ \Delta_c \equiv c_H - c_L \Rightarrow \pi_L = \Delta_c \tfrac{1}{2} \end{aligned} \tag{1.9}$$

For now assume that this is also the game reversal profit.

Under this setting we can analyze to what extent collusion is preferable to the two firms.

Collusion is preferable to the high cost type firm H if,

$$\begin{aligned} &\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta\pi^n}{1-\delta} \Rightarrow \frac{\left(p^c - c_H\right)\tfrac{1}{2}}{1-\delta} \geq \left(p^d - c_H\right) + \frac{\delta\left(c_H - c_H\right)\tfrac{1}{2}}{1-\delta} \\ &\Updownarrow \\ &\delta \leq \tfrac{1}{2} \end{aligned} \tag{1.10}$$

Thus, the condition for collusion for the high cost firm is the same as under symmetry and the same logic applies when considering the high cost type firm.

Collusion is preferable for the low cost type firm L if,

$$\begin{aligned}
&\frac{\pi^{c}}{1-\delta} \geq \pi^{d} + \frac{\delta\pi^{n}}{1-\delta} \Rightarrow \frac{\left(p^{c}-c_{L}\right)\frac{1}{2}}{1-\delta} \geq \left(p^{d}-c_{L}\right) + \frac{\delta\left(c_{H}-c_{L}\right)\frac{1}{2}}{1-\delta} \\
&\Updownarrow \\
&\left(p^{c}-c_{L}\right)\tfrac{1}{2} - (1-\delta)\left(p^{d}-c_{L}\right) \geq \delta\left(c_{H}-c_{L}\right)\tfrac{1}{2} \\
&\Updownarrow \\
&\left(p^{c}-c_{L}\right)\tfrac{1}{2} - \left(p^{d}-c_{L}\right) \geq \delta\left(c_{H}-c_{L}\right)\tfrac{1}{2} - \delta\left(p^{d}-c_{L}\right) \\
&\Updownarrow \\
&\delta \leq \frac{\left(p^{c}-c_{L}\right)\frac{1}{2} - \left(p^{d}-c_{L}\right)}{\frac{1}{2}\left(c_{H}-c_{L}\right) - \left(p^{d}-c_{L}\right)} = \frac{\left(p^{c}-c_{L}\right)\frac{1}{2}}{p^{d}-\frac{1}{2}\Delta_{c}}
\end{aligned} \tag{1.11}$$

In the case of the low cost type firm the incentives to collude depend not only on its perspective on profits, but also on its relative market power, which is captured by Δ_c.

For very large values of Δ_c the discount factor becomes very large even beyond its defined properties implying that no matter the firm's perspective on profits, the firm would never prefer to collude with a firm or firms with less market power than itself. For large values of Δ_c that respect the discount factor's properties, the discount factor $\delta \to 1$ indicating the same conclusion as just mentioned. Thus, a fairly small difference in market power can induce the more potent firm to engage in collusion, but generally we can say that asymmetry provide far less incentives to collude than symmetry.

If the firm with market power colludes then it may lead to a more stable cartel than if the firms were symmetric, since the low cost firm can make a much more credible punishment for any cheater with higher cost. The reason for this is that the low cost firm can set a price below the cheater's marginal cost and threaten its existence in the market. On the other hand, if the low cost firm decides to cheat then the high cost firm cannot punish the low cost firm.

6. Conclusion

The results indicate that extent to which the firms are symmetric is important to consider, when evaluating strategies of firms towards collusion.

Symmetry implies the same interest among firms towards collusion and generally requires that the firms have a long run perspective on profits.

More stable cartels seem to be linked to more asymmetric firms, but require that the firm with most market power is the 'ringleader' of the cartel.

7. References

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