The Demand Driven and the Supply-Sided Input-Output Models. Notes for the debate

AROCHE REYES, FIDEL and MARQUEZ MENDOZA, MARCO ANTONIO

UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO

January 2013

The Demand Driven and the Supply-Sided Input-Output Models. Notes for the debate

DRAFT

Fidel Aroche Reyes

Marco Antonio Marquez Mendoza[1]

Abstract: The demand-driven version of the open Input-Output model determines production as a function of final demand, given the production technology. On the contrary, in the supply-sided version, value added determines output and producers must induce sales in order to achieve a desired level of income. This latter version of the model has been criticised and even rejected on the bases of its implausibility, its difficult interpretation and its bizarre implications. This paper argues however that the logic of the supply-side model is not mathematically at odds with Leontief's arguments. Rejection of the model is a matter of theoretical interpretation.

Keywords: Input-Output model, demand-driven model, supply-driven model, Leontief, Ghosh, balanced growth

JEL classification: C67, B23

[1] Universidad Nacional Autónoma de México. E-mail address: aroche@unam.mx and antoniomrqz@gmail.com, respectively.
We are grateful to Prof. Dr. Joerg Beutel for useful comments and observations to an earlier version of this paper.

The demand-driven and the supply-sided Input-Output models. Notes for the debate

1. *Introduction*

Wassily Leontief's major contribution to economics is -no doubt- the formulation of the Input-Output (IO) model; he was awarded the Nobel Prize in 1973. Leontief first presented a closed model (Leontief, 1937), based on the hypothesis of the interdependence between sectors, which reaches two alternative and independent solutions, one for quantities and one for prices. Later on (Leontief, 1944) he published an open version which would prove to be the better-known IO standard model.

The latter is a workable open multisector account that determines production as a function of final demand, given the technology used by each sector; this model yields equilibrium results, which should also warrant optimality, because there are no reasons to expect that producers should choose technologies and output levels otherwise -even if the paper does not discuss such implication explicitly. That assumption is formalised as the non-substitution theorem by Georgescu-Roegen (1951) and Samuelson (1951). In the IO model each industry produces one homogeneous commodity, using one homogeneous technology, which also determines the proportions of inputs employed to produce each good. Technology is also crucial for the interrelation of sectors and, by the same token, it also conditions the shape of the economic structure. The latter showing the set of producing industries and their liaisons, determined by the exchange of goods to be used as inputs in the producing processes in each sector.

In 1958 Ambica Ghosh, from the Department of Applied Economics at the University of Cambridge, published "Input-Output Approach in an Allocation System" (Ghosh, 1958), presenting an alternative to Leontief's model, solved on the allocation of output, where the value of output depends on the value added vector. This version can be associated to a supply-sided economy, as coefficients are calculated on the revenues that each sector derives from supplying goods to its intermediate and final consumers. According to Ghosh (1958) the model would be useful to analyse centrally planned economies, as well as systems dominated by monopolisitic market structures and in general, economies constrained by scarce resources (as opposed to Keynesian frameworks, limited by final demand). Ghosh claims that in those environments the allocation of outputs would be a more complex task and his alternative model would be more useful.

Nevertheless, Ghosh's proposal did not receive a warm welcome and there was not significant discussion around, until María Augustinovics (1970) presented an empirical application. Later on forward linkages[2] have been often calculated from the perspective of the allocation of outputs and the supply side of the IO model (e.g. Jones, 1976; Bulmer-Thomas, 1982). Advocates argue that it is a supply-sided index; however, earlier applications used the coefficients matrix to determine both forward and backward linkages on the technical coefficients (e.g. Chenery and Watanabe, 1958; Hazari, 1970; Laumas, 1976). Other authors have disputed the rationality of indicators derived from a controvertible supply-sided model (e.g. McGilvray, 1977). Ghosh's formulation has also received attention in regional analysis and energy models (e.g., Bon, 1988; Giarratani, 1976 and 1980).

The discussion on Ghosh's supply-sided alternative formulation re-emerges every now and then to the present day, focusing on its applications, its meaning and even its plausibility; (Bon, 1986 and 1988; Chen and Rose, 1986 and 1991; Dietzenbacher, 1997; de Mesnard, 2007 and 2009; Guerra and Sancho, 2011a and 2011b; Oosterhaven, 1988), but no consensus has emerged so far. Most authors question the model rationality, which does not seem to comply with reality; presumably real world economies would follow a demand-driven logic. However, if that is the case, arguments against should not be limited to Ghosh's contribution, but should reach the whole supply-sided economics. And then, it should be

2 Forward linkages measure the relative capacity of each sector to induce the use of its output as input by other producers; backward linkages measure the relative ability of each sector to use other sectors' output as inputs (Bulmer-Thomas, 1982).

acknowledged that in the referred article, Ghosh does not subscribe that theoretical line. He was rather concerned with empirical problems that, in his opinion, demand-sided approaches do not contemplate. On the other hand it will be argued that -when criticising Ghosh- some of the referred authors reach not very accurate conclusions, but have misled the debate.

The purpose of this paper is twofold, first to analyse a few issues concerning Ghosh's supply-sided model and then, to assess some ideas that have been at the bases of that discussion, besides from reconsidering some of the arguments that various authors have advanced. This paper maintains that despite debatable logic and general disapproval, the latter is similar to the standard demand-driven Leontief's open model (in the mathematical sense) so, the supply-sided version can be seen as a formal extension to the better known and more agreeable demand-determined one. Whether Ghosh's model is used in applied economics is a matter of interpretation. The nature of the IO framework is such that both solutions are parallel (stretching the analogy); therefore, it is not easy to find solid arguments to explain why a valid proposition in one model finds its correspondent invalid in the other, except by introducing further assumptions.

The rationality of the supply-driven model would support various IO applications that lay on the distribution coefficients matrix, such as the aforementioned forward linkages. Maybe it would be necessary to appeal to authors such as J.B. Say, in order to understand the meaning of supply-sided models but certainly, such interpretation takes the IO model away from more accepted perspectives, based on demand-driven economics that modern theory takes for granted. On the other hand, accepting the supply-side model reinforces the notion that the IO framework is useful to study a variety of empirical problems from different theoretical perspectives, including those opposed to demand-sided economics. The IO model has proved to be useful to analyse a variety of empirical phenomena and (as a tool) need not be attached in principle to a single theoretical perspective. Clearly, results need to be interpreted from coherent theoretical notion.

The remaining of the paper is organised as follows: Section 1 discusses the IO model and shows the connections between the demand- and the supply- sided models, using no other considerations than those used by Leontief (1928, 1936, 1937 and 1944). Section 2 presents the solution by Ghosh, as an extension to the former; Section 3 presents the main ideas in the debate about the supply side model and its interpretations, for which a few numerical exercises are included. Finally a few remarks are discussed in the fourth Section.

2. *The Input-Output Model*

An economic system is defined as a set of interdependent industries[3]; each one identified by a productive process that consumes produced commodities as inputs in given proportions, in order to produce one particular homogeneous good by means of a technological relation. Disregarding non-produced merchandises in the system, each good is produced in one industry only. Consumption and investment can be also taken as economic activities that demand inputs to produce outputs –such as factors- through some production technology; the latter are also useful in the productive processes. Then, economy is a closed circular system (Leontief, 1937; von Neumann, 1936; Sraffa, 1960; Walras, 1874). On the contrary, if non-produced goods and factors exist and they are available for productive and consumptive activities, the system is open (Leontief, 1944; Marx, 1885). Exogenous variables -such as final demand or value added- determine the level of activity in open models, on given technological relationships.

The IO model defines an n-dimensional space, of the n produced goods, demanded both as inputs and final demand goods, the former are linearly transformed into n produced goods, by the n industries that define the economic system; those industries employ n productive techniques, observing strict constant returns to scale with zero rates of substitution between inputs (it is a short term scheme). It can be postulated that agents use the most efficient technologies within

[3] In this paper we use the words industry and sector as synonyms.

the set of all possible ones, as the model omits any explicit discussion on the choice of technology. Thence, the system remains in equilibrium, as long as prices persist and technical coefficients are constant. In order to complete the circular flow of the economy, production is transformed into revenue for all agents, which changes once again into demands of all kinds (Aroche, 1993).

Industries are numbered 1, 2, ..., *i, j*, ..., *n*; those exchange goods, valued $z_{ij} = p_i q_{ij} \geq 0$ in amounts (q_{ij}), determined by the consuming sector, at given equilibrium prices (p_i). Therefore, ordering those transactions conveniently, a square matrix can be arranged, $\mathbf{Z} = [z_{ij}]$ (Leontief, 1936). Adding up over the columns of **Z,** results in a row vector of the value of the inputs that each industry requires (and demands) in production; conversely, summing up on the rows, one gets the value of the goods that each industry (*i*) distributes among the rest of the producers.

In an open model, **Z** is a square matrix showing the exchange of produced goods between industries; demand for non-produced goods appears in a (second) rectangular array of the 1, ..., *g* different types of factors employed by the *n* industries in the system. Adding up over the columns of the latter matrix, yields a row vector of value added (**v**'). Besides, the various types of agents (1, ..., *m*) that own those primary inputs, consume the *n* produced goods outside the productive processes, as final demand, which can be arranged in a (third) rectangular matrix of the *n* sectors and the *m* types of agents. Summing up over the rows of this array results in a column vector of final demand (**f**). Adding up the sum of the supplies of goods to other producers and final demand agents yields the revenues of each industry; conversely, the demand for produced inputs plus primary inputs for each sector results in a row vector of industry expenditures. Revenues equal expenditures and the value of sectoral supply equals that of sectoral demand, i.e. no industry makes profits and each factor receives equilibrium income[4].

[4] If individual industries are not in equilibrium, transfers between them should be allowed. In any case, the system as a whole must comply with that equilibrium condition. For example, empirical national accounts often show that sectors not always meet the equilibrium conditions, but the economy as a whole must do so.

Following Leontief's reasoning two equations represent the model in its open version, although as stated above, Leontief (1944, 1986) concentrated his attention on the first one:

1. $\mathbf{Z}\iota + \mathbf{f} = \mathbf{x}$
2. $\iota'\mathbf{Z} + \mathbf{v}' = \mathbf{x}'$

ι is the sum vector, $\mathbf{x}$ is the (column) vector of outputs accounted by sectoral revenue and $\mathbf{x}'$ is the (row) vector of outputs accounted by sectoral expenditures. Those equations represent two sides of the same phenomenon, that of production. On the one hand, the IO model is demand-driven, one can assume that output is infinitely elastic to final demand and there are no scarce factors or sticky prices that impede any adjustment as needed to reach equilibrium; on the other, the model is supply-sided and revenues are explained by the generation of value added. Output is infinitely elastic to factor revenues; consumers of intermediate inputs and of final goods absorb as much output as producers offer[5], otherwise equilibrium is not warranted. Both equations are independent, but can be linked when output becomes factor incomes and, conversely, when value added is transformed into final demand.

Both value added and final demand are exogenous in the open IO model, therefore transforming one into another is exogenous as well. For that reason, those variables cannot be determined simultaneously as it happens in a general equilibrium model (Debreu, 1959) and the above equations cannot be solved simultaneously (Schummann, 1990). Consequently, those equations are not dual (as it is the case in a general equilibrium model). Moreover, as said above, Leontief (1944) discusses equation (1) only; supply and prices are beyond his interest,

[5] In order to understand that reasoning it may be useful to appeal to Say's law (Say, 1841, p. 141), but in the context of the IO model such is not necessary, if (as above) the equations are taken as accounting expressions.

despite the fact that in the 1937 closed version Leontief offered a solution to prices in the first place, which was neither simultaneous nor dual to that of quantities; those variables are solved through independent processes.

Solving any modern version of the general equilibrium model, from von Neumann (1937) to Arrow and Debreu (1954) and beyond, means determining two vectors at the same time, one for prices and one for quantities, which are dual one another. Von Neumann (1937) suggested using Brower's fixed point theorem for the task and determined also a uniform rate of growth for all sectors, while more modern variations use Kakutani's simplified fixed point theorem and do not discuss balanced growth. Leontief solves his open demand model following a different route. The supply solution can be reached through a mathematically similar process, as Ghosh showed. As a first step, both equations above will be rewritten in proportions (or coefficients):

$$3. \qquad \mathbf{Ax} + \mathbf{f} = \mathbf{x}$$

$$4. \qquad \mathbf{x'E} + \mathbf{v'} = \mathbf{x'}$$

As usual, $\mathbf{A} = \{a_{ij}\} = \{z_{ij}/x_j\}$ is the technical coefficients matrix, i.e., the proportion of each good *i* that each industry *j* uses in as input to produce a homogenous product. Matrix $\mathbf{E} = \{\mathbf{e_{ij}}\} = \{z_{ij}/x_i\}$ shows the proportions that each industry *i* sells to every other industry *j* out of its total output. In short, coefficients a_{ij} and e_{ij} are proportions of the sectoral expenditure (x_j) and sectoral revenue (x_i), respectively. As it is well known, **A** is technically determined, whereas **E** is not: from the viewpoint of the producer it is reasonable to say that the technology determines the list and proportions of inputs she employs, while there is no theoretical explanation of the amounts or proportions that suppliers sell to each consumer, who plays the active role; it is also unimportant for the seller whether her product is used as an input or as a final demand good.

Matrices **A** and **E** are square, semipositive and non-singular; besides, they share the associated eigenvalues. In a word, matrices **A** and **E** are similar, because they result from two similar production models; both linearly transform the space of produced goods into a space of produced goods by different means: intermediate consumption and the distribution of inputs. Moreover, the sum of each column of **A** and each row of **E** are less than unity, because each industry use goods as inputs in lesser value than that they produce and –at the same time- the value of total supply of each produced good is larger than the value of the goods absorbed by other producers as inputs. As a result, the economic system produces surplus and -in each model- either final demand goods and value added, (Nikaido, 1970).

The solutions to the above equations are:

$$5. \quad (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{f} = \mathbf{L}\,\mathbf{f} = \mathbf{x}$$

$$6. \quad \mathbf{v}'\,(\mathbf{I} - \mathbf{E})^{-1} = \mathbf{v}'\,\mathbf{H} = \mathbf{x}'$$

Those expressions determine, first, the level of total production necessary to satisfy final demand, **f** and, second, the level of output necessary to generate the desired level of value added. **L** is called Leontief or the multipliers matrix; its entries show the direct and indirect (total) requirements of inputs produced by *i* per unit of output produced by industry *j*. Analogously, the entries of matrix **H** show the direct and indirect sales that sector *j* must encourage to every other sector *i*, so that **v**' is attainable. Else, if there is shortage of good *j*, its increased supply will be demanded by other sectors in a natural way. These two models together imply that the productive process follows a circular logic, when a proportion of output returns to the productive sphere as inputs, required in the production.

3. *Ambica Ghosh's Model*

Ambica Ghosh´s (1958) model is shown by equations (2), (4) and (6); as already said, Ghosh suggests that Leontief's or his alternative formulation are similarly valid, under different institutional conditions. Neither Ghosh or Leontief consider optimality; nevertheless, one curious point of the paper is the surmise that it is possible to find non-optimal resource allocations that nevertheless maximise welfare, by maximising the employment of labour, regardless of its productivity (p.59). Perhaps Ghosh's preoccupation could be rephrased saying that the central planner could have the goal of maximising labour employment, regardless of any other concern (perhaps lowering wages); alternatively the model could be built assuming different rationality conditions.

Further, Ghosh postulates that in economies with surplus of factors technical coefficients ($\mathbf{a}_{ij}$) might be unstable, whereas the proportions of distribution ($\mathbf{e}_{ij}$) are not. That amounts to saying that one can find continuous technical change in the economy, while the allocation of outputs remains. More recently, Chen and Rose (1986) have postulated the opposite: since matrix A is technically determined, it is stable; while there are no theoretical grounds to justify the stability of matrix E.

From the viewpoint of the model as shown above, both matrices are subject to analogous weaknesses, unless further assumptions are accepted. If only one matrix changed, the similarity between matrices **A** and **E** would be broken. On the contrary, according to Leontief (1944), in the short run matrix **A** is fixed, it is possible to perform experiments assuming changes in final demand while technology is given (there are no reasons to expect changes in **E**); alternatively, following Ghosh's assumptions if **v** changes, **E** is fixed (as should be **A**) so that their similarity is maintained. It is not possible that allocation coefficients change on their own, keeping demand coefficients (or *vice versa*), unless the model does not comply with the principle of proportionality, on which Leontief (1937) bases the whole IO model originally. According to that principle, when one technical coefficient changes in the long run, one sector's sales to another will also change.

Leontief (1944) does not mention that principle, because it deals with the demand side only, but the logic of the construction of the model allows one to expect that it remains valid. No coefficient in any matrix (**A** or **E)** can change independently, unless the whole economic system changes as well.

3. The Dutch Connection

J. Oosterhaven (1988) claims that within the logic of Ghosh's model it is feasible to increase output in some sectors while keeping value added static. "... The Ghoshian model takes demand for granted, i.e., demand is supposed to be perfectly elastic (...) local consumption or investment reacts perfectly to any change in supply, and that purchases are made, e.g., of cars without gas (*sic*.) and factories without machines ..." (p. 207). The author concludes that the model is thus implausible; for him, it is unrealistic to assume that demand may be infinite elastic. That takes him also to reject the proportionality principle, since he accepts that supply may be elastic to demand. Nevertheless, if one accepts that supply may change to satisfy demand, the latter must also be elastic when supply changes and equilibrium is maintained.

Next, Gruver (1989) criticizes that in the supply-driven model no input is essential, so every input can be substituted by any other one. It is interesting to consider in that respect that Leontief does not consider the origins of value (Leontief, 1928). Its theory is based upon average costs of production that equal prices. Indeed no input is more important than any other and when technical change exists, in the long run coefficients, costs and relative prices may change in any direction.

Further, Dietzenbacher (1997) explains that in the IO model, when production grows in one sector, no other industry needs increasing value added, also that the model by Ghosh is similar to the standard IO price model. Such a conclusion, he claims, attends Oosterhaven's critique. According to Dietzenbacher, much of the confusion regarding the supply-driven model derives from its understanding as determining quantities (p. 631) when, he concludes, it determines prices. Probably based on the dual solutions encountered in the general equilibrium model, one for quantities and one for prices, Leontief's production model has been identified as a quantities model, lacking a price dual formulation (despite the explicit solution discussed in Leontief,1937). However, as it has been stated above, both the open IO demand-driven model and the supply-driven version determine output. The duality condition is beyond their scope. The price model that has been accepted for long is (Miller and Blair, 2009):

$$(7) \qquad \mathbf{p} = (\mathbf{I} - \mathbf{A}')^{-1}\,\mathbf{v}$$

The founding assumption of equation (7) is that the price level in each sector depends on the direct plus indirect costs of primary inputs, given the technology used in the system as a whole (matrix **A**). Once again, we need to remark that this equation can be solved independently from equation (5), because they are not dual. Returning to Dietzenbacher's interpretation of equation (6) as equivalent to Leontief price model[6], it would imply that matrix **E** is "equivalent" to matrix **A**'. If the term "equivalent" means "equal", it should be noted that those arrays are in general unequal, unless **Z** is symmetrical and **A** = **E**' or **E** = **A**'.

Louis de Mesnard (2009) re-examines the consistency of the supply-driven scheme, splitting both the demand- and the supply- driven formulations into quantities (physical) and prices models. That procedure disregards that even if a physical inputs matrix was attainable, no mathematical operation would be possible, making the model useless. For example, the amounts of inputs needed to produce one good could be expressed in grams, litres or meters, according to their

[6] "… the equivalence of the supply-driven input-output model and the Leontief price model can also be shown in another, surprisingly simple manner … Post multiplying both sides of Equation (9) with x_0 and using $B_0 = x_0^{-1} A_0 x_0$ yields $x_1' = v_1' (I - B_0)^{-1}$, which is exactly the supply-driven input-model in Equation (6)." (Dietzenbacher, 1997, p. 634).

nature. Technical coefficients are, on the contrary, proportions of outputs ($x_j = p_i q_i$).

Nevertheless, de Mesnard correctly concludes that Ghosh's is not the dual to Leontief's model, and also that the supply-sided provides poor and uninteresting solutions if compared to the demand model. No explicit explanation is made to support that point, however, but such position is of course valid, if some particular theory is chosen to understand the IO model. As a result, Louis de Mesnard finds that it is unreasonable to assume that buyers are forced to buy as much as a producer decides to offer, but as it has been suggested in this paper, maybe the equations representing the IO model can be taken as accounting arrays and avoid intricate discussions.

Guerra and Sancho (2011) present interesting considerations on Ghosh's model and show alternative closure possibilities in order to explore whether it is possible to make the model plausible. The authors support Oosterhaven's rejection to the supply-sided model on the grounds that it is not realistic. Beyond that, it has been shown in this paper that both the supply- and demand-driven models derive straight from the transactions IO table.

What happens when final demand changes in one sector in Leontief's demand-driven model? The immediate reply is that output changes in that sector proportionately and thus, provokes changes in the demand for inputs of that sector as well; causing changes in the production of the industries that supply inputs to the initial activity, as demand expands or contracts. In turn, it is expected that output changes in every sector. According to the multiplier analysis it is expected that resources are available at every moment to carry out any level of production, determined by demand. In that exercise it is also expected that technical coefficients remain, but there is no question on the allocation proportions; in principle, there are no reasons to expect them to change. Intermediate demand coefficients are stable because there is no reason for either the technology or the structure of the system to change, but output in all the sectors will increase or decrease in a magnitude explained by the multipliers and the initial final demand modification. If coefficients change, multipliers cannot be estimated.

Nevertheless, Oosterhaven (1988) and de Mesnard (2009) find that it is not sensible to carry on an analogous analysis in the supply-driven model; consumers cannot be forced to absorb any amount of production. However, according to equation (6) and following the assumptions of the model, if factorial income increases in one sector (v_j) and there are no obstacles for the system to return to equilibrium, that sector would increase its output and the needed extended sales are generated automatically and producers' revenue must expand, in order to afford the extra amounts of inputs required to grow their own production. The extended output induces other sectors to expand their own output. No industry should face difficulties to hire the necessary extra factors, or to find consumers willing to demand the new production. There is no question about the profitability of the increased production: the model does not mention it, but there are no reasons to argue that it will change. The former assumes that production in each sector is constrained by the availability of inputs, therefore, as soon as one input is available in bigger quantities, growth is a natural result. In fact this is the idea behind forward linkages (Bulmer-Thomas, 1982).

The IO model is static and maybe that is one major drawback, which has limited its development and application. It is also an equilibrium system and changing one coefficient may cause changes in the output of whole structure (Shintke and Stäglin, 1988); Leontief (1937) offers a detailed study of such possibility. The main preoccupation of the model in the early days was the analysis of sectoral interdependence; then if one or a few coefficients change, it also changes the way sectors interrelate. Therefore, the only scheme admissible to consider the possibility of growth in the IO model is that of a balanced rate. When a sector expands faster or more slowly than the rest, the system faces disequilibrium and unbalances. Exercises of the kind considered in the previous two paragraphs are valid only as bounded simulations to measure impacts of exogenous moves in a system that eventually returns to equilibrium; otherwise the technical coefficients matrix is unattainable.

Some numerical exercises

Quite a few authors have explored the numerical relationships between Leontief's demand driven and Ghosh's supply-sided models and between matrices **A** and **E**. Chen and Rose (1986) explain that it is empirically interesting to investigate whether changes in the **E** matrix (keeping array **A** fixed) or changing **A** (while keeping **E** fixed) are consistent with the simulated demand or value added increases or decreases. This has been defined as the problem of stability. They conclude that the models are jointly stable if the original growth (positive or negative) does not cause "much difference" in the changing matrix. In any case, it is clear that the analysis is symmetrical for both models and when all sectors grow at a balanced rate would be the only case when both models comply with the joint stability condition. Besides, by the proportionally principle one can expect that empirical models will be jointly stable.

Oosterhaven (1988) and Dietzenbacher (1997) perform similar analysis and conclude that joint stability can only be expected if sectoral growth is uniform. Oosterhaven derives two expressions for stability, which should be complied simultaneously:

$$\mathbf{A}_{t+1} = \hat{\mathbf{e}}\mathbf{A}_t\hat{\mathbf{e}}^{-1}$$

and

$$\mathbf{E}_{t+1} = \hat{\mathbf{e}}^{-1}\mathbf{E}_t\hat{\mathbf{e}}$$

where ê is the relative growth in total sectoral output and subindex *t+1* refers to the simulated matrix after final demand or value added grows. Those expressions mean that $\mathbf{A}_{t+1} = \mathbf{A}_t$ and $\mathbf{E}_{t+1} = \mathbf{E}_t$, since both arrays are premultiplied and postmultiplied by a diagonal matrix and its inverse[7].

A random numerical example may be useful to understand the models and the stability problem. Let a three-sector economy be represented by the following:

Sector	1	2	3	Intermediate Demand	Final Demand	x
1	58	75	98	231	262	493
2	123	342	198	663	168	831
3	178	215	343	736	164	900
Intermediate Consumption	359	632	639			
Value Added	134	199	261			
x	493	831	900			

Matrix $\mathbf{A_0}$:

0,118	0,090	0,109
0,249	0,412	0,220
0,361	0,259	0,381

and $\mathbf{E_0}$:

0,118	0,152	0,199
0,148	0,412	0,238
0,198	0,239	0,381

Let final demand vector grow at a uniform rate of 10% ($\mathbf{f_1}$):

288,2
184,8
180,4

Which gives rise to the new output vector $\mathbf{x_1}$ 10% bigger ($\mathbf{x_1} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f_1}$):

542,30
914,10
990,00

[7] Given matrices **A** and **B**, $\mathbf{B}\,\mathbf{B}^{-1} = \mathbf{I}$, the identity matrix; therefore, $\mathbf{BAB}^{-1} = \mathbf{A}$

The new transactions table $\mathbf{Z}_1$ (**A**) is also 10% bigger than the original ($\mathbf{Z_1} = \mathbf{Ax_1}$):

63,8	82,5	107,8
135,3	376,2	217,8
195,8	236,5	377,3

The whole system grows 10%; matrix $\mathbf{A} = \mathbf{A_1}$ and $\mathbf{E} = \mathbf{E_1}$. The model complies with the joint stability condition. If value added grows in 10% for each sector, the new vector **v'** is:

147,4
218,9
287,1

the corresponding new output vector equals $\mathbf{x_1}$. Likewise, the new exchange matrix $\mathbf{Z}(\mathbf{E})_1$ will be equal to $\mathbf{Z}(\mathbf{A})_1$:

63,8	82,5	107,8
135,3	376,2	217,8
195,8	236,5	377,3

This system is jointly stable in general. Uniform changes in final demand or value added do not mean changes in either technical or allocation coefficients. Moreover, the system's behaviour is similar when final demand or value added changes. One feature is that the above exercise assumes a uniform rate of growth.

In order to explore the problem of stability, the former example is modified and final demand in sector 2 (only) grows 16%, giving rise to the following final demand vector ($\mathbf{f_1}$):

262
194,88
164

Correspondingly, the output vector ($\mathbf{x_1}$) is:

503,27
892,96
931,90

i.e., each sector's total output expands at a different rate, due to the multipliers; the vector of expansion rates is:

2,1
7,5
3,5

The new transactions table $\mathbf{Z_1}$ (= $\mathbf{A_0x_1}$) is also bigger than the original; each column grows at the same rate as the sectoral output:

59,21	80,59	101,47
125,56	367,50	205,02
181,71	231,03	355,16

Matrix $\mathbf{A_1}$ equals the original one $\mathbf{A_0}$. Matrix $\mathbf{E_1}$ changes, even if differences are not "large":

0,118	0,160	0,202
0,141	0,412	0,230
0,195	0,248	0,381

In the supply-sided model, let sector 2 value added grow 16% as well. The new value added vector is:

134	230,84	261

The corresponding total output vector is ($\mathbf{x'} = \mathbf{v'}(\mathbf{I} - \mathbf{E})^{-1}$:

513,09	904,39	934,71

Each sector 1 expands 4.1%, 8.8% and 3.9%. The new exchange table ($\mathbf{Z_1} = \mathbf{x_1'E_0}$) is:

60,36	78,06	101,99
133,86	372,21	215,49
184,86	223,29	356,23

The latter gives rise to a non-changing matrix $\mathbf{E_1}$ and to a new matrix $\mathbf{A_1}$:

0,118	0,086	0,109
0,261	0,412	0,231
0,360	0,247	0,381

which is not significantly different from the original $\mathbf{A_0}$. It is interesting to note that if final demand and value added in sector 2 are allowed 10%, the joint stability is strictly complied. Miller and Blair (2009) present an example where sectors grow at a different rate each and the demand driven model gives rise to a stable matrix **A**, but changing matrix **E**, conversely in the supply-driven model. However, if all three sectors grow at the same rate, matrices **A** and **E** in either version of the model remain.

In a word, the principle of proportionality ensures that both models are stable. It is well known that the IO is an equilibrium model. Bon (1986) suggests that those imbalances between the simulated matrices can be used to assess whether sectors are constrained by final demand or by the availability of inputs. Both the demand- and the supply-sided models are useful to study the economic system.

4. *Final remarks*

This paper has reconsidered the discussion on Ghosh's supply-sided IO model, which re-emerges every now and then in the discussion as an oddity; for that purpose we have reconsidered many basic features of the IO framework.

Indeed, we have seen that some arguments against the supply formulation include inaccurate assumptions and have reached incorrect conclusions on its functioning; some other have made implicit assumptions and expected much more from Ghosh's model than it can deliver; finally some critics have argued that the model is not the symmetric construction to the demand-driven and well accepted Leontief's version, ignoring the principles on which the latter built the early closed version in 1937. The IO is an equilibrium model of production, based upon the interdependence of the various industries that constitute the economic system and therefore, when one piece changes, equilibrium is in peril, unless the whole system changes, as we read in the 1937 Leontief's paper.

Ghosh claims that his version is useful to understand economic systems constrained by scarce resources, where the allocation of outputs would be a more complex task. Nevertheless, the author did not advocate for supply-sided economics on theoretical grounds; for him the conditions existing in less developed and centrally planned economies made it irrelevant discussing on insufficient demand to ensure full employment. That conclusion has not been addressed in the discussion, i.e., centrally planned economies are no longer relevant, but less developed countries may still face resource scarcities. If different economies face different problems, perhaps different tools of analysis are needed for the applied discussion.

The supply sided model has also found practical applications and forward linkages have been often calculated from the perspective of the allocation of outputs. An extended notion amongst practitioners (despite criticism from outside) is that the IO model is flexible so that it may accommodate various theoretical perspectives and it is useful to study a variety of empirical problems. The supply-sided extension can be useful to extend both cases.

Leontief developed the demand-sided version of his IO model; Ghosh suggested the supply-sided one. The former is clearly more robust, because it is related to technology, i.e., the proportions or coefficients derive from the technology that producers employ. Distribution amongst consumers explains the latter; supply coefficients are not based on anything reliable. Nevertheless, both models are symmetric one another (in the sense discussed in this paper) and the demand-driven side gives support to the supply-sided. Both models are mathematically similar and, relying on the interdependence principle, disturbing demand coefficients has consequences on supply and vice versa.

The two equations that represent the IO model are not dual one another and they do not correspond the quantities/prices duality encountered in the general equilibrium model. In the IO formulation final demand and value added are exogenous variables and one cannot become the other. That metamorphosis can only happen beyond the scope of the model.

The numerical exercises included in this paper have also shown how the models are related and also that the joint stability principle that has been used as an argument against the supply sided version is insufficient to reject Ghosh's proposal. If the demand-side model is acceptable, there are no mathematical or logic arguments to reject the supply sided one, unless the interdependence principle is mistaken. The choice between the demand and the supply driven models is a matter of further considerations.

References

Aroche Reyes F. (1993) "Endogenous Prices for Input-Output Models: A Note" *Economic Systems Research* Vol. 5, No. 4, pp. 365-376

Arrow K. J. and Debreu G. (1954) "Existence of an Equilibrium for a Competitive Economy" *Econometrica* Vol. 22 No. 3 pp. 265-290

Augustinovics M. (1970) "Methods of International and Intertemporal Comparison of Structure" in A. P. Carter and Brody A. (eds.) *Contributions to Input-Output Analysis*, Amsterdam: North Holland, pp. 249-269

Bon R. (1986) "Comparative Stability Analysis of Demand-side and Supply-side Input-Output Models" *International Journal of Forecasting,* vol. 2, pp. 231-235

Bon R. (1988) "Supply-Side Multirregional Input-Output Models" *Journal of Regional Science* vol. 28, pp. 41-50

Bulmer-Thomas V. (1982) *Input-Output Analysis in Developing Countires: Sources and Methods*. Chichester: Wiley

Cassel G. (1918) *Theoretische Sozialökonomie*, Leipzig: C. F. Winter

Chen C. Y. and Rose A. (1986) *The Joint Stability of Input-Output Production and Allocation Coefficients* Regional Research Institute, West Virginia University. Research Paper No. 8617 http://www.rri.wvu.edu/pdffiles/wp8617.pdf (August 30, 2012)

Chenery H. and T. Watanabe (1958) "International Comparisons of the Structure of Production" *Econometrica,* Vol. 26, pp. 487-521

de Mesnard L. (2009) "Is the Ghosh Model Interesting" *Journal of Regional Science* Vol. 49, No. 2, pp. 361-372.

Dietzenbacher E. (1997) "In Vindication of the Ghosh Model: A Reinterpretation as a Price Model" *Journal of Regional Science* Vol. 37, No. 4, pp. 629-651

Georgescu-Roegen N. (1951) "Some Properties of a Generalized Leontief Model" Koopmans T. (editor) *Activity Analysis of Production and Allocation* New York: John Willey and Sons; London: Chapman and Hall, Cowles Commission for Research in Economics

Giarratani F. (1976) "Application of an Interindustry Supply Model to Energy Issues" *Environment and Planning* Vol. 8, pp. 447-454

Ghosh A. (1958) "Input-Output Approach in an Allocation System" *Economica* New Series, Vol. 25, No. 97, pp. 58-64

Gruver G. (1989) "On the Plausibility of the Supply-Driven Input-Output Model: A Theoretical Basis for Input-Coefficient Change" *Journal of Regional Science* Vol. 29, pp. 441-450

Hazari R. B. (1970) "Empirical Identification of Key Sectors in the Indian Economy" *The Review of Economics and Statistics,* Vol. 52, No. 3

Jones L. (1976) "The Measurement of Hirschman Linkages" *Quarterly Journal of Economics,* Vol. XC, No. 2

Laumas P. (1976) “The Weighting Problem in Testing the Linkage Hypothesis” *Quarterly Journal of Economics,* Vol. XC, No. 2

Leontief W. (1936) “Quantitative Input and Output Relations in the Economic System and the United States.” *The Review of Economics and Statistics* Vol. XVIII No. 3 pp. 105-125

Leontief W. (1937) “Interrelation of Prices, Output, Savings and Investment. A Study in Empirical Application of the Economic Theory of General Interdependence.” *The Review of Economics and Statistics* Vol. XIX No. 3 pp. 109-132

Leontief W. (1944) “Output, Employment, Consumption and Investment” *Quarterly Journal of Economics,* Vol. 58, No.2, pp. 290-314

Leontief W. (1986) *Input-Output Economics*, New York: Oxford University Press

Marx K. (1885) *Das Kapital, Kritik der politischen Ökonomie Bd.2: Der Zirkulationsprozeß des Kapitals*. Translated into Spanish as *El Capital, Crítica de la Economía Política,* Tomo II, Vol. 5*, El Proceso de Circulación del Capital,* México: Siglo XXI Editores, 1979

McGilvray J. (1981) “Linkages, Key Sectors and Development Strategy” in Leontief W. (ed.) *Structure, System and Economic Policy,* Cambridge: Cambridge University Press

Nikaido H. (1970) *Introduction to Sets and Mappings in Modern Economics*, Amsterdam: North Holland

Miller R., Blair P. (2009) *Input-Output Analysis: Foundations and Extensions*, Cambridge: Cambridge University Press

von Neumann J. (1937) „Über ein öikonomisches Gleichungssystem und eine Verallgemeinerung des Brouwersschen Fixpunksatzes.“ *Ergebnisse eines mathematischen Kolloquiums,* pp 73-83. Translated into Spanish as “Un modelo de equilibrio económico general.” in Segura J. y Rodríguez C. (eds.) *La economía y sus textos*, Madrid: Taurus, 1998.

Oosterhaven J. (1988) “On the Plausibility of the Supply-Driven Input-Output Model” *Journal of Regional Science* Vol. 28, No. 2, pp. 203-217

Say J. B. (1841) *Traité d'économie politique*, 6e éd, Osnabrück: O. Zeller

Samuelson Paul (1951) “Abstract of a Theorem concerning Substitability in Open Leontief Models” Koopmans T. (editor) *Activity Analysis of Production and*

Allocation New York: John Willey and Sons; London: Chapman and Hall, Cowles Commission for Research in Economics

Schumann J. (1990) "On Some Basic Issues of Input-Output Economics: Technical Structure, Prices, Imputations, Structural Decomposition, Applied General Equilibrium" *Economic Systems Research* Vol. 2, No. 3

Schinkte, J. and R. Stäglin (1988): "Important Input Coefficients in Market Transactions' Tables and Production Flow Tables", in M. Ciaschini (ed.), *Input-Output Analysis: Current Developments*, Chapman and Hall, London

Sraffa P. (1960) *Production of Commodities by Means of Commodities* Translated into Spanish as *Produccion de mercancías por medio de mercancías: Preludio a una critica de la teoría económica*, Barcelona: Oikos-Tau, 1982

Walras L. (1874) *Elements d'economie politique pure, ou, theorie de la richesse sociale*, Lausanne: F. Rouge (1889)