Teamwork Efficiency and Company Size

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Abstract

We study how ownership structure and management objectives interact in determining the company size without assuming information constraints or explicit costs of management. In symmetric agent economies, the optimal company size balances the returns to scale of the production function and the returns to collaboration efficiency. For a general class of payoff functions, we characterize the optimal company size, and we compare the optimal company size across different managerial objectives. We demonstrate the restrictiveness of common assumptions on effort aggregation (e.g., constant elasticity of effort substitution), and we show that common intuition (e.g., that corporate companies are more efficient and therefore will be larger than equal-share partnerships) might not hold in general.

JEL: D2, J5, L11, D02.

Keywords: team; partnership; effort complementarities; firm size

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1 Introduction

Many human activities benefit from collaboration. For instance, writing papers in Economics with a coauthor is often much more efficient and fun than writing them *solo*. But it is very infrequent that an activity benefits from the universal participation of the whole human population—a moderate finite group suffices for almost every purpose. So what determines the size of the productive company? When do the gains from cooperation balance out the costs of overcrowding? Williamson (1971) notes:

> The properties of the firm that commend internal organization as a market substitute would appear to fall into three categories: incentives, controls, and what may be referred to broadly as "inherent structural advantages."

We concentrate on the inherent structural advantages of groups of different sizes. We study a model of collaborative production that demonstrates that the answer critically depends on the properties of the production function in a very specific way. Our main contribution is to summarize a generic but hard-to-use *effort aggregation* function that maps the agents' individual efforts to the aggregated effort spent on production with a simpler *teamwork efficiency* function that measures the comparative efficiency of a team of N workers against one worker. We demonstrate that many tradeoffs arising from employing different managerial criteria can be characterized by the interplay of the production function, which transforms aggregated effort into output, and the teamwork efficiency function. For instance, to determine what company size maximizes the effort chosen by the company's employees, one needs to study the balance between the returns to teamwork efficiency and the behavior of the marginal productivity of the total effort.

We compare the predictions for two types of *companies*:

team: workers determine their effort independently, and the product is split evenly; and

firm: the residual profit claimant sets the effort level with the optimal contract.

We attempt to make as few assumptions about the shape of production functions as possible, which makes us give up the chance to obtain closed-form solutions. However, we are able to obtain comparative static results regarding the change in the optimal size of the firm due to changes in marginal costs of effort, ownership structure (going from a worker-owned to capitalist-owned firm and back), and managerial criteria (maximizing individual effort versus maximizing surplus per worker). We demonstrate that the difference in the sizes chosen by different owners under different managerial criteria are governed by the direction of change in the elasticity of the production function, and therefore results obtained under the assumption of constant elasticities are misleading. The premise that elasticities are constant is natural in parametric estimation, but, as we show, assuming constant elasticities rules out economically significant behavior.

We assume away monitoring, transaction and management costs, direct and indirect, to guarantee that they do not drive our results. We believe they are an important part of the reason why firms exist, but they are complementary to the forces we discuss, and their effects have been extensively studied. Our point is that even in the absence of these costs, there still might be a reason for cooperation—and a reason to limit cooperation. Ignoring most issues about incentives and controls allows us to obtain strong predictions, providing an opportunity to test for the comparative importance of incentives in organizations empirically[1]. Our framework allows one to make judgements about the direction of change in the company's size due to changes in the institutional organization based upon the values of elasticities of certain functions, which can be recovered from the empirical observations.

We now review the relevant literature. In Section 2, we introduce the model and solve for the effort choice in both the team and the firm. In Section 3, we discuss how to identify the optimal size of the company. The conclusion follows. The mathematical appendix contains proofs and elaborates on the characterization of the teamwork efficiency function.

[1]See Bikard et al. (2013) as an example in team efficiency estimation. That paper also contains a vast review of other empirical papers estimating collaboration effects, i.e. in writing comic books, Broadway musicals and research papers.

1.1 Literature Review

The paper contributes to two strands of the literature. The moral hazard in teams literature was introduced by Holmstrom (1982), who showed that provision of effort in teams will be generally suboptimal due to externalities in effort levels and the impossibility of monitoring individual efforts perfectly. Legros and Matthews (1993) showed that the problem of deviation from efficient level effort might be effectively mitigated if sharing rules are well-designed.[2] Kandel and Lazear (1992) suggest peer pressure to mitigate the $1/N$ *effect*: the increase of the number of workers lowers the marginal payoff from higher effort. When the firm gets larger, they argue, the output is divided between a larger quantity of workers, while they bear the same individual costs. Hence, the effort of each worker should decrease as firms grow larger, and the peer pressure should compensate for that decrease. Adams (2006) showed that the $1/N$ effect may not occur if the efforts of workers are complementary enough. Because he uses a CES production function, the determinant of sufficient complementarity is the value of the elasticity of substitution. Particulary, this means that it's efficient to either always increase the firm size or to always decrease the firm size. By generalizing, we in this paper obtain a nontrivial optimal company size.

This allows us to contribute to the firm size literature too. Theories of firm boundaries are classified as technological, organizational and institutional (cf Kumar et al. (1999)). The technological theories explain the firm size by the productive inputs and ways the valuable output is produced. Basically, there are five technological factors that are taken into account in describing the firm size: market size, gains from specialization, management control constraints, limited workers' skills, loss of coordination. For example, Adam Smith explained the firm size by benefits from specialization limited by the market size. By his logic, workers can specialize and invest in a narrower range of skills, hence economizing on the costs of skills. Becker and Murphy (1992) focus on the tradeoff between specializa-

[2] Winter (2004) argues that, frequently, the uniform split of surplus is not necessarily a good outcome. We keep treating workers equally for analytical tractability.

tion and coordination costs. The larger the firm, the larger the costs of management to put them together to produce the valuable output.

Williamson (1971), Calvo and Wellisz (1978) and Rosen (1982) use loss of control for explaining the firm size. Williamson points out that the size of a hierarchical organization may be limited by loss of control, assuming the intentions of managers are not fully transmitted downwards from layer to layer. Calvo and Wellisz (1978) show that the effect of the problem is largely dependent on the structure of monitoring. If the workers do not know when the monitoring occurs, the loss of control doesn't hinder the firm size, while it may if the monitoring is scheduled. Rosen (1982) highlights the tradeoff between increasing returns to scale in management and the loss of control. As highly qualified managers foster the productivity of their workers, able managers should have larger firms. However, the attention of managers is limited, hence having too many workers results in loss of control and decreases the productivity of their team substantially. The optimal firm size in this model is when the value produced by the new worker is less than the losses due to attention diverted from his teammates.

In this literature, Kremer (1993) is the paper closest to ours, because this is one paper that obtains the optimal size of the firm based solely on the firm's production function. This paper focuses on the tradeoff between specialization and probability of failure associated with low skill of workers. He assumes that the the value of output is directly proportional to the number of tasks needed to produce it. A larger number of workers—and hence tasks tackled—allows for the production of more valuable output, but each additional worker is a source of risk of spoiling the whole product. Hence, the size of the firm is explained by the probability of failure by the workers, which correlates with the worker's skill.

Acemoglu and Jensen (2013) analyze a problem similar to ours. In this paper, agents pariticipate in an *aggregative* game, where the payoff of each agent is a function only of the agent himself and of the aggregate of the actions of all agents, and they establish existence and comparative statics results for this type of games; Nti (1997) does a similar analysis for contests. In our game, we allow general interactions, but under certain assumptions we can summarize these interactions in a similar way, which does not depend on additive separability. Also, Acemoglu and Jensen (2013) and Nti (1997) study comparative statics for this general class of games with respect to the number of players, whereas we go a step beyond, looking at the optimal number of players from the perspectives of different managerial objectives. Jensen (2010) establishes the existence of pure strategy Nash equilibrium in aggregative games but does not explore the symmetry of the equilibrium or the comparative statics.

2 The Model

In this part, we will introduce the model of endogenous effort choice by the company workers as a reaction to the size of the company. We will define the equilibrium, determine how the amount of effort responds to the change in the company size N, and obtain some comparative statics results.

Company workers contribute effort for production. Efforts $\{e_1, ..., e_N\}$ are transformed into *aggregated effort* by the *effort aggregator* function:

$$g(e_1, ..., e_N|N), \tag{1}$$

where $g(\cdot|N)$ changes with N. The aggregated effort is then used for production via $f(\cdot)$, the *production function*[3]. Exercising effort lowers the utility of a team member by the effort cost $c(e)$. Obviously, the choice of effort depends upon other members' effort choice.

[3]This does not have to be a production function. If, for instance, $g(\cdot)$ delivers the amount of effort spent, $q(g)$ delivers the quantity produced from employing g efforts, and $P(q)$ is the inverse demand function, $f(g) \equiv q(g)P(q(g))$ would be the revenue function, which does not have to be concave. We omit this discussion for brevity, and keep calling $f(\cdot)$ the production function.

THE TEAM members split the fruits of their efforts equally. The worker's problem in the team is therefore to choose effort e to maximize

$$u(e|e_2, ..., e_N, N) = \frac{1}{N} f\left(g(e, e_2, ..., e_N|N)\right) - c(e). \tag{2}$$

THE FIRM of size N, in line with the literature, acknowledges the strategic complementarities between workers' efforts, and provides each worker with a contract that makes this worker implement the first best effort level. We assume that the residual claimant collects all the surplus; results do not change if the residual claimant only collects a fixed proportion of the surplus, with the rest of the surplus going to the government, to employees as a fixed transfer, or to pestilence. Workers face the same effort aggregator function and production function.

We introduce a number of assumptions in order to obtain useful characterizations.

Assumption 1. *$f(\cdot)$ is strictly increasing and continuously differentiable degree 2.*

This is a technical assumption on the production function. We do not require for now that $f(\cdot)$ have decreasing returns to scale.

Assumption 2. *$g(\cdot|N)$ is symmetric in e_i, twice continuously differentiable, strictly increasing in each argument, concave in one's own effort, and homogenous*[4] *of degree 1 with respect to $\{e_1, ..., e_N\}$. Normalize $g(1|1)$ to 1.*

This assumption states that the identities of workers do not matter, only the amount of effort does. We will be using this assumption extensively, since we will be considering symmetric equilibria.

[4] Homogeneity of degree of exactly 1 is not a very restrictive assumption: if one has $g(\cdot)$ which is homothetic of degree γ, one can use $\tilde{g}(\cdot) = g(\cdot)^{1/\gamma}$ and $\tilde{f}(x) = f(x^\gamma)$. They produce the same composition, but $\tilde{g}(\cdot)$ is homogenous degree 1.

One of the consequences of this assumption is that $g'_1(e_1, e_2, .., e_N|N)$ is homogenous degree 0. This, in turn, implies that in a symmetric outcome

$$g''_{11}(e, e, .., e|N) + g''_{11}(e, e, .., e|N) + ... + g''_{1N}(e, e, .., e|N) = 0 \Leftrightarrow$$
$$g''_{11}(e, e, .., e|N) = -(N-1)g''_{1i}(e, e, .., e|N) \quad \forall i \in \{2..N\}, \tag{3}$$

which by concavity in one's own effort means that in symmetric outcomes, not necessarily everywhere, efforts of members are strategic complements.

Assumption 3. *$c(\cdot)$ is increasing, convex, twice differentiable, $c(0) = c'(0) = 0$.*

This immediately implies that every team member exerts a positive amount of effort, since $f(g(\cdot))$ is assumed to be strictly increasing at zero.

Example 1. *(based on Adams, 2006) Let $g(e_1, .., e_N|N) = \left(\sum_{i=1}^N e_i^\rho\right)^{1/\rho}$, $f(x) = x^\alpha$, $c(x)$ is increasing, twice differentiable and concave, and $c'(e)e^{1-\alpha}$ is increasing*[5]*. Therefore, agent 1 solves*

$$\max_{e_1} \frac{1}{N}\left(\sum_{i=1}^N e_i^\rho\right)^{\alpha/\rho} - c(e_1),$$

which, assuming a symmetric outcome, produces $e_1 = ...e_N = e^*(N) = z(N^{\frac{\alpha-2\rho}{\rho}})$, where $z(x)$ is the inverse of $c'(x)x^{1-\alpha}/\alpha$, an increasing function. Hence, $e^*(N)$ is increasing in N if and only if $\rho \in (0, \alpha/2)$, and this implies that the effort aggregator needs to be closer to Cobb-Douglas case to have effort increasing in the team size.

Even for a well-behaved aggregation function like CES it is hard to obtain a well-defined $\text{argmax}_N e^*(N)$, and this is even harder for other maximands, like the utility of a representative agent. This goes against the data: most companies operate with a limited workforce, whatever is the maximand they pursue. In order to understand better what kind of function can deliver nontrivial predictions (neither 1 nor $+\infty$), we need to charac-

[5]Particularly, $\alpha \leq 1$ suffices.

terize the changes in $e^*(N)$. The first-order condition of the worker's problem is

$$f'(g(e_1,...,e_N)|N))g_1'(e_1,...,e_N|N)/N - c'(e_1) = 0. \tag{4}$$

Solving the first-order condition is sufficient to solve for the maximum when

$$f''(g(e_1,...,e_N)|N))(g_1'(e_1,...,e_N|N))^2/N + f'(g(e_1,...,e_N)|N))g_1''(e_1,...,e_N|N)/N - c''(e_1) < 0 \tag{5}$$

for every $\{e_2,...,e_N\}$. Denote $\varepsilon_q(x) = q'(x)x/q(x)$, the elasticity of $q(\cdot)$ with respect to x. By dividing the second-order condition by the first-order condition and multiplying by e_1, with a slight abuse of notation one can obtain

$$\varepsilon_{f'}(g(e_1,...,e_N|N))\varepsilon_g(e_1,...,e_N|N) + \overbrace{\varepsilon_{g_1'}(e_1,...,e_N|N)}^{<0} - \varepsilon_{c'}(e_1) < 0, \tag{6}$$

which will hold whenever (5) holds.

Assumption 4. (5) *holds for every* $\{e_1,...,e_N\}$ *for every* N*.*

This holds when $f(\cdot)$ features decreasing returns to scale, and the aggregator function $g(\cdot)$ is concave in each argument. Alternatively, one can require that $c(\cdot)$ is convex enough.

2.1 Effort Choice in a Team: Equilibrium Outcome

The equilibrium is a collection of efforts of agents $\{e_1^*,..e_N^*\}$ such that each worker i solves his problem (2) subject to treating efforts of other peers as given:

$$e_i^* = \text{argmax}_e \frac{1}{N} f\left(g(e, e_{-i}^*|N)\right) - c(e),$$

where e_{-i}^* denotes values of $\{e_1^*,..,e_N^*\}$ omitting the value of e_i^*.

Assumption 5. *A unique symmetric equilibrium with nonzero efforts exists.*[6]

[6]We can obtain this assumption as a result by imposing additional assumptions on $f(\cdot)$ and $g(\cdot)$, like supermodularity and Inada conditions. The pure strategy equilibrium exists because the game we consider

Let $e^*(N)$ be the function that solves

$$f'(g(e^*(N),..,e^*(N)|N))g_1'(e^*(N),..,e^*(N)|N)/N = c'(e^*(N)). \tag{7}$$

Homogeneity of degree 1 for $g(\cdot)$ helps us to study the behavior of $e^*(N)$. Define

$$h(N) \equiv g(1,..,1|N).$$

This function represents the efficiency of coworking. Observe that

$$h(N) = \frac{eg(\overbrace{1,1,1,..,1}^{N \text{ times}}|N)}{eg(1|1)} = \frac{g(\overbrace{e,e,e,..,e}^{N \text{ times}}|N)}{g(e|1)};$$

that is, $h(N)$ measures how much more efficient is the team of agents compared to a single person, holding effort level unchanged. Henceforth we will call it *the teamwork efficiency function*. For instance, if it is linear, the working team is as efficient as its members applying the same effort separately. By Euler's rule and symmetry of $g(\cdot)$,

$$h(N) = (h(N)e)'_e = (g(e,e,..,e|N))'_e = g_1'(e,..,e)+g_2'(e,..,e)+..+g_N'(e,..,e) = Ng_1(e,..,,e|N).$$

Therefore, (7) can be rewritten as

$$f'(e^*(N)h(N))h(N)/N^2 = c'(e^*(N)). \tag{8}$$

The equation (8) is the incentive constraint that defines $e^*(N)$ as a function of N.

here is a potential game; see Monderer and Shapley (1996), Dubey et al. (2006) and Jensen (2010). To secure the existence and uniqueness of the symmetric outcome, one needs additional assumptions on $f \circ g(\cdot)$, $c(\cdot)$, direct (concavity) or indirect (profit single-crossing, compactness of strategy space), but such outcomes are clearly quite common. We opt to avoid the discussion of restrictiveness of these additional assumptions, and concentrate on the interesting case.

2.2 Effort Choice in a Firm: First Best

Following Holmstrom (1982), we assume that the residual claimant provides the employees with contracts that implement the first-best choice of effort.

Assumption 6. *The first-best choice of effort is positive and symmetric.*

The residual claimant would choose the effort size $e^P(N)$ to implement by maximizing

$$\max_{e_1,..e_N} f(g(e_1, e_2, .., e_N|N)) - \sum_{i=1}^{N} c(e_i),$$

which, assuming a symmetric outcome, leads to the first-order condition

$$f'(e^P(N)h(N))h(N)/N = c'(e^P(N)). \tag{9}$$

The solution of (9), $e^P(N)$, is larger than the solution of (8), $e^*(N)$. The reason is that in equilibrium, the marginal payoff to the individual effort does not take into account the complementarities provided to other workers. Even if the product $f(\cdot)$ were not split N ways, but instead were non-rivalrous,[7] the additional $1/N$ in the marginal benefit of the team worker would persist.

2.3 Second-Order Conditions and Uniqueness

Equation (6), the second-order condition of (8), in the symmetric equilibrium can be rewritten as

$$\varepsilon_{f'}(e^*(N)h(N))\frac{1}{N} + \overbrace{\varepsilon_{g_1'}(e^*(N), .., e^*(N)|N)}^{<0 \text{ because (3)}} - \varepsilon_{c'}(e^*(N)) < 0. \tag{10}$$

This is because $\varepsilon_g(e^*(N), .., e^*(N)|N) = \frac{(h(N)/N)e^*(N)}{e^*(N)h(N)} = \frac{1}{N}$. Let

$$\varepsilon_{f'}(e^*(N)h(N)) - \varepsilon_{c'}(e^*(N)) < 0 \tag{11}$$

[7] For non-rivalrous goods, consumption by one agent does not prevent or worsen the consumption of the same unit of good by another agent. Think of coauthoring a paper: the fact of eventual publication contributes to both authors in a similar amount as if there was only one author, at least in the opinion of some promotion committees.

hold; then (10) is satisfied automatically. If $c(x)$ is *more convex* than $f(y)$ at every $x \geq y$, this condition is satisfied. Similar math is used to compare the risk-aversity of individuals: for every $u(x)$, $\varepsilon_{u'}(x)$ is just the negative of Arrow-Pratt measure of relative risk aversion.

The second-order condition for (9) is

$$f''(e^P(N)h(N))h^2(N)/N - c''(e^P(N)) < 0,$$

which, after dividing by the first-order condition, can be rewritten as

$$\varepsilon_{f'}(e^P(N)h(N)) - \varepsilon_{c'}(e^P(N)) < 0. \tag{12}$$

Observe that it is very similar to (11): the effort level in the argument is different.

Result 1. *If $\varepsilon_{f'}(x)$ is weakly decreasing, $\varepsilon_{f'}(x) < \varepsilon_{c'}(x)$, and $h(N) \geq 1$,* (11) *and* (12) *are satisfied.*

Second-order conditions hold at maxima automatically, but if they hold everywhere, the solution of the corresponding FOC has to be unique. Result 1 thus provides sufficient conditions for the uniqueness of the pure strategy outcome.

$\varepsilon_{f'}(x)$ being decreasing has the following interpretation. When $\varepsilon_{f'}(x)$ is constant and equal to α, it means that $f'(x) = Kx^\alpha + C$, which makes $f(x)$ a power function unless $\alpha = -1$, in which case $f'(x) = K \ln x + C$, where K and C are integration constants. The decreasing $\varepsilon_{f'}(x)$ implies "lower power", or "less convexity" of $f(\cdot)$ at larger arguments.

3 The Optimal Size of the Company

Algebraically, the problem of the optimal firm size with distinct nonatomary agents lies in the discreteness of the firm size. However, using homogeneity and the function $h(N)$, we alleviated this mathematical problem. With differentiable $h(N)$, we can take derivatives with respect to N, and expect $e^*(N)$ and $e^P(N)$ defined with (8) and (9) to be continuous and differentiable.

In order to conduct the comparative statics with respect to N, we will apply the usual implicit function apparatus. For now, $h(N)$ has only been defined for $N \in \{1, 2, 3, ...\}$. With a heroic leap of faith, we extend the definition of $h(N)$ to real positive semi-axis.[8] We postpone the discussion of how to choose a continuous $h(N)$ if one only wields $g(\cdot)$ to Appendix A.1.

Knowing how the workers of the company of size N choose their effort, we can characterize the consequences of various company managerial objectives on its hiring policy.

Assumption 7. *The Problems we study are* single-peaked, *that is, there is a unique interior maximum point, and the derivative of every Problem's Lagrangean is strictly positive below that point, and strictly negative above that point.*

The omitted caveats (multiple local maxima, etc) do not improve the understanding.

3.1 Team Size That Maximizes Effort

In this subsection, we will introduce the apparatus we use to make statements about the optimal size of the company. This subsection is crucial to understanding the further analysis. We therefore keep the analysis in this part very explicit. Other problems will be dealt with in a similar fashion, therefore we relocate the repetitive parts to the Appendix.

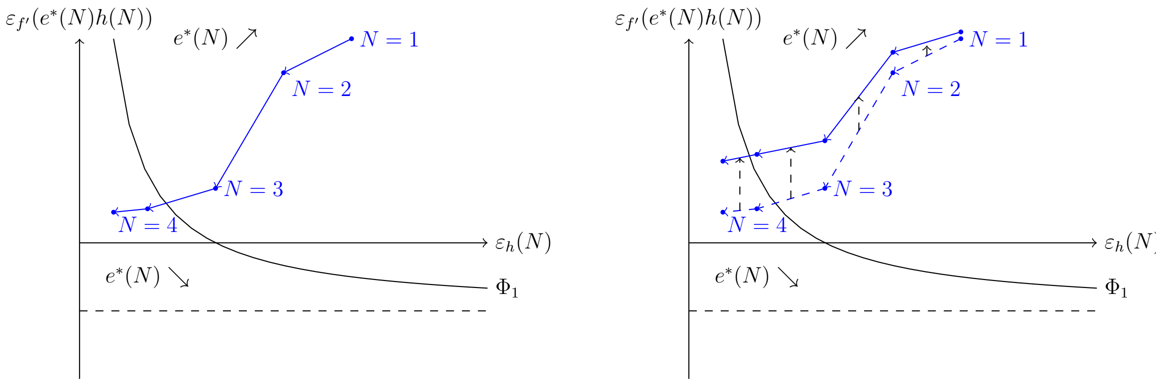
From (8) one can deduce $e^*(N)$, well-defined over $N \in \mathbb{R}_+$, continuous and differentiable.

Problem 1. *Characterize* $N_1 = arg \max_N e^*(N)$.

Take elasticities with respect to N on both sides of (8) to get:

$$\varepsilon_{f'}(e^*(N)h(N)) \left[\varepsilon_{e^*}(N) + \varepsilon_h(N)\right] + \varepsilon_h(N) - 2 = \varepsilon_{c'}(e^*(N))\varepsilon_{e^*}(N).$$

[8]For $g(e_1, e_2, ..e_N|N) = \sqrt{e_1^2 + .. + e_N^2 + \alpha \sum_{i \neq j} e_i e_j}$, $\alpha \in [0, +\infty)$ yields $h(N) = \sqrt{\alpha N^2 + (1-\alpha)N}$, with $\varepsilon_h(N) = 1 - \frac{1-\alpha}{2\alpha N + (1-\alpha)}$, an increasing function of N when $\alpha < 1$ and a decreasing function when $\alpha > 1$. Many papers impose an ad hoc $g(\cdot)$; Kremer (1993) argues for Cobb-Douglas, Rajan and Zingales (1998) goes for linear additive; see Dubey et al. (2006), p. 86 and Jensen (2010), p. 16 for other examples.



(a) In $(\varepsilon_h, \varepsilon_{f'})$ space

(b) Result 2 logic

Figure 1: The choice of N to maximize effort in a team; and the Result 2 logic

Solve this to obtain

$$\varepsilon_{e^*}(N) = \frac{\varepsilon_h(N)\left(\varepsilon_{f'}(e^*(N)h(N)) + 1\right) - 2}{\varepsilon_{c'}(e^*(N)) - \varepsilon_{f'}(e^*(N)h(N))}. \tag{13}$$

From (13) one can immediately see that the N that maximizes $e^*(N)$ has to satisfy

$$\varepsilon_h(N)\left(\varepsilon_{f'}(e^*(N)h(N)) + 1\right) = 2. \tag{14}$$

The denominator of (13) is positive: it is a second-order condition of the effort choice problem, (11). Therefore, whenever $\varepsilon_h(N)\left(\varepsilon_{f'}(e^*(N)h(N)) + 1\right) > 2$, $e^*(N)$ is increasing in N, and otherwise it is decreasing in N.

In the space of $(x, y) = (\varepsilon_h(\cdot), \varepsilon_f(\cdot))$, Equation (14) simplifiesto:

$$\Phi_1 = \{(x, y) | x\,(y+1) = 2.\}$$

Solving out the equilibrium will produce a function $e^*(N)$, and therefore a sequence of values of $(\varepsilon_h(N), \varepsilon_{f'}(e^*(N)h(N))$. We depict an example of this path on Figure 1. Denote

$$\Gamma_1 = ((\varepsilon_h(N), \varepsilon_{f'}(e^*(N)h(N))) | \text{Equation (8) holds}).$$

For the sequence depicted on the Figure 1, one can observe that $e^*(N)$ is increasing at $N \leq 3$, and decreasing for $N \geq 4$. Therefore, the optimal "continuous" N (denote it N_1) is between 3 and 4, and the integer N that delivers the maximum effort is either 3 or 4.

The assumption that $g(\cdot)$ is CES makes $\varepsilon_h(N)$ constant; the assumption that $f'(\cdot)$ is a power function makes $\varepsilon_{f'}(\cdot)$ constant. Example 1 predicts that whether $e^*(N)$ is increasing or decreasing everywhere depends upon the elasticity of substitution of $g(\cdot)$ precisely because, in the world of Example 1, $f(x) = x$ and $g(\cdot)$ is CES. Γ_1 is a single point in these assumptions. Therefore, in order to have a nontrivial prediction about the optimal effort size, one needs either a decreasing $\varepsilon_h(N)$, or a decreasing $\varepsilon'_f(\cdot)$, or both. Obtaining values in the general case in inherently complicated, but one, however, can make comparative statics predictions without knowing the precise specification of relevant functions.

Result 2. *When $\varepsilon_{f'}$ is decreasing, an increase (decrease) in the marginal costs of effort leads to an increase (decrease) in N_1. When $\varepsilon_{f'}$ is increasing, an increase (decrease) in the marginal costs of effort leads to a decrease (increase) in N_1.*

Even without knowing the precise values of elasticities, one can obtain useful results. Assumptions like concavity of f can restrict the economically important behavior:

Example 2. *(based on Rajan and Zingales, 1998, Lemma 2, p. 398) Let $g(e_1, ..e_N|N) = \sum_{i=1}^N e_i$, and let $f(x)$ be concave. Then*

$$\varepsilon_{f'}(x) = \frac{f''(x)x}{f'(x)} < 0, \qquad h(N) = N \Rightarrow \varepsilon_h(N) = 1,$$

and therefore, for every N, $(\varepsilon_h(N), \varepsilon_{f'}(e^*(N)h(N))) < (1, 1)$, no matter what $c(\cdot)$ is. The individual effort decreases with N for every N.

Our results extend to the case when intersections are multiple in a manner similar to the way that comparative statics with multiple equilibria are treated. We will concentrate on the single-crossing case for brevity.

3.2 Firm Size That Maximizes Effort

We will assume that when the firm designs a contract, it tries to implement the first-best, which takes into account the agents' complementarities in $g(\cdot)$. If a social planner were choosing the effort for the agents, his FOC would suggest a higher effort for a given N (see the discussion of the $1/N$ effect on p. 11). Since $c'(\cdot)$ is increasing, this immediately implies that $e^P(N) \geq e^*(N)$, with equality at $N = 1$, and therefore the effort-maximizing sizes of a firm and a team do not have to coincide.

Problem 2. *Characterize* $N_2 = \textit{arg}\max_N e^P(N)$.

The first-order condition[9] becomes

$$\varepsilon_h(N)\left(\varepsilon_{f'}(e^P(N)h(N)) + 1\right) = 1. \tag{15}$$

Again, if the left-hand side is larger than the right-hand side, the effort is increasing in N, and the reverse holds when the left-hand side is smaller than 1. The change of the managerial objective affects multiple components of the optimal size problem:

- The threshold that governs when the firm is big enough, Φ_1, is now replaced by

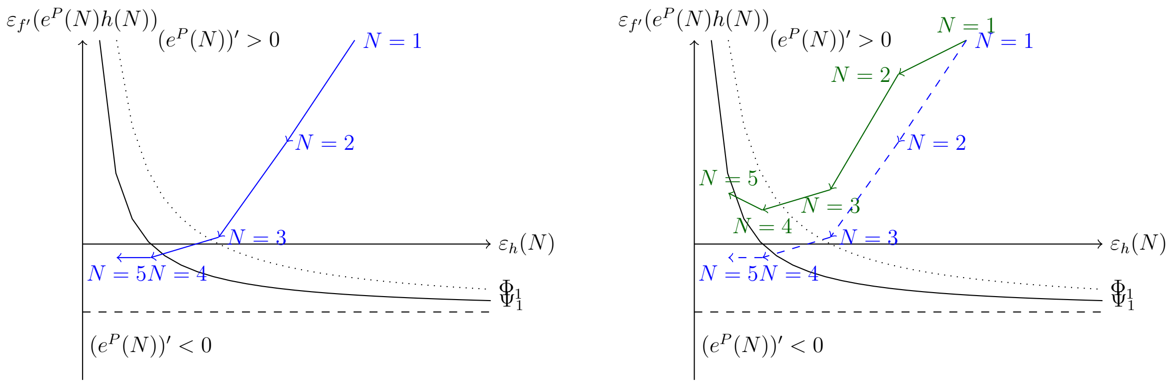
  $$\Psi_1 = \{(x, y)|x\,(y + 1) = 1\}.$$

  The reason why 2 in the definition of Φ_1 is replaced with 1 in the definition of Ψ_1 is exactly because the marginal $1/N$ effect, that appeared because individual marginal benefit did not include the benefits provided to the other participants, went away.

- Since $e^P(N) > e^*(N)$ for almost every level of N, the values of $\varepsilon'_f(e^P(N)h(N)) \neq \varepsilon'_f(e^*(N)h(N))$, unless $f(\cdot)$ is a power function in the relevant domain.

Figure 2a demonstrates the difference, assuming that $\varepsilon'_f(\cdot)$ is a decreasing function. Since $h(N)$ did not change, abscissae are the same for different values of N for both Φ_1

[9] See Appendix for derivation of solutions for Problems 2-5.



(a) If $e^*(N)$ were equal to $e^P(N)$

(b) $e^*(N) < e^P(N)$, and $\varepsilon_{f'}(x)$ is increasing near N_1

Figure 2: Choosing N to maximize effort, the firm case

and Ψ_1. One can see that two effects are at odds: since the threshold is further away, larger firms become more efficient. However, the change in $\varepsilon_{f'}(\cdot)$ due to higher efforts for each firm size might lower the optimal firm size.

Result 3. *If $\varepsilon_{f'}(x)$ is weakly increasing, firms that maximize employees' effort will be larger than teams that choose their team size to maximize the efforts of the members.*

Proof. See Appendix. □

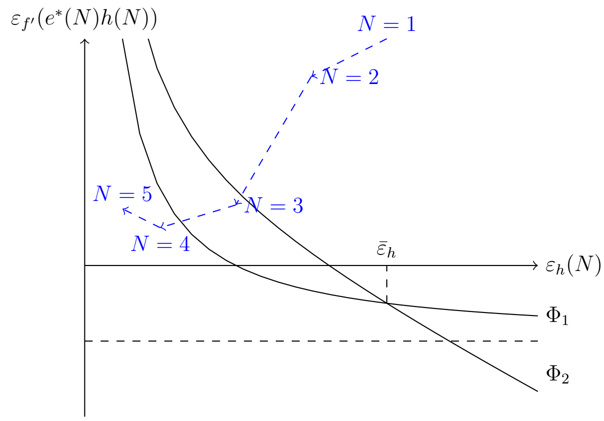
3.3 Team Size That Maximizes Utility

Does it make sense to invite more members to join the team? If this increases the utility of other team members, absolutely. Therefore, the team size that maximizes the utility of a member of the team is the team size that would emerge if teams were free to invite or expel members.

Problem 3. *Characterize $N_3 = arg\max_N \frac{1}{N} f(h(N)e^*(N)) - c(e^*(N))$.*

N_3 should solve the following first-order condition:

$$\varepsilon_f(e^*(N)h(N))\left(\varepsilon_h(N) + \frac{N-1}{N}\varepsilon_{e^*}(N)\right) = 1. \tag{16}$$



Note: Below both graphs both efforts and profits increase as the size of the firm goes larger. Above both graphs both efforts and profits decrease with N. Between graphs, when $\varepsilon_h(N) < \bar{\varepsilon}_h$, efforts increase with N, but profits decrease; the reverse holds when $\varepsilon_h(N) > \bar{\varepsilon}_h$.

Figure 3: Choosing N to maximize individual utility

Again, at values of N where the left-hand side is larger (smaller) than 1, the utility is increasing (decreasing) in N. Let Φ_2 be the set of locations where (16) holds with equality. This line, evaluated at $N = N_1$, is plotted over Γ_1 and Φ_1 on Figure 3.

One can immediately see that:

- There is a unique intersection of Φ_1 and Φ_2, and it happens at $\bar{\varepsilon}_h = {}^1/_{\varepsilon_f(e(N_1)h(N_1))}$.
- The path of Γ_1 intersects Φ_1 above $\Phi_1 \bigcap \Phi_2$ if and only if $N_1 < N_3$. In general, when two different maximands are used, different answers are to be expected, but our result makes issues clearer: the only thing necessary to establish whether $N_1 < N_3$ is the value of $\varepsilon_h(N_1)$ and $\varepsilon_f(e^*(N_1)h^*(N_1))$.

Result 4. *If $\varepsilon_f(x)$ is increasing (decreasing), $\varepsilon_{f'}(x) + 1 > (<)\ \varepsilon_f(x)$, and therefore N_3 is larger (smaller) than N_1.*

Therefore, if the elasticity of $f(\cdot)$ at the size of the team chosen by team members N_3 is too small, it is likely that the team will be too large to implement high efforts ($N_3 > N_1$). For instance, in teaching, many lecturers assign home assignments for group work. Some lecturers use fixed group sizes, other lecturers allow students to form groups of their choosing. If higher effort is desirable (for instance, because effort in the classroom is valuable on the labor market, which is not fully understood by students), it might be a good idea to restrict the group size, against the complaints of students. If the elasticity of $f(\cdot)$ at N_1 is larger than $\varepsilon_{f'}(\cdot)+1$ at the same N_1, students will yearn for increase of the size of the group, and they will complain that the required group size is too large otherwise.

3.4 Firm Size That Maximizes Utility

This could be a problem for an employee-owned firm, where the residual claimant collects zero profit and is only necessary to punish deviators for violations of the optimal contract.

Problem 4. *Characterize* $N_4 = \text{arg}\max_N \frac{1}{N} f(h(N)e^P(N)) - c(e^P(N))$.

At N_4, the following holds (see Appendix for derivation):

$$\varepsilon_f(e^P(N)h(N))\varepsilon_h(N) = 1 \tag{17}$$

When $\varepsilon_f(e^P(N)h(N))\varepsilon_h(N) > 1$, the utility of each member of the firm is increasing with the size of the firm, and the utility is decreasing otherwise.

One can see the difference of (15) and (17): they have to be equal only when $\forall x, \varepsilon_f(x) = \varepsilon_{f'}(x) + 1$, which implies that $f(x)$ is the power function.

Result 5. *If* $\varepsilon_f(x)$ *is increasing (decreasing),* $\varepsilon_{f'}(x) + 1 > (<)\ \varepsilon_f(x)$*, and therefore* N_4 *is larger (smaller) than* N_2*.*

Proof. See Appendix. □

This Result helps to establish why people do not work efficiently in different environments. The problem is not so much in the returns to scale of the production function, the relevant threshold is the comparison of the first and second derivativeÑŃ of the production functionĐś which boils down to whether the elasticity of the production function is locally increasing or decreasing. Those employee-owned companies, whose employees feel that they would be more motivated and would work harder had they had more collaborators, have $\varepsilon_f(e^P(N)h(N)) < \varepsilon_{f'}(e^P(N)h(N)) + 1$, their production function is locally more concave than the power function.

Results for other managerial objectives can be obtained in a similar fashion: for instance, a residual claimant that collects a fixed proportion of the total surplus of the firm will employ *more* than N_4 workers as long as (12) holds. We reserve these for the future research.

3.5 The Quagmire Of Constant Elasticities

The previous analysis showed that at least one of two elasticities cannot be constant in order to obtain a well-defined optimal company size. However, even holding one of two elasticities constant can mislead. In the following example, we assume that $\varepsilon_h(N)$ is decreasing from a large enough value to 0, whereas the production function is a power function.

Example 3. *Let $f(x) = x^\alpha$ and $c(e) = e^\beta$. Let $\beta > \alpha > 0$, then relevant Assumptions and* (11) *are satisfied. For general but convenient $h(\cdot)$, where $\varepsilon_h(\cdot)$ is decreasing, the first-best $e^P(N)$ chosen by the firm satisfies*

$$\alpha(e^P(N)h(N))^{\alpha-1}\frac{h(N)}{N} = \beta(e^P(N))^{\beta-1} \Rightarrow$$

$$e^P(N) = \exp\left[\frac{\ln\alpha - \ln\beta}{\beta-\alpha} + \frac{\alpha}{\beta-\alpha}\ln h(N) - \frac{1}{\beta-\alpha}\ln N\right].$$

The effort size $e^*(N)$ chosen by the members of the team satisfies

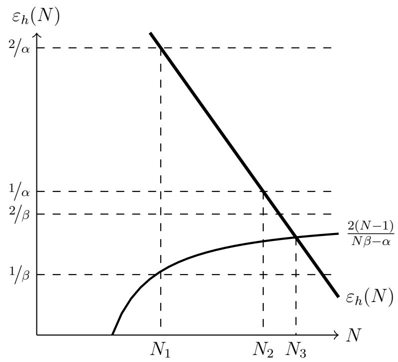
$$\alpha(e^*(N)h(N))^{\alpha-1}\frac{h(N)}{N^2} = \beta(e^*(N))^{\beta-1} \Rightarrow$$

$$e^*(N) = \exp\left[\frac{\ln\alpha - \ln\beta}{\beta-\alpha} + \frac{\alpha}{\beta-\alpha}\ln h(N) - \frac{2}{\beta-\alpha}\ln N\right].$$

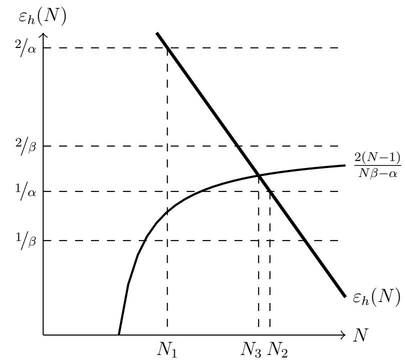
Let us order firm sizes chosen with different managerial objectives. When $\varepsilon_h(N)$ is decreasing,

1. *N_1, the team size that maximizes the effort when the effort level is chosen simultaneously and independently, satisfies $\varepsilon_h(N_1) = 2/\alpha$;*

2. *N_2, the firm size that maximizes the effort when the effort level is chosen according to the first best, satisfies $\varepsilon_h(N_2) = 1/\alpha$;*



(a) When $\beta > 2\alpha$



(b) When $\beta < 2\alpha$

NOTE: $N_2 = N_4$ in both cases because $f(\cdot)$ is a power function.

Figure 4: Ordering solutions from Example 3

3. *N_3, the team size that maximizes the team member's utility when the effort level is chosen simultaneously and independently, solves $\varepsilon_h(N) = \frac{2(N-1)}{N\beta-\alpha}$, right-hand size of which is monotone, and converges to $2/\beta$ from below.*

4. *N_4, the firm size that maximizes the utility per worker*[10] *when the effort level is chosen according to the first best, satisfies $\varepsilon_h(N_4) = 1/\alpha$;*

Example 3 supplies the following intuition for different maximands (see Figure 4):

1 & 2 Effort-maximizing size of the firm is larger than the effort-maximizing size of the team. This is a consequence of $f(\cdot)$ being a power function (see Result 3), and does not have to hold in general.

1 & 3 The company size chosen by the team when the effort level is chosen individually is smaller than the company size chosen to maximize the effort size. This is not a general result, but a consequence of a strong connection between $\varepsilon_f(\cdot) = \alpha$ and $\varepsilon_{f'}(\cdot) = \alpha - 1$. Compare (14) and (16): when N is such that (8) is satisfied, (16) suggests that the utility of each participant would go up if the size of the team went down.

[10]This coincides with revenue per worker if the first best contract provides 0 utility to the worker.

2 & 4 The size of the firm that maximizes employees' utilities is maximizing their effort as well. This is not a general result, but a direct consequence of $f(x) = x^\alpha$: conditions (15) and (17) coincide algebraically.

3 & 4 When a self-organized team becomes incorporated, it might become larger or smaller. If $2\alpha < \beta$, then the incorporated firm becomes smaller than the team. Otherwise, the firm can become larger, but only when $N_3 > 1/(2 - \beta/\alpha)$.

This exercise demonstrates many spurious findings arising simply from the desire of closed form solutions. Some of the strong predictions are generalizable, but most are a consequence of the power function assumptions.

4 Conclusion

In this paper, we stepped away from the common assumptions about production functions to study the effects of scale on the optimal size of a company from many perspectives. Our contribution is to circumvent the inherent discontinuity in hiring when complementarities are important. We found ways to characterize the effects of changes in the management of the company, like incorporation of a partnership, or going from private to public, on hiring or firing, and whether employers's effort will suffer from overcrowding or from insufficient specialization. We found that teams do not have to be larger or smaller than firms that use the same production function. The analytic framework we suggest is very general, and can be modified to include uncertainty, non-trivial firm ownership (for instance, one worker can be the claimant to the residual profit, with nontrivial implications on the effort choice), non-trivial wage schedules (for instance, imperfect observability of effort, total or individual, can call for the design of the optimal wage schedule), or profit-splitting schemes from cooperative game theory like the Shapley value.

The homogeneity of workers is important in our analysis. We have obtained results for heterogenous workforce, where some workers are capable (can choose a positive effort value), and incapable (those who can only choose zero effort). We can show that it might be the case that the incapable workers are employed along with capable ones: this happens if the effort aggregation function is such that the employment of an extra person provides teamwork efficiency externalities for the capable workers, whereas additional effort from the hired capable person would diminish the productivity of other capable employees.

A Mathematical Appendix

Solution of Problem 1 in text, on page 13.

Solution of Problem 2 To choose the firm size that maximizes the level of effort, take the derivative of both sides of

$$f'(e^P(N)h(N))h(N)/N = c'(e^P(N))$$

with respect to N. The values of N where $(e^P(N))' = 0$ will be the one we are looking for. The derivative looks like

$$f''(e^P(N)h(N))[h(N)(e^P(N))' + h'(N)e^P(N)]h(N)/N + f'(e^P(N)h(N))[h'(N)/N - h(N)/N^2] =$$

$$= c''(e^P(N))(e^P(N))'.$$

Divide by the first-order condition to obtain

$$\frac{f''(e^P(N)h(N))[h(N)(e^P(N))' + h'(N)e^P(N)]h(N)/N + f'(e^P(N)h(N))[h'(N)/N - h(N)/N^2]}{f'(e^P(N)h(N))h(N)/N} =$$

$$= \frac{c''(e^P(N))(e^P(N))'}{c'(e^P(N))}.$$

Rearrange to obtain

$$\left[\frac{c''(e^P(N))e^P(N)}{c'(e^P(N))} - \frac{f''(e^P(N)h(N))h(N)e^P(N)}{f'(e^P(N)h(N))}\right]\frac{(e^P(N))'N}{e^P(N)} = \frac{h'(N)N}{h(N)}\left[1 + \frac{f''(e^P(N)h(N))}{f'(e^P(N)h(N))}\right] - 1.$$

Rewrite:

$$\varepsilon_{e^P}(N) = \frac{\varepsilon_h(N)\left(\varepsilon_{f'}(e^P(N)h(N)) + 1\right) - 1}{\varepsilon_{c'}(e^P(N)) - \varepsilon_{f'}(e^P(N)h(N))}.$$

When $\varepsilon_h(N)\left(\varepsilon_{f'}(e^P(N)h(N)) + 1\right) > 1$, effort increases with the size of team, and effort decreases otherwise.

Solution of Problem 3 To choose the team size that maximizes utility, solve

$$\max_N \frac{1}{N} f\left(h(N)e^*(N)\right) - c(e^*(N)),$$

where $e^*(N)$ is such that (8) holds. The first-order condition is:

$$f'(e^*(N)h(N))\left(e^*(N)h'(N) + (e^*(N))'h(N)\right)/N - f(e^*(N)h(N))/N^2 - c'(e^*(N))(e^*(N))' <> 0,$$

with a $>$ sign when the utility of each team member is increasing in the membership size, with a $<$ when the utility of each member is decreasing in the membership size, and with equality at optimum. Substitute (8):

$$f'(e^*(N)h(N))\left(e^*(N)h'(N) + (e^*(N))'h(N)\right)/N - f(e^*(N)h(N))/N^2 - \\ \left(f'(e^*(N)h(N))h(N)/N^2\right)(e^*(N))' <> 0.$$

Group variables and divide by $f(e^*(N)h(N))/N^2 > 0$ to obtain

$$\frac{f'(e^*(N)h(N))(e^*(N)h(N))}{f(e^*(N)h(N))}\left(\frac{e^*(N)h'(N)N + (e^*(N))'h(N)(N-1)}{(e^*(N)h(N))}\right) - 1 <> 0,$$

$$\varepsilon_f(e^*(N)h(N))\left(\varepsilon_h(N) + \frac{N-1}{N}\varepsilon_{e^*}(N)\right) - 1 <> 0.$$

Solution of Problem 4 To maximize the utility of each member of the team when their effort is imposed to deliver the first best outcome, the size of the firm should be chosen to solve

$$\max_N f(e^P(N)h(N))\frac{1}{N} - c(e^P(N)),$$

subject to (9). The first-order condition of this problem is

$$f'(f(e^P(N)h(N)))[e^P(N)h'(N)+h(N)(e^P(N))']\frac{1}{N}-\frac{1}{N^2}f(e^P(N)h(N))-c'(e^P(N))(e^P(N))' <> 0.$$

Divide by $f(e^P(N)h(N))/N^2$ and rearrange to obtain

$$\frac{1}{f(e^P(N)h(N))/N^2}\left(\varepsilon_f(e^P(N)h(N))\varepsilon_h(N)-1\right) <> 0. \tag{18}$$

Result 1. For every level of effort e,

$$\varepsilon_{f'}(eh(N)) < \varepsilon_{f'}(e) < \varepsilon_{c'}(e).$$

Using the effort levels implied by either equilibrium outcome or first best completes the proof. □

Lemma 1. *Let $\tilde{e}(N) > e(N)$. If $\varepsilon_{f'}(\cdot)$ is weakly decreasing (increasing), the effort-maximizing team size under $\tilde{e}(N)$ is lower (higher) than the effort maximizing team size for $e(N)$.*

Lemma 1. Let N_1 and $\tilde{N}_1$ be solutions to team effort maximizing problems with effort functions $e(N)$ and $\tilde{e}(N)$ respectively. If $\varepsilon_{f'}(\cdot)$ is weakly decreasing, since $e(N) < \tilde{e}(N)$

$$\varepsilon_h(\tilde{N}_1)\left(\varepsilon_{f'}(e(\tilde{N}_1)h(\tilde{N}_1))+1\right)-2 \geq \varepsilon_h(\tilde{N}_1)\left(\varepsilon_{f'}(\tilde{e}(\tilde{N}_1)h(\tilde{N}_1))+1\right)-2=0.$$

Since we assumed that the problem is single-peaked, this implies that the effort is increasing with N for $e(N)$ at $N=\tilde{N}_1$, or that $N_1 > \tilde{N}_1$. The result for increasing $\varepsilon_{f'}(\cdot)$ is proven similarly. □

Result 2. Suppose the marginal costs decrease to $\tilde{c}'(x) \leq c'(x)$ for any x. Consider symmetric equilibrium efforts $e(N)$ for the initial problem and $c(\cdot)$ costs, and $\tilde{e}(N)$ under modified costs $\tilde{c}(\cdot)$. By necessary conditions $e(N)$ and $\tilde{e}(N)$ solve (7) with marginal cost functions

$c'(x)$ and $\tilde{c}'(x)$ respectively. Therefore,

$$f'(e(N)h(N))h(N)/N^2 - \tilde{c}'(e(N)) \geq 0 = f'(\tilde{e}(N)h(N))h(N)/N^2 - \tilde{c}'(\tilde{e}(N)).$$

This, combined with second order conditions and single crossing, implies $\tilde{e}'(N) \geq e(N)$. Applying Lemma 1, we obtain the result. $\square$

Result 3. Let $\tilde{N}_1$ solve

$$\varepsilon_h(\tilde{N}_1)\left(\varepsilon_{f'}(e^P(\tilde{N}_1)h(\tilde{N}_1)) + 1\right) - 2 = 0.$$

Then $\tilde{N}_1 \leq N_2$ by single-peakedness assumption for Problem 1. Moreover, by Lemma 1, $\tilde{N}_1 \geq N_1$ as $e^P(N) \geq e^*(N)$ for each N. Hence, $N_2 \geq \tilde{N}_1 \geq N_1$. $\square$

Result 4. Observe that

$$(\varepsilon_f(x))' = (\varepsilon_{f'}(x) + 1 - \varepsilon_f(x))\frac{\varepsilon_f(x)}{x}.$$

Since $f(\cdot)$ is an increasing function, the first part of the statement is proven. The second part of the statement follows immediately from evaluating (16) at N_1. $\square$

Result 5. $\varepsilon_f(x) \geq \varepsilon_{f'}(x) + 1$ means

$$\varepsilon_f(e^P(N_2)h(N_2))\varepsilon_h(N) - 1 \geq (\varepsilon_{f'}(e^P(N_2)h(N_2)) + 1)\varepsilon_h(N) - 1 = 0$$

Workers' utility increases at N_2, hence by single-peakedness assumption $N_2 \leq N_4$. $\square$

A.1 The Choice of $h'(\cdot)$

If one knows $f(\cdot)$, $h(\cdot)$, and $c(\cdot)$, one can conduct the analysis above. However, $h'(N)$ is not a fundamental, at least not in non-integer values. It suffices to know $h(N)$ to evaluate e^*, e^P, ε_f, $\varepsilon_{f'}$ and ε_c at integer Ns. The optimum characterizations, however, depend upon

$h'(N)$ as well. $h'(N)$ values at integer points would suffice, since optimization requires checking whether the value of the elasticity of $h(\cdot)$ is above or below a certain threshold. How can one choose the value of $h'(N)$ at integer points if one only knows $h(N)$ at integer points? Obviously, arbitrary choices of $h'(N)$ can position the points all over the space of $(\varepsilon_h, \varepsilon_{f'})$. One can impose a refinement over the possible derivatives of $h(N)$:

$$h'(N) \in [\min(h(N+1) - h(N), h(N) - h(N-1)), \max(h(N+1) - h(N), h(N) - h(N-1))] \, . \tag{19}$$

To connect integer points, assume that between two neighboring integers, $h'(N)$ is monotone. This implies that extrema of $h(N)$ are only at integer points. Obviously, this preserves concavity, convexity and monotonicity, had $h(N)$ defined over integers had these properties. This limitation helps a lot in characterizing the optimal paths. Consider Figure 5, which is similar to Figure 3, but instead of *points* along the path of Γ_1, we plot *sets* for every value of $\varepsilon_{f'}(e^*(N)h(N))$ that is consistent with some value of $h'(N)$ restricted by (19) at integer values, and then impose monotonicity for $h(\cdot)$ across the path to connect the integer values. On Figure 5, one can see that the intersection with Φ_1 happens between $N = 3$ and $N = 4$, whereas for Φ_2 intersection with Γ_1 happens between $N = 4$ and $N = 5$. Therefore, for $f(\cdot)$ and $g(\cdot)$ behind Figure 5, the self-organizing team will be too large to maximize efforts.

The reverse problem of obtaining $g(\cdot)$ if one knows $h(\cdot)$ but not $g(\cdot)$ is surprisingly easy.

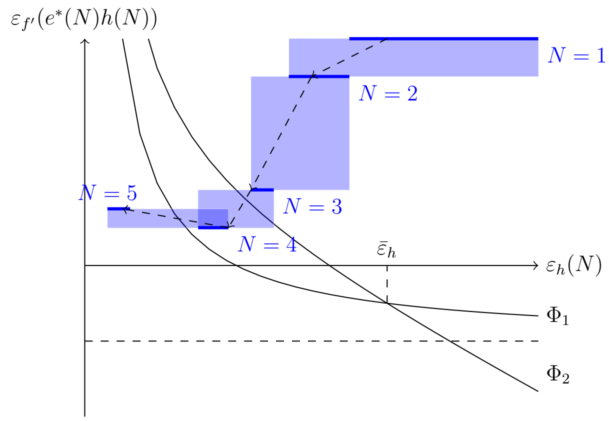
Result 6. *For every $h(N)$,*

$$g(e_1, .., e_N|N) = h(N) \left(e_1 e_2 ... e_N\right)^{1/N}$$

and

$$g(e_1, .., e_N|N) = h(N)/N^{1/\rho} \left(\sum_{i=1}^{N} e_i^{\rho}\right)^{1/\rho}$$

for $\rho < 1$ have properties necessary to apply the analysis above.



NOTE: The solid lines represent the possible values for the path Γ_1 at integer Ns under the restriction of (19). Shaded region represent possible places for the path of Γ_1 over non-integer values of N. Arrows follow a sample path.

Figure 5: Applying restriction (19) to characterize N_1 when continuous $h(\cdot)$ is not available.

Proof. It is straightforward to see that, for $g(e_1, ..e_N) = h(N)(e_1 e_2 ... e_N)^{1/N}$, one obtains

$$g(1, 1, .., 1|N) = h(N)(1 \times 1 \times 1 \times .. \times 1)^{1/N} = h(N),$$

and homogeneity degree 1 is trivial. Since the function is Cobb-Douglas conditional on N, $g_i'(\cdot|N) = \frac{1}{N}\frac{g(\cdot|N)}{e_i} > 0$ and $g_{ii}'' = -\frac{N-1}{N^2}\frac{g(\cdot|N)}{e_i^2} < 0$. Therefore, Assumption 1 is satisfied. The CES case is proven similarly. □

This result emphasizes the comparative importance of $h(N)$ over the complementarities in $g(\cdot)$: many different families of $g(\cdot)$ functions can supply mathematically identical $h(N)$ functions. $g(\cdot)$ should provide enough complementarity for effort choice problem to have a unique solution. The marginal effects of effort complementarity are less important than scale effects of teamwork for the question of the efficient firm size. This, of course, is a consequence of homogeneity of $g(\cdot)$.

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