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Abstract

We investigate the efficiency of the Dickey-Fuller (DF) test as a tool to examine the convergence hypothesis. In doing so, we first describe two possible outcomes, overlooked in previous studies, namely Loose Catching-up and Loose Lagging-behind. Results suggest that this test is useful when the intention is to discriminate between a unit root process and a trend stationary process, though unreliable when used to differentiate between a unit root process and a process with both deterministic and stochastic trends. This issue may explain the lack of support for the convergence hypothesis in the aforementioned literature.

Keywords: Divergence, Loose Catching-up/Lagging-behind, Convergence, Deterministic and Stochastic Trends.
JEL classification: C32; O40

1 Introduction

Bernard and Durlauf (1996) showed that the cross-sectional notion of convergence is weaker than the time-series notion of convergence. Since then, the empirical literature on time-series convergence (hereinafter τ-convergence) has focused on differentiating between three cases out of all the possible results that the different testing procedures are able to identify: catching-up, convergence

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and divergence; whereas *catching-up* or *stochastic convergence* (the weaker definition of convergence) is the case where the logarithmic difference in per capita income between two economies is related to a trend stationary process; *convergence* or *deterministic convergence* (the stronger definition of convergence) is associated to a constant mean stationary process. Finally, *divergence* is linked to a process that contains a unit root.

Time-series evidence has not been completely supportive of the convergence hypothesis. Studies that used the DF test have found it difficult to reject the null hypothesis of unit root (see, for example, Carlino and Mills (1993), Oxley and Greasley (1995), Loewy and Papell (1996), Li and Papell (1999), and Lee, Lim, and Azali (2005) amongst others). As has been shown (Perron 1989), the effectiveness of a unit root test decreases significantly in the presence of structural breaks. Hence, in order to find evidence in favor of this hypothesis, researchers have employed tests that allow structural breaks at the intercept, on the slope of the trend function, or both. Nevertheless, these tests provide mixed results. Bernard and Durlauf (1995) investigate this issue using cointegration techniques; they look for similar long-run trends in per capita output—either stochastic or deterministic, finding no evidence in favor of convergence in 15 OECD economies.

Regardless of the methodological procedure followed, little attention has been paid to the correct sign estimation of the parameter of the deterministic trend. This is extremely important because if such a parameter is positive, rather than a steady reduction in per capita income difference, we observe a constant increment in such disparity.

We assert that the lack of support for the convergence hypothesis may be due to two factors: firstly, the range of outcomes so far considered is incomplete; given the empirical evidence, it is necessary to clearly identify that outcome occurring most frequently in this literature, namely the simultaneous presence of stochastic and deterministic trends in the series under analysis. Secondly, the limited effectiveness of the standard DF methodology to differentiate between certain potential results—specifically, between a unit root process and a process with deterministic and stochastic trends.

The article is organized as follows: Section 2 lists the relevant Data Generating Processes (DGP) included in τ-convergence literature. Section 3 analyzes the asymptotic efficiency of the DF test in estimating both the sign and estimated value of the parameter associated with the determinist trend. Section 4 presents a Monte Carlo exercise to evaluate the performance of this test in finite samples. Finally, the main conclusions are presented in Section 5.

2 Relevant DGPs in τ-convergence

All pertinent DGPs are summarized in Table 1. DGP 1 is associated with divergence, i.e. the case where the logarithmic difference in per capita income between two economies follows a random walk; DGP 2 is interpreted as convergence: the series under analysis is mean stationary; DGP 3 is related to the systematic narrowing (widening) of the per capita income gap if the sign of the deterministic trend estimator is negative (positive), i.e. catching-up and lagging-behind respectively.

	Interpretation	Representation
1	Divergence	$y_t = y_{t-1} + u_{yt}$
2	Convergence	$y_t = \mu_y + u_{yt}$
3	Catching-up or Lagging-behind*	$y_t = \mu_y + \beta_y \cdot t + u_{yt}$
4	Loose Catching-up or Loose Lagging-behind**	$y_t = Y_0 + \mu_y \cdot t + \sum_{i=1}^{t} u_{yi}$

* Catching-up: where $\beta_y < 0$ and Lagging-behind: where $\beta_y > 0$.
** Loose Catching-up: where $\mu_y < 0$ and Loose Lagging-behind: where $\mu_y > 0$.
The process u_{yt} satisfies the conditions stated by Phillip's(1986) lemma, p.369 with variance σ_y^2, long-run variance λ^2, and autocovariances ρ_{yi} for $i = 1, 2, \ldots$

Table 1: Relevant DGPs in τ-convergence analysis.

The cases implied by DGP 4 (when the series displays a stochastic and a deterministic trend simultaneously) have been mentioned only to a limited extent in this literature. Such process represents a weaker notion of catching-up or lagging-behind. Indeed, *loose catching-up* (*loose lagging-behind*) suggests that the poorer economy is erratically, though inexorably catching up (lagging behind) if the sign of the deterministic trend estimator is negative (positive). The dominance of a deterministic trend over a stochastic is a well-established fact[1]; hence, finding evidence of both trends indicates an inevitable reduction—or increase—in income differences in the long-run.

3 The performance of the DF test

The standard methodology to test for convergence using time-series is the DF framework. In this case, the relevant auxiliary regression includes a constant and a deterministic trend, as in Equation 1[2]:

$$\Delta y_t \quad = \quad \alpha + \delta y_{t-1} + \beta T + U_t \tag{1}$$

The various possible outcomes that result from this test are shown in Table 2. Each result is presented alongside its corresponding process:

[1] See for example Hasseler (2000).
[2] Generally, the specification of the DF regression includes lags of Δy_t to alleviate potencial autocorrelation, as well as dummy variables to incorporate structural breaks. Nevertheless, we do not focus on these issues since we consider it important to evaluate the performance of the DF under its simplest specification, prior to carrying out such an analysis.

δ	β	Process
$=0$	$=0$	Divergence
<0	$=0$	Convergence
<0	<0	Catching-up
<0	>0	Lagging-behind
$=0$	<0	Loose Catching-up
$=0$	>0	Loose Lagging-behind

Table 2: Interpretation of the DF test.

It has been proved that the DF test correctly identifies a unit root when the series are generated by DGPs 1, 3 and 4. However, the test fails to produce an adequate estimate of the linear trend parameter (DGPs 3 and 4).

Remark 1 *Let y_t be generated by DGP 3, and be used to estimate regression (1). Hence, the estimated parameter, $\hat{\beta}$, does not converge to its true value, β_y:*

$$\begin{aligned}
\hat{\beta} &\xrightarrow{p} \beta_y \frac{\sigma_y^2 - \rho_{y1}}{\sigma_y^2} \\
T^{-\frac{1}{2}} t_{\hat{\beta}} &\xrightarrow{p} \left[\frac{\sigma_y^2 - \rho_{1y}}{\sigma_y^2 + \rho_{1y}}\right]^{\frac{1}{2}}
\end{aligned}$$

Remark 1 asserts that the test asymptotically identifies the parameter sign but does not correctly estimate the parameter value. On the one hand, the accurate sign identification allows differentiation between Catching-up and Lagging-behind. On the other, the imprecise parameter estimation can be regarded as a minor drawback, given that τ–convergence analysis is statistical in nature and not explicitly tied to a particular growth theory. Hence, the parameter lacks an economic interpretation. Nevertheless, the performance of the DF when the relevant alternative is DGP 4 deserves further attention.

Proposition 1 *Let y_t be generated by DGP 4, and be used to estimate regression (1). Hence, the estimated parameter, $\hat{\beta}$, collapses; its associated t-statistic does not diverge, and both asymptotic expressions contain nuisance parameters (σ_y^2 and λ^2):*

$$\begin{aligned}
T\hat{\beta} &\xrightarrow{d} \mu_y \frac{\lambda^2 \left[\int \omega \left(\omega(1) + 3\int r\omega\right) - \frac{3}{2}\left(\omega(1)\int r\omega + \left(\int \omega\right)^2\right) + \frac{1}{8}\omega(1)^2\right] + \frac{1}{8}\sigma_y^2}{\lambda^2 \left(\int \omega\right)^2 - \frac{1}{4}\int \omega^2 - 3\int r\omega \left(\int \omega - \int r\omega\right)} \\
t_{\hat{\beta}} &= O_p(1)
\end{aligned}$$

Proof: see Appendix

The nuisance parameters may modify the shape of the distribution of $t_{\hat{\beta}}$. This causes some combinations of nuisance parameter values to significantly increase the probability of accepting the null of no significance when in fact it is false (error type II). This issue is illustrated in Figure 1, for the chosen parameter values approximately half of the cases fall outside the standard $(-2,\ 2)$ null acceptance zone.

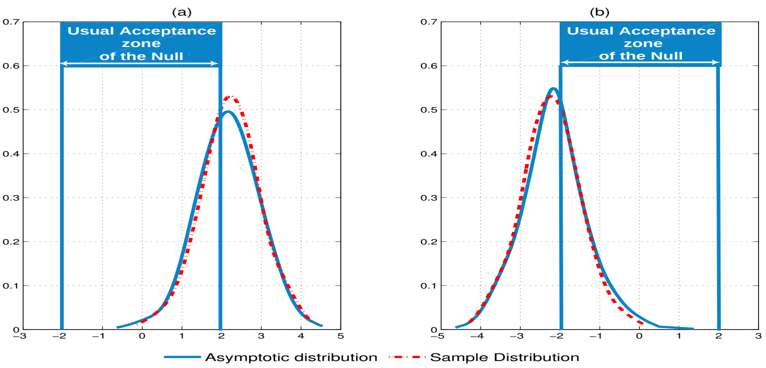

Figure 1: Asymptotic and Sample non-parametric estimated distributions of $t_{\hat{\beta}}$ under: (a) Loose Lagging-behind ($\mu_y = 0.77$) and (b) Loose Catching-up ($\mu_y = -0.07$); $u_{yt} \sim iid\mathcal{N}(0,1)$; $T = 10,000$

If the true DGP of the difference in per capita income contains both deterministic and stochastic trends, the standard significance test of parameter β under-rejects the null hypothesis. In other words, we would tend to erroneously conclude that two economies are diverging when in fact they are, though somewhat sluggishly, catching up or lagging behind. This is a serious mistake given that loose catching-up (loose lagging-behind) is in fact asymptotically equal to catching-up (lagging-behind), whilst its economic significance is entirely different from divergence.

4 Monte Carlo evidence

We present a Monte Carlo study in which the finite sample behavior of the DF test is analyzed where the variable under examination contains both deterministic and stochastic trends. Table 3 shows the proportion of times (for 10,000 replications) in which the test correctly identifies the DGP, as well as the sign of the deterministic trend parameter. Sample size, T, ranges from 25 to 250; the

μ_y	Sample Size									
	No autocorrelation, $\rho = 0.0$					Autocorrelation, $\rho = 0.7$				
	25	50	100	150	250	25	50	100	150	250
-1.50	0.53	0.56	0.56	0.57	0.57	0.50	0.54	0.56	0.56	0.56
-1.25	0.54	0.55	0.56	0.57	0.57	0.47	0.52	0.54	0.55	0.56
-1.00	0.52	0.55	0.57	0.57	0.57	0.43	0.48	0.54	0.53	0.56
-0.75	0.51	0.55	0.55	0.57	0.57	0.38	0.44	0.48	0.52	0.54
-0.50	0.48	0.52	0.54	0.57	0.57	0.32	0.36	0.41	0.45	0.49
-0.25	0.34	0.41	0.48	0.51	0.54	0.25	0.27	0.29	0.32	0.36
0.25	0.34	0.41	0.49	0.53	0.54	0.25	0.26	0.29	0.31	0.35
0.50	0.46	0.52	0.55	0.56	0.56	0.32	0.36	0.41	0.45	0.49
0.75	0.51	0.56	0.55	0.56	0.57	0.37	0.44	0.49	0.52	0.54
1.00	0.52	0.55	0.56	0.57	0.57	0.43	0.50	0.53	0.55	0.55
1.25	0.53	0.56	0.56	0.57	0.57	0.47	0.53	0.54	0.56	0.56
1.50	0.54	0.56	0.56	0.56	0.57	0.50	0.53	0.56	0.56	0.57

Table 3: The DF test and the correct identification of the Loose Catching–up/Lagging–behind DGP.

noise is assumed to be either standard normal or autocorrelated, $AR(1)$ with $\rho_{1y} = 0.7$.

Results indicate a poor performance of the test; its effectiveness increases the larger the absolute value of parameter μ_y, the larger the sample size and the smaller the autocorrelation. The Monte Carlo experiment suggests that the overwhelming evidence of divergence in this literature may be due to the fact that this test fails to differentiate between divergence and loose catching-up (loose lagging-behind).

5 Conclusions

The lack of empirical support for the convergence hypothesis may be due to two factors: 1) the failure of previous studies to give due importance to the case where the difference in per capita income contains both a determinist and a stochastic trend—the situation defined in this study as loose catching-up or loose lagging-behind; 2) the poor performance of the DF test when analyzing series with this characteristic. These circumstances may have led practitioners to erroneously conclude that two economies are diverging when they are, in fact, catching up or lagging behind, though somewhat wearily.

We are aware that current empirical studies make extensive use of more sophisticated tests procedures that allow for the possible existence of structural breaks. Their results, indicating rejection of the convergence hypothesis, should also be taken with caution because the shortcomings of the DF may likewise be applicable to these tests.

A Appendix

Proof of Proposition 1. The expressions needed to compute the asymptotic value of $t_{\hat{\beta}}$ are:

$$\begin{aligned}
\sum y_{t-1} &= Y_0 T + \mu_y \sum t - \mu_y T + \underbrace{\sum \xi_{y,t-1}}_{O_p\left(T^{\frac{3}{2}}\right)} \\
\sum y_{t-1} t &= Y_0 \sum t + \mu_y \sum t^2 - \mu_y \sum t + \underbrace{\sum \xi_{y,t-1} t}_{O_p\left(T^{\frac{5}{2}}\right)} \\
\sum \Delta y_t &= \mu_y T + \underbrace{\sum u_{y,t}}_{O_p\left(T^{\frac{1}{2}}\right)} \\
\sum \Delta y_t t &= \mu_y \sum t + \underbrace{\sum u_{y,t} t}_{O_p\left(T^{\frac{3}{2}}\right)}
\end{aligned}$$

$$\begin{aligned}
\sum y_{t-1}^2 &= Y_0^2 + \mu_y^2 \sum t^2 + \mu_y^2 T + \underbrace{\sum \xi_{y,t-1}^2}_{O_p(T^2)} + 2Y_0\mu_y \sum t - 2Y_0\mu_y T + ... \\
&\quad 2Y_0 \sum \xi_{y,t-1} - 2\mu_y^2 \sum t + 2\mu_y \sum \xi_{y,t-1} t - 2\mu_y \sum \xi_{y,t-1} \\
\sum \Delta y_t y_{t-1} &= Y_0 \mu_y T + \mu_y^2 \sum t - \mu_y^2 T + \mu_y \sum \xi_{y,t} + Y_0 \sum u_{y,t} + ... \\
&\quad \mu_y \sum u_{y,t} t - \mu_y \sum u_{y,t} + \underbrace{\sum u_{y,t} \xi_{y,t-1}}_{O_p(T)} \\
\sum \Delta y_t &= \mu_y^2 T + \underbrace{\sum u_{y,t}^2}_{O_p(T)} + 2\mu_y \sum u_{y,t}
\end{aligned}$$

where $\xi_{y,t} = \sum_{i=1}^{t} u_{y,i}$ and all the other summations range from 1 to T. The orders in probability can be found in Phillips (1986), Phillips and Durlauf (1986) and Hamilton (1994). These expressions can be written in *Mathematica 4.1* code; the software computes the asymptotics of the classical OLS formula $(X'X)^{-1}X'Y$ as well as the asymptotic value of the variance estimator: $\widehat{\sigma}_u^2 = T^{-1}\sum_{t=1}^{T} \widehat{u}_t^2$ where

$$X'X = \begin{pmatrix} T & \sum y_{t-1} & \sum t \\ \sum y_{t-1} & \sum y_{t-1}{}^2 & \sum y_{t-1} t \\ \sum t & \sum y_{t-1} t & \sum t^2 \end{pmatrix}$$

and,

$$Y = \begin{pmatrix} \sum \Delta y_t \\ \sum \Delta y_t y_{t-1} \\ \sum \Delta y_t t \end{pmatrix}$$

As indicated previously, the proof was achieved with the aid of *Mathematica 4.1* software[3].

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[3]The corresponding code is available at:
http://www.ventosa-santaularia.com/VSG_07a.zip