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Abstract

Whilst the existence of a unit root implies that current shocks have permanent effects, in the long run, the simultaneous presence of a deterministic trend obliterates that consequence. As such, the long-run level of macroeconomic series depends upon the existence of a deterministic trend. This paper proposes a formal statistical procedure to distinguish between the null hypothesis of unit root and that of unit root with drift. Our procedure is asymptotically robust with regard to autocorrelation and takes into account a potential single structural break. Empirical results show that most of the macroeconomic time series originally analysed by Nelson and Plosser (1982) are characterized by their containing both a deterministic and a stochastic trend.

Keywords: Unit Root, Deterministic Trend, Trend Regression, R^2.
JEL Classification: C12, C13, C22.

1 Introduction

The influential paper by Nelson and Plosser (1982) (hereinafter NP) triggered a considerable amount of research into the unit-root hypothesis on both the empirical and the theoretical fronts. Since then, an impressive and increasingly complex array of unit-root tests has been available in the literature, many of which were applied first to the original NP dataset.

The significance of the debate lies in the effects of the stochastic shocks. Whenever macroeconomic time series contain a unit root, random shocks have a permanent effect

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on the series. However, in the long run, the effects of these shocks will be reduced if the series also contains a deterministic trend.

The existence of a deterministic trend is also important for the limit distribution of the unit root tests, since the distribution changes depending on the specification of the deterministic component. Moreover, even though the existence of a deterministic trend is more important for the long-run level of the series, there is a bias towards the accurate analysis of the existence of a unit root, i.e. while most unit-root test procedures include a deterministic trend regressor in their analysis, many of these do not formally assess the performance of such estimate when there is evidence of unit root. Indeed, Ventosa-Santaulària and Gómez (2007) proved that it is incorrect to carry out standard hypothesis testing on the deterministic trend parameter estimated with Dickey-Fuller (DF)-type tests when there is a unit root since the limiting distribution of its t-statistic is neither asymptotically normal with unit variance nor nuisance-parameter-free when the innovations are not i.i.d.

This implies that anyone interested in estimating the deterministic rate of growth of a macroeconomic variable may find it difficult to perform such a task; although seemingly straightforward, it becomes nontrivial when the series contains a unit root. In this case, there is neither a reliable nor a simple tool available with which to carry out such estimation.

This paper proposes a formal statistical procedure to distinguish between the null hypothesis of unit root without drift and that of unit root with drift, with and without a structural break [Note that the model under the alternative hypothesis of our test corresponds to Perron's Model B under the null hypothesis; see Perron (1989, p. 1364)]. Our work is in line with those that developed unit-root tests which also consider a drift and a structural break under the null hypothesis; see, for example, Perron (1989), Perron (1997), Carrion–i–Silvestre and Sansó (2006), Kim and Perron (2009), Carrion-i Silvestre, Kim, and Perron (2009), among others.[1] Nevertheless, these do not

[1]This is not a common specification; for example, the popular Zivot and Andrews test allows for breaks

focus on the estimation and hypothesis testing on the drift and the potential structural break associated with it, but rather on the parameter associated with the autoregressive term. Therefore, we believe that our procedure complements these unit-root tests because it formally concentrates on examining the presence of a deterministic trend and a single structural break once there is evidence of a stochastic trend.[2]

In the empirical section, we enter the debate concerning the statistical properties of the macroeconomic series of NP. When characterizing the series, we utilize a longer span—updated to 1988—in order to benefit from the asymptotic properties of our procedure. In addition, we contrast our results with those of Perron (1997) and Carrion–i–Silvestre and Sansó (2006), who proposed unit-root tests that allow for a drift and a break under the null hypothesis.

The article is organized as follows: in Section 2, we present a concise summary of the best-known papers that analyse NP's series. In Section 3, we derive the asymptotic distribution of the new test under the null hypothesis, as well as under the relevant alternative hypothesis, and tabulate the critical values for different levels. Section 4 presents a Monte Carlo exercise to evaluate the performance of this test in finite samples. Section 5 presents the empirical results for the NP dataset, whilst conclusions are drawn in Section 6.

2 Literature Review

In this section, we briefly review the main findings of well-known papers that analyze the unit-root hypothesis for the historical time series of NP.

In their seminal study, NP analyzed 14 US macroeconomic time series using Dickey and Fuller (1979) unit-root test and failed to reject the null hypothesis of nonstationarity in all

only under the alternative hypothesis.

[2] All the unit-root tests so far mentioned consider a drift under the null hypothesis, consequently, if it cannot be rejected, the conclusion is that the series contains both a deterministic and a stochastic trend. Nevertheless, the procedure only focuses on the parameter associated with the autoregressive term parameter.

but one of the series, i.e. unemployment. Kwiatkowski, Phillips, Schmidt, and Shin (1992) complemented existing unit-root tests by proposing a new procedure with trend stationarity as the null hypothesis. They argued that the typical way in which this issue is tested—unit root as the null hypothesis—causes the null hypothesis to be accepted unless there is remarkable evidence against it; they could not actually reject the null hypothesis of trend stationarity for unemployment, real per capita GNP, employment, GNP deflator, wages and money stock. Perron (1989) extended the standard DF procedure by adding dummy variables to allow for the presence of a one-time change in the level or in the slope of the trend function under the alternative hypothesis or both. The results showed that when the Great Depression and the first oil crisis in 1973 are treated as points of structural change in the economy, it is possible to reject the null hypothesis of unit root in favor of broken-trend stationary process—he could not reject the null hypothesis in only 3 of the 14 series: CPI, velocity and bond yield. The assumption that the location of the break is known a priori was criticized by several authors, particularly Christiano (1992), who argued that the choice of the break date is, in most cases, correlated with the data. As a result, formal statistical test procedures capable of determining breakpoints endogenously were proposed to test the unit-root hypothesis. Zivot and Andrews (1992) proposed a Perron—type sequential test-applying his methodology for each possible break date in the sample—applying his methodology for each possible break date in the sample—that maximizes the evidence against the null hypothesis of nonstationarity. They found less support in favor of broken-trend stationarity than had Perron, rejecting the null hypothesis in only 7 of the original 14 series. Perron (1997) reconsidered his 1989 work by allowing endogenous breakpoint determination. Most of the results in Perron (1989) were confirmed, although mixed results were found for real per capital GNP, money stock and GNP deflator.

3 Identification of a deterministic trend in the presence of a stochastic trend

Ventosa-Santaulària and Gómez (2007) proved that the DF-type test procedure may fail to correctly identify the presence of a deterministic trend if the series also contains a stochastic one.[3] We propose an alternative procedure that can be used once there is evidence in favor of unit root. Particularly, we are interested in distinguishing between:

- Driftless Unit Root:

$$\mathcal{H}_0 : \quad y_t = Y_0 + \underbrace{\xi_{yt}}_{b} \tag{1}$$

- Unit Root with drift:

$$\mathcal{H}_a : \quad y_t = Y_0 + \underbrace{\mu_y t}_{a} + \underbrace{\xi_{yt}}_{b} \tag{2}$$

where $\xi_{yt} = \sum_{i=1}^{t} u_{yi}$; u_{yi} represents the innovations and obeys the (general-level) conditions stated in Phillips (1986, p. 313) and the underbraced components are interpreted as (a) Deterministic Trend, and (b) Stochastic Trend.

To distinguish between $\mathcal{H}_0$ and $\mathcal{H}_a$, we will use the following auxiliary regression:

$$y_t \quad = \quad \gamma + \tau t + v_t \tag{3}$$

3.1 The case without structural breaks

If y_t is a unit root with drift process, then:

[3]The inference drawn from the t-ratio associated with the deterministic trend is misleading because it does not follow a standard distribution.

Proposition 1 *Let y_t be generated by equation (1), and be used to estimate regression (3). Hence, the associated R^2:*

1. $R^2 \xrightarrow{d} 1 - \frac{\Omega}{\int \omega^2 - \left(\int \omega\right)^2}$ *for* $\mu_y = 0$
2. $R^2 = 1 - O_p\left(T^{-1}\right) \xrightarrow{p} 1$ *for* $\mu_y \neq 0$

where $\Omega = \int \omega^2 - 4\left(\int \omega\right)^2 + 12 \int \omega \int r\omega - 12\left(\int r\omega\right)^2$*. The* $O_p\left(T^{-1}\right)$ *term is* $\frac{12 - \Omega\sigma_{lr}^2}{\mu_y^2}$ *and* σ_{lr}^2 *is the long-run variance of* u_{yt}*.*

Proposition 1 implies that under $\mathcal{H}_0$, R^2 converges to a non-degenerate and non-standard distribution and is always less than one, whereas under the relevant alternative hypothesis, R^2 converges in probability to one. We computed the asymptotic distribution and estimated its shape non-parametrically (see Figure 1). The critical values are also computed by simulating the asymptotic distribution. Actually, we simulated such expression $100,000$ times and obtained the relevant quantiles of the distribution (see Table 1):

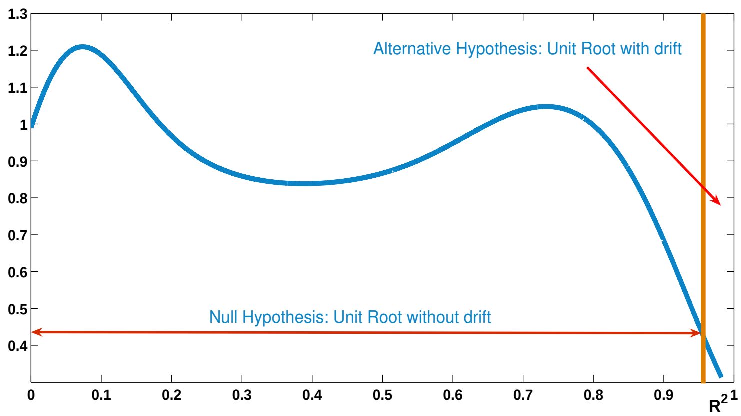

Figure 1: R^2 test asymptotic distribution under $\mathcal{H}_0$

Level (α)	10%	5%	2.5%	1%
Critical Values at α level:	0.84	0.89	0.92	0.94

Table 1: Asymptotic critical values for the R^2 test

3.2 The case with structural breaks

All the asymptotics presented so far are made under the assumption that there are no breaks in the series. Nevertheless, the vast literature concerning this issue favors the hypothesis that structural breaks do occur occasionally in most economic series. Therefore, our previous approach is generalized to allow a one-time change in the deterministic rate of growth, that is, our proposal accounts for one structural break that affects only the slope of the time trend [Perron's Model B under the null hypothesis (Perron, 1989, p. 1364)].

In doing so, we first show that the test, as originally proposed, no longer works correctly.[4] Secondly, we modify the test regression to enable it to control for a possible break. Thirdly, we propose an algorithm that correctly identifies the break and thus recovers the power of the test.

Assume now that the Data Generating Process (DGP) of y_t is given by equation (4):

$$\begin{aligned} y_t &= \mu_y + \theta_y DU_{yt} + y_{t-1} + u_{yt} \\ &= Y_0 + \mu_y t + \theta_y DT_{yt} + \xi_{yt} \end{aligned} \tag{4}$$

where DU_{yt} is a step dummy, that is, $DU_{yt} = \mathbf{1}(t > T_{b_y})$, where $\mathbf{1}(\cdot)$ is the indicator function, T_{b_y} is the unknown date of the break in y, $\lambda = T_{b_y}/T$, and $DT_{yt} = \sum_{j=1}^{t} DU_{yj}$ is the deterministic trend structural break.

Running the test regression (3) on DGP (4) leads to erroneous inference. The R^2

[4]Without any loss of generality we will assume that y_t has a single break. The asymptotics for multiple breaks are analogous.

statistic behaves differently under the alternative hypothesis than has been previously stated. In fact, R^2 does not converge to one under the alternative hypothesis, so the two hypotheses become indistinguishable. There is a total loss of power. This result is summarized in Proposition 2:

Proposition 2 *Let y_t be generated by equation (4), and be used to estimate regression (3). Hence, the associated R^2:*

$$R^2 \xrightarrow{d} 1 - O_p(1) < 1$$

We may override this problem by running the test regression (5) on DGP (4) with a correct specification of the break location:

$$y_t = \gamma + \tau t + \pi DT_{yt} + v_t \tag{5}$$

The results stated in the previous section are once again valid, that is, R^2 converges to one in probability under the alternative hypothesis, as stated in Proposition 3. Note that under the null hypothesis it is assumed that there is neither a drift nor a structural break:

Proposition 3 *Let y_t be generated by equation (4), and be used to estimate regression (5). Hence, the associated R^2:*

1. $R^2 \xrightarrow{d} 1 - O_p(1)$ *for* $\mu_y = \theta_y = 0$
2. $R^2 = 1 - O_p\left(T^{-1}\right) \xrightarrow{p} 1$ *for* $\mu_y \neq 0$ *and* $\theta_y \neq 0$
3. $\hat{\pi} \xrightarrow{p} \theta_y$ *for* $\mu_y \neq 0$ *and* $\theta_y \neq 0$
4. $t_{\hat{\pi}} = O_p(T)$ *for* $\mu_y \neq 0$ *and* $\theta_y \neq 0$

As proved in the appendix, the asymptotic expressions under the null and the alternative hypotheses are far more complicated than those obtained in Proposition 1. In particular, the limiting distribution under the null hypothesis depends upon the location of the break.

Nevertheless, if we run a test regression (5) on DGP (4) with an incorrect specification of the break location, as in equation (6), the test will fail again. Let $T^I_{b_y} \neq T_{b_y}$, i.e., $T^I_{b_y}$ denote an incorrect break date.

$$y_t = \gamma + \theta t + \tau DT^I_{yt} + v_t \tag{6}$$

The test statistic, R^2, does not converge to one under the alternative hypothesis. This is stated in Proposition 4.

Proposition 4 *Let y_t be generated by equation (4), and be used to estimate regression (6). Hence, the associated R^2:*

$$R^2 \xrightarrow{d} 1 - O_p(1) < 1$$

Finally, if we include a break in the test regression and apply it to a series generated by a DGP that does not have one, such test still works. Asymptotically, it does not matter if a non-existent break is included:

Proposition 5 *Let y_t be generated by equation (1), and be used to estimate regression (6). Hence, the associated R^2:*

1. $R^2 \xrightarrow{d} 1 - O_p(1) < 1$ *for* $\mu_y = \theta_y = 0$
2. $R^2 \xrightarrow{p} 1$ *for* $\mu_y \neq 0$ *and* $\theta_y = 0$
3. $\hat{\pi} = O_p\left(T^{-\frac{1}{2}}\right)$ *for* $\mu_y = \theta_y = 0$
4. $T^{-\frac{1}{2}} t_{\hat{\pi}} \xrightarrow{d} \Psi$ *for* $\mu_y = \theta_y = 0$

where Ψ is an unknown-nuisance-parameter-free distribution.

Given that our test statistic, R^2, is asymptotically maximized when the break date is correctly specified and there is no loss of power when we search for an inexistent break, it is possible to design a "break-finder" algorithm by running equation (6) sequentially and allowing the break location to change along the sample. Eventually, if there is indeed a break, R^2 will be maximized whenever $T^I_{b_y}$ falls in the correct location and will thus be equal to T_{b_y}. More precisely, the break date is obtained by maximizing (minimizing) the R^2 (sum of squared residuals, SSR):

$$\hat{T}_{b_y} = arg\, max_{T_{b_y} \in [\varepsilon T, (1-\varepsilon)T]} R^2(\hat{T}_{b_y})$$

where $\hat{T}_{b_y}$ is the estimated break date and $\varepsilon = 0.05$ is the trimming parameter.

It is important to note that, under the alternative hypothesis, we have not yet established that our estimation method provides a consistent estimate of the break point. Nevertheless, we can make use of Perron and Zhu (2005) (PZ, hereinafter) results to assert that this requirement is met since our estimation procedure matches one of their cases [our DGP under the alternative hypothesis corresponds to PZ's model I.a; refer to equation (1) and assumption 2, pp. 69-70].[5] PZ's findings allow us to ensure that, under the alternative hypothesis, our test consistently estimates the break date.[6]

Under the null hypothesis there is no break but the auxiliary regression includes one (located at $T^I_{b_y}$). The asymptotic distribution under the null hypothesis is a function of the—known—location of the break relative to the total sample ($\hat{\lambda} = \hat{T}_{b_y}/T$). New critical values that allow us to carry out hypothesis testing for given values of $\hat{\lambda}$ are thus tabulated in Table 2. These were computed for different break locations, $\hat{\lambda} = 0.10, 0.15, 0.20, 0.25, \ldots, 0.85, 0.90$.

We also computed the distribution of $\frac{t_{\hat{\pi}}}{\sqrt{T}}$. It contains no unknown nuisance parameters,

[5] PZ prove that a break fraction estimated by minimizing the SSR converges in probability—at a rate of $O_p(T^{1/2})$—to the true break fraction, whenever there is such a break [see Theorem 3 (1), p. 75]. Moreover, they prove that the estimate associated to the trend break converges—at a rate of $O_p(T^{1/2})$—to the true parameter if and only if, the break fraction is correct, which it is, given PZ's previous result (refer to theorem 6 1(a), pp. 79). This implies that the break date estimate's rate of convergence is fast enough to be considered as known

[6] Kim and Perron (2009) and Carrion-i Silvestre, Kim, and Perron (2009) also minimized the SSR; in both cases the argument is analogous: the test statistic is a function of the estimated break point and it has the same limit distribution as if the true break point would have been employed.

$\hat{\lambda}$	LEVEL			
	10%	5%	2.5%	1%
0.10	0.87	0.91	0.93	0.95
0.15	0.88	0.91	0.94	0.96
0.20	0.88	0.92	0.94	0.96
0.25	0.89	0.92	0.94	0.95
0.30	0.89	0.93	0.95	0.96
0.35	0.89	0.93	0.95	0.96
0.40	0.90	0.93	0.95	0.96
0.45	0.90	0.93	0.95	0.96
0.50	0.90	0.93	0.95	0.96
0.55	0.90	0.93	0.95	0.96
0.60	0.90	0.93	0.95	0.96
0.65	0.89	0.92	0.94	0.96
0.70	0.89	0.92	0.94	0.96
0.75	0.88	0.92	0.94	0.96
0.80	0.88	0.92	0.94	0.96
0.85	0.87	0.91	0.94	0.96
0.90	0.87	0.91	0.93	0.96

Table 2: Break location and asymptotic critical values for the R^2 test.

Note: the critical values are obtained from the simulation of the asymptotic distribution of the test statistic under the null hypothesis. Number of replications: $20,000$; the simulation of the Brownian motions is made exactly as in Perron (1989, p. 1375). Matlab code available upon request to the authors.

such as σ_{lr}^2. Nevertheless, there is a known nuisance parameter—the estimated break location ($\hat{\lambda}$)—that alters this distribution. Therefore, we obtained critical values for different break locations with which to test the null hypothesis: $\hat{\pi} = 0$;[7] these critical values[8] appear in Table 3.

An example of the distribution of $\frac{t_{\hat{\pi}}}{\sqrt{T}}$ under the null hypothesis of non-significance is shown in Figure 2. The specified break location is $\hat{\lambda} = 0.45$.

[7]The t-ratio associated with this parameter must be normalized by $T^{\frac{1}{2}}$ in order to attain the asymptotic distribution under $\mathcal{H}_0$. Under the alternative hypothesis, the t-ratio diverges at rate T, so the square-root normalization factor does not impede its divergence; in fact, under the alternative hypothesis, $\frac{t_{\hat{\pi}}}{\sqrt{T}} = O_p\left(T^{\frac{1}{2}}\right)$.

[8]The test is double-tailed; notice that the—non-standard—distribution appears to be symmetric. Note also that the break date can be treated as known (under the alternative hypothesis) because of the same arguments stated for the R^2 statistic.

$\hat{\lambda}$	Level 10%	5%	2.5%	1%
0.10	±0.66	±0.78	±0.88	±1.01
0.15	±0.84	±1.00	±1.15	±1.33
0.20	±0.98	±1.18	±1.36	±1.58
0.25	±1.13	±1.36	±1.58	±1.85
0.30	±1.22	±1.48	±1.72	±2.02
0.35	±1.30	±1.59	±1.85	±2.16
0.40	±1.37	±1.66	±1.93	±2.31
0.45	±1.41	±1.71	±2.01	±2.39
0.50	±1.42	±1.72	±2.02	±2.39
0.55	±1.41	±1.70	±2.01	±2.38
0.60	±1.31	±1.66	±1.95	±2.32
0.65	±1.31	±1.61	±1.87	±2.20
0.70	±1.24	±1.50	±1.74	±2.00
0.75	±1.11	±1.33	±1.55	±1.83
0.80	±0.98	±1.15	±1.37	±1.60
0.85	±0.83	±1.00	±1.15	±1.33
0.90	±0.67	±0.79	±0.90	±1.04

Table 3: Asymptotic critical values for $\frac{t_{\hat{\pi}}}{\sqrt{T}}$.

Note: the critical values are obtained from the simulation of the asymptotic distribution of the test statistic under the null hypothesis [$\hat{\pi} = 0$]. Number of replications: $20,000$; the simulation of the Brownian motions is made exactly as in Perron (1989, p. 1375). Matlab code available upon request to the authors.

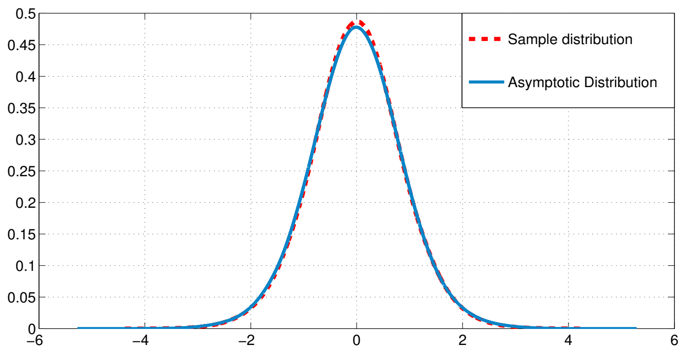

Figure 2: Asymptotic distribution of $\frac{t_{\hat{\pi}}}{\sqrt{T}}$ under the null hypothesis

4 Finite-sample properties of the test

We present a Monte Carlo study to analyze the finite-sample effectiveness of the test. In each case, the number of replications is $1,000$. Firstly, we evaluate the test performance when no structural breaks are present in the data and the algorithm does not search for breaks. Figure 3 shows the effect of autocorrelation[9] on the behavior of the test statistic for different values of the drift. This figure shows that autocorrelation has only a marginal effect (for a 10% level); the power of the test decreases slightly as ρ approaches one. As the sample size increases, from $T = 75$ to $T = 500$, the area with low power shrinks, although the gain in power seems to be relatively small. Furthermore, there is a logical loss of power around the zero-valued drift, where the null hypothesis is actually true.

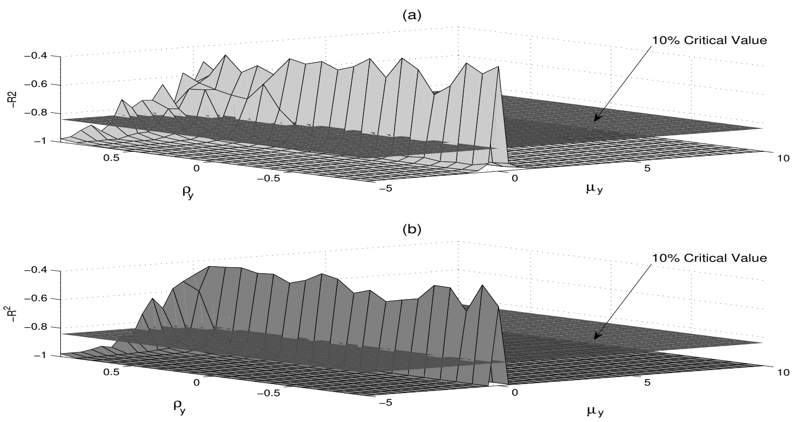

Figure 3: R^2 test-statistic in the presence of autocorrelation and for different values of the drift; (a) $T = 75$ obs. and (b) $T = 500$ obs.

More accurate Monte Carlo exercises are shown in Tables 4 and 5, in which the rejection rates of the null hypothesis for some selected parameter values, sample sizes and statistical significance levels, are shown, these being: ρ = 0.0, 0.25, 0.5, 0.7 and 0.9; T

[9]The underlying error sequence is assumed to be $AR(1)$, where ρ ranges from -0.95 to 0.95.

= 100, 150, 250, 500, 1,000; and α= 1%, and 5%, respectively.

Results show that the test is proficient for samples as small as one hundred observations. Where the DGP is a unit root without drift [Panels (a) of Tables 4 and 5], rejection rates are as low as the significance levels for low values of the autocorrelation coefficient (less than 0.5). In these cases, autocorrelation distortions may be assumed to be unimportant. For values of the autocorrelation coefficient above 0.70, level distortions are important for sample sizes below 250. Where the DGP is a unit root with drift [Panels (b)], the power of the test decreases when the drift approaches to zero and autocorrelation is above 0.50. Although our test is asymptotically immune to autocorrelation, the Monte Carlo experiments show that such immunity is not perfect in finite samples, yet does work well for low levels of autocorrelation.

Secondly, we compare our test with that of Dickey and Fuller (1981) (hereinafter DF81). DF81 specified a procedure to test the joint null hypothesis of unit root and the non-significance of the deterministic regressors, in particular, the drift.

A comparison between DF81 and our test is not straightforward, since the R^2 test presupposes that there is already evidence of unit root and focuses on testing the significance of the deterministic components. However, our test may serve as a complement when DF81 rejects the null hypothesis, as in those cases illustrated by Panels (b) of Tables 6 and 7. When the underlying process is a unit root with drift, DF81 systematically rejects the null hypothesis because it is half false [see Panels (b) of Tables 6 and 7].

Furthermore, the Monte Carlo experiment reveals that the level distortions caused by the presence of autocorrelation are more severe in the DF81 test [see Panel (a) of Table 6] than in the R^2 test [see Panels (a) of Tables 4 and 5]. Of course, Dickey-Fuller's auxiliary regression can be adapted to control for autocorrelation; however, there is the issue of selecting the number of lags to consider. We therefore applied Ng and Perron (1995) lag's selection strategy (see Table 7). Controlling for autocorrelation definitively

Panel (a)							
DGP	Parameters		Sample Size				
	μ_y	$\rho_{y,1}$	100	150	250	500	1,000
U.R. No Drift	0	0.00	0.011	0.009	0.010	0.010	0.010
		0.25	0.013	0.011	0.011	0.011	0.010
		0.50	0.015	0.015	0.013	0.011	0.010
		0.70	0.029	0.020	0.014	0.013	0.013
		0.90	0.082	0.054	0.035	0.019	0.016
Panel (b)							
U.R. With Drift	3	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	1.000	1.000	1.000	1.000	1.000
		0.50	1.000	1.000	1.000	1.000	1.000
		0.70	0.999	1.000	1.000	1.000	1.000
		0.90	0.707	0.776	0.891	0.984	0.999
	0.75	0.00	0.993	0.999	1.000	1.000	1.000
		0.25	0.928	0.984	0.999	1.000	1.000
		0.50	0.660	0.823	0.948	0.998	1.000
		0.70	0.315	0.415	0.606	0.869	0.986
		0.90	0.139	0.128	0.123	0.170	0.280
	-0.75	0.00	0.993	0.999	1.000	1.000	1.000
		0.25	0.932	0.983	0.999	1.000	1.000
		0.50	0.659	0.815	0.953	0.998	1.000
		0.70	0.320	0.425	0.612	0.869	0.988
		0.90	0.139	0.122	0.120	0.163	0.290
	-3	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	1.000	1.000	1.000	1.000	1.000
		0.50	1.000	1.000	1.000	1.000	1.000
		0.70	0.999	1.000	1.000	1.000	1.000
		0.90	0.697	0.771	0.889	0.983	0.999

Table 4: Rejection rates of the R^2 test. The case with no break (level: $\alpha = 0.01$)

decreases the level distortions, however, it reduces the power of the test for high values of ρ (above 0.50) and for low absolute values of the drift.[10]

Thirdly, we assess the performance of the test when it searches for a single break in the series. Tables 8 and 9 show the rejection rates of the null hypothesis when two different DGPs are analyzed at the 1% and 5% levels. Panel (a) of each table—when the DGP is unit root without drift—demonstrates that the test performs satisfactorily,

[10]The Matlab code of the Monte Carlo experiment is available upon request to the authors.

Panel (a)							
DGP	Parameters		Sample Size				
	μ_y	$\rho_{y,1}$	100	150	250	500	1,000
U.R. No Drift	0	0.00	0.049	0.049	0.052	0.047	0.048
		0.25	0.053	0.054	0.052	0.049	0.051
		0.50	0.065	0.060	0.055	0.054	0.052
		0.70	0.087	0.074	0.064	0.057	0.054
		0.90	0.178	0.131	0.100	0.076	0.067
Panel (b)							
U.R. With Drift	3	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	1.000	1.000	1.000	1.000	1.000
		0.50	1.000	1.000	1.000	1.000	1.000
		0.70	1.000	1.000	1.000	1.000	1.000
		0.90	0.850	0.916	0.970	0.999	1.000
	0.75	0.00	0.999	1.000	1.000	1.000	1.000
		0.25	0.991	0.999	1.000	1.000	1.000
		0.50	0.868	0.949	0.994	0.999	1.000
		0.70	0.540	0.670	0.832	0.972	0.999
		0.90	0.270	0.247	0.261	0.337	0.527
	-0.75	0.00	0.999	1.000	1.000	1.000	1.000
		0.25	0.988	0.999	1.000	1.000	1.000
		0.50	0.857	0.948	0.993	0.999	1.000
		0.70	0.537	0.664	0.831	0.973	0.999
		0.90	0.258	0.246	0.258	0.346	0.523
	-3	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	1.000	1.000	1.000	1.000	1.000
		0.50	1.000	1.000	1.000	1.000	1.000
		0.70	1.000	1.000	1.000	1.000	1.000
		0.90	0.863	0.914	0.972	0.999	1.000

Table 5: Rejection rates of the R^2 test. The case with no break (level: $\alpha = 0.05$)

particularly when the inference is drawn based on a 1% level; rejection rates under the null hypothesis are fairly low even for small samples when autocorrelation is low. Panel (b) of each table—when the DGP is unit root with drift—shows high rejection rates of the null hypothesis in both cases, i.e. when the DGP has no break, and when it does. Nevertheless, it is noticeable that positive autocorrelation may have a considerable negative effect on the power of the test in relatively small samples, i.e., those with fewer than 150 observations.

Panel (a) No lags included							
DGP	Parameters		Sample Size				
	μ_y	$\rho_{y,1}$	100	150	250	500	1,000
U.R. No Drift	0	0.00	0.034	0.038	0.045	0.041	0.048
		0.25	0.058	0.055	0.060	0.066	0.062
		0.50	0.138	0.141	0.149	0.152	0.158
		0.70	0.289	0.292	0.311	0.302	0.301
		0.90	0.606	0.591	0.587	0.586	0.583
Panel (b) No lags included							
U.R. With Drift	3	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	1.000	1.000	1.000	1.000	1.000
		0.50	1.000	1.000	1.000	1.000	1.000
		0.70	1.000	1.000	1.000	1.000	1.000
		0.90	0.995	0.999	1.000	1.000	1.000
	0.75	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	0.999	1.000	1.000	1.000	1.000
		0.50	0.980	0.998	1.000	1.000	1.000
		0.70	0.880	0.949	0.992	1.000	1.000
		0.90	0.703	0.725	0.796	0.884	0.965
	-0.75	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	0.999	1.000	1.000	1.000	1.000
		0.50	0.981	0.998	1.000	1.000	1.000
		0.70	0.873	0.949	0.993	1.000	1.000
		0.90	0.698	0.738	0.782	0.885	0.969
	-3	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	1.000	1.000	1.000	1.000	1.000
		0.50	1.000	1.000	1.000	1.000	1.000
		0.70	1.000	1.000	1.000	1.000	1.000
		0.90	0.995	0.999	1.000	1.000	1.000

Table 6: Rejection rates of Dickey-Fuller's (1981) joint test: the case with no break. Lags not included, level: $\alpha = 0.05$

5 Empirical results for Nelson and Plosser series

The purpose of this section is twofold. Firstly, we use our new test to review the statistical properties of the NP series. We apply the popular Zivot and Andrews (1992) test, since our test is properly used only after a unit-root test has been employed. If the former fails to reject the null hypothesis of nonstationarity, then our test can be used.[11]

[11] Although Zivot and Andrews's (1992) test does not allow for a structural break under the null hypothesis of unit root, Vogelsang and Perron (1998) argue on pp. 1092-1093 that: "asymptotic results—**[assuming a break under the null hypothesis]**—were shown to provide poor approximations to finite sample distribu-

Panel (a) Lag selection strategy: Ng and Perron (1995)							
DGP	Parameters		Sample Size				
	μ_y	$\rho_{y,1}$	100	150	250	500	1,000
U.R. No Drift	0	0.00	0.047	0.050	0.056	0.048	0.050
		0.25	0.051	0.042	0.048	0.050	0.052
		0.50	0.054	0.047	0.048	0.049	0.052
		0.70	0.055	0.051	0.052	0.049	0.054
		0.90	0.062	0.056	0.061	0.059	0.052
Panel (b) Lag selection strategy: Ng and Perron (1995)							
U.R. With Drift	3	0.00	0.999	1.000	1.000	1.000	1.000
		0.25	0.997	1.000	1.000	1.000	1.000
		0.50	0.985	0.999	1.000	1.000	1.000
		0.70	0.893	0.986	1.000	1.000	1.000
		0.90	0.206	0.312	0.613	0.983	1.000
	0.75	0.00	0.988	1.000	1.000	1.000	1.000
		0.25	0.936	0.996	1.000	1.000	1.000
		0.50	0.589	0.849	0.991	1.000	1.000
		0.70	0.192	0.329	0.631	0.964	1.000
		0.90	0.073	0.079	0.091	0.149	0.310
	-0.75	0.00	0.988	0.999	1.000	1.000	1.000
		0.25	0.938	0.995	1.000	1.000	1.000
		0.50	0.585	0.848	0.993	1.000	1.000
		0.70	0.195	0.336	0.628	0.965	0.999
		0.90	0.074	0.077	0.094	0.138	0.320
	-3	0.00	0.999	1.000	1.000	1.000	1.000
		0.25	0.997	1.000	1.000	1.000	1.000
		0.50	0.986	0.999	1.000	1.000	1.000
		0.70	0.897	0.984	1.000	1.000	1.000
		0.90	0.207	0.313	0.608	0.979	1.000

Table 7: Rejection rates of Dickey-Fuller's (1981) joint test: the case with no break. Lag selection strategy: Ng and Perron (2005), level: $\alpha = 0.05$

Secondly, we contrast our results with those obtained by unit-root tests which include a break under the null hypothesis: Perron (1997) and Carrion–i–Silvestre and Sansó (2006) [hereinafter $P97$ and $CS06$, respectively]. $P97$ and $CS06$ proposed models that differ on whether or not there is a break under the null hypothesis and the type of break. The specific models that they selected in their empirical applications inhibits the

tions for trend breaks of the magnitudes typically encountered in practice. Indeed, for typical shifts in slope the asymptotic distributions obtained assuming no break under the unit-root null hypothesis provide adequate approximations of finite sample distributions."

Panel (a)							
DGP	Break	$\rho_{y,1}$	Sample Size				
			100	150	200	250	500
U.R. No Drift	NO	0.00	0.028	0.016	0.018	0.022	0.016
		0.25	0.036	0.036	0.030	0.032	0.024
		0.50	0.058	0.038	0.032	0.024	0.026
		0.70	0.068	0.040	0.042	0.030	0.028
		0.90	0.280	0.168	0.090	0.064	0.034
Panel (b)							
U.R. With Drift	NO	0.00	0.978	0.996	1.000	1.000	1.000
		0.25	0.828	0.958	0.994	1.000	1.000
		0.50	0.552	0.728	0.866	0.988	1.000
		0.70	0.334	0.400	0.508	0.762	0.970
		0.90	0.314	0.236	0.166	0.176	0.236
	YES	0.00	0.994	0.998	1.000	1.000	1.000
		0.25	0.928	0.996	1.000	1.000	1.000
		0.50	0.694	0.842	0.962	0.998	1.000
		0.70	0.424	0.492	0.662	0.908	0.994
		0.90	0.342	0.288	0.248	0.262	0.400

Table 8: Rejection rates of the R^2 test. The case with break[a] (Level: $\alpha = 0.01$; trimming: $\varepsilon = 0.05$)

[a] In all cases, there is a grid search for a break; $\mu_y = 2.7$, $\sigma_y = 5$, and, if the DGP contains a break: $\theta_y = 1.05$, $\lambda = 0.5$.

comparison with the results of our test.[12] Therefore, we compute their results choosing those models that allow us to make the fairest comparison with our test.[13] In particular, we apply Perron´s model B[14] to all the series under analysis; this model is referred as the "changing growth model". Under the null hypothesis, it permits a change in the trend function without any change in the level at the time of the break. We also apply the model that $CS06$ denominated $\Theta_{5,1}(\lambda)$. This particular specification allows a slope shift under the null hypothesis.

We employ the NP series updated to 1988 by Herman van Dijk that can be found in the JBES 1994 dataset archives; we expect the longer span to make the results more

[12] We previoulsly warned that a comparison between our test and any root test is not straightforward, given that our test assumes that there is evidence of unit root and tests the significance of the drift, whilst $P97$ and $CS06$ test the unit root hypothesis.

[13] We used the test statistics, $t^*_\alpha(3)$ of $P97$ and $\Theta_{5,1}(\lambda)$ of $CS06$; both allow for a change in the time trend [Model B in Perron's (1989) notation]. The Matlab code is available upon request.

[14] See equations (3a) and (3b) in Perron (1997).

Panel (a)							
DGP	Break	$\rho_{y,1}$	Sample Size				
			100	150	200	250	500
U.R. No Drift	NO	0.00	0.102	0.064	0.106	0.076	0.098
		0.25	0.132	0.128	0.126	0.096	0.120
		0.50	0.156	0.136	0.100	0.120	0.110
		0.70	0.222	0.172	0.130	0.114	0.112
		0.90	0.450	0.344	0.270	0.194	0.136
Panel (b)							
U.R. With Drift	NO	0.00	0.998	1.000	1.000	1.000	1.000
		0.25	0.952	0.994	1.000	1.000	1.000
		0.50	0.762	0.904	0.976	1.000	1.000
		0.70	0.548	0.626	0.718	0.934	1.000
		0.90	0.544	0.438	0.366	0.374	0.462
	YES	0.00	1.000	1.000	1.000	1.000	1.000
		0.25	0.988	0.996	1.000	1.000	1.000
		0.50	0.860	0.964	0.996	1.000	1.000
		0.70	0.618	0.734	0.870	0.992	1.000
		0.90	0.516	0.470	0.458	0.452	0.590

Table 9: Rejection rates of the R^2 test. The case with break[a] (Level: $\alpha = 0.05$; trimming: $\varepsilon = 0.05$)

[a] In all cases, there is a grid search for a break; $\mu_y = 2.7$, $\sigma_y = 5$, and, if the DGP contains a break: $\theta_y = 1.05$, $\lambda = 0.5$.

reliable. The data are annual and all the series are in natural logs except for that of bond yield.

The results in Table 10 show that for all variables except real GNP, nominal GNP and real per capita GNP, there is insufficient evidence to reject the null hypothesis of unit root. Therefore, these three variables can be considered broken-trend stationary series. The remaining variables in Table 10 are appropriate candidates for the procedure developed in this paper, since there is evidence in favor of unit root. We are able to reject the null hypothesis of driftless unit root for all the series except CPI, velocity, bond yield and stock prices.

For the remainder—industrial production, employment, deflator, nominal wages, real wages and money stock—there is evidence to affirm that these are governed by a deterministic trend (the drift) in the long run. Moreover, our test detected two significant

Series	ZA[a]	Break location	R^2	Break location	$P97$ $t^*_\alpha(3)$	$CS06$ $\Theta_{5,1}(\lambda)$
Real GNP	-5.542**	1934	—	—	1930	1978***
Nominal GNP	-5.734***	1930	—	—	1939	1928***
Real per capita GNP	-5.860***	1939	—	—	1930	1937
Industrial Production	-4.939	1919	0.988***	1901	1897	1917***
Employment	-4.748	1930	0.972***	1906	1904	1943***
GNP deflator	-3.813	1930	0.921**	1965***	1953	1918***
CPI	-2.284	1931	0.726	—	1953	1871***
Wages	-4.889	1930	0.967***	1940	1943	1920***
Real Wages	-3.653	1972	0.978***	1973**	1971	1973***
Money Stock	-4.451	1929	0.986***	1970	1970	1978***
Velocity	-4.030	1930	0.927	—	1936	1928***
Bond Yield	-4.191	1954	0.857	—	1954	1931**
Stock Prices	-4.476	1954	0.873	—	1942*	1928***

Table 10: Extended NP data set

[a] Zivot and Andrews's (1992) t-statistic associated with autoregressive term, Model (C).
Trimming: $\varepsilon = 0.05$; Breaks allowed: level and trend; lags selected by the Akaike Information Criterion.
The symbols *, **, and *** denote rejection of the null hypothesis at 10%, 5%, and 1% level, respectively.

structural breaks in the deterministic trend of real wages (1973) and deflator (1965). Results in the Monte Carlo section show that the test loses some power in the presence of positive autocorrelation for sample sizes below 200. Notwithstanding, the test still has enough power to reject the null hypothesis in all but four cases. Furthermore, the combined results of $P97$ and $CS06$ tests for the series, industrial production, employment, GNP deflator, wages, real wages and money stock, can be interpreted and reconciled as follows. For all these series, the $P97$ test does not reject the null hypothesis of unit root, whereas the $CS06$ test does reject the null; the $CS06$ test rejects the null because one or more of the constraints related to the slope or the slope shift are not met, and not necessarily because of the absence of a unit root. These results imply the presence of a unit root and the absence of a drift/drift and shift, among others. Since our test also rejects the null hypothesis, we can conclude that all these series contain both, a deterministic and a stochastic trend. Moreover, besides the deterministic trend, our test shows that the GNP deflator and real wages also have a structural break in the deterministic rate of growth. The application of our test further refines the results of those of $P97$ and $CS06$ tests. For example, the model $\Theta_{5,1}(\lambda)$ of $CS06$ tests under the null the joint validity of several parameter restrictions—besides unit root; therefore, if the null hypothesis is rejected, it is not possible to tell which of the constraints are not true. By using our test, it is possible to draw inference about the deterministic components, specifically, the deterministic trend or the structural break associated with it.

6 Concluding remarks

This work aims to complement unit-root literature by proposing a new and simple methodology that provides a correct assessment of the deterministic trend when there is evidence of unit root. Our procedure contributes by increasing the degree of precision in the inference drawn from unit-root tests that consider drift and break under the null hypothesis. For these tests, it is impossible to evaluate whether both the drift and the break are simultaneously present whenever the null of nonstationarity cannot be rejected, whereas our methodology provides a simple and reliable approach to executing this task.

The importance of such an assessment relies on the fact that existing unit-root tests fail to correctly estimate the existence of the deterministic trend under the null hypothesis of unit root; therefore, the literature lacks a reliable tool with which to estimate the deterministic rate of growth of a series when a stochastic trend exists. The procedure is simple and its implementation straightforward; furthermore, it facilitates the interpretation of the dynamics of the macroeconomic and financial time series.

The new procedure is shown to be asymptotically robust with regard to autocorrelation, and to have reasonable power for sample sizes of practical interest. We considered the possibility of a single structural break in the deterministic trend and derived the asymptotic distribution of both the R^2 statistic as well as the t-statistic associated with the structural break parameter estimated under the null hypothesis of no break.

The empirical results show that most of the NP series extended up to 1988—with the exception of CPI, velocity, bond yield and stock prices—are characterized by their containing a deterministic trend. The results of Perron (1997) test using his "changing growth" model are in line with ours since there is not enough evidence against the unit-root hypothesis in all cases but one. For these variables, our test clarifies that there is a deterministic trend besides the unit root.

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A Appendix

Proof of Propositions 1-5. We present a guide on how to obtain the order in probability of one combination of DGP and specification, namely DGP (1) and specification (4). The expressions needed to compute the asymptotic value of R^2 are:

$$
\begin{aligned}
\sum y_t &= Y_0 T + \mu_y \sum t + \underbrace{\sum \xi_{y,t-1}}_{O_p\left(T^{\frac{3}{2}}\right)} \\
\sum y_t t &= Y_0 \sum t + \mu_y \sum t^2 + \underbrace{\sum \xi_{y,t-1} t}_{O_p\left(T^{\frac{5}{2}}\right)} \\
\sum y_t^2 &= Y_0^2 T + \mu_y^2 \sum t^2 + \underbrace{\sum \xi_{y,t-1}^2}_{O_p(T^2)} + 2Y_0 \mu_y \sum t + ... \\
& \quad 2Y_0 \sum \xi_{y,t-1} + 2\mu_y \sum \xi_{y,t-1} t \\
\sum t &= \frac{1}{2}\left(T^2 + T\right) \\
\sum t^2 &= \frac{1}{6}\left(2T^3 + 3T^2 + T\right)
\end{aligned}
$$

where $\xi_{y,t} = \sum_{i=1}^{t} u_{y,i}$ and all other summations range from 1 to T. The orders in probability can be found in Phillips (1986), Phillips and Durlauf (1986) and Hamilton (1994). These expressions were written in *Mathematica 4.1* code; the software computes the asymptotics of the classical OLS formula $(X'X)^{-1}X'Y$ as well as the asymptotic value of the variance estimator: $\widehat{\sigma}_u^2 = T^{-1}\sum_{t=1}^{T} \widehat{u}_t^2$ where.

$$
X'X = \begin{pmatrix} T & \sum t \\ \sum t & \sum t^2 \end{pmatrix}
$$

and,

$$Y = \begin{pmatrix} \sum y_t \\ \sum y_t t \end{pmatrix}$$

The code in this case[15] is represented below. To understand it, a brief glossary is required:

Character	Represents
A	Y_0
K	μ_y
B	$\sum \xi_{y,t-1}$
C	$\sum \xi^2_{y,t-1}$
D	$\sum \xi_{y,t-1} t$
St	$\sum t$
$St2$	$\sum t^2$

Table 11: glossary of the Mathematica Code

$\mathbf{ClearAll; St} = \frac{1}{2} * (T^2 + T); \mathbf{St2} = \frac{1}{6} * (2 * T^3 + 3 * T^2 + T);$

$\mathbf{Sy} = A * T + K * \mathbf{St} + B * T^{1.5};$

$\mathbf{Sy2} = A^2 * T + K^2 * \mathbf{St2} + C * T^2 + 2 * A * K * \mathbf{St} + 2 * A * B * T^{1.5}$

$+2 * K * D * T^{2.5};$

$\mathbf{Syt} = A * \mathbf{St} + K * \mathbf{St2} + D * T^{2.5};$

$\mathbf{Mx} = (\begin{matrix} T & \mathbf{St} \\ \mathbf{St} & \mathbf{St2} \end{matrix});$

iMx = Inverse[Mx];

R1 = Extract[iMx, {1, 1}]; R2 = Extract[iMx, {1, 2}];

R3 = Extract[iMx, {2, 1}]; R4 = Extract[iMx, {2, 2}];

R40 = Factor[R4];

R4num = Numerator[R40];

R4den = Denominator[R40];

[15]As indicated previously, the proof was achieved with the aid of *Mathematica 4.1* software. The corresponding code for the other results is available upon request.

$\mathrm{K15} = \mathrm{Exponent}[\mathrm{R4num}, T];$

$\mathrm{K16} = \mathrm{Exponent}[\mathrm{R4den}, T];$

$\mathrm{R4num2} = \mathrm{Limit}[\mathrm{Expand}[\mathrm{R4num}/T^{\mathrm{K15}}], T \to \infty];$

$\mathrm{R4den2} = \mathrm{Limit}[\mathrm{Expand}[\mathrm{R4den}/T^{\mathrm{K16}}], T \to \infty];$

$\mathrm{R42} = \mathrm{Factor}[\mathrm{Expand}[(\mathrm{R4num2}/\mathrm{R4den2}) * \frac{T^{\mathrm{K15}}}{T^{\mathrm{K16}}}]];$

$\mathrm{P10} = \mathrm{Factor}[\mathrm{Expand}[\mathrm{R1} * \mathrm{Sy} + \mathrm{R2} * \mathrm{Syt}]];$

$\mathrm{P20} = \mathrm{Factor}[\mathrm{Expand}[\mathrm{R3} * \mathrm{Sy} + \mathrm{R4} * \mathrm{Syt}]];$

$\mathrm{P21num} = \mathrm{Numerator}[\mathrm{P20}];$

$\mathrm{K3} = \mathrm{Exponent}[\mathrm{P21num}, T];$

$\mathrm{Bnum} = \mathrm{Limit}[\mathrm{Expand}[\mathrm{P21num}/T^{\mathrm{K3}}], T \to \infty];$

$\mathrm{P22den} = \mathrm{Denominator}[\mathrm{P20}];$

$\mathrm{K4} = \mathrm{Exponent}[\mathrm{P22den}, T];$

$\mathrm{Bden} = \mathrm{Limit}[\mathrm{Expand}[\mathrm{P22den}/T^{\mathrm{K4}}], T \to \infty];$

$\mathrm{Bpar} = \mathrm{Factor}[\mathrm{Expand}[(\mathrm{Bnum}/\mathrm{Bden}) * \frac{T^{\mathrm{K3}}}{T^{\mathrm{K4}}}]];$

$\mathrm{P40} = \mathrm{Factor}\left[\mathrm{Expand}\left[\mathrm{Sy2} + \mathrm{P10}^2 * T + \mathrm{P20}^2 * \mathrm{St2}\right.\right.$

$\left.\left.-2 * \mathrm{P10} * \mathrm{Sy} - 2 * \mathrm{P20} * \mathrm{Syt} + 2 * \mathrm{P10} * \mathrm{P20} * \mathrm{St}\right]\right];$

$\mathrm{P41num} = \mathrm{Numerator}[\mathrm{P40}];$

$\mathrm{K7} = \mathrm{Exponent}[\mathrm{P41num}, T];$

$\mathrm{U2num} = \mathrm{Factor}[\mathrm{Limit}[\mathrm{Expand}[\mathrm{P41num}/T^{\mathrm{K7}}], T \to \infty]];$

$\mathrm{P42den} = \mathrm{Denominator}[\mathrm{P40}];$

$\mathrm{K8} = \mathrm{Exponent}[\mathrm{P42den}, T];$

$\mathrm{U2den} = \mathrm{Factor}[\mathrm{Limit}[\mathrm{Expand}[\mathrm{P42den}/T^{\mathrm{K8}}], T \to \infty]];$

$\mathrm{Su2} = \mathrm{FullSimplify}[\mathrm{Factor}[\mathrm{Expand}[(\mathrm{U2num}/\mathrm{U2den}) * \frac{T^{\mathrm{K7}}}{T^{\mathrm{K8}}}]]];$

$\mathrm{P50} = \mathrm{Factor}[\mathrm{Expand}[\mathrm{P40}/(\mathrm{Sy2} + T * (\frac{\mathrm{Sy}}{T})^2 - 2 * (\frac{\mathrm{Sy}}{T}) * \mathrm{Sy})]];$

$\mathrm{P51num} = \mathrm{Numerator}[\mathrm{P50}];$

$\mathrm{K1} = \mathrm{Exponent}[\mathrm{P51num}, T];$

$\mathrm{Rcnum} = \mathrm{Factor}[\mathrm{Limit}[\mathrm{Expand}[\mathrm{P51num}/T^{\mathrm{K1}}], T \to \infty]];$

$\mathrm{P52den} = \mathrm{Denominator}[\mathrm{P50}];$

$\mathrm{K2} = \mathrm{Exponent}[\mathrm{P52den}, T];$

$\mathrm{Rcden} = \mathrm{Factor}[\mathrm{Limit}[\mathrm{Expand}[\mathrm{P52den}/T^{\mathrm{K2}}], T \to \infty]];$

$\mathrm{Rc} = \mathrm{FullSimplify}[\mathrm{Factor}[\mathrm{Expand}[(\mathrm{Rcnum}/\mathrm{Rcden}) * \frac{T^{\mathrm{K1}}}{T^{\mathrm{K2}}}]]]$