The Synthesis of Economic Law, Evolution, and History

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Abstract

It has long been criticized that history is almost entirely absent from orthodox economics. This deficiency is due to the fact that equilibrium and time make an odd couple. Because equilibrium is one of the crucial hard-core propositions of the research program it cannot be abandoned. This impedes the treatment of time in a methodologically acceptable manner. The orthodox approach is based on indefensible axioms which are in this paper replaced by objective structural axioms. This enables the synthesis of timeless economic laws, randomness, and goal-oriented human action, which are the essential elements of a formally consistent historical account.

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1 A question of time

> History dependence stares us in the face ..., but it is not the stuff of pure theory. (Hahn, 1991, p. 48)

It has been realized and criticized by the American Institutionalists, the German Historical School, Marxists and Heterodoxy in general that history is almost entirely absent from orthodox economics (Hodgson, 2001). Part of the explanation lies in the fact that theoretical economics primarily deals with laws and these are independent from historical time, like Archimedes' Law of the Lever.

What strongly contributed to the uneasiness about the treatment of time was the shift of emphasis in the wider scientific environment from physics to biology and evolution. Veblen famously articulated the turning point in 1898 with the question: Why is economics not an evolutionary science?

The better part of the explanation, however, lies in the fact that equilibrium and time make an odd couple. Since equilibrium is seen as an indispensable ingredient of every economic model, time and dynamic analysis never got out of the background. It is widely acknowledged, though, that simultaneous adaptation or, alternatively, the long run are of little relevance to the real world.

The core problem, the synthesis of timeless law, evolution and history, is one of economic methodology or theory building. The ultimate reason why this synthesis did not happen until recently lies in the unwavering adherence to the axiomatic foundations of standard economics. These were already in place in Veblen's days, yet have only later been articulated and rigorously formalized.

> As with any Lakatosian research program, the neo-Walrasian program is characterized by its hard core, heuristics, and protective belts. Without asserting that the following characterization is definitive, I have argued that the program is organized around the following propositions: HC1 *economic agents have preferences over outcomes*; HC2 *agents individually optimize subject to constraints*; HC3 *agent choice is manifest in interrelated markets*; HC4 *agents have full relevant knowledge*; HC5 *observable outcomes are coordinated, and must be discussed with reference to equilibrium states.* By definition, the hard-core propositions are taken to be true and irrefutable by those who adhere to the program. "Taken to be true" means that the hard-core functions like axioms for a geometry, maintained for the duration of study of that geometry. (Weintraub, 1985, p. 147), original emphasis

Equilibrium and optimization are firmly cemented in the hard core. If the diagnosis is correct that these two concepts in combination maneuver economics beyond time and reality the logical conclusion is that the axiomatic core has to be changed.

This amounts to a paradigm shift. Axioms are indispensable to build up a theory that epitomizes formal and material consistency. The fatal flaw of the standard approach is that at least two hard-core propositions are unacceptable for cogent methodological reasons.

Orthodoxy seemingly has a strong formal basis that, in the final analysis, is indeed unacceptable. Heterodoxy has not yet agreed upon any axiomatic foundation at all and is therefore formally lost in the wilderness. This is the main reason why Heterodoxy could not develop a serious alternative since Veblen's wake-up call. Without a superior alternative at hand Heterodoxy has, willy-nilly, to make compromises with the obsolete paradigm.

The conceptual consequence of the present paper is to completely discard the conventional hard-core propositions and to take objective-structural axioms as the formal point of departure. This is the first step to bring past, present and future, that is, reality, back to economics.

In the following, Sections 2 to 5 first provide the new formal foundations with the set of four structural axioms and a couple of definitions from profit to the quantity of money. The well-defined formal premises represent the evolving consumption economy with all flows and stocks. This elementary economy is governed solely by structural and stochastic laws. Human agents are at first absent. In Section 6 the propensity function as general formal representation of human behavior is introduced. This marks the transition from evolution to history. In Section 7 the First Economic Law, pure randomness, and directed randomness are applied to the labor market. It is exemplarily shown how from the interaction of these three elements economic history emerges. Section 8 concludes.

2 Build higher, dig deeper

> The procedure of the axiomatic method, as it is expressed here, amounts to *a deepening of the foundations* of the individual domains of knowledge – a deepening that is necessary for every edifice that one wishes to expand and to build higher while preserving its stability. (Hilbert, 2005, pp. 1107-1109), original emphasis

We now advance from behavioral axioms as formal incarnation of homo oeconomicus to structural axioms as formal incarnation of the evolving economic system. Human beings are thereby moved to the analytical periphery. This amounts to a *decoupling* of behavioral assumptions and the axiomatic method. This does not mean that human behavior is ignored, it means that it is moved to another place in the formal structure. Because any proposition about human behavior is vague and uncertain it cannot serve as an axiom. To build on a behavioral axiom is to build on sand. This is the fatal methodological flaw of conventional economics.

2.1 Axioms

The new formal foundations of theoretical economics define the interdependencies of the real and nominal variables that constitute the monetary economy.

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N. Nothing is implied at this stage about who owns the shares.

$$Y = WL + DN \quad |t \tag{1}$$

The period counter t runs from 0, the initial period, to ∞. The coupling with normal calendar time requires that the initial period is equated with the emergence of the monetary economy in historical time. There is no need to fixate this concrete historical event here.

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function. Geometrically the 2nd axiom is a ray from the coordinate origin that tracks underlying discontinuous non-linearities; it does not contain any implicit assumption about increasing or decreasing returns.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment, no foreign trade, and no government.

The period values of the axiomatic variables are formally connected by the familiar growth equation, which is added as the 4th axiom.

$$\begin{gathered} Z_t = Z_{t-1}\left(1+\ddot{Z}_t\right) \\ \text{with} \quad Z \leftarrow W, L, D, N, R, P, X, \ldots \end{gathered} \tag{4}$$

The path of the representative variable Z_t is then determined by the initial value Z_0 and the rates of change $\ddot{Z}_t$ for each period:

$$Z_t = Z_0\,(1+\ddot{Z}_1)\,(1+\ddot{Z}_2)\ldots(1+\ddot{Z}_t) = Z_0\prod_{t=1}^{t}(1+\ddot{Z}_t)\,. \tag{5}$$

For a start it is assumed that the elementary axiomatic variables vary at random. This produces an evolving economy. The respective probability distributions of the change rates are given in general form by:

$$\begin{array}{cc} Pr\left(l_W \leq \ddot{\dot{W}} \leq u_W\right) & Pr\left(l_R \leq \ddot{\dot{R}} \leq u_R\right) \\ Pr\left(l_L \leq \ddot{\dot{L}} \leq u_L\right) & Pr\left(l_P \leq \ddot{\dot{P}} \leq u_P\right) \\ Pr\left(l_D \leq \ddot{\dot{D}} \leq u_D\right) & Pr\left(l_X \leq \ddot{\dot{X}} \leq u_X\right) \\ Pr\left(l_N \leq \ddot{\dot{N}} \leq u_N\right) & |t. \end{array} \tag{6}$$

The four axioms, including (6), constitute a stochastic simulation. It is, of course, also possible to switch to a completely deterministic rate of change for any variable and any period. The structural formalism does not require a preliminary decision between determinism and indeterminism.

2.2 Well-defined mathematical objects

So far the argument has been entirely abstract. Before we can run a simulation, though, concrete assumptions about the initial conditions and the upper (u) and lower (l) bounds of the probability distributions have to be made. This is the point where we need input from experience. We know from observation for instance that productivity changes lie normally between, say, 5 percent and 0 percent per period. But it may happen that the rate of change is -100 percent in case a plant burns down or is cut off from the power supply or is paralyzed by a software bug or something else of this sort. In order to bring the simulation as close as possible to reality, we take the probability distribution from experience, and in order to make it simple, we at first exclude all kinds of accidents.

We know that probability distributions may change over time and that accidents do happen. What we do not know is the exact date and extent of a possible accident in the future. This is what Keynes famously called uncertainty.

> The sense in which I am using the term [uncertainty] is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention … About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know. (Keynes, 1937, p. 214)

One has to be careful here. It is, of course, possible to simulate, for example, the breakdown of productivity as it usually happens in a large scale catastrophe. What we do not know is when and where a catastrophe happens. The same holds for the introduction of a newly invented product. This is what uncertainty refers to. It does not mean that we cannot analyze the economic effects of unforeseeable events. However, because of the predominance of normality, these kinds of events are at first put aside.

A simulation yields a scenario and not a prediction. Each scenario is fully determined, explicit, and traceable in every detail. A simulation as defined by the four structural axioms and the probability distributions is a well-defined mathematical object just like a system of equations. While they are formally on the same footing both mathematical objects yield different kinds of outputs: the system of equations yields a solution vector which refers to an instant in time, a simulation yields a bundle of paths. This bundle has a counterpart in reality.

The upper (u) and lower (l) bounds of the respective probability distributions are, for a start, taken to be symmetrical around zero. This produces a drifting or stationary economy as a limiting case of the growing economy. There is no need at this early stage to discuss the merits and demerits of different probability distributions. Eq. (6) represents the general stochastic case which in the limit $u - l \to 0$ shades into determinism.

The four axioms generate at every run an outcome like that shown in Figure 1 which is the archetype of the evolving monetary economy. The evolution is not distorted by any external restrictions or hindrances. These have to be dealt with separately.

The evolving consumption economy is a well-defined mathematical object which contains no subjective elements and is fully defined with four axioms and a set of probability distributions. Also, it contains no occult forces that push or pull the economy towards a definite end state. There is nothing in the underlying formalism of Figure 1 that fits the definition of an equilibrium.

2.3 Assumptions

What has to be avoided for compelling methodological reasons is assumptionism. It should be obvious that it is illegitimate to take assumptions like equilibrium, perfect competition, decreasing returns, optimization, etc. into the premises. The set of axioms including (6) constitutes the *minimum* of premises. The paths in Figure 1 are, for the beginning, entirely independent. If we suspect that there are indeed relations between the path variables either over time or across variables or both then the respective hypotheses have to be explicitly introduced and consistently integrated into the formal frame. The structural axiom set lends itself to further concretion.

Making assumptions is not the same thing as assumptionism. When we define probability distributions for the future we make assumptions. Without these assump-

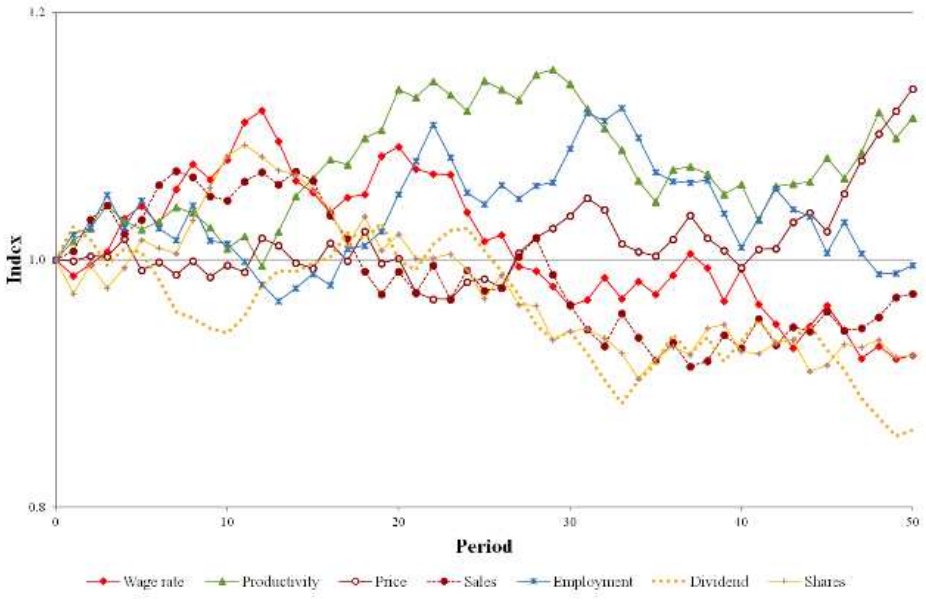

Figure 1: The evolving consumption economy consists of the entirely independent random paths of the seven elementary axiomatic variables (shown here) and the paths of composed and defined variables

tions we cannot run a simulation. The assumptions determine the range of possible rates of change for each variable. The axioms and the probability distributions for all variables taken together define the space of possible outcomes which changes with time. The question whether this space expands forever, stabilizes eventually, contracts again, or oscillates leads to the theory of stochastic processes. There is no need to go further into this direction here because we know that pure randomness is disturbed by intentional human action. Pure randomness is, however, of importance as a limiting case.

Utility maximization is a quite different kind of assumption. It relates to human behavior and it contains the vacuous concept marginal utility. Assumptionism introduces physical or psychological nonentities and thereby creates a parallel world. A scenario and a parallel world are different things. A scenario can come true and there is a chance to verify/falsify it. A parallel world is enclosed in itself and one can only speculate about it without ever coming to an end. General equilibrium is a case in point.

The economic content of the four axioms is perfectly transparent. The point to emphasize is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit.

Thus far we have avoided three major methodological pitfalls: (i) to take equilibrium into the premises, (ii) to take utility maximization into the premises, (iii) to confound profit and distributed profit.

2.4 Definitions

Income categories

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (7) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{7}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

Given the paths of the elementary variables, the development of the composed variables is also determined. From the random paths of employment L and wage rate W follows the path of wage income Y_W. Likewise follows from the paths of dividend D and number of shares N the path of distributed profit Y_D. From the 1st axiom then follows the random path of total income Y as a compound of four random paths.

Key ratios

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{8}$$

A sales ratio $\rho_X = 1$ indicates that the quantity bought/sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \tag{9}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to total income Y, in other words, that the household sector's budget is balanced.

We define the factor cost ratio as:

$$\rho_F \equiv \frac{W}{PR} \quad |t. \tag{10}$$

A factor cost ratio $\rho_F = 1$ indicates that the nominal value of one hour's labor input W is equal to the value of output PR which implies that profit per hour, respectively per unit of output, is zero.

We define the distributed profit ratio as:

$$\rho_D \equiv \frac{DN}{WL} \quad |t. \tag{11}$$

The distributed profit ratio may, for instance, assume a value between zero and 10 percent.

3 The First Economic Law

> According to Schmoller, it was wrong to derive economic laws of nature from human rules of behavior and to speak of a natural economic order. (Klant, 1988, p. 97)

3.1 Period Core

Economists, like everybody else, have taken their idea of a law from Newtonian physics. This idea involves causality and determinism. However, it has always been problematic whether these concepts are directly applicable in economics. If not, a special version of law has to be developed. This version has to be consistent with the structural axiom set.

With the help of the key ratios, the first three axioms are now consolidated to one single equation:

$$\rho_F \frac{\rho_E}{\rho_X} (1+\rho_D) = 1 \quad |t \tag{12}$$

The Period Core determines the interdependencies of the measurable structural key ratios for each period. The factor cost ratio ρ_F summarizes the internal conditions of the firm. A value of $\rho_F < 1$ signifies that the real wage is lower than the productivity or, in other words, that unit wage costs are lower than the price, or in still other words, that the value of output exceeds the value of input. In this case the profit per unit is positive. Then we have the conditions in the product market. An expenditure ratio $\rho_E < 1$ indicates that consumption expenditures are lower than income in the period under consideration and a value of $\rho_X < 1$ of the sales ratio means that the quantity sold is less than the quantity produced or, in other words, that the product market is not cleared. One case is special, that is, with $\rho_E = 1$ *and* $\rho_X = 1$ the budget is balanced and the product market is cleared in period t. This case is analytically most convenient but rarely, if ever, to be found in the real world.

The Period Core is general and fundamental. It covers the key ratios about the firm, the product market, and the income distribution and determines their mutual interdependencies. It holds in each period from $t = 0$ to $t \to \infty$.

Why is eq. (12) a law? Because if we go out and measure the four ratios in our simple economy and insert the values on the left side the result will (my prediction) *always* be unity on the right hand side. This is comparable to measuring the three angles of a triangle. The sum will always turn out to be 180° degrees or π – provided we live in a 'flat' world where the Euclidean axioms apply. Otherwise, we have to turn to non-Euclidean axioms. The result of the measurement confirms the axioms. Likewise, if the calculation of the ratios according to (12) yields unity the structural axiom set is confirmed. Since the measurement has not yet been carried out eq. (12) formally represents, strictly speaking, a tentative structural law. Note in passing that the first measurement of the angles of a triangle on a greater scale had to wait until Gauss, i.e. more than 2,000 years. Tentative laws are used all the time.

The Period Core is timeless and deterministic but there is nothing in it that fits the common sense notion of causality. If one ratio changes then one or all other ratios must change, but we cannot say in which feasible combination. Vice versa, if we have measured three ratios exactly we can calculate and 'predict' the fourth with high precision, i.e. with a tiny measurement error. In practical terms: if we can control three ratios we can determine the fourth exactly. This is, in general terms, what we expect from a law.

3.2 Path Core

The paths are given in a convenient form as abbreviation of (5):

$$Z_t = Z_0 \Pi_{Zt}. \tag{13}$$

The period value of each variable is now replaced by its development until period t. From the period core (12) and (13) then follows:

$$\rho_{F0}\Pi_{Ft}\,\frac{\rho_{E0}\Pi_{Et}}{\rho_{X0}\Pi_{Xt}}\,(1+\rho_{D0}\Pi_{Dt}) = 1. \tag{14}$$

The Path Core (14) describes the evolution of the whole system from the initial period to $t \to \infty$ as a combination of the paths of the four key ratios. All path operators Π have the value 1 for $t = 0$. Equation (14) thus boils down to:

$$\underbrace{\Pi_{Ft}}_{\text{Firm}}\ \underbrace{\frac{\Pi_{Et}}{\Pi_{Xt}}}_{\text{Market}}\ \underbrace{\frac{1+\rho_{D0}\Pi_{Dt}}{1+\rho_{D0}}}_{\text{Distribution}} = 1. \tag{15}$$

When the initial value ρ_{D0} in (15) is conveniently determined nothing but the *rates of change* for each elementary variable remain as explananda. Structural axiomatization thus directly leads to a theory of change. The Path Core is the most economical expression of the first four axioms. As a purely formal relationship it

must always be satisfied independently of the actual formulation of any particular economic model. Given the structural axiom set as an agreed upon formal starting point, different approaches can only differ in the *explanation* of the rates of change. The preliminary explanation consists of pure randomness and is formally embodied in (6). The at any time possible refutation of randomness then points the way to an underlying non-random relationship. If there is a behavioral law we will find it. The preliminary explanation has the methodological advantage that it is self-correcting. If, on the other hand, randomness cannot be refuted then we are already at the end of the analytical flagpole and (6) has to be accepted as an irreducible property of the economic system.

The first four axioms including (6) formally represent the entirety of *possible* paths of the consumption economy. One of the possibilities is eventually realized as the factual evolution of the economic system. When we take the present period as reference point then all rates of change prior to the present are known and can be inserted into (15), all future rates are produced with the help of the random number generator. Thus, in the course of time the random rates of change are replaced by the realized rates and the Path Core meticulously documents the development of the economy up to the present. That is the historical segment. The Path Core is a hybrid of realized, actual, and simulated change, i.e. of past, present, and future.

The price at the beginning of the present period, for example, is determined by the initial value and the rates of change up to the present period, that is, in Veblen's terminology, by cumulative causation. When we have an explanation for the rate of change of every foregoing period then we have an explanation for the price at the beginning of the present period.

Of course, real economies are much more complex than the pure consumption economy. In order to cover the greater part of real world phenomena, the structural axiomatic framework therefore has to be differentiated and extended.

4 The Profit Law

> Unfortunately Smith has no explanation for profits. (Obrinsky, 1981, p. 492)

Total profit consists of monetary and nonmonetary profit. Here we are at first concerned with monetary profit. Nonmonetary profit is treated at length in (2012).

The business sector's monetary profit/loss in period t is defined with (16) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:

$$Q_m \equiv C - Y_W \quad |t. \tag{16}$$

Because of (3) and (7) this is identical with:

$$Q_m \equiv PX - WL \quad |t. \tag{17}$$

This form is well-known from the theory of the firm.

From (16) and (1) follows:

$$Q_m \equiv C - Y + Y_D \quad |t \tag{18}$$

or, using the definitions (9) and (11),

$$Q_m \equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \quad |t. \tag{19}$$

The four equations (16) to (19) are formally equivalent and show profit under different perspectives. The Profit Law (19) tells us that total monetary profit is zero if $\rho_E = 1$ and $\rho_D = 0$. Profit or loss for the business sector as a whole depends on the expenditure and distributed profit ratio and nothing else (for details see 2013). Total income Y is the scale factor. The development of monetary profit is directly coupled to the Path Core (15).

It is a unique fact of the history of economic thought that neither Classicals, nor Walrasians, nor Marshallians, nor Keynesians, nor Marxians, nor Institutionialists, nor Monetary Economists, nor Austrians, nor Sraffaians, nor Evolutionists, nor Game theorists, nor Econophysicists ever came to grips with profit. (Desai, 2008), (Tómasson and Bezemer, 2010), (Kakarot-Handtke, 2014). The Profit Law (19) fully replaces orthodox as well as heterodox profit theories.

5 Completing the picture

In direct formal lineage follow from the first four axioms a host of derived variables and paths.

Retained profit

Once profit has come into existence for the first time (that is: logically – a historical account is an entirely different matter) the business sector has the option to distribute or to retain it. This in turn has an effect on profit. This effect is captured by (18) but it is invisible in (16). Both equations, though, are formally equivalent.

Retained profit Q_{re} is defined for the business sector as a whole as the difference between profit and distributed profit in period t:

$$Q_{re} \equiv Q_m - Y_D \quad \rightarrow \quad Q_{re} \equiv C - Y \equiv (\rho_E - 1)\, Y \quad |t. \tag{20}$$

Retained profit is, due to (18), equal to the difference of consumption expenditures and total income.

Monetary saving

The household sector's monetary saving is given as the difference of income and consumption expenditures (for nonmonetary saving see 2012):

$$S_m \equiv Y - C \equiv (1 - \rho_E)\, Y \quad |t. \tag{21}$$

In combination with (20) follows:

$$Q_{re} \equiv -S_m \quad |t. \tag{22}$$

Monetary saving and retained profit always move in opposite directions. This is the Special Complementarity. It says that the complementary notion to saving is negative retained profit; positive retained profit is the complementary of dissaving. There is no such thing as an equality of saving and investment in the consumption economy.

Stock of money

If income is higher than consumption expenditures the household sector's stock of money increases. The change in period t is defined as:

$$\Delta \bar{\mathsf{M}}_{\mathbf{H}} := Y - C := Y\,(1 - \rho_E) \quad |t. \tag{23}$$

The alternative identity sign := indicates that the definition refers to the monetary sphere. There no change of stock if the expenditure ratio is unity.

The stock of money $\bar{\mathsf{M}}_{\mathbf{H}}$ at the end of an arbitrary number of periods $\bar{t}$ is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{\mathsf{M}}_{\mathbf{H}t} \equiv \sum_{t=1}^{t} \Delta \bar{\mathsf{M}}_{\mathbf{H}t} + \bar{\mathsf{M}}_{\mathbf{H}0}. \tag{24}$$

The interrelation between the expenditure ratio and the households sector's stock of money, is then given by:

$$\bar{\mathsf{M}}_{\mathbf{H}t} \equiv \sum_{t=1}^{t} Y_t \,(1-\rho_{Et}) \quad \text{if} \quad \bar{\mathsf{M}}_{\mathbf{H}0} = 0. \tag{25}$$

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta\bar{\mathsf{M}}_{\mathbf{B}} := C - Y \quad |t. \tag{26}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{\mathsf{M}}_{\mathbf{B}t} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{B}t} + \bar{\mathsf{M}}_{\mathbf{B}0}. \tag{27}$$

The development of the stock of money follows without further assumptions from the axioms and is determined by variations of the elementary variables P, X, W and L. While the stock of money can be either positive or negative the quantity of money is always positive and given by:

$$\bar{\mathsf{M}}_t \equiv \left| \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_t \right| \quad \text{if} \quad \bar{\mathsf{M}}_0 = 0. \tag{28}$$

The quantity of money follows either from (25) or from (27).

Stock of products

The change of the stock of – durable – products in period t is defined as the excess between output O and the quantity bought X by the households:

$$\Delta\bar{O} \equiv O - X \equiv O\,(1-\rho_X) \quad |t. \tag{29}$$

The stock at the end of an arbitrary number of periods $\bar{t}$ is given as the numerical integral of all previous stock changes plus the initial endowment:

$$\bar{O}_t \equiv \sum_{t=1}^{t} \Delta\bar{O}_t + \bar{O}_0. \tag{30}$$

The resulting interrelation between the sales ratio and the stock is given by

$$\bar{O}_t \equiv \sum_{t=1}^{t} O_t \,(1-\rho_{Xt}) \quad \text{if} \quad \bar{O}_0 = 0. \tag{31}$$

The development of the stock of products depends on ρ_X, and that of the stock of money on ρ_E. Both are directly coupled with the Period Core (12).

A closer look at the definitions of profit (19), retained profit (20), saving (21), stock of money (23), and stock of products (29) shows that these variables depend on the dimension-free ratios ρ_E, ρ_X, and ρ_D. If these three ratios are fixated all other variables are determined except for the scale factor. This means, for instance, that if ρ_E has been chosen by the households, which amounts to the realization of the optimal intertemporal consumption plan, they cannot choose the stock of money because it is already determined by the structural interrelations. Analogously, if ρ_E and ρ_D are given the business sector cannot choose the profit maximum. One has to take care not to over-determine the formalism with behavioral assumptions.

6 From evolution to history

> That is why Descartes said that history was not a science – because there were no general laws which could be applied to history. (Berlin, 2002, p. 76)

The evolutionary economy is governed by two types of laws: the Period Core (12) as structural law and the stochastic laws which are incorporated in (6). Human beings are at first absent. The question about the existence of behavioral laws is deliberately postponed. Economic evolution is a pure random process and the sophisticated tools that have been customized for the analysis of stochastic processes can be applied. This helps to focus at first on the objective systemic interrelations and to establish a benchmark. This benchmark is a spontaneous process in the mechanical sense and it is needed to demonstrate how order can possibly emerge out of chaos or pure randomness. This is the crucial question since Adam Smith and it has been left unanswered by equilibrium economics.

> It is good to have [the technically best study of equilibria], but perhaps the time has now come to see whether it can serve in an analysis of how economies behave. The most intellectually exciting question of our subject remains: is it true that the pursuit of private interest produces not chaos but coherence, and if so, how is it done? (Hahn, 1984, p. 102)

It is quite clear that human beings make a huge difference in comparison to a pure random economy. The question to ask changes to, loosely speaking, can spontaneous individual human behavior produce order out of chaos or randomness? The next task is to consistently integrate economic agents into the structural axiomatic framework. The most general proposition about agents is that their actions are goal-oriented. This is the point to start with.

Formally, for every variable of the structural axiomatic formalism there exists a reference or target variable. The realized values of the variables are superimposed by desired values. In a sense, the real world is duplicated by a desired world:

$$Z \to Z^{\theta}. \tag{32}$$

Let Z stand for the stock of money then Z^{θ} stands for the desired stock. Or, let Z stand for profit then Z^{θ} stands for the profit target. This may or may not be the profit maximum. The question how different economic agents set their respective targets must be left open for the moment. To assume that agents maximize utility or profit would be premature. We simply have no certain knowledge about behavior at the moment.

What we can say with certainty is that there are three logical configurations for the relation between the actual value of a variable and the target value:

$$Z - Z^{\theta} \gtreqless 0. \tag{33}$$

The actual value of a variable may be greater than, equal to, or less than the target value. This is the economic situation. The agent's action depends alone on whether the deviation from the target value is positive, zero, or negative. There is no need for the agent to measure the deviation precisely, what is needed is only the sign.

$$sgn\left(Z - Z^{\theta}\right) \to +, 0, - \text{ resp. } 1, 0, -1. \tag{34}$$

What is needed next is an instrument variable $\mathbb{Z}$. If, for example, the actual stock of products is higher than the target stock, then it is plausible that the firm lowers the price in order to sell off. In this case, the price is the instrument variable. The general formal relationship is given by:

$$\underbrace{(-1, 0, 1)_{\mathbb{Z}_t}}_{\text{Direction}} = sgn\left(Z_{t-1} - Z^{\theta}_{t-1}\right). \tag{35}$$

If the deviation of the actual value from the target value yields a $+$ sign then the sign of the instrument variable $\mathbb{Z}$ in the current period is here negative, i.e. -1. If the actual value is on target, the signum function yields 0. If the actual variable is below target, then the sign of the instrument variable is positive, i.e. 1. In brief, the signum function delivers the direction of change of the instrument variable. There are only two directions: up and down. Eq. (35) is the general expression of the intentionality of human action. It does not demand any sophisticated calculations from the agent. Indeed, it is the weakest possible behavioral assumption.

In the example above it holds: if the sign of the deviation is positive then the sign of the direction of change is negative, and vice versa. This is not an immutable law

but a plausible assumption. There is no such thing as a behavioral law. For other instrument variables the combination of signs may be alternatively $(1,0,-1)$.

The determination of the target value involves expectations. This means that target values can change fast and that they are generally more volatile than the actual values. For our present purposes it is not necessary to occupy ourselves with the determination of targets, hence they are without further explanation taken as given. It is important to keep in mind that (35) is the formal expression of a behavioral assumption that is based on more or less reliable observation and second-guessing the agents. It is at the moment not based on behavioral experiments or established certain knowledge. Eq. (35) is general and covers more specific assumptions like profit maximization. It is therefore possible to integrate other approaches as limiting cases into the structural axiomatic framework.

The magnitude of the change of the instrument variable is a random variable. With this we overcome the initial lack of exact knowledge.

Thus, the directed random change of the instrument variable consists of two elements: (i) direction, which depends on the deviation of the actual value from the target value, and (ii), magnitude, which depends on a plausible set of discrete random rates of change. For our simulations the concrete numbers are taken from the worksheet random number generator and adapted. The stochastic change vector in period t is accordingly given by:

$$\dddot{\mathbb{Z}} = \underbrace{(-1,0,1)}_{\text{Direction}} \underbrace{Pr\,(0 \leq \dddot{\mathbb{Z}} \leq x)}_{\text{Magnitude}} \quad |t. \tag{36}$$

This equation – the propensity function – delivers the rates of change for all variables, among them the elementary variables of the structural axiomatic set. These rates are fed into the four ratios of the Path Core (15); they replace the pure random rates of change. This is the synthesis of economic law and history. The development of an economy with a defined structure depends on human action and randomness. The agents determine the real path. In a sense they 'chose' one path from the infinity of possible paths – but only in retrospect.

The simulation shows how the system behaves, that is, how the agents make economic history. Whether the system grows or shrinks or oscillates or heads towards a steady state is not known in advance.

The idea, or hope, or promise is since Adam Smith that the collective outcome of goal-oriented individual human action is the best of all possible outcomes. All possible outcomes for an arbitrary period t in the future are defined with the axiom set and the propensity function.

The strongest motivator of general equilibrium theory has been to deliver the proof that individual optimizing behavior produces the best collective outcome.

> General equilibrium theory, for all its twentieth-century complexity, is nothing more than the mathematical elaboration of Smith's eighteenth-century metaphor. (Nadal, 2004, p. 181)

What has to be criticized is that, with putting equilibrium into the premises, the cart has been put before the horse. If something like an equilibrium exists then it must show up in the simulation at some period t.

7 Synthesis: Making the history of the labor market

> The fundamental problem of both the theoretical and historical social sciences is *to explain and understand events in terms of human actions and social situations*. (Popper, 1994, p. 166), original emphasis

When we impose directionality upon the Period Core (12) and define employment as the dependent variable then the First Economic Law turns to:

$$L = \frac{DN}{\frac{\rho_X}{\rho_E} PR - W} \quad |t. \tag{37}$$

Actual employment L depends on seven variables, i.e. D, N, P, R, W, ρ_X, ρ_E. This is quite different from the determination of employment by supply-function–demand-function–equilibrium. The desired employment or labor supply L^θ is taken to be a random variable. Full employment is defined as $L = L^\theta$. The actual economic situation is given by $L - L^\theta \gtreqless 0$. The question is: how does full employment come about?

In principle, all variables on the right hand side of (37) can be used as instrument variables. We choose here the wage rate and simplify matters by keeping the other variables constant; ρ_E and ρ_X are set to unity. The familiar intuition is that a falling wage rate boots employment. A closer look at eq. (37) makes it clear that the familiar intuition, which stems from partial analysis, is wrong. The structural law tells us that only an increasing wage rate boosts employment. To be sure, an objective structural law like (37) always overrules subjective behavioral functions like supply and demand.

The propensity function reads:

$$\begin{aligned} &\text{(i)} \quad (-1,0,1)_{Wt} = sgn\left(L_{t-1} - L^\theta_{t-1}\right) \\ &\text{(ii)} \quad \dddot{W}_t = (-1,0,1)_t \, Pr\left(0 \leq \dddot{W} \leq x\right)_t. \end{aligned} \tag{38}$$

The upper part of (38) says that the sign, i.e. the direction of change of the wage rate in period t, depends on whether there was over- or under-employment in the previous period. In the case of over-employment, i.e. $L_{t-1} - L^{\theta}_{t-1} > 0$, the sign is negative, that is, the business sector reduces the wage rate, and vice versa in the case of under-employment, i.e. if $L_{t-1} - L^{\theta}_{t-1} < 0$. Part (ii) combines the direction with a random rate of change. This random rate assumes values between 0 and x percent, which is the symmetrical upper or lower bound depending on the positive or negative sign of the direction of the change vector (ii). In combination, the two parts of (38) define the elementary behavioral dependency which says: if you see unemployment in the economy immediately increase the wage rate by a random percentage rate, and likewise for all other possible situations. It is assumed for the moment that no exogenous factors restrict this process.

The labor supply follows a random walk (5) which is defined by the initial value and

$$Pr\left(l_L \leq \dddot{L^{\theta}} \leq u_L\right) \quad |t. \tag{39}$$

With all other instrument variables constant eqs. (37), (38), and (39) produce the exemplary labor market scenario as shown in Figure 2.

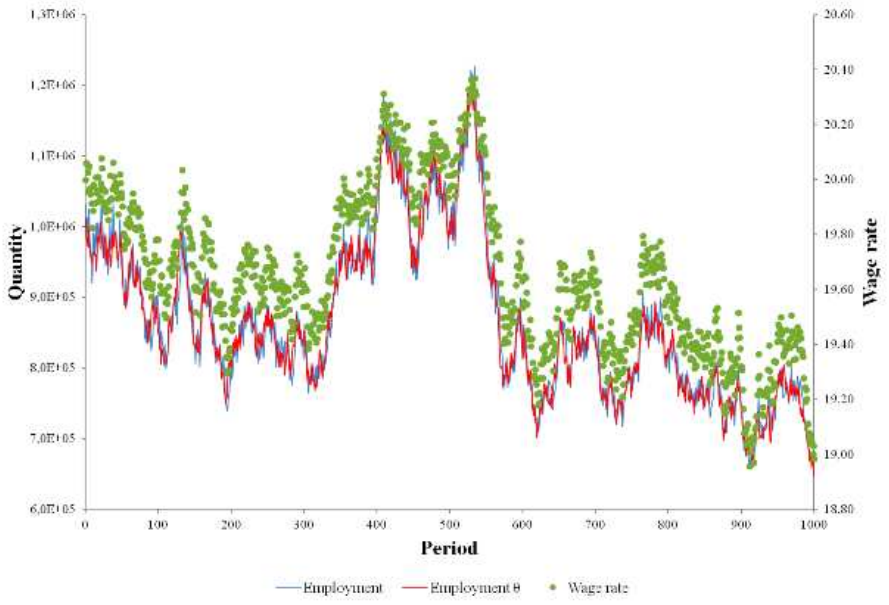

Figure 2: Continuous labor market clearing in the structural axiomatic consumption economy with randomly varying labor supply and the wage rate as instrument variable

The directed random adaptation works reliably, at least for the time span of observation which covers 1,000 periods. The employment path L follows the random full employment path L^{θ}. The difference between the two paths measures under- or over-employment. The product market is cleared by assumption because of $\rho_X = 1$;

and the household sector's budget is balanced because of $\rho_E = 1$. These conditions can be relaxed without affecting the main conclusion. The relation between wage rate and employment is pro-cyclic. This is a testable result.

The elementary consumption economy with budget balancing and market clearing can, in principle, exist for an indefinite time. Profit is positive and because of $\rho_E = 1$, equal to distributed profit, that is, by assumption constant. Full employment is – approximately – feasible. The usual behavioral assumptions, i.e. utility and profit maximization, are not applied but replaced by the propensity function. The wage rate is used as an instrument variable in order to speedily absorb the randomly varying labor supply. This works satisfactorily and makes the history of the labor market a success story. If the business sector spontaneously reacts with a wage rate increase to unemployment then we are justified in saying that in the market economy the labor market clears spontaneously.

8 Conclusion

It has long been criticized that history is almost entirely absent from orthodox economics. This deficiency is due to the fact that equilibrium, the ingredient of all orthodox models, and time make an odd couple. Because equilibrium is one of the crucial hard-core propositions of the orthodox research program it cannot be abandoned. This hitherto hindered, and still makes it impossible, to formally integrate time in a methodologically acceptable manner.

The orthodox approach is based on indefensible axioms which are in the present paper replaced by objective structural axioms. The set of four structural axioms constitutes an evolving consumption economy. The interaction of flows and stocks over time is transparent, the logical implications are testable in principle.

The main results of the paradigm shift are:

- the new formal foundations constitute a stochastic simulation as a well-defined mathematical object,
- the consistent structural axiomatic formalism represents an evolving consumption economy that is governed exclusively by structural and stochastic laws,
- goal-oriented human behavior is in a general form captured by the propensity function, which connects the actual economic situation with the directed change of an instrument variable,
- exogenous disturbances at first excluded, the synthesis of timeless economic laws, randomness, and goal-oriented human action makes economic history.

References

Berlin, I. (2002). *Freedom and Its Betrayal*. London: Chatto Windus.

Desai, M. (2008). Profit and Profit Theory. In S. N. Durlauf, and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics Online*, pages 1–11. Palgrave Macmillan, 2nd edition. URL http://www.dictionaryofeconomics.com/article?id=pde2008_P000213.

Hahn, F. H. (1984). *Equilibrium and Macroeconomics*. Cambridge, MA: MIT Press.

Hahn, F. H. (1991). The Next Hundred Years. *Economic Journal*, 101(404): 47–50.

Hilbert, D. (2005). Axiomatic Thought. In W. Ewald (Ed.), *From Kant to Hilbert. A Source Book in the Foundations of Mathematics*, volume II, pages 1107–1115. Oxford, New York, NY: Oxford University Press. (1918).

Hodgson, G. M. (2001). *How Economics Forgot History. The Problem of Historical Specificity in Social Science.* London, New York, NY: Routledge.

Kakarot-Handtke, E. (2012). Primary and Secondary Markets. *Levy Economics Institute Working Papers*, 741: 1–27. URL http://www.levyinstitute.org/publications/?docid=1654.

Kakarot-Handtke, E. (2013). Confused Confusers: How to Stop Thinking Like an Economist and Start Thinking Like a Scientist. *SSRN Working Paper Series*, 2207598: 1–16. URL http://ssrn.com/abstract=2207598.

Kakarot-Handtke, E. (2014). The Three Fatal Mistakes of Yesterday Economics: Profit, I=S, Employment. *SSRN Working Paper Series*, 2489792: 1–13. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2489792.

Keynes, J. M. (1937). The General Theory of Employment. *Quarterly Journal of Economics*, 51(2): 209–223. URL http://www.jstor.org/stable/1882087.

Klant, J. J. (1988). The Natural Order. In N. de Marchi (Ed.), *The Popperian Legacy in Economics*, pages 87–117. Cambridge: Cambridge University Press.

Nadal, A. (2004). Freedom and Submission. Individuals and the Invisible Hand. In F. Ackerman, and A. Nadal (Eds.), *The Flawed Foundations of General Equilibrium*, pages 181–201. London, New York, NY: Routledge.

Obrinsky, M. (1981). The Profit Prophets. *Journal of Post Keynesian Economics*, 3(4): 491–502. URL http://www.jstor.org/stable/4537615.

Popper, K. R. (1994). *The Myth of the Framework. In Defence of Science and Rationality.*, chapter Models, Instruments, and Truth, pages 154–184. London, New York, NY: Routledge.

Tómasson, G., and Bezemer, D. J. (2010). What is the Source of Profit and Interest? A Classical Conundrum Reconsidered. *MPRA Paper*, 20557: 1–34. URL http://mpra.ub.uni-muenchen.de/20557/.

Weintraub, E. R. (1985). Joan Robinson's Critique of Equilibrium: An Appraisal. *American Economic Review, Papers and Proceedings*, 75(2): 146–149. URL http://www.jstor.org/stable/1805586.