Labor Unions, Directed Technical Change and Cross-Country Income Inequality

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Abstract

This study explores the macroeconomics effects of labor unions in a two-country model of directed technical change in which the market size of each country determines the incentives for innovation. We find that an increase in the bargaining power of a wage-oriented union leads to a decrease in employment in the domestic economy. This result has two important implications on innovation. First, it reduces the rates of innovation and economic growth. Second, it causes innovation to be directed to the foreign economy, which in turn causes a negative effect on domestic wages relative to foreign wages in the long run. We also calibrate our model to data in the US and the UK. We find that the degree of unions' wage preference must be stronger in the UK than in the US in order for the calibrated economies to replicate the simultaneous decrease in labor income share and unemployment in the two countries. We also explore the quantitative implications of labor unions on social welfare and relative wage across countries. In summary, our calibrated model is able to explain about half of the decrease in relative wage between the US and the UK from 1980 to 2007.

JEL classification: O30, O43, E24, J51
Keywords: economic growth, R&D, labor unions, income inequality

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1 Introduction

This study explores the macroeconomics effects of labor unions in an open-economy model of directed technical change. We consider a two-country model in which the market size of each country determines the incentives for innovation. Our model differs from other models of directed technical change in the literature by featuring labor unions that bargain with employers over wages and employment. Within this growth-theoretic framework, we find that an increase in the bargaining power of a wage-oriented union leads to a decrease in employment in the domestic economy. In contrast, an increase in the bargaining power of an employment-oriented union leads to an increase in employment. Empirical studies[1] often find that unionization has a negative effect on employment, which is consistent with our result under a wage-oriented union. This result has two important implications on innovation. First, by decreasing employment, an increase in the bargaining power of a wage-oriented union reduces the rates of innovation and economic growth. This theoretical implication is consistent with empirical studies[2] that find negative effects of unions on innovation and growth. Second, by decreasing employment and the market size of the domestic economy, an increase in the bargaining power of a wage-oriented union causes innovation to be directed to the foreign economy, which in turn causes a negative long-run effect on domestic wages relative to foreign wages. In the long run, this negative effect of directed technical change on relative wage income across countries would dominate the positive effect of labor unions on wages if the elasticity of substitution between domestic and foreign goods is sufficiently large.

We also calibrate our model to data in the US and the UK to provide a quantitative analysis. Figure 1a plots the HP-filtered trends of labor income share of GDP from 1980 to 2007 in the US and the UK.[3] This figure shows a well-documented stylized fact that the labor share of income has gradually declined since the early 1980's. Figure 1b plots the HP-filtered trends of unemployment rates in the two countries.[4] This figure shows that unemployment has also gradually declined in these two countries until 2007.[5] We calibrate our model to compute the degree of unions' wage preference and the decrease in their bargaining power that enable the model to replicate this simultaneous decrease in labor income share and unemployment in the two countries. We find that the degree of unions' wage preference must be stronger in the UK than in the US in order for the calibrated economies to match the data. We also explore the quantitative implications on social welfare and relative wage across the two countries. In summary, our calibrated model is able to explain about half of the decrease in relative wage between the US and the UK from 1980 to 2007. Finally, we find that both countries gain from the decrease in unions' bargaining power, but the welfare improvement in the UK is greater than that in the US due to changes in relative wage income.

[1] See for example Montgomery (1989), Blanchflower *et al.* (1991), Nickell and Layard (1999) and Krol and Svorny (2007).

[2] See for example Connolly *et al.* (1986), Hirsch and Link (1987), Acs and Audretsch (1988), Carmeci and Mauro (2003) and Bradley *et al.* (2013).

[3] Data source: OECD Annual Indicators on Unit Labour Costs.

[4] Data source: US Bureau of Labor Statistics, and UK Office for National Statistics.

[5] We do not consider data after 2007 because of the financial crisis.

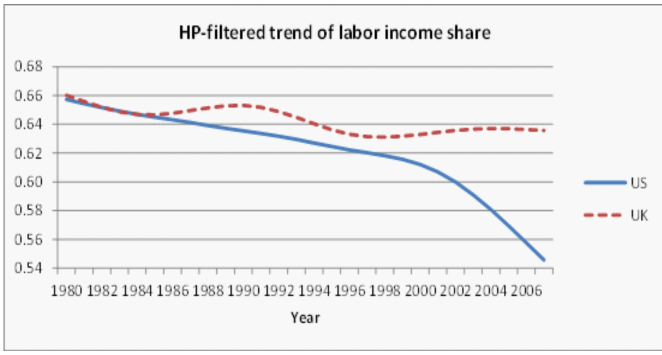

Figure 1a

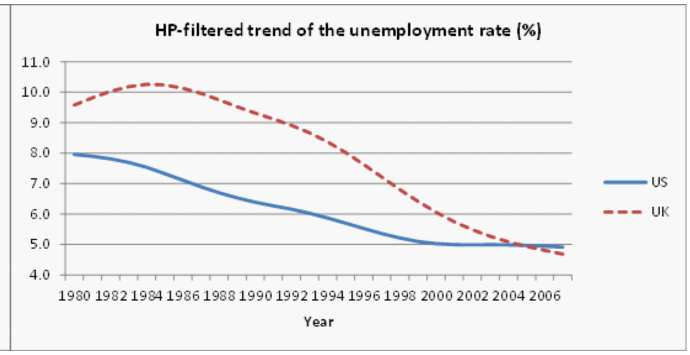

Figure 1b

This study relates to the literature on labor unions. Early studies in this literature focus on the formulation of labor unions' objective function; see for example Oswald (1985) for a survey. We follow a common approach in the literature to specify a Stone-Geary union objective function over wages and employment. Pemberton (1988) provides a microeconomic foundation for this union objective function as "the outcome of an internal bargain between the leadership and membership" in a managerial union. Our study relates most closely to a recent branch of this literature that explores the effects of labor unions on innovation and economic growth. Palokangas (1996) is an early study in this literature, and subsequent studies by Palokangas (2000, 2004), Boone (2000), Lingens (2003, 2007) and Chang *et al.* (2014) also analyze the effects of labor unions in R&D-based growth models. Palokangas (1996, 2000, 2004) finds that increasing the bargaining power of labor union serves to increase economic growth, whereas Boone (2000) finds that labor union dampens economic growth. Lingens (2003, 2007) finds that labor union has both positive and negative effects on growth. Chang *et al.* (2014) also find that labor union has both a negative growth effect via unemployment and a positive growth effect via endogenous market structure, and these two effects exactly offset each other leaving an overall neutral effect on growth. Our theoretical model is able to replicate (via an alternative mechanism) the above results that increasing the bargaining power of labor unions can have a positive effect on growth and innovation (under an employment-oriented union), a negative effect on growth and innovation (under a wage-oriented union) and a neutral effect on growth and innovation (when the union is neither wage nor employment oriented).[6] More importantly, in addition to analyzing the effects of labor unions on the *level* of innovation in a closed economy as is common in the literature, our study also explores the effects of labor unions on the *direction* of innovation and wage income inequality across countries, complementing the abovementioned studies in the literature. Furthermore, unlike previous studies that focus on qualitative analysis, we calibrate our model to aggregate data to explore the quantitative implications of labor unions on the macroeconomy. Aghion *et al.* (2001) also considers the relationship between labor unions and directed technical change. They analyze the endogenous

[6] Chang *et al.* (2007) also find that unions' wage preference determines the effects of their bargaining power on economic growth, but they consider an AK-type growth model in which economic growth is driven by capital accumulation. Our study complements their interesting analysis by exploring the effects of labor unions in an R&D-based growth model of directed technical change.

formation of labor unions but take innovation as an exogenous change in productivity parameters, whereas our study provides a complementary analysis by taking the existence of labor unions as given and exploring their effects on *endogenous* directed technical change.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 provides analytical results. Section 4 conducts a quantitative analysis. The final section concludes. Proofs are relegated to the appendix.

2 The model

In this section, we consider an open-economy version of the R&D-based growth model originated from the seminal study by Romer (1990).[7] In the model, there are two countries: Home and Foreign. Final goods are produced by combining intermediate goods from the two countries via a standard CES Armington aggregator. Intermediate goods in each country are produced using domestic labor and differentiated monopolistic inputs. The number of varieties of these differentiated inputs increases over time as a result of R&D. In each country, there is an economy-wide labor union that bargains with an economy-wide federation representing employers to determine wage and employment, which in turn determines the market size of each country. As a result, changes in employment in a country potentially affect the direction of innovation across countries as in the model of directed technical change in Acemoglu (1998, 2002).[8] The novelty of this study is that we explore the effects of labor unions in the Acemoglu model and focus on the direction of innovation across countries instead of sectors.

2.1 Household

In the Home country h, there is a representative household, which has the following lifetime utility function:

$$U^h = \int_0^\infty e^{-\rho t} \ln c_t^h dt, \tag{1}$$

where c_t^h denotes consumption of final goods at time t, and $\rho > 0$ is the subjective discount rate. The household maximizes utility subject to the following asset-accumulation equation:[9]

$$\dot{a}_t^h = r_t a_t^h + w_t^h l_t^h + b_t^h \left(L^h - l_t^h\right) - \tau_t^h - c_t^h. \tag{2}$$

a_t^h is the amount of financial assets (i.e., the equity shares of firms that generate monopolistic profits) owned by the household in the Home country, and r_t is the real interest rate.[10] w_t^h is the

[7] See also Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) for other seminal studies on the R&D-based growth model and Gancia and Zilibotti (2005) for a survey of this literature.

[8] Other influential studies on directed technical change include Acemoglu (2003), Acemoglu and Zilibotti (2001), Gancia and Bonfiglioli (2008) and Gancia and Zilibotti (2009).

[9] We also impose the usual no-Ponzi-game condition that requires the household's lifetime budget constraint to be satisfied.

[10] r_t is not indexed by a country superscript because we consider a global financial market. Our derivations are robust to any distribution of financial assets across the two countries. One special case is that all domestic (foreign) firms are owned by the domestic (foreign) household.

wage rate in the Home country. L^h is the inelastic supply of labor, and l_t^h is employment. Therefore, $L^h - l_t^h$ is unemployment, and the unemployment rate is $u_t^h \equiv 1 - l_t^h/L^h$. b_t^h is unemployment benefit, and τ_t^h is a lump-sum tax levied by the government on the household. From standard dynamic optimization, the Euler equation is[11]

$$\frac{\dot{c}_t^h}{c_t^h} = r_t - \rho. \tag{3}$$

There are analogous conditions and variables with a superscript f in the Foreign country.

2.2 Final goods

The production of final goods is perfectly competitive, and they are freely traded across countries. Final goods are produced with the following CES Armington aggregator:

$$Y_t = \left[\gamma(X_t^h)^{(\varepsilon-1)/\varepsilon} + (1-\gamma)(X_t^f)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}, \tag{4}$$

where X_t^h and X_t^f are respectively intermediate goods produced in the Home and Foreign countries. $\varepsilon > 1$ is the elasticity of substitution between the two types of intermediate goods,[12] and γ determines their relative importance. We choose final goods as the numeraire, and the standard price index of final goods is

$$1 = \left[\gamma^\varepsilon \left(P_t^h\right)^{1-\varepsilon} + (1-\gamma)^\varepsilon (P_t^f)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}, \tag{5}$$

where we have set the price of final goods to one. P_t^h and P_t^f are respectively the price of X_t^h and X_t^f. The conditional demand functions for intermediate goods are

$$X_t^h = \left(\frac{\gamma}{P_t^h}\right)^\varepsilon Y_t, \tag{6}$$

$$X_t^f = \left(\frac{1-\gamma}{P_t^f}\right)^\varepsilon Y_t. \tag{7}$$

2.3 Intermediate goods and labor union

There is a unit continuum of firms producing intermediate goods in the each country. The production function of X_t^h is given by

$$X_t^h = (l_t^h)^\alpha \int_0^{N_t^h} [x_t^h(i)]^\beta di, \tag{8}$$

[11] Also, the transversality condition requires $r_t > \dot{a}_t^h/a_t^h$, which holds on the balanced growth path given the log utility function and $\rho > 0$.

[12] See Broda and Weinstein (2006) for empirical evidence on $\varepsilon > 1$.

where l_t^h is the employment of labor and $x_t^h(i)$ is differentiated input $i \in [0, N_t^h]$. We also impose the following parameter restrictions: $\alpha, \beta \in (0,1)$ and $\alpha + \beta < 1$. Here we follow the formulation in Chang *et al.* (2007) to assume decreasing returns to scale,[13] allowing the firms to have positive profit in order to facilitate the bargaining process between the employers' federation and the labor union. The profit function of the representative firm is

$$\Pi_t^h = P_t^h X_t^h - w_t^h l_t^h - \int_0^{N_t^h} p_t^h(i) x_t^h(i) di, \tag{9}$$

where $p_t^h(i)$ is the price of $x_t^h(i)$. The firm chooses $x_t^h(i)$ to maximize Π_t^h. The conditional demand function for $x_t^h(i)$ is

$$p_t^h(i) = \beta P_t^h (l_t^h)^\alpha [x_t^h(i)]^{\beta-1}. \tag{10}$$

Departing from the usual treatment without labor union, we follow previous studies, such as Palokangas (1996), Lingens (2007) and Chang *et al.* (2007), to consider an economy-wide labor union that bargains with an economy-wide federation representing employers to determine wage w_t^h and employment l_t^h. We consider a *closed shop* union under which only union members are eligible for employment. As in Pemberton (1988) and Chang *et al.* (2007), we consider a managerial union whose objective is influenced by the union leaders' desire for a larger membership and the members' desire for a higher wage. Formally, the union's objective is given by a standard Stone-Geary function:

$$O_t^h = (w_t^h - b_t^h)^{\omega^h} (l_t^h)^{\lambda^h}. \tag{11}$$

The parameter $\omega^h > 0$ measures the weight that the union places on workers' incremental wage income from employment (i.e., wage minus unemployment benefit). The parameter $\lambda^h > 0$ measures the weight that the union places on membership. For simplicity, we normalize λ^h to unity and use ω^h to measure the weight that the union places on wage relative to membership; i.e., we focus on $\lambda^h = 1$ for the rest of the analysis.[14] When $\omega^h > 1$ ($\omega^h < 1$), we refer to the union as being wage-oriented (employment-oriented).

The employers' federation and the labor union bargain over w_t^h and l_t^h. The generalized Nash bargaining function is

$$\max_{w_t^h, l_t^h} B_t^h = (O_t^h)^{\theta^h} (\Pi_t^h)^{1-\theta^h}, \tag{12}$$

where the parameter $\theta^h \in (0,1)$ measures the bargaining power of the labor union relative to the employers. The bargaining solutions are

$$\frac{\partial B_t^h}{\partial w_t^h} = 0 \Leftrightarrow \frac{(w_t^h - b_t^h) l_t^h}{\Pi_t^h} = \frac{\omega^h \theta^h}{1 - \theta^h}, \tag{13}$$

$$\frac{\partial B_t^h}{\partial l_t^h} = 0 \Leftrightarrow \frac{w_t^h l_t^h - \alpha P_t^h X_t^h}{\Pi_t^h} = \frac{\theta^h}{1 - \theta^h}. \tag{14}$$

There are analogous conditions for w_t^f and l_t^f in the Foreign country.

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[13] This can be justified by the presence of a fixed factor input owned by the firms.

[14] Our results are robust to $\lambda^h > 0$ (derivations available upon request), but some of the expressions become more complicated in this case.

2.4 Differentiated inputs

In each country, there is a continuum of industries producing differentiated inputs $i \in [0, N_t^h]$. Each differentiated input i is produced by a monopolist who owns a patent on the production technology. For simplicity, we follow Acemoglu (2002) to assume that these differentiated inputs are produced using final goods. In particular, one unit of final goods produces one unit of differentiated input. The profit function of the monopolist in industry i is given by

$$\pi_t^h(i) = p_t^h(i)x_t^h(i) - x_t^h(i) = \beta P_t^h(l_t^h)^\alpha [x_t^h(i)]^\beta - x_t^h(i), \tag{15}$$

where the second equality uses (10). Differentiating (15) with respect to $x_t^h(i)$, we find the familiar profit-maximizing price of $x_t^h(i)$ given by $p_t^h(i) = 1/\beta$. Substituting $p_t^h(i) = 1/\beta$ into (10) and (16) yields

$$x_t^h(i) = [\beta^2 P_t^h(l_t^h)^\alpha]^{1/(1-\beta)} \equiv x_t^h, \tag{16}$$

$$\pi_t^h(i) = \left(\frac{1-\beta}{\beta}\right) x_t^h(i) = \left(\frac{1-\beta}{\beta}\right) [\beta^2 P_t^h(l_t^h)^\alpha]^{1/(1-\beta)} \equiv \pi_t^h. \tag{17}$$

There are analogous conditions for x_t^f and π_t^f in the Foreign country.

2.5 R&D

There is a continuum of entrepreneurs investing in R&D in each country. The invention of a new variety of differentiated inputs in either country requires μ units of final goods. We consider the lab-equipment R&D specification in Rivera-Batiz and Romer (1991). In particular, the innovation process in the Home country is given by

$$\dot{N}_t^h = R_t^h/\mu, \tag{18}$$

where R_t^h is the amount of final goods devoted to R&D in the Home country. Suppose we denote v_t^h as the value of an invention in the Home country. Free entry in the R&D sector implies

$$(v_t^h - \mu)\dot{N}_t^h = 0. \tag{19}$$

The familiar Bellman equation is

$$r_t = \frac{\pi_t^h + \dot{v}_t^h}{v_t^h}, \tag{20}$$

Intuitively, the Bellman equation equates the interest rate to the asset return per unit of asset, where the asset return is the monopolistic profit π_t^h plus any potential capital gain $\dot{v}_t^h$. There are analogous conditions for v_t^f in the Foreign country.

2.6 Government

In each country, there is a government that determines unemployment benefit and levies a lump-sum tax on the household to balance the fiscal budget. The balanced-budget condition in the Home country is

$$\tau_t^h = b_t^h \left(L^h - l_t^h \right). \tag{21}$$

To be consistent with balanced growth, we assume that unemployment benefit b_t^h is proportional to the value of domestic output $P_t^h X_t^h$; i.e., $b_t^h = \bar{b}^h P_t^h X_t^h$, where $\bar{b}^h > 0$ is a policy parameter. There are analogous conditions for τ_t^f and b_t^f in the Foreign country.

2.7 Decentralized equilibrium

An equilibrium is a time path of allocations $\{c_t^h, c_t^f, Y_t, X_t^h, X_t^f, x_t^h(i), x_t^f(i), l_t^h, l_t^f, R_t^h, R_t^f\}$, prices $\{r_t, w_t^h, w_t^f, P_t^h, P_t^f, p_t^h(i), p_t^f(i), v_t^h, v_t^f\}$ and fiscal policies $\{\tau_t^h, \tau_t^f, b_t^h, b_t^f\}$ such that the following conditions hold at each instance of time:

- the representative household in the Home country chooses $\{c_t^h\}$ to maximize utility taking $\{r_t, w_t^h, b_t^h, \tau_t^h\}$ as given;
- the representative household in the Foreign country chooses $\{c_t^f\}$ to maximize utility taking $\{r_t, w_t^f, b_t^f, \tau_t^f\}$ as given;
- perfectly competitive final-goods firms produce $\{Y_t\}$ to maximize profit taking prices $\{P_t^h, P_t^f\}$ as given;
- intermediate-goods firms in the Home country produce $\{X_t^h\}$ to maximize profit taking prices $\{P_t^h, p_t^h(i)\}$ as given;
- intermediate-goods firms in the Foreign country produce $\{X_t^f\}$ to maximize profit taking prices $\{P_t^f, p_t^f(i)\}$ as given;
- in the Home country, an economy-wide federation representing intermediate-goods firms bargains with an economy-wide labor union to determine $\{w_t^h, l_t^h\}$;
- in the Foreign country, an economy-wide federation representing intermediate-goods firms bargains with an economy-wide labor union to determine $\{w_t^f, l_t^f\}$;
- monopolistic firms in the Home country produce differentiated inputs $\{x_t^h(i)\}$ and set $\{p_t^h(i)\}$ to maximize profit;
- monopolistic firms in the Foreign country produce differentiated inputs $\{x_t^f(i)\}$ and set $\{p_t^f(i)\}$ to maximize profit;
- R&D firms in the Home country choose $\{R_t^h\}$ to maximize profit taking $\{r_t, v_t^h\}$ as given;
- R&D firms in the Foreign country choose $\{R_t^f\}$ to maximize profit taking $\{r_t, v_t^f\}$ as given;

- the market-clearing condition for final goods holds such that $Y_t = R_t^h + R_t^f + N_t^h x_t^h + N_t^f x_t^f + c_t^h + c_t^f$;
- the government in the Home country balances its fiscal budget given by $\tau_t^h = b_t^h \left(L^h - l_t^h\right)$;
- the government in the Foreign country balances its fiscal budget given by $\tau_t^f = b_t^f (L^f - l_t^f)$.

Here we first solve for the equilibrium levels of employment in the two countries. Substituting (8) and (10) into (9) yields the profit function of intermediate goods in the Home country given by

$$\Pi_t^h = (1-\beta) P_t^h (l_t^h)^\alpha \int_0^{N_t^h} [x_t^h(i)]^\beta di - w_t^h l_t^h = (1-\beta) P_t^h X_t^h - w_t^h l_t^h. \tag{22}$$

Substituting (22) into the bargaining solution in (14) yields

$$w_t^h l_t^h = \left[\alpha + \theta^h (1-\alpha-\beta)\right] P_t^h (l_t^h)^\alpha \int_0^{N_t^h} [x_t^h(i)]^\beta di = \left[\alpha + \theta^h (1-\alpha-\beta)\right] P_t^h X_t^h, \tag{23}$$

where labor income share $w_t^h l_t^h / P_t^h X_t^h = \alpha + \theta^h (1-\alpha-\beta)$ is increasing in the union's bargaining power θ^h. Then, substituting (23) into (22) yields

$$\Pi_t^h = (1-\theta^h)(1-\alpha-\beta) P_t^h (l_t^h)^\alpha \int_0^{N_t^h} [x_t^h(i)]^\beta di = (1-\theta^h)(1-\alpha-\beta) P_t^h X_t^h. \tag{24}$$

Substituting (23), (24) and $b_t^h = \bar{b}^h P_t^h X_t^h$ into the bargaining solution in (13) yields the equilibrium level of employment in the Home country given by

$$l_t^h = \frac{\alpha + (1-\omega^h)\theta^h (1-\alpha-\beta)}{\bar{b}^h} \equiv l^h, \tag{25}$$

where employment l^h is decreasing in the union's wage preference ω^h and is ambiguous with respect to its bargaining power θ^h depending on the value of ω^h. We impose parameter restrictions to ensure $l_t^h \in (0, L^h)$. By analogous inference, the equilibrium level of employment in the Foreign country is given by

$$l_t^f = \frac{\alpha + (1-\omega^f)\theta^f (1-\alpha-\beta)}{\bar{b}^f} \equiv l^f. \tag{26}$$

We also impose parameter restrictions to ensure $l_t^f \in (0, L^f)$. Equations (25) and (26) give the equilibrium levels of employment regardless of whether the economy is on or off the balanced growth path.

Next we derive the relative wage across countries. Combining (6) and (7) yields the relative demand function for intermediate goods given by

$$\frac{P_t^h}{P_t^f} = \frac{\gamma}{1-\gamma} \left(\frac{X_t^h}{X_t^f}\right)^{-1/\varepsilon}. \tag{27}$$

Substituting (16) into (8) yields the production of intermediate goods X_t^h given by

$$X_t^h = (l^h)^{\alpha/(1-\beta)} (\beta^2 P_t^h)^{\beta/(1-\beta)} N_t^h. \tag{28}$$

Substituting (28) and the analogous condition for X_t^f into (27) yields

$$\frac{P_t^h}{P_t^f} = \left(\frac{\gamma}{1-\gamma}\right)^{\frac{(1-\beta)\varepsilon}{\beta+(1-\beta)\varepsilon}} \left(\frac{l^h}{l^f}\right)^{-\frac{\alpha}{\beta+(1-\beta)\varepsilon}} \left(\frac{N_t^h}{N_t^f}\right)^{-\frac{(1-\beta)}{\beta+(1-\beta)\varepsilon}}, \tag{29}$$

which determines the relative price of X_t^h and X_t^f. Substituting (16) into (23) yields the equilibrium wage rate in the Home country given by

$$w_t^h = \beta^{2\beta/(1-\beta)} \left[\alpha + \theta^h(1-\alpha-\beta)\right] (P_t^h)^{1/(1-\beta)} (l^h)^{-\frac{1-\alpha-\beta}{1-\beta}} N_t^h. \tag{30}$$

Combining (30) and the analogous expression for w_t^f yields an expression for relative wage across countries. If we substitute the relative price in (29) into this expression, we would obtain the equilibrium relative wage given by

$$\frac{w_t^h}{w_t^f} = \frac{\alpha+\theta^h(1-\alpha-\beta)}{\alpha+\theta^f(1-\alpha-\beta)} \left(\frac{\gamma}{1-\gamma}\right)^{\frac{\varepsilon}{\beta+(1-\beta)\varepsilon}} \left(\frac{l^h}{l^f}\right)^{-\frac{\alpha+\beta+(1-\alpha-\beta)\varepsilon}{\beta+(1-\beta)\varepsilon}} \left(\frac{N_t^h}{N_t^f}\right)^{\frac{(1-\beta)(\varepsilon-1)}{\beta+(1-\beta)\varepsilon}}, \tag{31}$$

which is decreasing in relative employment l^h/l^f but increasing in relative technology N_t^h/N_t^f. Equation (31) gives the equilibrium relative wage regardless of whether the economy is on or off the balanced growth path. Proposition 1 characterizes the dynamic property of N_t^h/N_t^f.

Proposition 1 *There is a unique and globally stable steady-state equilibrium value of N_t^h/N_t^f. When N_t^h/N_t^f is below (above) this steady-state value, N_t^h/N_t^f increases (decreases) until N_t^h/N_t^f reaches the steady state, at which point N_t^h and N_t^f grow at the same rate.*

Proof. See the appendix. ■

We now derive the *steady-state* equilibrium value of N_t^h/N_t^f. In the long run, innovation takes place in both countries; therefore, we have $v_t^h = \mu$, which in turn implies that $\dot{v}_t^h = 0$. Substituting $\dot{v}_t^h = 0$ into (20), we obtain the equilibrium value of an invention in the Home country given by

$$v^h = \frac{\pi^h}{r} = \left(\frac{1-\beta}{\beta}\right) \frac{[\beta^2 P^h (l^h)^\alpha]^{1/(1-\beta)}}{r}, \tag{32}$$

where the second equality follows from (17). Combining (32) with the analogous expression for v^f, we obtain the relative value of inventions across countries given by

$$\frac{v^h}{v^f} = \left(\frac{P^h}{P^f}\right)^{1/(1-\beta)} \left(\frac{l^h}{l^f}\right)^{\alpha/(1-\beta)} = 1, \tag{33}$$

where the second equality follows from $v^h = v^f = \mu$. Combining (29) and (33) yields the *steady-state* equilibrium value of N_t^h/N_t^f given by

$$\frac{N_t^h}{N_t^f} = \left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon} \left(\frac{l^h}{l^f}\right)^{\alpha(\varepsilon-1)}, \tag{34}$$

which is increasing in relative employment l^h/l^f capturing the market-size effect on the direction of innovation across countries. We substitute (34) into (31) to derive the *steady-state* equilibrium relative wage given by

$$\frac{w_t^h}{w_t^f} = \frac{\alpha + \theta^h(1-\alpha-\beta)}{\alpha + \theta^f(1-\alpha-\beta)} \left(\frac{\gamma}{1-\gamma}\right)^{\varepsilon} \left(\frac{l^h}{l^f}\right)^{\alpha(\varepsilon-1)-1}, \tag{35}$$

which is increasing in relative employment l^h/l^f if and only if the substitution elasticity ε is greater than $(1+\alpha)/\alpha$.

Finally, Lemma 1 derives the steady-state equilibrium growth rate, which is monotonically increasing in the equilibrium levels of employment $\{l^h, l^f\}$. Therefore, whenever labor unions affect employment, they also affect economic growth in the same way.

Lemma 1 *The steady-state equilibrium growth rate of consumption is given by*

$$g = \frac{(1-\beta)\beta^{(1+\beta)/(1-\beta)}}{\mu} \left[\gamma^{\varepsilon} \left(l^h\right)^{\alpha(\varepsilon-1)} + (1-\gamma)^{\varepsilon} \left(l^f\right)^{\alpha(\varepsilon-1)}\right]^{\frac{1}{(1-\beta)(\varepsilon-1)}} - \rho. \tag{36}$$

Proof. See the appendix. ■

3 Macroeconomic effects of labor unions

In this section, we explore the macroeconomic implications of labor unions. In particular, we are interested in the effects of a labor union becoming more wage oriented (i.e., ω^h increases) and having more bargaining power relative to employers (i.e., θ^h increases). In Section 3.1, we analyze the effects of increasing ω^h.[15] In Section 3.2, we analyze the effects of increasing θ^h.

3.1 Wage preference of labor unions

Equation (25) shows that an increase in the union's wage preference ω^h leads to a decrease in employment l^h in the Home country. Intuitively, as the union in the Home country becomes more wage oriented, it demands a higher wage and depresses labor demand. Given that employment l^f in the Foreign country is independent of ω^h, an increase in ω^h leads to a decrease in relative employment l^h/l^f, which in turn increases relative wage w_t^h/w_t^f across countries in the short run (i.e., for a given N_t^h/N_t^f) as shown in (31). This short-run increase in relative wage is partly due to the direct effect of the decrease in relative employment and partly due to an indirect effect via the increase in relative price P_t^h/P_t^f as shown in (29). In the long run, N_t^h/N_t^f decreases to a lower steady-state value as shown in (34) because the decrease in relative employment l^h/l^f changes the relative market size of the two countries and causes innovation to be directed towards the Foreign country. This negative market-size effect partly offsets and may even dominate the positive price

[15] The effects of increasing unemployment benefit $\overline{b}^h$ are the same as increasing ω^h.

effect on relative wage w_t^h/w_t^f. Equation (35) shows that the overall effect of ω^h on w_t^h/w_t^f would be negative if and only if the substitution elasticity ε is sufficiently large (i.e., $\varepsilon > (1+\alpha)/\alpha$). Finally, from Lemma 1, we see that the decrease in employment reduces economic growth in the long run. We summarize these results in Proposition 2. Figure 2 plots the transitional path of w_t^h/w_t^f in response to a permanent increase in ω^h at time t.

Proposition 2 *As the labor union becomes more wage oriented, employment in the domestic economy decreases. This decrease in employment increases the wage rate in the domestic economy relative to the foreign economy in the short run. In the long run, the decrease in employment in the domestic economy causes innovation to be directed to the foreign economy, which causes a negative effect on the relative wage. The overall effect on the relative wage in the long run is negative if and only if the substitution elasticity ε is greater than a threshold given by $(1+\alpha)/\alpha$. Finally, the effect on the steady-state equilibrium rate of economic growth is negative.*

Proof. See the appendix. ■

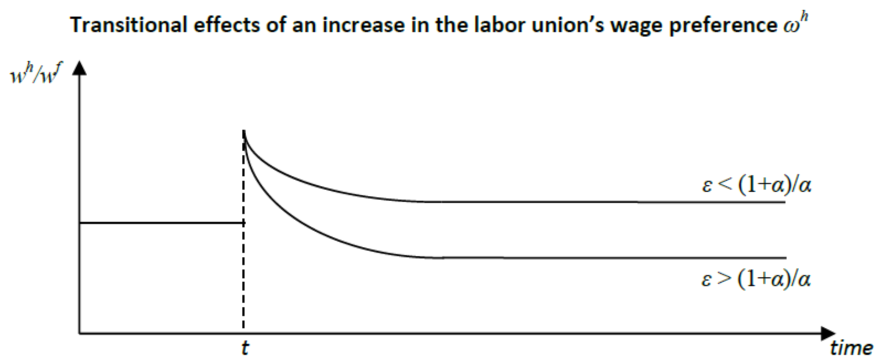

Figure 2

3.2 Bargaining power of labor unions

In this subsection, we first consider the case of a wage-oriented union (i.e., $\omega^h > 1$). Equation (25) shows that an increase in the union's bargaining power θ^h leads to a decrease in employment l^h if and only if the union is wage oriented. This decrease in employment l^h has a positive effect on the wage rate in the Home country as shown in (30). Furthermore, an increase in the union's bargaining power increases the share of output that goes to wage income as shown in (23). These two positive effects on w_t^h lead to an unambiguous increase in relative wage w_t^h/w_t^f in the short run (i.e., for a given N_t^h/N_t^f) as shown in (31). However, in the long run, the decrease in employment l^h exerts a negative market-size effect on w_t^h/w_t^f. This negative market-size effect would dominate the abovementioned positive effects if the elasticity of substitution between Home and Foreign intermediate goods is sufficiently large; i.e.,[16]

$$\varepsilon > 1 + \frac{\omega^h}{\omega^h - 1}\left[\frac{1}{\alpha + \theta^h(1-\alpha-\beta)}\right] \equiv \bar{\varepsilon}. \tag{37}$$

The transitional path of w_t^h/w_t^f in response to an increase in ω^h is similar to Figure 2, except that the threshold of ε is now given by $\bar{\varepsilon}$ instead of $(1+\alpha)/\alpha$. Finally, from Lemma 1, we see that the decrease in employment reduces economic growth in the long run. We summarize these results in Proposition 3.

Proposition 3 *When a wage-oriented union has more bargaining power, employment in the domestic economy decreases. This decrease in employment increases the wage rate in the domestic economy relative to the foreign economy in the short run. In the long run, the decrease in employment in the domestic economy causes innovation to be directed to the foreign economy, which causes a negative effect on the relative wage. The overall long-run effect of increasing a wage-oriented union's bargaining power on the relative wage is negative if and only if the substitution elasticity ε is greater than a threshold given by $\bar{\varepsilon}$ in (37). Finally, the effect on the steady-state equilibrium rate of economic growth is negative.*

Proof. See the appendix. ■

Next we consider an employment-oriented union (i.e., $\omega^h < 1$). In this case, (25) shows that an increase in the union's bargaining power θ^h raises employment l^h in the domestic economy. Although this increase in employment causes a negative effect on relative wage w_t^h/w_t^f as shown in (31), the increase in θ^h still increases relative wage w_t^h/w_t^f in the short run. This positive short-run effect of ω^h on relative wage w_t^h/w_t^f is due to the increase in the share of output that goes to wage income as shown in (23). As for the long-run effect of the union's bargaining power θ^h on relative wage w_t^h/w_t^f, an increase in θ^h further increases relative wage under an employment-oriented union because the market-size effect works in favor of increasing w_t^h/w_t^f due to the increase in employment l^h. Finally, from Lemma 1, we see that this increase in employment also stimulates economic growth in the long run. We summarize these results in Proposition 4. Figure 3 plots the transitional path of w_t^h/w_t^f in response to a permanent increase in the bargaining power θ^h of an employment-oriented union at time t.

Proposition 4 *When an employment-oriented union has more bargaining power, employment in the domestic economy increases. This increase in employment causes a negative effect on the wage rate in the domestic economy. However, the increase in the union's bargaining power also increases the share of output that goes to labor income. The overall effect on the relative wage is positive in the short run. In the long run, the increase in employment causes innovation to be directed to the domestic economy, which leads to an additional positive effect on the relative wage. Therefore, the overall long-run effect of increasing an employment-oriented union's bargaining power on the relative wage is always positive. Finally, the effect on the steady-state equilibrium rate of economic growth is also positive.*

[16] It is useful to note that $\bar{\varepsilon} > (1+\alpha)/\alpha$ given an upper bound imposed on ω^h to ensure $l^h > 0$ in (25).

Proof. See the appendix. ■

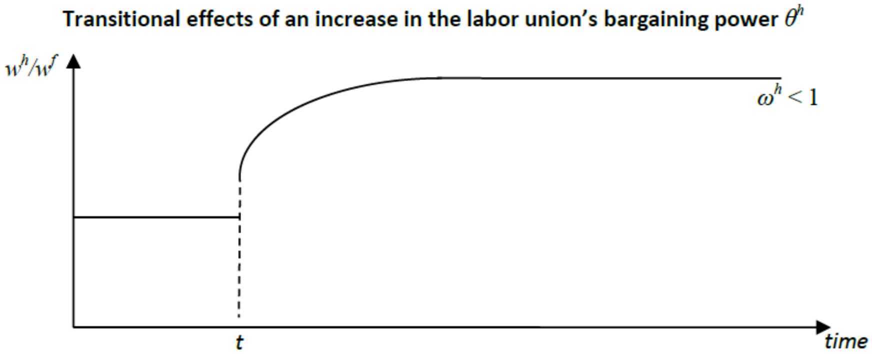

Figure 3

4 Quantitative analysis

In this section, we calibrate our model to data in the US and the UK to see if it can replicate the simultaneous decrease in labor income share and unemployment in Figure 1. In particular, we assume that the decrease in labor income share is due to a decrease in workers' bargaining power $\{\theta^h, \theta^f\}$.[17] Then, we compute the implied values of the unions' wage preference $\{\omega^h, \omega^f\}$ that enable the model to deliver the observed decrease in the unemployment rates in the two countries. Finally, we also explore the quantitative implications on social welfare and relative wage across the two countries.

The model features the following parameters $\{\rho, \varepsilon, \alpha, \beta, \mu, \gamma, L^h, L^f, \bar{b}^h, \bar{b}^f, \omega^h, \omega^f, \theta^h, \theta^f\}$. We define the US as the Home country h and the UK as the Foreign country f. We follow Acemoglu and Akcigit (2012) to consider a standard value for the discount rate $\rho = 0.05$. As for the elasticity of substitution between Home and Foreign intermediate goods, we set ε to 3.5, which is within the range of empirical estimates in Broda and Weinstein (2006). For the parameters in the intermediate-goods production function (8), we set the degree of labor intensity α to 0.5 and the intensity of intermediate goods β to 0.3. It is useful to note that the parameter α is the *lower bound* of labor income share given by $\alpha + \theta^h(1 - \alpha - \beta)$ in the model. Given that labor income share has fallen to as low as 0.54 in the US in recent years, we consider 0.5 to be a reasonable value of α. As for the R&D productivity parameter μ, we calibrate it using the long-run average growth rate in the two economies, and g is about 2%. As for the share parameter γ in the final-goods production function (4), we calibrate it using the relative wage of the two countries, and w_t^h/w_t^f is 1.45 in 1980.[18] We calibrate the policy parameters $\{\bar{b}^h, \bar{b}^f\}$ using data on the average value of

[17]For example, Kristal (2013) uses industry-level data to show that the decrease in labor income share in the US since the early 1980's is due to the decrease in unionization and workers' bargaining power; see also Kristal (2010) and Judzik and Sala (2013) for evidence based on country-level data from a sample of countries including the UK and the US.

[18]Data source: Penn World Table, and OECD Annual Indicators on Unit Labour Costs.

unemployment benefits as a share of GDP in the two countries.[19] To match the unemployment rates in the early 1980's, we calibrate the values of $\{L^h, L^f\}$.[20] We use the trend values of labor income share in the two countries in 1980 and 2007 to calibrate respectively the values of $\{\theta^h, \theta^f\}$ in 1980 and 2007. Finally, we calibrate the values of $\{\omega^h, \omega^f\}$ so that the calibrated economies replicate the observed decrease in the unemployment rates from 8% to 5% in the US and from 10% to 5% in the UK. Table 1 summarizes the calibrated parameter values.

Table 1: Calibrated parameter values

ρ	ε	α	β	μ	γ	L^h	L^f	$\bar{b}^h$	$\bar{b}^f$	ω^h	ω^f	θ^h	θ^f
0.05	3.5	0.5	0.3	9.67	0.51	137.1	43.7	0.38%	0.88%	1.13	1.96	$0.8 \rightarrow 0.2$	$0.8 \rightarrow 0.7$

Figure 1a shows that labor income shares in the US and the UK were about the same at 0.66 in 1980. By 2007, labor income share in the US falls to 0.54, whereas labor income share in the UK decreases only slightly to 0.64. Our model provides the following structural interpretation on this empirical pattern: the bargaining power of workers falls by a much larger degree in the US than in the UK, as indicated in Table 1. Figure 1b shows that unemployment rates in the two countries fall to a similar value of 5%. The fact that unemployment decreases in response to a decrease in the bargaining power of workers implies that unions are wage-oriented (i.e., $\{\omega^h, \omega^f\} > 1$).[21] Furthermore, the degree of wage orientation must be larger in the UK in order for its unemployment rate to fall by a larger magnitude despite the smaller decrease in its workers' bargaining power. Table 1 shows that the degree of wage preference ω^f in the UK is 1.96, which is larger than $\omega^h = 1.13$ in the US. Under these values of $\{\omega^h, \omega^f\}$, decreasing the union's bargaining power θ^h from 0.8 to 0.2 causes the unemployment rate to decrease from 8% to 5% in the US, whereas decreasing θ^f from 0.8 to 0.7 causes the unemployment rate to decrease from 10% to 5% in the UK.

In the rest of this section, we simulate the effects of decreasing unions' bargaining power $\{\theta^h, \theta^f\}$ on relative wage and welfare. Using (1), we can express the representative household's lifetime welfare on the balanced growth path as

$$U^j = \frac{1}{\rho}\left(\ln c_0^j + \frac{g}{\rho}\right), \tag{38}$$

where $j \in \{h, f\}$. To derive the balanced-growth level of consumption in each country, we need to make an assumption on the distribution of assets across countries. Following the standard treatment in the literature,[22] we assume home bias in asset holding under which domestic (foreign) firms are owned by the domestic (foreign) household. Under this assumption, Lemma 2 derives the following expression for $\ln c_0^j$ and U^j, where the term N_0^j in (39) is determined by (34):

[19] Data source: OECD Dataset on Social Expenditure.

[20] The calibrated value of 3.1 for L^h/L^f is slightly less than the relative labor-force size of about 4 in the 1980's.

[21] In the case of employment-oriented unions, decreasing their bargaining power would lead to lower employment and higher unemployment.

[22] See for example Dinopoulos and Segerstrom (2010).

Lemma 2 *Under home bias in asset holding, the steady-state welfare function is given by*

$$U^j = \frac{1}{\rho}\left\{\ln N_0^j + \ln\left[\frac{\mu}{\beta}\left[\rho(1+\beta)+g\right]\right] + \frac{g}{\rho}\right\}. \tag{39}$$

Proof. See the appendix. ■

We consider the same calibrated parameter values and the same changes in $\{\theta^h, \theta^f\}$ as before. The decrease in unions' bargaining power $\{\theta^h, \theta^f\}$ leads to an increase in employment $\{l^h, l^f\}$, which in turn increases the long-run growth rate from 2% to 2.18%.[23] This positive growth effect benefits the two countries equally. However, the decrease in l^h/l^f (recall that unemployment falls by more in the UK than in the US) causes innovation to be directed to the UK and gives rise to a decrease in N^h/N^f, which affects the two countries differently due to changes in relative wage income. In the data, relative wage w_t^h/w_t^f decreases from 1.45 in 1980 to 1.03 in 2007,[24] whereas w_t^h/w_t^f decreases from 1.45 to 1.22 in our simulation, which explains about half of the decrease in relative wage in the data. Finally, we also simulate the welfare changes in the two countries.[25] We find that as a result of the increase in employment and growth, welfare improves in the US, and the welfare gain is equivalent to a permanent increase in consumption of 5.94%. The welfare improvement in the UK is even more significant at 8.89% due to the increase in wage income relative to the US. We summarize these results in Table 2.

Table 2: Simulated effects on $\{w_t^h/w_t^f, U^h, U^f\}$

	Data	Model
Relative wage between the US and the UK in 1980	1.45	1.45
Relative wage between the US and the UK in 2007	1.03	1.22
Welfare changes in the US	n/a	5.94%
Welfare changes in the UK	n/a	8.89%

5 Conclusion

In this study, we have explored the macroeconomics effects of labor unions in an open-economy model of directed technical change. We find that the effects of labor unions on employment, innovation and economic growth are theoretically ambiguous and depend on their wage preference. In the case of the US and the UK, wage-oriented unions seem to fit the data better by enabling the model to replicate the observed decrease in labor income share, unemployment and relative wage

[23] For example, Carmeci and Mauro (2003) estimate dynamic panel regressions using data on OECD countries and find that *decreasing* labor union density indeed has a positive effect on long-run growth.

[24] From the Penn World Table, we obtain PPP-adjusted real income per worker. Then, we use OECD data on labor income share to compute real wage income per worker in the two countries. OECD also provides direct data on average annual wages, according to which relative wage of the two countries decreases from 1.30 in 1990 to 1.17 in 2007; unfortunately, earlier data is not available.

[25] We express welfare changes as equivalent variation in annual consumption.

across countries. In this case, decreasing the bargaining power of unions stimulates employment and economic growth, as some empirical studies (discussed in the introduction) tend to find. However, in other cases, decreasing the bargaining power of unions could reduce employment and growth if the unions are employment-oriented instead. These theoretical findings suggest that there is no one-size-fit-all policy when it comes to reforming existing labor market institutions, such as labor unions, and policymakers should make an effort to understand the country-specific or even industry-specific effects of labor unions.

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Appendix

Proof of Proposition 1. If N_t^h/N_t^f is smaller than its unique steady-state value in (34) (i.e., $N_t^h/N_t^f < [\gamma/(1-\gamma)]^{\varepsilon}\left(l^h/l^f\right)^{\alpha(\varepsilon-1)} \equiv \eta$), then $\pi_t^h > \pi_t^f$ must hold, and hence, $v_t^h = v_t^f = \mu$ cannot hold, noting (17) and (29). In fact, one can show that so long as $N_t^h/N_t^f < \eta$, $\pi_t^h/r_t = v_t^h = \mu$ and $v_t^f < \mu$, implying $\dot{N}_t^h > 0$ and $\dot{N}_t^f = 0$. Following Acemoglu (1998) and Acemoglu and Zilibotti (2001), we have demonstrated that only one type of innovation takes place off the steady state. Furthermore, the economy converges to the steady state and arrives there in finite time. In the steady state, N_t^h/N_t^f is constant over time implying that N_t^h and N_t^f grow at the same rate. An analogous argument can be applied to the case of $N_t^h/N_t^f > \eta$. ■

Proof of Lemma 1. By (32) and $v^h = \mu$, the interest rate is given by

$$r = \frac{\pi^h}{\mu} = \left(\frac{1-\beta}{\beta}\right)\frac{\left[\beta^2 P^h \left(l^h\right)^{\alpha}\right]^{1/(1-\beta)}}{\mu}. \tag{A1}$$

Using (A1), the Euler equation (3) becomes

$$\frac{\dot{c}_t^h}{c_t^h} = \left(\frac{1-\beta}{\beta}\right)\frac{\left[\beta^2 P^h \left(l^h\right)^{\alpha}\right]^{1/(1-\beta)}}{\mu} - \rho \equiv g. \tag{A2}$$

(5) can be rewritten as

$$P^h = \left[\gamma^{\varepsilon} + (1-\gamma)^{\varepsilon}\left(P^h/P^f\right)^{\varepsilon-1}\right]^{\frac{1}{\varepsilon-1}}. \tag{A3}$$

Substituting (34) into (29) leads to $P^h/P^f = \left(l^h/l^f\right)^{-\alpha}$. Applying this to (A3) and substituting the resulting expression into (A2), we can obtain (36). ■

Proof of Proposition 2. It is shown in (25) that an increase in ω^h causes a decrease in employment l^h, which increases w_t^h/w_t^f in the short run (i.e., taking N_t^h/N_t^f as given) through (31). In the long run, the decrease in l^h reduces N_t^h/N_t^f (i.e., innovation to be directed towards the Foreign country) given $\varepsilon > 1$; see (34). Finally, as shown in (35), this results in a reduction in w_t^h/w_t^f if and only if $\alpha(\varepsilon-1) > 1$, which is equivalent to $\varepsilon > (1+\alpha)/\alpha$. Finally, for the effect of ω^h on long-run growth, use Lemma 1. ■

Proof of Proposition 3. Under a wage-oriented union, we have $\omega^h > 1$. We first address the short run by taking N_t^h/N_t^f as given in (31). Using this fact and (25), we can rewrite (31) as

$$\ln(w_t^h/w_t^f) = \ln[\alpha+\theta^h(1-\alpha-\beta)] - \frac{\alpha+\beta+(1-\alpha-\beta)\varepsilon}{\beta+(1-\beta)\varepsilon}\ln\left[\alpha+(1-\omega^h)\theta^h(1-\alpha-\beta)\right], \tag{A4}$$

where we have omitted some exogenous terms for simplicity. Differentiating (A4) yields

$$\frac{\partial\ln(w_t^h/w_t^f)}{\partial\theta^h} = \frac{1-\alpha-\beta}{\alpha+\theta^h(1-\alpha-\beta)} + \frac{\alpha+\beta+(1-\alpha-\beta)\varepsilon}{\beta+(1-\beta)\varepsilon}\frac{(\omega^h-1)(1-\alpha-\beta)}{\alpha+(1-\omega^h)\theta^h(1-\alpha-\beta)}, \tag{A5}$$

which is positive given $\omega^h > 1$. In the rest of this proof, we address the long run by considering (35). In an analogous manner, we can derive from (35)

$$\ln(w_t^h/w_t^f) = \ln[\alpha + \theta^h(1-\alpha-\beta)] + [\alpha(\varepsilon-1)-1]\ln\left[\alpha + (1-\omega^h)\theta^h(1-\alpha-\beta)\right], \tag{A6}$$

where some exogenous terms are omitted again for simplicity. Differentiating (A6) yields

$$\frac{\partial \ln(w_t^h/w_t^f)}{\partial\theta^h} = \frac{1-\alpha-\beta}{\alpha+\theta^h(1-\alpha-\beta)} - [\alpha(\varepsilon-1)-1]\frac{(\omega^h-1)(1-\alpha-\beta)}{\alpha+(1-\omega^h)\theta^h(1-\alpha-\beta)}. \tag{A7}$$

From (A7), it can be shown that $\partial \ln(w_t^h/w_t^f)/\partial\theta^h < 0$ if and only if (37) holds. Finally, for the effect of θ^h on long-run growth, use Lemma 1. ■

Proof of Proposition 4. Under an employment-oriented union, we have $\omega^h < 1$. First, we consider the short-run effect by examining (A4) and (A5). In (A5), $\partial \ln(w_t^h/w_t^f)/\partial\theta^h$ is monotonically increasing in ω^h and takes a strictly positive value at $\omega^h = 0$. Therefore, $\partial \ln(w_t^h/w_t^f)/\partial\theta^h$ is always positive for any $\omega^h > 0$. Then, we examine the long-run effect using (A7). If $\alpha(\varepsilon-1) > 1$, then $\partial \ln(w_t^h/w_t^f)/\partial\theta^h > 0$ must hold because $\omega^h < 1$. If $\alpha(\varepsilon-1) < 1$, then $\partial \ln(w_t^h/w_t^f)/\partial\theta^h$ in (A7) is monotonically increasing in ω^h and takes a strictly positive value at $\omega^h = 0$. Therefore, $\partial \ln(w_t^h/w_t^f)/\partial\theta^h$ is always positive for any $\omega^h \in (0,1)$. Finally, for the effect of θ^h on long-run growth, use Lemma 1. ■

Proof of Lemma 2. Under the assumption of home bias in asset holding, one can rewrite (2) as

$$v_t^h \dot{N}_t^h = r_t v_t^h N_t^h + \Pi_t^h + w_t^h l_t^h - c_t^h, \tag{A8}$$

where $v_t^h = \mu$ and hence $\dot{v}_t^h = 0$. On the balanced growth path, we can solve (A8) for c_t^h:

$$c_t^h = \pi_t^h N_t^h + \Pi_t^h + w_t^h l_t^h - \mu g N_t^h, \tag{A9}$$

using $r_t v_t^h = \pi_t^h$ and $\dot{N}_t^h = g N_t^h$. Using (17), we obtain

$$\pi_t^h N_t^h = \beta^{(1+\beta)/(1-\beta)}(1-\beta)\left[(l^h)^\alpha P^h\right]^{\frac{1}{1-\beta}} N_t^h. \tag{A10}$$

Here it holds that

$$(l^h)^\alpha P^h = \left[\gamma^\varepsilon (l^h)^{\alpha(\varepsilon-1)} + (1-\gamma)^\varepsilon \left(l^f\right)^{\alpha(\varepsilon-1)}\right]^{\frac{1}{\varepsilon-1}} = \left[\frac{\mu(g+\rho)}{\beta^{(1+\beta)/(1-\beta)}(1-\beta)}\right]^{1-\beta}, \tag{A11}$$

where the first equality comes from (A3) with $P^h/P^f = \left(l^h/l^f\right)^{-\alpha}$ and the second equality comes from (36). Using (23) and (24), we obtain

$$\Pi_t^h + w_t^h l_t^h = (1-\beta) P_t^h X_t^h = \beta^{2\beta/(1-\beta)}\left[(l^h)^\alpha P^h\right]^{1/(1-\beta)} N_t^h, \tag{A12}$$

where we have used (28) for the second equality. Substituting (A10)-(A12) into (A9) yields

$$c_t^h = \mu\left[\frac{(1+\beta)\rho + g}{\beta}\right] N_t^h. \tag{A13}$$

Substituting (A13) into (38) yields (39). Finally, c_t^f can be derived analogously. ■