# Asymmetric Pricing Caused by Collusion

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**Abstract**

In many markets, empirical evidence suggests that positive production cost shocks are transmitted more quickly and fully to final prices than negative ones. This article explains asymmetric price adjustment caused by firms imperfectly colluding on supra-competitive price levels. While positive cost shocks are transmitted instantaneously, negative price adjustments only occur once aggregate market demand turns out unexpectedly low. In equilibrium, this can be supported whenever demand is sufficiently stable, and negative cost shocks are not too large.

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# 1 Introduction

A vast body of empirical evidence documents that positive production cost shocks tend to be transmitted more quickly and fully to final prices than negative ones. For example, in a large sample of 77 consumer and 165 producer goods, Peltzman (2000) finds that *asymmetric price adjustment* (or *rockets and feathers*) can be observed in more than two thirds of the markets he examined. Moreover, a multitude of individual empirical studies confirm asymmetric price adjustment in markets related to retail and wholesale gasoline, certain agricultural products, and banking.[1]

Ever since the seminal paper of Borenstein et al. (1997), collusion has been mentioned as one likely cause of the phenomenon. However, apart from few specific exceptions which will be discussed below, no rigorous model of collusive asymmetric price adjustment to common (market-wide) production-cost shocks has been provided. The aim of this paper is to fill this gap in the literature.

To this end, I provide a simple model of asymmetric price adjustment caused by firms imperfectly colluding on supra-competitive price levels. The main mechanism, which is inspired by an informal discussion in Borenstein et al. (1997), works as follows. In the considered oligopolistic market, firms would like to coordinate their prices on high levels, but a multiplicity of equilibria and the prohibition of overt collusion renders coordination on arbitrary price levels impossible. Instead, the firms use downward cost shocks as coordinating mechanism. Whenever a negative cost shock hits the markets, they use the previous period's price as focal point for collusion, which lets them achieve supra-competitive profits during low-cost periods. Clearly, as this only requires a passive pricing behavior of firms, the aforementioned coordination problem can be avoided. On the other hand, the same logic does not prevail for positive cost shocks. Whenever a positive cost shock occurs, the firms have no interest in sticking to a low price level, and increase their prices immediately. Asymmetric price transmission results.

However, according to the above mechanism, negative cost shocks would never be transmitted to final prices if the firms' collusive scheme worked perfectly. This would be counterfactual to the rockets-and-*feathers* pattern, which describes slowly falling prices following negative cost shocks. I resolve this issue by introducing informational frictions. In particular, I assume that due to substantial transaction costs (e.g., because of spatial distance or opportunity costs), the firms

[1] E.g., see Tappata (2009) for further references.

find it impractical to effectively monitor their rivals' prices.[2] Instead, with a lag of one period, they observe their own demand, which provides an imperfect signal about the other firms' past pricing. Hence, much in the spirit of Green and Porter (1984) and Tirole (1988), the firms can only discourage profitable deviations (in the sense of undercutting the collusive price) by punishing unusually low demand.

Now, in order to explain downward price *adjustment*, I model each firm's demand as a confounded signal of the other firms' pricing *and* a random, unobservable aggregate demand level. This ensure that collusion must eventually break down on the equilibrium path, as very low demand levels have to be punished in order to make collusion sustainable. It follows that price adjustments to negative cost shocks occur with a delay. This is in contrast to positive cost shocks, which, by the previous argument, are transmitted instantaneously.

My main findings are as follows. First, in order for an equilibrium of this type to be sustainable, it is necessary that each firm's own demand provides a sufficiently precise signal about the other firms' pricing. This is always the case if the variance of the aggregate market demand is sufficiently low. Second, given that the first property is satisfied, a sufficient equilibrium condition is that the size of the negative cost shock is not too large. Third, for any given probability distribution of aggregate demand, asymmetric-pricing equilibria must break down as the number of firms in the market grows large, firms highly discount future profits, or the low-cost state becomes less and less persistent. And finally, a downward price adjustment similar to the one described in the empirical rockets-and-feathers literature can be generated when one considers the case of multiple independently operating submarkets.

The theoretical literature on asymmetric price transmission caused by collusion is scarce.[3] To the best of my knowledge, the earliest article was given by Damania and Yang (1998), who set up a model of asymmetric price adjustment to firm-specific (idiosyncratic) *demand* shocks. The intuition is that if a firm is in an implicit collusive agreement and experiences a negative demand shock that is not faced (and observed) by other firms, it might be reluctant to reduce its output price, as this may trigger a punishment phase. Because the reverse logic does not hold for positive demand shocks, asymmetric price adjustment to demand shocks may be implied.

[2] All of the model's main qualitative results prevail if the firms can only sometimes observes their rivals' prices, as long as this happens with sufficiently low probability. Further details can be obtained from the author upon request.

[3] Many other explanations for asymmetric pricing have been proposed. These include consumer search costs (Yang and Ye (2008), Tappata (2009), Lewis (2011), Cabral and Fishman (2012)), menu costs (Ball and Mankiw (1994)), lags in adjustment of production and finite inventories (Borenstein et al. (1997)), habit formation and consumption inertia (Xia and Li (2010)), and Edgeworth price cycles that merely resemble asymmetric pricing (Eckert (2002)).

However, their article cannot explain asymmetric price adjustment to *market-wide cost* shocks, which is the principal focus of the empirical literature.

In an attempt to model the German electricity spot market, Wölfing (2008) considers the case of collusive asymmetric price transmission in supply-function equilibrium. The market structure Wölfing considers is special, as firms have to submit supply-functions rather than set prices directly. On top of the limited applicability of this setup to traditional markets, the model cannot endogenously generate negative price adjustment on the equilibrium path. This is because firms have to coordinate on the fraction of a cost shock that is submitted to final prices in each period, and there is no reason why they should not collude perfectly (up to some maximal incentive compatible level). This is in contrast to the present article, which endogenously explains price transmission as coordination failure that must inevitably happen on the equilibrium path.

The most closely related theoretical work is given by Sherman and Weiss (forthcoming). In order to match their empirical setting of a large outdoor market in Jerusalem, they model a specific market structure in which a horizontally differentiated "isolated" firm competes with several homogeneous "rival" firms that compete à la Bertrand, and may engage in implicit collusion. The crucial difference to the present model is a perfect observability of demand and prices, which gives rise to contrasting empirical predictions. For example, colluding "rival" firms may instantaneously decrease their prices when costs decrease or aggregate market demand increases, as under some parameter configurations, this implies that the maximally collusive scheme cannot be supported anymore. In contrast, my model predicts downward price adjustment to negative cost shocks as the result of punishment phases on the equilibrium path, which only happen following severe negative demand shocks. Moreover, I impose less structure on the random demand distribution, and do not consider an asymmetric market structure. Due to their different motivation and partly opposing testable predictions, both models should be viewed as complimentary to each other.

Out of the ample empirical literature on asymmetric price adjustment, a number of studies report a link between the estimated market power of firms and the degree of asymmetric price adjustment in their market. For the retail gasoline market, these studies include Deltas (2008), Verlinda (2008), and Balmaceda and Soruco (2008). For example, analyzing a wide panel of state-level average retail prices for 48 US-American states, Deltas (2008) finds a significant correlation between average retail markups (as a proxy for market power) and the severity of asymmetric price adjustment. Similar results, based on proximity to rival stations and brand identity as measures for market power, are reported by Verlinda (2008), who uses a disaggregated panel of station-level retail gasoline prices in Southern California. A comparable price-response asymmetry can also be found in the banking sector. Analyzing the response of consumer deposit interest rates to changes in the market interest rate, Hannan and Berger (1991) and Neumark and Sharpe (1992) document that markets with a more concentrated banking sector are prone to a higher degree of asymmetric pricing. In particular, the researchers find that deposit interest rates rise *slower* following an increase in the market interest rate if the market concentration is high. This is the interest-rate analogue to the more traditional setting where prices rise faster than they fall facing negative cost shocks. As market power typically facilitates collusion, all of the mentioned articles suggest that collusion may play a non-negligible role in explaining the rockets-and-feathers pattern.

The remainder of this article is structured as follows. Section 2 introduces the model setup and solves for the unique symmetric equilibrium of the stage game with arbitrary production cost. In Section 3, a simple asymmetric-pricing strategy combination for an infinitely repeated, dynamic version of the stage game (with fluctuating costs and demand) is constructed. Moreover, necessary and sufficient conditions for equilibrium existence are provided. Section 4 extends the baseline model of Section 3 to the case of multiple separated submarkets, allowing for a more realistic pattern of the pricing asymmetry. Section 5 concludes. Technical proofs are relegated to the appendix.

## 2 Model Setup and Equilibrium of the Stage Game

Consider a market with $N$ profit-maximizing and risk neutral firms which compete over prices $p_i$ (of some single homogeneous good produced) in a dynamic environment. Importantly, the firms can never directly observe their rivals' prices, both in the current and all bygone periods. Instead, with a lag of one period, firms observe their own demand, which provides an imperfect signal about their competitors' past pricing.

Time is discrete, with $t = 1, 2, ....$ In each period, all $N$ firms face a common marginal cost $c_t$. For simplicity, I follow the majority of the theoretical literature on asymmetric price adjustment by assuming that there are two cost states $c_H$, $c_L$, with $c_H > c_L$. These costs follow a two-state Markov chain, where $\mathbb{P}(c_{t+1} = c_H | c_t = c_H) = \rho_H \in (0, 1)$, $\mathbb{P}(c_{t+1} = c_L | c_t = c_H) = 1 - \rho_H$, $\mathbb{P}(c_{t+1} = c_L | c_t = c_L) = \rho_L \in (0, 1)$, and $\mathbb{P}(c_{t+1} = c_H | c_t = c_L) = 1 - \rho_L$. Firms discount future profits with a common discount factor $\delta \in (0, 1)$.

The demand side is characterized by a continuum of identical consumers with a random total mass $\tilde{\theta}_t$ (henceforth called "aggregate demand") that is drawn from a stationary probability distribution $F(\theta) := \mathbb{P}(\tilde{\theta}_t \leq \theta)$, where $\mathbb{E}(\tilde{\theta}_t) = 1$, in each period. $F$ is assumed to be twice continuously differentiable over its support $[0, \overline{\theta}]$, where $\overline{\theta} > 1$ may be infinite.[4] Moreover, for every $\theta \in (0, \overline{\theta})$, $f(\theta) := F'(\theta) > 0$, which implies that there are no gaps in the aggregate-demand distribution. As with prices, the firms are unable to observe $\tilde{\theta}_t$ directly. Further conditions on $F$ will be discussed later in the analysis.

The consumers always prefer buying over not buying and the total market demand is perfectly inelastic at each point in time. The (random) demand of firm $i$ if it prices at $p_i$ and all other firms price at some vector $\boldsymbol{p_{-i}} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_N)$ is given by

$$\tilde{D_i} = \tilde{\theta} * s_i(p_i; \boldsymbol{p_{-i}}), \tag{1}$$

where $s_i(p_i; \boldsymbol{p_{-i}})$ is a function that maps a vector of prices to a market share $s_i \in [0, 1]$ of the aggregate demand $\tilde{\theta}$. Since I will only consider symmetric equilibria in pure strategies, it is sufficient to characterize $s_i(p_i; p, ..., p) := s_i(p_i; \boldsymbol{p})$, where $p$ is a price that is commonly chosen by all firms other than $i$, as well as $s_j(p; \boldsymbol{p_{(p_i)}})$, where $\boldsymbol{p_{(p_i)}}$ denotes the price vector in which the $N-2$ firms other than $j$ and $i$ price at firm $j$'s price $p$, and firm $i \neq j$ charges $p_i$. In order to minimize technicalities, I focus on a linear demand specification that can be seen as special case of the well-known "spokes model" of non-localized spatial competition provided by Chen and Riordan (2007). In particular, let the firms' market shares be given by[5]

$$s_i(p_i; \boldsymbol{p}) = \frac{1}{N} - \alpha(p_i - p) \qquad \text{if } p_i \in \left[p - \frac{N-1}{\alpha N}, p + \frac{1}{\alpha N}\right], \tag{2}$$

$$s_j(p; \boldsymbol{p_{(p_i)}}) = \frac{1 - s_i(p_i; \boldsymbol{p})}{N-1}, \tag{3}$$

[4] Imposing zero as lower bound of the demand distribution implies that in any given period, the firms' demand may be arbitrarily low. Later in the analysis, this will guarantee that all negative cost shocks must be transmitted eventually, although prices may be very sticky downward.

[5] In the relevant variant of the spokes model, $N$ firms are located at the endpoints of different line segments that have a common origin. The consumers are uniformly distributed along these segments, with the disutility of purchasing at any given firm being proportional to the distance to the firm (consumers have to travel along the line segments). Moreover, each consumer may only choose between purchasing at their "preferred" firm (which is closest) and *one* random firm out of the $N-1$ (equally distant) other firms. The considered market-share function follows if $N-1$ firms charge a common price $p$, and a single firm $i$ charges some arbitrary price $p_i$ (as long as $p_i$ is not too low – in the original spokes model, $s_i(p_i; \boldsymbol{p})$ can never exceed $\frac{2}{N}$).

where $\alpha = \alpha(N) > 0$ may depend on $N$.[6]

This specification summarizes the following ideas. First, if all firms price at some common price level $p$, they split the aggregate market demand evenly. Second, if a firm unilaterally deviates from a common price level, it receives a higher (lower) market share than its rivals if, and only if, it prices lower (higher) than them. In the linear setup, the strength of the marginal market-share response is given by $\alpha > 0$ everywhere (where $\alpha$ may depend on $N$). And third, given a unilateral deviation of firm $i$, all other firms share the residual demand evenly.[7]

Using this setup, it is straightforward to derive firm $i$'s (unique) best response to any price vector $p$ that is commonly chosen by all other firms. In a symmetric equilibrium, this best response must be equal to $p$. Doing so, I find that the unique symmetric stage-game equilibrium price is given by

$$p^*(c) = c + \frac{1}{\alpha N}, \tag{4}$$

with associated equilibrium profits of

$$\pi^*(c) = \pi^* = \frac{1}{\alpha N^2}. \tag{5}$$

Note that the equilibrium price and profits decrease with the "degree of competition" $\alpha$ and the number of firms $N$. In the limit as either $\alpha$ or $N$ goes to infinity, each firm prices at marginal cost and makes a profit of zero. Moreover, the equilibrium price shifts one to one with the cost level $c$, while the equilibrium profits are independent of it.[8] A direct implication is that a cost shock of size $\Delta c$ is fully transmitted if and only if the firms' prices also shift by $\Delta c$.

Finally, suppose that all firms price at some supra-competitive price level $\hat{p} = p^*(c) + \Delta$, where $\Delta > 0$. Then, firm $i$'s incentive to marginally deviate is given by

$$\frac{\partial}{\partial p_i}\left[(p_i - c)s_i(p_i; \hat{\boldsymbol{p}})\right]\Big|_{p_i=\hat{p}} = -\alpha\Delta < 0.$$

[6]If $p_i < p - \frac{N-1}{\alpha N}$, firm $i$'s price is so low relative to the other firms' price $p$ that it captures the whole market: $s_i = 1$. On the other hand, if $p_i > p + \frac{1}{\alpha N}$, firm $i$'s price is so high that it does not attract any consumers: $s_i = 0$.

[7]All of the models' main results carry over to the case of non-linear demand as long as these properties are preserved, under the additional assumptions that $\frac{\partial s_i(p_i;\boldsymbol{p})}{\partial p_i}\Big|_{p_i=p} = -\alpha(N) < 0 \quad \forall i, p$, and that a stage-game equilibrium exists for $c \in \{c_L, c_H\}$. The first additional assumption means that each firm's market share response following a marginal deviation from a common price vector $\boldsymbol{p}$ is constant in the price level $p$. This is consistent with markets in which consumers only care about *absolute* price savings.

[8]The latter two features are preserved in the non-linear-demand case as long as $\frac{\partial s_i(p_i;\boldsymbol{p})}{\partial p_i}\Big|_{p_i=p} = -\alpha(N) < 0 \quad \forall i, p$ (see also the previous footnote).

Thus, each firm has an incentive to (marginally) undercut, and this incentive increases in the competition intensity $\alpha$ and the premium over the competitive price level $\Delta$.

# 3 Equilibrium of the Dynamic Game

In the stage game, each firm has an incentive to lower its price, starting from a collusive price level $\hat{p} > p^*(c)$. The goal of this section is to provide a necessary and sufficient condition for collusion on supra-competitive price levels to be feasible, given the stochastic nature of aggregate market demand and costs, as well as the firms' inability to directly observe their rivals' prices.

Unfortunately, due to the various "folk theorems" that have been proven in the literature (see, e.g., Fudenberg and Maskin (1986)), it is well-known that any repeated game gives rise to an infinite number of subgame-perfect equilibria, provided that the players' (common) discount factor is sufficiently close to one. In order to proceed with the analysis, I ignore equilibria in which the firms coordinate on "arbitrary" price levels that have never been played in previous periods, and which do not correspond to any stage-game equilibrium. While this assumption is restrictive, it seems plausible that – in the absence of any communication – firms would find it difficult to select an arbitrary equilibrium out of the infinitely many equilibria that can be played. On the other hand, continuing to charge the same price after a negative cost shock has happened is a simple and intuitive way of increasing any firm's profit, provided that its rivals do not adapt their prices either. In some sense, past price levels provide a natural focal point for collusion.[9]

Hence, in what follows, I will focus on equilibria where the firms employ a simple mechanism in order to enforce collusion on supra-competitive past price levels. Once a negative cost shock hits the market such that the marginal production cost drops from $c_H$ to $c_L$, the firms keep pricing on the supra-competitive price level $p^*(c_H)$ as long as their demand exceeds some critical threshold $k$ in each period. If the firms' demand falls short of $k$, they enter a punishment phase in which they charge the Nash-equilibrium price $p^*(c_L)$ of the low-cost stage game until the next opportunity for coordination arises.

More precisely, I consider symmetric equilibria in which each firm plays the following strategy.

1. Price at $p^*(c_H)$ whenever $c = c_H$. (High-Cost Phase H)

[9] See Schelling (1960) for a seminal treatment of focal points.

2. If $c = c_L$ and demand has exceeded $k$ in every period since $c$ last switched from $c_H$ to $c_L$, price at $p^*(c_H)$. (Collusive Phase C)

3. If $c = c_L$ and demand has fallen short of $k$ in some period since $c$ last switched from $c_H$ to $c_L$, price at $p^*(c_L)$. (Punishment Phase P)

If an equilibrium of such a structure can be found, it must exhibit asymmetric price transmission. A downward cost shock from $c_H$ to $c_L$ is not transmitted instantaneously to final prices, as the firms keep pricing on $p^*(c_H)$ until demand turns out unexpectedly low. Only in that case (which takes at least one period, as demand is observed with a lag), a punishment phase is entered in which the firms reduce their prices to the equilibrium level $p^*(c_L)$ of the stage game with low costs. On the other hand, upward cost shocks from $c_L$ to $c_H$ are either transmitted immediately (if the firms are currently in the punishment phase and price competitively), or not at all (if the firms are currently in the collusive phase, i.e., a downward cost shock has never been transmitted). In particular, if the low cost state is sufficiently persistent such that an upward cost shock typically happens when the collusive phase has already ended, the well-documented rockets-and-feathers pattern emerges.[10]

I will now start to analyze equilibria of the described type. First, note that each firm's behavior is clearly optimal in both the high-cost phase and punishment phase. Given that all other firms price at $p^*(c_H)$ in the high-cost state ($p^*(c_L)$ in the low-cost state) no matter what happens (and given that an individual firm cannot influence when the high-cost or punishment phase ends), a firm can do no better than by playing the stage-game best response $p^*(c_H)$ ($p^*(c_L)$) itself.

The non-trivial part of the suggested strategy-combination is the collusive phase. As collusion on the supra-competitive price level $p^*(c_H)$ in the low-cost state $c_L$ should be sustainable, each firm has to be deterred from profitably undercutting its rivals (and obtaining a larger market share). As the firms are unable to observe their rivals' price choices directly, the simplest way to do so is by adequately punishing unexpectedly low demand. Doing so, the demand threshold $k$ must necessarily be chosen in such a way that each firm's expected increase in profits by marginally undercutting $p^*(c_H)$ is exactly offset by an expected loss of profits due to a higher probability of collusion to end.

[10] Moreover, from an outside perspective, the high-cost state is always associated with high prices, whereas the low-cost state is only sometimes associated with low prices.

Let $r(p_i; \boldsymbol{p}; k)$ denote the probability that any firm $j$'s demand exceeds $k$, given that all firms $j \neq i$ price at $p$ and firm $i$ prices at $p_i$. It is then easy to see that[11]

$$r(p_i; \boldsymbol{p}; k) = 1 - F\left(\frac{(N-1)k}{1 - s_i(p_i; \boldsymbol{p})}\right), \tag{6}$$

whereas

$$r(p; \boldsymbol{p}; k) = 1 - F(Nk). \tag{7}$$

As $s_i(p_i; \boldsymbol{p})$ decreases in $p_i$ and is equal to $\frac{1}{N}$ for $p_i = p$, it can be seen that $r(p_i; \boldsymbol{p}; k) < r(p; \boldsymbol{p}; k)$ for $p_i < p$ (as long as $k > 0$, that is, any positive punishment threshold is used). Hence, a firm that deviates downward from the collusive price level $p^*(c_H)$ does in fact decrease the probability of collusion to be continued in each period. It is crucial to characterize how $k$ must be chosen in order to ensure that a marginal deviation form the collusive price does not pay.

Next, denote by $\Pi_i^H$, $\Pi_i^C(p_i)$, and $\Pi_i^P$ firm $i$'s expected discounted profit stream (given the proposed strategy for all other firms) in the high-cost phase, collusive phase, and punishment phase, respectively. Note that firm $i$'s expected discounted profit stream of the collusive phase $\Pi_i^C(p_i)$ has firm $i$'s collusive-phase price $p_i$ as argument.[12] Only if firm $i$'s total expected discounted profit is maximized for $p_i = p^*(c_H)$, the proposed strategy-combination forms an equilibrium.

The following recursive equations then define firm $i$' expected discounted profit stream in each of the three regimes (where $\pi_i(p_i)$ is a short notation for $(p_i - c_L)s_i(p_i; \boldsymbol{p^*(c_H)})$ and $r(p_i)$ is a short notation for $r(p_i; \boldsymbol{p^*(c_H)}; k)$).

$$\begin{aligned}
\Pi_i^H &= \pi^* + \rho_H \delta \Pi_i^H + (1 - \rho_H)\delta \Pi_i^C(p_i) & (8)\\
\Pi_i^C(p_i) &= \pi_i(p_i) + \rho_L \left[ r(p_i)\delta \Pi_i^C(p_i) + (1 - r(p_i))\delta \Pi_i^P \right] + (1 - \rho_L)\delta \Pi_i^H & (9)\\
\Pi_i^P &= \pi^* + \rho_L \delta \Pi_i^P + (1 - \rho_L)\delta \Pi_i^H. & (10)
\end{aligned}$$

The first and third of these equations have a similar structure. The expected discounted profit stream of the high-cost phase (punishment phase) is given by the sum of the phase's expected stage-game profit and the one-time discounted expected continuation profit. With probability $\rho_H$ ($\rho_L$), costs stay the same in the high-cost state (low-cost state), which gives rise to an expected continuation profit that is equal to the initial expected discounted profit stream.

[11] The first equation follows because $r(p_i; \boldsymbol{p}; k) := \mathbb{P}\left(\tilde{\theta} * s_j(p; \boldsymbol{p_{(p_i)}}) > k\right) = \mathbb{P}\left(\tilde{\theta} * \frac{1 - s_i(p_i; \boldsymbol{p})}{N-1} > k\right) = 1 - F\left(\frac{(N-1)k}{1 - s_i(p_i; \boldsymbol{p})}\right)$. This directly implies $r(p; \boldsymbol{p}; k) = 1 - F(Nk)$, as $s_i(p; \boldsymbol{p}) = \frac{1}{N}$.

[12] Since the collusive phase is stationary, it suffices to consider one single price $p_i$ that firm $i$ chooses in every period of that phase.

With probability $1-\rho_H$ $(1-\rho_L)$, costs switch to the other state, which leads to an expected continuation profit that is equal to the expected discounted profit stream of the collusive phase (high-cost phase).

The second equation has the following interpretation. The expected discounted profit stream of the collusive phase, given that firm $i$ prices at $p_i$ in each stage of that phase, can be written as the sum of the expected stage-game profit of pricing at $p_i$ (while all other firms stick to the plan of pricing at $p^*(c_H)$) and the one-time discounted expected continuation profit. This continuation profit has two parts. With probability $\rho_L$, costs stay low. Then, depending on whether the previous period's demand has exceeded $k$ or not (which happens with probability $r(p_i)$ and $1-r(p_i)$, respectively), the expected continuation profit is either given by the initial expected discounted profit stream, or the expected discounted profit stream of the punishment phase. With probability $1-\rho_L$, costs switch to the high state. Then, the expected continuation profit is equal to the expected discounted profit stream of the high-cost phase.[13]

Figure 1 provides a graphical representation of the underlying dynamical system if firm $i$ prices at $p_i$ in every period of the collusive phase.

It was already discussed above that the firms only face a non-trivial pricing decision when the game is in the collusive phase. Clearly, continuing to price at $p^*(c_H)$ in the collusive phase is a best response to all other firms' strategies if, and only if, $p^*(c_H)$ is a global maximizer of $\Pi_i^C(p_i)$. Solving the above system of equations, the following lemma can be stated.

**Lemma 1.** *Firm i's expected discounted profit stream in the collusive phase, given that all other firms price according to the proposed strategy, can be written as*

$$\Pi_i^C(p_i) = \frac{\pi^*}{1-\delta} + \frac{(1-\delta\rho_H)(1-\delta\rho_L)}{(1-\delta)\left[1+\delta-\delta(\rho_H+\rho_L)\right]} \times \frac{\pi_i(p_i)-\pi^*}{1-\delta\rho_L r(p_i)}. \tag{11}$$

The interpretation to Lemma 1 is straightforward: the expected discounted profit stream of pricing at $p_i$ in every period of the collusive phase is given by the expected discounted profit stream $\frac{\pi^*}{1-\delta}$ of receiving the (competitive) stage-game equilibrium profit in every period, *plus* the expected excess profit over the competitive profit in collusive periods, $\pi_i(p_i)-\pi^*$, properly discounted.

[13] A different way of writing down equation (9) is as follows: $\Pi_i^C(p_i) = \int_0^{\frac{(N-1)k}{1-s_i(p_i)}} \left[\pi_i(p_i)\tilde{\theta} + \rho_L\delta\Pi_i^P + (1-\rho_L)\delta\Pi_i^H\right] f(\tilde{\theta})d\tilde{\theta} + \int_{\frac{(N-1)k}{1-s_i(p_i)}}^{\overline{\theta}} \left[\pi_i(p_i)\tilde{\theta} + \rho_L\delta\Pi_i^C(p_i) + (1-\rho_L)\delta\Pi_i^H\right] f(\tilde{\theta})d\tilde{\theta}$, where the bound $\frac{(N-1)k}{1-s_i(p_i)}$ is the necessary aggregate demand level that is needed for sustained collusion, given $p_i$ and $k$. As $r(p_i) = 1-F\left(\frac{(N-1)k}{1-s_i(p_i;\boldsymbol{p})}\right)$, it is easy to see that both formulations are equivalent.

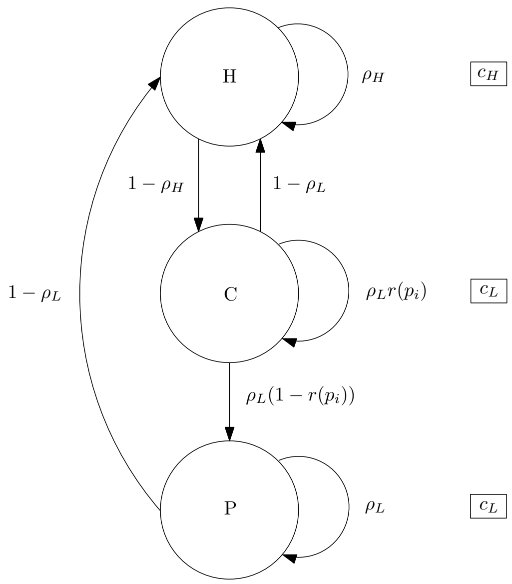


Figure 1: Depiction of the dynamical system that is implied if firm $i$ charges $p_i$ in the collusive phase, given the proposed strategy-combination of all other firms. Transition probabilities are found next to the arrows indicating a state change.

Examining equation (11), it is apparent that $\Pi_i^C(p_i)$ reaches its global maximum at the value of $p_i$ that maximizes $\hat{\Pi}_i(p_i) := \frac{\pi_i(p_i)-\pi^*}{1-\delta\rho_L r(p_i)}$. In order for the proposed strategy-combination to form an equilibrium, this maximum must be reached at $p^*(c_H)$. A necessary condition for this is that the derivative of $\hat{\Pi}_i(p_i)$, evaluated at $p^*(c_H)$, is equal to zero. Carrying out the corresponding calculation, one arrives at the following proposition.

**Proposition 1.** *(Necessary condition) In order for the proposed strategy-combination to form an equilibrium, the demand threshold $k$ must be chosen such that $\phi := Nk$ satisfies*

$$h(\phi) := (N-1)\left[1-\delta\rho_L+\delta\rho_L F(\phi)\right]-\delta\rho_L\phi f(\phi)=0. \tag{12}$$

The intuition to equation (12) is a simple marginal-cost marginal-benefit tradeoff. If a firm marginally deviates downward from the collusive price level $p^*(c_H)$, it makes a higher-stage game profit in expectation (as the best response to all other firms pricing at $p^*(c_H)$ is to price *lower* than $p^*(c_H)$), but this comes at the cost of a higher probability of collusion to end after each period, which decreases the expected length of collusive-phases with supra-competitive profits.

In particular, as the marginal cost of undercutting the collusive price level is proportional to $\phi f(\phi)$, one can see that the above first order condition can only be satisfied for adequately chosen demand thresholds $\phi^* = Nk^*$ if the probability density of aggregate market demand is sufficiently large somewhere in its distribution. Only if that is the case, the probability of sustained collusion following a marginal price decrease can be reduced by so much (when choosing $k$ appropriately) that the firms are successfully discouraged from deviating.

In fact, examining $h'(\phi) = \delta\rho_L\,[(N-2)f(\phi) - \phi f'(\phi)]$ and noting that $h(0) > 0$, it is apparent that the necessary condition can never be fulfilled if $f$ is non-increasing ($F$ is weakly concave). This rules out some common cumulative distribution functions, including the uniform, exponential, and Pareto distribution. The interpretation is that these distribution functions provide too weak signals about the firms' pricing in order to discourage marginal deviations. No matter how the demand threshold $k$ is chosen, firms can never be deterred from profitably undercutting the collusive price level, as doing so reduces the probability of sustained collusion by too little.

Note moreover that for any given aggregate-demand distribution $F$, it directly follows from equation (12) that as $N$ increases without bound or $\delta\rho_L$ decreases towards zero, the first order condition must eventually be violated. Hence, an asymmetric-pricing equilibrium of the analyzed structure can only be supported if there are not too many firms in the market, firms do not highly discount future profits, and the low-cost state is sufficiently persistent.

So far, only a necessary condition in order to allow the collusive price level $p^*(c_H)$ to be a *local* extremum of the expected discounted profit stream of the collusive phase has been provided. However, in order to make pricing at $p^*(c_H)$ a best response to the other firms' strategies, it has to hold that this price is a *global* maximizer of firm $i$'s expected discounted profit stream in the collusive phase. The following proposition provides a sufficient condition for that.

**Proposition 2.** *(Sufficient Condition) The proposed strategy combination forms an equilibrium ($p^*(c_H)$ is a global maximizer of $\hat{\Pi}_i(p_i)$) if a solution to the necessary condition in equation (12) exists, and $\Delta c := c_H - c_L$ is sufficiently small.*[14] *In particular, the former is true whenever the variance of aggregate market demand is sufficiently low, that is, $Var(\tilde{\theta}) < (\frac{3}{6+16\frac{N-1}{\delta\rho L}})^2$.*

Thus, asymmetric pricing equilibria exist whenever the variance of aggregate market demand is low relative to the market parameters $N$, $\delta$ and $\rho_L$, given that the size of the negative cost shock is not too large. In particular, the sufficient bound for the variance becomes less stringent (larger) for a lower number of firms, a higher discount factor, and a higher persistence of the low-cost state. Moreover, it can be seen that a wide range of plausible distribution functions for modeling stochastic aggregate market demand, e.g. the Log-normal, Gamma, Beta, Log-logistic,

[14]Explicit sufficient conditions on $\Delta c$ can be found at the end of the proof of the proposition.

and Weibull distribution, give rise to the existence of asymmetric-pricing equilibria, provided that their variance is sufficiently low. This is because all of these distribution functions can be normalized in such a way that their expectation is set to one, with a free parameter governing their variance.

The intuition to the above proposition is twofold. First, a sufficiently low variance of aggregate market demand guarantees that marginal deviations from the collusive price level are not profitable when the demand threshold $k$ is set properly, as the probability of sustained collusion decreases by too much. And second, also larger deviations from the collusive price level do not pay if the size of the cost shock is sufficiently small. This is because, for a small negative cost shock, the collusive price level lies close to the (new) competitive price level, implying that large deviations from the collusive level cannot pay.

Having established the existence of an asymmetric-pricing equilibrium under suitable model parameters, it is now possible to quantify the *degree* of asymmetry in price adjustment. For this, note that for any solution $\phi^* = Nk^*$ of equation (12) that does in fact constitute an equilibrium, the probability of the collusive phase to end, conditional that costs remain low, is given by $F(\phi^*)$ in each period. Thus, following a persistent negative cost shock, the expected number of periods until prices adjust from $p^*(c_H)$ to the lower competitive level of $p^*(c_L)$ is given by

$$L(\phi^*) = \frac{1}{F(\phi^*)} > 1.^{15} \tag{13}$$

On the other hand, by construction, positive cost shocks are transmitted instantaneously, given that the firms were pricing at the competitive level $p^*(c_L)$ before.

Finally, the implicit equation (12) also allows for comparative statics with respect to the firms' discount factor $\delta$ and the persistence of the low-cost state $\rho_L$. The following proposition is a direct consequence of the implicit function theorem.

**Proposition 3.** *A marginal increase in $\delta$ or $\rho_L$ leads to a more pronounced asymmetry in price transmission if, and only if, $h'(\phi^*)$ is negative.*

Proposition 3 shows that the effect of a marginal increase in $\delta$ or $\rho_L$ is directly related to the sign of $h'(\phi^*)$. This sign is ambiguous, as in the case of multiple equilibria, it may depend on the chosen equilibrium demand threshold. In particular, it can be the case that as firms become more patient or negative cost shocks become more persistent, negative cost shocks are

[15] This expectation follows from a well-known property of geometrically distributed random variables.

transmitted more quickly to final prices.[16] Also, it should be noted that comparative statics with respect to $N$ cannot be provided for general demand distributions $F$, as the implicit function theorem does not work for discrete parameters.

# 4 Multiple Submarkets

In the baseline model developed in Sections 2 and 3, I considered the case of a single oligopolistic market in which a small number of firms engages in imperfect collusion on supra-competitive price levels. Under suitable model parameters, the proposed trigger-sales strategy combination forms an equilibrium in which asymmetric price transmission to cost shocks is the outcome. However, the baseline model is counterfactual to the rockets-and-*feathers* pattern in the sense that once collusion breaks down, all prices adapt fully and abruptly to the lower competitive level. In contrast, empirical evidence documents slowly declining prices after the occurrence of negative cost shocks.

The purpose of this section is to reconcile the theoretical model with the patterns that are found in the data. The main argument is that the price series that are typically studied in the literature are not station-specific, but reflect average retail prices in a large market, which may be comprised of several independently operating local submarkets. Hence, due to the stochastic nature of demand, collusion on supra-competitive price levels may persist longer in some submarkets than others. This implies that time series of market-wide average retail prices should be slowly declining following negative cost shocks.

Let $M \geq 1$ denote the total number of locally separated submarkets, with $N_m$ and $\alpha_m$ ($m \in \{1, ..., M\}$) referring to the total number of firms $N_m$ and competition intensity $\alpha_m$ in submarket $m$, respectively. The discount factor $\delta$ is assumed to be constant across submarkets.

Moreover, for simplicity, let the random aggregate-demand variable $\tilde{\theta}_m$ be distributed independently across submarkets, with $\Pr\left(\tilde{\theta}_1 \leq \theta_1 \wedge ... \wedge \tilde{\theta}_M \leq \theta_M\right) = \prod_{m=1}^{M} F_m(\theta_m)$ in each period (the probability distribution of the stochastic demand variable is allowed to differ across submarkets).

The stochastic cost process for the whole market follows the same rules as in the baseline model. That is, the market-wide production costs stick to the high-cost level $c_H$ (low-cost level $c_L$) with probability $\rho_H$ ($\rho_L$) after each period.

[16]It has to be noted though that in all numerical simulations I undertook, equilibria with positive $h'(\phi^*)$ were accompanied by equilibria with negative $h'(\phi^{**})$, for some $\phi^{**} < \phi^*$. Hence, as equilibria with a lower $\phi$ give rise to higher expected firm profits, equilibria with positive $h'(\phi)$ were always Pareto-dominated.

Now, given this setup, submarket $m$ will face a first order condition of

$$(N_m - 1)\left[1 - \delta\rho_L + \delta\rho_L F_m(\phi)\right] - \delta\rho_L \phi f_m(\phi) = 0.$$

Submarkets in which at least one solution $\phi_m^*$ to the above equation exists will find it optimal to stick to the collusive price level $p_m^*(c_H) := c_H + \frac{1}{\alpha_m N_m}$ following a negative cost shock, given that $c_H - c_L$ is not too large relative to the respective submarket's parameters.

Suppose $L \leq M$ of the submarkets are in a collusive equilibrium (using some specific demand threshold $N_m k_m^* = \phi_m^*$), and without loss of generality, label them with $1, ..., L$. Then, following a *persistent* negative cost shock, the probability that any subset $S \subseteq \{1, ..., L\}$ of these firms will still charge the high price $p_m^*(c_H)$, $t$ periods after a negative cost shock has happened, is given by $\prod_{m \in S} \left(1 - F_m(\phi_m^*)\right)^t$. In particular, this shows that the probability that any given subset $S$ of the colluding firms will still price high $t$ periods after a persistent negative cost shock is strictly decreasing in $t$.

Unfortunately, in the case of heterogeneous submarkets, it is not practical to explicitly derive the corresponding probability $H(l, t)$ that some number of firms $l \leq L$ continue to price collusively $t$ periods after a persistent negative cost shock has happened. For analytical purposes, I will subsequently consider the case in which all of the $M \geq 1$ submarkets are identical and characterized by a common probability $\gamma := 1 - F(\phi^*)$ that collusion is continued after each period of the collusive phase. In turn, this implies that the probability that any given submarket will still be in the collusive phase $t$ periods after a persistent negative cost shock has happened is equal to $\gamma^t$.

Then, given that each submarket is independent from all others, the probability $J(m, t)$ that exactly $m \leq M$ of all submarkets will still be in the collusive phase $t$ periods after a negative cost shock has occurred follows a binomial distribution, where

$$J(m, t) := \binom{M}{m} (\gamma^t)^m (1 - \gamma^t)^{M-m}. \tag{14}$$

By a well-known property of binomially distributed random variables, the expected number of firms who continue to price collusively $t$ periods after a persistent negative cost is given by $M\gamma^t$, which decreases exponentially in $t$. Hence, the expected average retail price of the whole market, $t$ periods after a persistent negative cost shock has happened, can easily be calculated. The following proposition highlights this finding.

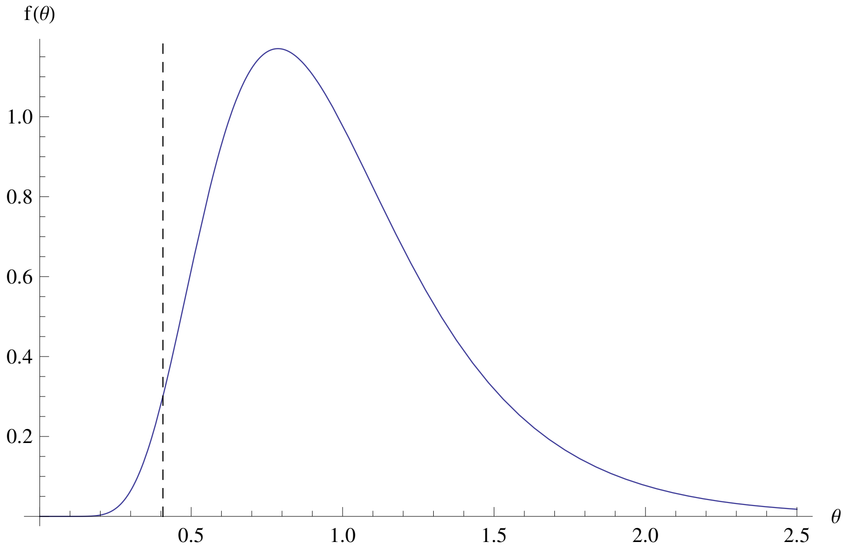


Figure 2: Density function of a log-normal distribution with mean-parameter $-0.08$ and standard-deviation parameter 0.4, implying a mean of 1. Given the selected parameters, the aggregate demand threshold $\phi^*$ is located at the dashed line.

**Proposition 4.** *Suppose the whole market is comprised of $M \geq 1$ independently operating submarkets, each characterized by a probability $\gamma > 0$ that collusion is continued after each period of the collusive phase. Then, the expected average retail price of the whole market, $t$ periods after a persistent negative cost shock has occurred, is given by*

$$p^*(c_H)\gamma^t + p^*(c_L)(1 - \gamma^t). \tag{15}$$

*For large $M$, a smooth transition from the collusive price level $p^*(c_H)$ to the new competitive price level $p^*(c_L)$ can be observed.*

In the following, I will present a numerical simulation of the extended model. For this, let the parameters of the whole market be given by $M = 50$, $c_H = 20$, $c_L = 16$, $\delta = 0.9999$, $\rho_H = \rho_L = 0.98$. Moreover, for each submarket, let $N = 4$, $\alpha = 0.05$, and $F(\theta)$ log-normal with mean-parameter $-0.08$ and standard-deviation parameter 0.4. The latter implies a mean of the aggregate market demand random variable of 1 (as required by the model) and a standard deviation of roughly 0.417. See Figure 2 for a depiction of the corresponding probability density function $f(\theta)$.

It is now easy to see that $p^*(c_H) = 25$ and $p^*(c_L) = 21$. Also, one can verify numerically that $\phi^* = Nk^* = 0.406724$ is a solution to the first order condition stated in Proposition 1. As

$p^*(c_H)$ is also a global maximizer of $\Pi_i^C(p_i)$ for $\phi = \phi^*$,[17] this implies that asymmetric pricing can be observed in equilibrium. In fact, if all firms stick to the punishment threshold $\phi^*$, there is a probability of $1 - F(0.406724) \approx 0.979771$ that collusion is continued after each period of the collusive phase.

Figure 3 depicts a simulation of the outlined market for a length of 500 periods ("days").[18] The well-documented rockets-and-feathers pattern can clearly be discerned.

# 5 Conclusion

In a wide range of markets, positive production cost shocks are transmitted more quickly and fully to final prices then negative ones. This article provides a simple model of asymmetric price transmission caused by firms imperfectly engaging in tacit collusion. In the model, negative cost shocks are only transmitted to final prices once collusion breaks down. This happens when an unobservable aggregate-demand variable turns out unexpectedly low, which typically occurs with a delay. On the other hand, positive cost shocks are transmitted instantaneously, as the firms have no interest in sticking to lower than competitive prices.

By considering a simple trigger-sales strategy according to which firms punish unusually low demand, I prove that asymmetric-pricing equilibria exist whenever the variance of aggregate market demand is sufficiently low *and* the size of negative cost shocks is not too large. Conversely, I show that the considered equilibrium can only exist if there are not too many firms in the market, low-cost states are relatively persistent, and the firms discount the future by not too much. Moreover, in order to discourage marginal deviations, it should not always be the case that low aggregate demand levels are more probable than high levels. Since all of these features can be examined empirically, a rich array of testable predictions is generated.

Future research might extend the simple model to a more general class of random cost processes, allow the firms to endogenously monitor their rivals' prices, or consider the case of asymmetric market shares. However, already the current model can generate pricing patterns that are close to the ones observed in the data, given that multiple separated submarkets are considered.

The most important agenda is hence to analyze the various predictions of the model empirically and contrast them with those of other theoretical models of asymmetric price adjustment. In particular, if the portrayed collusive mechanism causes the phenomenon, it should be observed

[17] A flexible Mathematica-code to perform numerical simulations like this can be obtained from the author upon request.

[18] The underlying R-code (alternatively, pseudo-code) can be obtained from the author upon request.

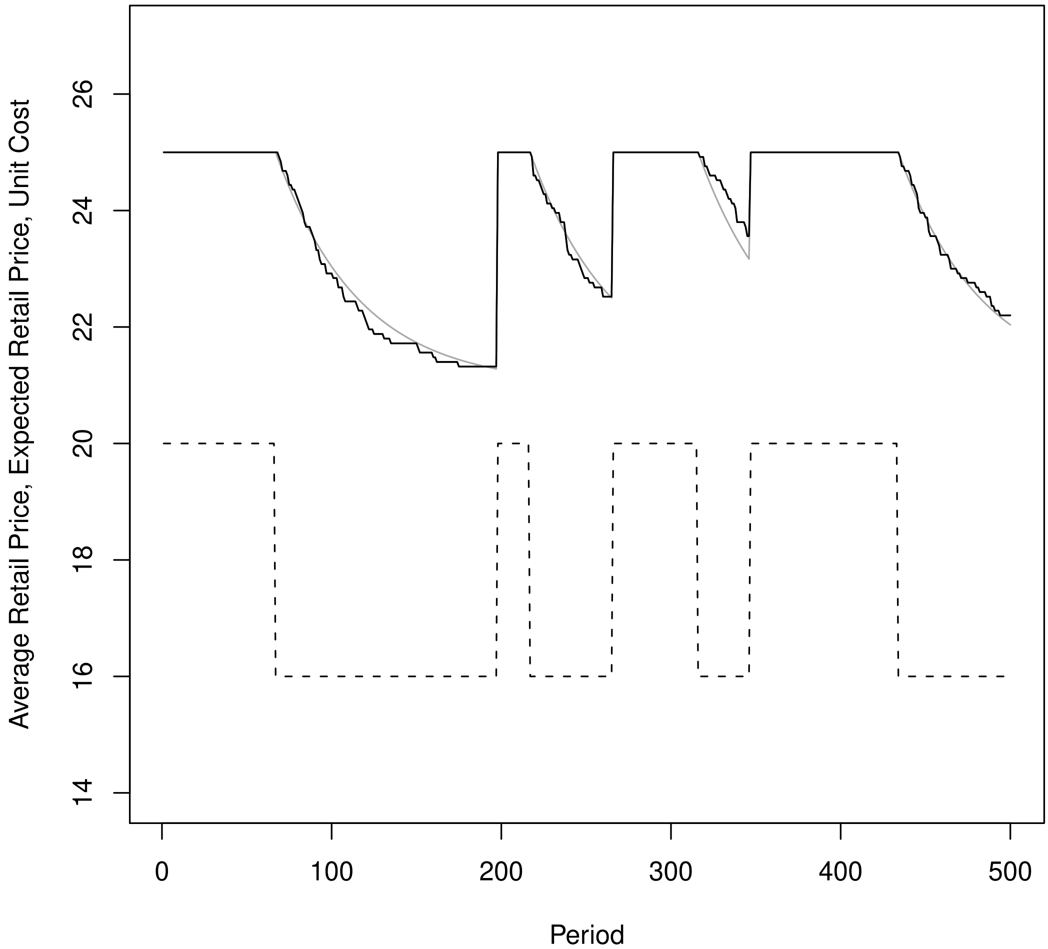


Figure 3: Market simulation for $T = 500$, $M = 50$, $c_H = 20$, $c_L = 16$, $\delta = 0.9999$, $\rho_H = \rho_L = 0.98$, and for each submarket, $N = 4$, $\alpha = 0.05$, $F(\theta)$ log-normal with mean-parameter $-0.08$ and standard-deviation parameter 0.4. The black solid (gray solid) [dashed] line represents the whole market's actual average retail price (expected average retail price) [marginal cost], respectively.

that prices tend to adjust downward in low-demand periods, and that markets with more stable aggregate demand, more persistent negative cost shocks, and fewer firms, are more likely to exhibit the rockets-and-feathers pattern.

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# 6 Appendix: Technical Proofs

*Proof of Lemma 1.* First, use equations (8) and (10) in order to solve for $\Pi_i^H$ and $\Pi_i^P$ as functions of $\Pi_i^C(p_i)$. This yields

$$\Pi_i^H = \frac{\pi^* + (1-\rho_H)\delta\Pi_i^C(p_i)}{1-\delta\rho_H}$$

and

$$\Pi_i^P = \frac{\pi^*[1+\delta-\delta(\rho_H+\rho_L)]}{(1-\delta\rho_L)(1-\delta\rho_H)} + \frac{(1-\rho_L)(1-\rho_H)\delta^2}{(1-\delta\rho_L)(1-\delta\rho_H)}\Pi_i^C(p_i).$$

Next, insert the above expressions into equation (9), isolate $\Pi_i^C(p_i)$ and multiply both sides with $(1-\delta\rho_L)(1-\delta\rho_H)$ in order to get

$$\begin{aligned}\Pi_i^C(p_i)\Big[(1-\delta\rho_L r(p_i))(1-\delta\rho_L)(1-\delta\rho_H) - \delta^3\rho_L(1-\rho_L)(1-\rho_H)(1-r(p_i)) \; - \\ \delta^2(1-\rho_L)(1-\rho_H)(1-\delta\rho_L)\Big] = \\ \pi_i(p_i)(1-\delta\rho_L)(1-\delta\rho_H) + \pi^*\delta\rho_L(1-r(p_i))\left[1+\delta-\delta(\rho_H+\rho_L)\right] \; + \\ \pi^*\delta(1-\rho_L)(1-\delta\rho_L).\end{aligned}$$

Simplify the squared brackets to the right of $\Pi_i^C(p_i)$ and add and subtract $\pi^*(1-\delta\rho_L)(1-\delta\rho_H)$ to the RHS to obtain

$$\begin{aligned}\Pi_i^C(p_i)\left\{(1-\delta)\left[1+\delta-\delta(\rho_L+\rho_H)\right](1-\delta\rho_L r(p_i))\right\} = \\ (1-\delta\rho_L)(1-\delta\rho_H)[\pi_i(p_i)-\pi^*] + \pi^*\delta\rho_L(1-r(p_i))\left[1+\delta-\delta(\rho_H+\rho_L)\right] \; + \\ \pi^*\delta(1-\rho_L)(1-\delta\rho_L) + \pi^*(1-\delta\rho_L)(1-\delta\rho_H).\end{aligned}$$

Collecting terms with $\pi^*$ in the RHS and simplifying, this further reduces to

$$\begin{aligned}\Pi_i^C(p_i)\left\{(1-\delta)\left[1+\delta-\delta(\rho_L+\rho_H)\right](1-\delta\rho_L r(p_i))\right\} \; = \\ (1-\delta\rho_L)(1-\delta\rho_H)[\pi_i(p_i)-\pi^*] + \pi^*\left[1+\delta-\delta(\rho_L+\rho_H)\right](1-\delta\rho_L r(p_i)),\end{aligned}$$

which directly implies the equation in the lemma. □

*Proof of Proposition 1.* Differentiating $\hat{\Pi}_i(p_i) := \frac{\pi_i(p_i)-\pi^*}{1-\delta\rho_L r(p_i)}$ with respect to $p_i$ and eliminating the positive denominator leads to the first order condition

$$\frac{\partial\pi_i(p_i;\boldsymbol{p^*(c_H)})}{\partial p_i}\left[1-\delta\rho_L r(p_i;\boldsymbol{p^*(c_H)};k)\right] + \delta\rho_L\left[\pi_i(p_i;\boldsymbol{p^*(c_H)})-\pi^*\right]\frac{\partial r(p_i;\boldsymbol{p^*(c_H)};k)}{\partial p_i} = 0,$$

which has to be satisfied for $p_i = p^*(c_H)$.

Inserting the definition of $\pi_i(p_i; \boldsymbol{p^*(c_H)}) = (p_i - c_L)s_i(p_i; \boldsymbol{p^*(c_H)})$, this can be reformulated to

$$\left[s_i(p_i; \boldsymbol{p^*(c_H)}) + (p_i - c_L)\frac{\partial s_i(p_i; \boldsymbol{p^*(c_H)})}{\partial p_i}\right][1 - \delta\rho_L r(p_i; \boldsymbol{p^*(c_H)}; k)] \ + \tag{16}$$
$$\delta\rho_L\left[\pi_i(p_i; \boldsymbol{p^*(c_H)}) - \pi^*\right]\frac{\partial r(p_i; \boldsymbol{p^*(c_H)}; k)}{\partial p_i} = 0,$$

One can now calculate that

$$\begin{aligned}\frac{\partial r(p_i; \boldsymbol{p^*(c_H)}; k)}{\partial p_i} &= \frac{\partial\left[1 - F\left(\frac{(N-1)k}{1-s_i(p_i;\boldsymbol{p^*(c_H)})}\right)\right]}{\partial p_i} \\ &= -f\left(\frac{(N-1)k}{1 - s_i(p_i; \boldsymbol{p^*(c_H)})}\right)\frac{(N-1)k}{[1 - s_i(p_i; \boldsymbol{p^*(c_H)})]^2}\frac{\partial s_i(p_i; \boldsymbol{p^*(c_H)})}{\partial p_i},\end{aligned}$$

where the first equality follows from equation (6). Evaluated at $p^*(c_H)$, this expression simplifies to

$$\frac{\alpha N}{N-1}Nkf(Nk).$$

Moreover, $r(p^*(c_H); \boldsymbol{p^*(c_H)}; k)$ is given by $1 - F(Nk)$, as was already stated in equation (7).

Evaluating equation (16) at $p_i = p^*(c_H)$ thus gives

$$\left[\frac{1}{N} + \left(\frac{1}{\alpha N} + c_H - c_L\right)(-\alpha)\right][1 - \delta\rho_L + \delta\rho_L F(Nk)] + \delta\rho_L\frac{c_H - c_L}{N}\frac{\alpha N}{N-1}Nkf(Nk) = 0.$$

Simplifying the squared bracket, canceling out the positive factor $\alpha(c_H - c_L)$, multiplying by $-(N-1)$ and setting $Nk = \phi$ finally yields the expression in the proposition. $\square$

In order to prove Proposition 2, it is convenient to state the subsequent lemma first. In all of what follows, let $u(p_i) := \pi_i(p_i) - \pi^*$ and $v(p_i) := 1 - \delta\rho_L + \delta\rho_L F\left(\frac{(N-1)k^*}{1-s_i(p_i;p^*(c_H))}\right)$, where $v(p_i) > 0$.

**Lemma 2.** *For any set of parameters $c_H$, $c_L$, $\alpha$, $\delta$, $\rho_L$, $N$, some price level $p_i \geq 0$ can only be a global maximizer of $\hat{\Pi}_i(p_i)$ if $u(p_i) > 0$ and $u'(p_i) < 0$.*

*Proof.* As $\hat{\Pi}_i(p^*(c_H)) = \frac{u(p^*(c_H))}{v(p^*(c_H))} = \frac{\Delta c}{Nv(p^*(c_H))}$ is strictly positive for any $\Delta c$, it is clear that only positive values of $u(p_i) = \pi_i(p_i) - \pi^*$ are candidates for a global maximizer of $\hat{\Pi}_i(p_i)$. Moreover, note that $\hat{\Pi}_i'(p_i)$ has the same sign has $u'(p_i)v(p_i) - u(p_i)v'(p_i)$. Hence, since $v(p_i)$ is unambiguously positive, $v'(p_i)$ is unambiguously negative, and $u(p_i)$ is unambiguously positive over the relevant range for global maximizers (by the previous observation), it has to hold that $u'(p_i)$ is strictly negative in order for $u'(p_i)v(p_i) - u(p_i)v'(p_i)$ to be non-positive, which must be the case for a global maximizer of $\hat{\Pi}_i(p_i)$. $\square$

*Proof of Proposition 2.* I will proceed in two steps. First, I will show that whenever $Var(\tilde{\theta}) < \left(\frac{3}{6+16\frac{N-1}{\delta\rho L}}\right)^2$, at least one solution $\phi^* = Nk^*$ to equation (12) exists. Second, I will prove that whenever a solution $\phi^*$ exists and the demand threshold $k$ is set accordingly, $p^*(c_H)$ must be a global maximizer of $\Pi_i^C(p_i)$, conditional that $\Delta c := c_H - c_L$ is sufficiently small.

For the first part, note first that by continuity of $F$ and $f$, a solution $\phi^*$ to the equation $h(\phi) = (N-1)\left[1 - \delta\rho_L + \delta\rho_L F(\phi)\right] - \delta\rho_L \phi f(\phi) = 0$ must exist whenever there exists some $\hat{\phi}$ such that $h(\hat{\phi}) < 0$, as $h(0) = (N-1)(1-\delta\rho_L) > 0$. Next, observe that the left part of $h(\phi)$, $(N-1)\left[1 - \delta\rho_L + \delta\rho_L F(\phi)\right]$, is bounded above by $N-1$. Hence, it suffices to show that

$$\exists \phi : \tilde{h}(\phi) := N - 1 - \delta\rho_L \phi f(\phi) < 0.$$

Now, from Chebyshev's inequality, it is known that for $z \geq 1$, at least $1 - \frac{1}{z^2}$ of the probability mass of *any* random variable must not be more than $z$ standard deviations away from the mean. Hence, if the aggregate-market-demand random variable $\tilde{\theta}$ has a standard deviation of $\sigma$, at least $1 - \frac{1}{z^2}$ of its probability mass must fall in the range $[1 - z\sigma, 1 + z\sigma]$.

As this interval has a length of $2z\sigma$, the average probability density in this interval must at least be given by $\frac{1-\frac{1}{z^2}}{2z\sigma}$. At worst, the *maximum* density in this interval is then equal to the average probability density (if all values in the interval have the same density), and therefore it must hold that

$$\max_{\phi \in [1-z\sigma, 1+z\sigma]} \phi f(\phi) \geq (1 - z\sigma)\frac{1 - \frac{1}{z^2}}{2z\sigma}.$$

Inserting this minimal maximum of $\phi f(\phi)$ into the condition from above, a solution to equation (12) is guaranteed whenever

$$N - 1 - \delta\rho_L (1 - z\sigma)\frac{1 - \frac{1}{z^2}}{2z\sigma} < 0.$$

The bound on the variance in the proposition then simply follows by inserting the simple (but generally not tight) value of $z = 2$ and rearranging for $\sigma$, which is the square root of the variance. This proves the first part of the statement.

For the second part, note that if $\phi^* = Nk^*$ solves equation (12) ($p^*(c_H)$ is a local extremum of $\hat{\Pi}_i(p_i) = \frac{u(p_i)}{v(p_i)}$), a sufficient condition for $p^*(c_H)$ to be a global maximizer of $\hat{\Pi}_i(p_i)$ is that this function is strictly concave over the (connected) range of all of its potential maximizers. Due to Lemma 2, this range is characterized by values of $p_i$ such that $u(p_i) > 0$ and $u'(p_i) < 0$.[19]

Next, it is easy to calculate that

$$\hat{\Pi}_i''(p_i) = \frac{\left[u''(p_i)v(p_i) - u(p_i)v''(p_i)\right] v(p_i)^2 - 2\left[u'(p_i)v(p_i) - u(p_i)v'(p_i)\right] v(p_i)v'(p_i)}{v(p_i)^4},$$

which has the same sign as

$$\begin{aligned}
&\left[u''(p_i)v(p_i) - u(p_i)v''(p_i)\right] v(p_i) - 2\left[u'(p_i)v(p_i) - u(p_i)v'(p_i)\right] v'(p_i) = \\
&-2\alpha v(p_i)^2 - u(p_i)v''(p_i)v(p_i) - 2u'(p_i)v(p_i)v'(p_i) + 2u(p_i)\left(v'(p_i)\right)^2 .
\end{aligned}$$

For any set of parameters, over the range of potential maximizers of $\hat{\Pi}_i(p_i)$, this expression is smaller than

$$-2\alpha v(p_i)^2 - u(p_i)\left[v''(p_i)v(p_i) - 2\left(v'(p_i)\right)^2\right],$$

which should be negative in order to guarantee strict concavity of $\hat{\Pi}_i(p_i)$ in the relevant region.

Hence, rearranging the last equation from above, a sufficient condition for $p^*(c_H)$ to be a global optimizer of $\hat{\Pi}_i(p_i)$ is that

$$u(p_i)\left[\left(\frac{v'(p_i)}{v(p_i)}\right)^2 - \frac{v''(p_i)}{2v(p_i)}\right] < \alpha \tag{17}$$

over the range of potential maximizers $p_i$, given the model parameters.

Now, fix any $\overline{s_i} \in \left(\frac{1}{N}, 1\right)$ and note that whenever $c_L$ is sufficiently close to $c_H$ ($\Delta c < \overline{\Delta c}(\overline{s_i}) := \frac{\overline{s_i} - \frac{1}{N}}{2\alpha}$), no firm will ever want to price so low that it obtains a market share larger than $\overline{s_i}$.[20]

As $F(\theta)$ is twice continuously differentiable, it is easy to see that for any $\overline{s_i} < 1$,

$$\overline{v}(\overline{s_i}) := \max_{p_i \in \left[s_i^{-1}(\overline{s_i}), s_i^{-1}(0)\right]} \left[\left(\frac{v'(p_i)}{v(p_i)}\right)^2 - \frac{v''(p_i)}{2v(p_i)}\right]$$

[19] The fact that this range is connected trivially follows from strict concavity of $u(p_i)$.

[20] The inequality in brackets is obtained by solving $s_i(p^D; p^*(c_H)) < \overline{s_i}$, where $p^D := \frac{1}{\alpha N} + \frac{c_H + c_L}{2}$ is the solution to the strictly concave program $\max_{p_i}(p_i - c_L)s_i(p_i; p^*(c_H))$.

must be finite and independent of $c_L$. Moreover, $u(p_i)$ is bounded above by $\max_{p_i} u(p_i) = \frac{\Delta c}{N} + \frac{\alpha(\Delta c)^2}{4}$, which can be made arbitrarily small as $c_L$ approaches $c_H$. Hence, given a fixed $\overline{s_i}$, equation (17) must be satisfied for all relevant $p_i$ if the following two conditions are met:

$$\begin{aligned} &\Delta c < \overline{\Delta c}\,(\overline{s_i}) \quad \text{and} \\ &\left[\frac{\Delta c}{N} + \frac{\alpha(\Delta c)^2}{4}\right] \overline{v}(\overline{s_i}) \leq \alpha. \end{aligned}$$

In particular, this can always be achieved if $c_L$ is sufficiently close to $c_H$. $\square$