# The Benefits of Costly Voting

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**Abstract**

We present a costly voting model in which each voter has a private valuation for their preferred outcome of a vote. When there is a zero cost to voting, all voters vote and hence all values are counted equally regardless of how high they may be. By having a cost to voting, only those with high enough values would choose to incur this cost. Hence, the outcome will be determined by voters with higher valuations. We show that in such a case welfare may be enhanced. Such an effect occurs even if the cost is wasteful when there is both a large enough density of voters with low values and the expected value of voters is high enough. If the cost is recouped such as with a poll tax, having a cost is always beneficially.

**JEL codes**: C70, D72.

**Keywords**: costly voting, externalities.

## 1 Introduction

"The object of our deliberations is to promote the good purposes for which elections have been instituted, and to prevent their inconveniences." (Edmund Burke as cited in Lakeman and Lambert, 1959, p. 19)

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Groups within society often have to make collective decisions. In order to reach correct social decisions, the valuations of all those affected by the decision should be aggregated. By leaving some out, a group may reach an incorrect decision. For example, take a committee that must decide an issue at a meeting. Each member has a certain private value to the results of the decision reached by the committee. The committee's social value of the decision is the sum of the individual private values and, hence, aggregation is necessary to reach the correct decision. This scenario fits many decision problems such as public good provision.

A common method to reach a decision is to have a vote.[1] Since each member of the committee has information that is relevant to the decision, we would normally think that ensuring all participate in voting would improve the final outcome. In fact, many countries (including Argentina, Australia, Belgium, and Greece) have compulsory voting to ensure inclusion.[2] There is, however, significant difference between aggregating private values and ensuring full participation of all voters.

In social valuation, the strength of preference counts. In voting, the options for expressing preference for any particular alternative are limited to either voting for it, or not voting for it (that is, vote for an alternative or abstain).[3] This means with voting it is not possible to demonstrate intensity of preferences.[4] One voter mildly in favor of an alternative exactly offsets another voter who is strongly opposed.

These observations suggest that there may be gains, in avoiding poor outcomes, by ensuring that voters who have only mild feelings about the alternatives are excluded. In some sense, if their vote counts for more than their strength of feeling then they may change the voting outcome in a detrimental way. Indeed, in Australia where voting is mandatory, "donkey votes" (those that simply were cast by order of a ballot) give a 1% edge to those listed first (see Orr, 2002, and King and Leigh, 2009). One way to exclude such voters is to ensure that there is a cost to voting that deters participation by those without strong preferences (or are not well informed) and hence achieve a socially-better outcome. This intuition is contrary to the widely held view that costly

[1] See Drexl and Klein (2013) and Gershkov et al. (2013).

[2] Enforcement ranges from fines (Australia) to disenfranchisement (Belgium) or making it difficult to obtain a passport or driver's license (see The Guardian, July 4, 2005).

[3] Among the rare exceptions are reality TV shows such as Pop Idol where individuals can vote more than once (and pay for each vote).

[4] As mentioned by Mueller (2003, page 104): "Majority rule records only these ordinal preferences for each individual on the issue pair. The condition for the Pareto optimality of the supply of the public goods requires information on the relative intensity of individual preferences."

voting is detrimental since it deters voting (and is a cost to those that do vote) leading to a paradox of why people vote (see Dhillon and Peralta, 2002, for an overview).

In this paper, we show that increasing costs even though they are wasteful may be beneficial using a model with a continuous distribution of values both when there is a fixed number of supporters for each outcome and when there is aggregate supporter uncertainty. We find that whether costly voting is superior depends upon both the expected value and the density of lower value voters. We also show that under aggregate supporter uncertainty, a government would never want to have mandatory voting by imposing fines or subsidizing voting but would wish to implement a poll tax (a charge for voting) if it is politically practical.

This analysis can be seen as the normative counterpart to the positive analyses of Bulkley et al. (2001) and Osborne et al. (2000). These papers establish that when voting is costly the outcome of the voting game will have an equilibrium in which only voters with high values (from the extremes) will participate. Again, at first sight it might appear that this is a bad outcome since it excludes moderate opinion. What we show is that instead it can be efficient to have precisely such an equilibrium.

Börgers (2000) asks a question in the spirit of our analysis. Namely, whether a reduction of the costs of voting can be damaging. While he graphically, shows such a possibility, in his model, enough of a reduction would always be beneficial, since while there is uncertainty for which alternative a voter prefers, there is no difference in intensity of preference for a particular alternative.[5] More recently, Börgers (2004) further analyzes this type of model to show that an equilibrium with costly voting is superior to both mandatory voting (still with voting costs) and random selection of a winner (with no voting). Thus, mandatory voting has the benefit of including all the available information (everyone votes) and selecting the best alternative, but at the highest cost. Random selection has the lowest cost but uses no information. The tradeoff between mandatory voting and random selection is voluntary voting. This causes only those with a low cost of voting to vote and is superior to the other options. Krasa and Polborn (2009) vary the Börgers model by allowing for

[5] It is possible to see how Börgers (2000) works with a simple numerical example: There are two voters, $V1$ and $V2$, and two candidates, $A$ and $B$. Each voter has a 50% chance of preferring each candidate and values their candidate winning at 1 (and the other at 0). If the cost of $V1$ voting is $0.5-\epsilon$ and $V2$ voting is $0.5-2\epsilon$, then both will vote and the total surplus will be $1.5-1+3\epsilon$. If costs increase such that the cost of $V1$ voting is $0.5+\epsilon$ and $V2$ voting is $0.5-\epsilon$, then only $V2$ would vote yielding a surplus of $1.5-0.5+\epsilon$. Of course, if voting costs drop to zero, both will vote and surplus will be 1.5.

ex-ante asymmetry of preferences over alternatives. They find that for a large enough number of voters, it is optimal to move towards mandatory voting from voluntary voting (by a penalty for not voting or a subsidy to voting).

In our paper, we add intensity of preferences over alternatives. Now in *contrast* to the Börgers model (and Krasa and Polborn), everyone voting no longer includes all information (it neglects intensity of preference) and thus sometimes does not select the best alternative. We find that even if we eliminate all costs to mandatory voting by setting the cost of voting to zero, it may not necessarily be superior. Note that unlike Ledyard (1981, 1984), we treat the alternatives as fixed (as does Börgers) and thus find that the equilibrium is inefficient.

To help understanding how we differ from the existing literature, let us return to our example of a committee voting on an issue. The committee can (1) make it mandatory to show up to the meeting (compulsory), (2) buy cookies for the meeting (inducement), (3) allow for electronic voting (zero cost), (4) schedule the meeting late at night (create cost), or (5) charge a fee to show up to the meeting (perhaps by means of additional work at the meeting). Börgers (2004) shows that (1) or (2) would not be worthwhile and Börgers (2000) shows (4) may be beneficially. We add to this by confirming those results in our model and in showing that surprisingly (3) may not be beneficial and that (5) will always be beneficial.[6] We also show (4) may be beneficial even if there is an equal number of supporters for each alternative.

Our paper also relates to the public good provision literature. Palfrey and Ledyard (1994, 1999, 2002) look at mechanisms including simple voting schemes for providing public goods. In Palfrey and Ledyard, the voting options are to provide or not provide a public good where provision could have a loss for those that have little value for it and have to pay for it. In this paper we have two options each with non-negative value and voting can potentially have a cost.

While less related, the Condorcet Jury literature models voting by a group of individuals with a common value over two alternatives (see Young, 1988). Krishna and Morgan (2012) show that as the cost of voting goes to zero, voluntary voting is the optimal mechanism. Ghoshal and Lockwood (2009) have combined the common value in the Condorcet Jury literature with the private value of alternatives and comparisons in Börgers (2004). They find that if the voters put a high weight

[6] Under the Börgers (2000, 2004) framework, there is no intensity to preference, so it is best to not have voting costs and thus (3) would always be beneficial. It may be that (4) is beneficial in Börgers (2000) since increasing voting costs may reduce the overall costs of voting since the numbers that vote would decrease.

on personal preferences then there is an inefficiently high voter turnout and in the case voters care more about the common aspect then there is an inefficiently low voter turnout. Information acquisition has been studied in the Condorcet Jury literature by allowing voters to buy information about the common feature of the alternatives (see Persico, 2004, Gerardi and Yariv, 2005).

The lobbying literature models a similar problem (see Austen-Smith and Wright, 1992, Baye, Kovenock and de Vries, 1993, Che and Gale, 1998, 2006, Kaplan and Wettstein, 2006). The method of reaching a group decision is by allowing would-be voters to send a signal of how much they care: by lobbying. With this method, we would expect that voters with strong preferences or special interest groups to have greater influence on the outcome than with voting. This is due to the ability of voters with more extreme preferences to send a stronger signal. Such undue influence is not necessarily harmful; lobbying may be welfare enhancing over voting since under voting the outcome can be determined by a large number of voters that do not strongly care about the outcome or vote without any information about the specific issues. In Chakravarty and Kaplan (2010), we determine under which conditions, a purely wasteful signal (which we call shouting) will lead to a more efficient solution than voting. In Chakravarty and Kaplan (2013), for purely private goods, we find the optimal allocation mechanism when only wasteful signals can be used and determine under which conditions making use of these signals is useful.

In the next section, we provide an illustrative example with two types and two voters which gives the basic intuition of our model. In section, 3, we examine our model when there is an equal number of voters supporting each option and the uncertainty in the model is in the strength of each voter's support for the option. Then in section 4, we analyze the case where, in addition to the uncertainty of the strength, there is uncertainty as to which alternative a voter supports. We conclude in section 5.

## 2 Illustrative Example

Here we provide basic example that captures the main intuition of our paper. There are two options, $A$ and $B$, and two voters: one prefers $A$ and the other prefers $B$. There is an 1/2 chance that a voter has utility of $v_\ell \equiv 1$ for his preferred option (and 0 for the other) and 1/2 chance of utility of $v_h \equiv 5$ for his preferred option (and 0 for other). If the cost of voting is 0, everyone votes and there is a tie that is broken randomly. Hence, the social surplus is the expected value of a preferred choice: $\frac{v_h+v_\ell}{2} = 3$.

Now say the cost of voting is $c \equiv \frac{2}{3}$. Suppose there is an equilibrium where only those with a high value will vote. In this case, if there is indeed a high value, then we are guaranteed that someone will value the winner $v_h = 5$. Otherwise (when both voters have low values), the winner will be worth $v_\ell = 1$. There is at least one high value 3/4 of the time. Thus, the surplus is $\frac{3}{4} \cdot 5 + \frac{1}{4} \cdot 1 = 4$. The total cost expended in voting is $c$ since each voter votes half the time and there are two voters. The social surplus is then $4 - c = 3\frac{1}{3}$. Thus, as long as this is indeed an equilibrium, then costly voting can be socially beneficial. To verify that this is an equilibrium, we must show that both voters find it in their individual interests to vote only with a high value. A voter by voting improves the chance of his preferred option winning by $\frac{1}{2}$ no matter what the other voter chooses to do (since by voting one's option either goes from a tie to a win or from losing to a tie). Hence, if one has a private value to an option of $v$, one votes if $\frac{v}{2} > c$. Thus, for this to be an equilibrium we must have $\frac{5}{2} \geq c \geq \frac{1}{2}$ (only the high-value voter should vote), which is satisfied by $c = 2/3$.

In this example, we compare two situations, one with costly voting and one with free voting, and find that, in fact, costly voting is superior. Hence, adding a cost to voting can be welfare enhancing. This captures the intuition behind this phenomena, namely, with costly voting, only those with high enough values would choose to incur the cost to voting, while with free voting, all values are counted equally regardless of how valuable they may be.

Notice that there are two important components of this model that enable costly voting to be superior to free voting. First, the difference between the high value and low value must be large enough. If instead of 5, the value were 3, then costly voting would be inferior. (Social surplus with costly voting would be $\frac{3}{4} \cdot 3 + \frac{1}{4} \cdot 1 - \frac{2}{3} = \frac{11}{6}$ and with free voting the social surplus would be 2.) Second, the probability of having a low value should be sufficiently large. For instance, if instead of 1/2 this was 1/4, then again costly voting would be inferior.

We see this further in Table 1 which provides a breakdown of the surplus in each of the four states of nature (the four possible outcomes of the two voters' values). We see under free voting the social surplus is the average of the two individual values since both voters vote and there is no cost of voting. Comparing this to the case where the cost of voting is 2/3, we see that there is no

| | **No cost** | | **Cost of $\frac{2}{3}$** | |
|---|---|---|---|---|
| | $v_\ell$ | $v_h$ | $v_\ell$ | $v_h$ |
| $v_\ell$ | $\mathbf{1}, \mathbf{1}$<br>$1$ | $\mathbf{1}, \mathbf{5}$<br>$3$ | $1, 1$<br>$1$ | $1, \mathbf{5}$<br>$5-\frac{2}{3}$ |
| $v_h$ | $\mathbf{5}, \mathbf{1}$<br>$3$ | $\mathbf{5}, \mathbf{5}$<br>$5$ | $\mathbf{5}, 1$<br>$5-\frac{2}{3}$ | $\mathbf{5}, \mathbf{5}$<br>$5-\frac{4}{3}$ |

Table 1: Social surplus of voting. Value of winning for each voter with those that vote in bold. Underneath is the social surplus including voting costs

difference when both voters have a low value $v_\ell$. When one voter has a low value and one voter has a high value, there is a net gain. The expected value of the winner goes up from 3 to 5 with a cost of voting of 2/3. However, when both voters have a high value, there is a net loss. The expected value of the winner is the same under both regimes, but both voters incur the cost of 2/3. The gains in the two states are higher the higher the difference between both values. The odds of both having high values relative to just one having a high value is reduced the higher the chance of a low type.

Let us examine the model with a general $v_\ell$ and $v_h$, and with $p$ denoting the probability of having a low value. By analysis similar to that in Table 1, the gains from voting is by having a voter with a high value instead of a low value in the upper right and lower left corners of the table. This gain is realized half the time since with free voting the voter with the high type will win the coin flip half the time. Hence, the difference is $(v_h - v_\ell)/2$. We are in these two boxes with chance $2p(1-p)$. In expectation, the gain is then $(v_h - v_\ell)p(1-p)$. Since each voter will only vote with a high type, the cost of voting is $2(1-p)c$. Dividing both the gain and cost by $(1-p)$, we see there is a net gain to voting if $(v_h - v_\ell)p \geq 2c$. This would be an equilibrium if $v_h/2 \geq c \geq v_\ell/2$. Again, we see that for there to exist a benefit of costly voting, relative to the voting cost, the difference between the high value and low value must be large enough as well as the probability of a low type. However, for the benefit it must also be an equilibrium for only a voter with a high value to vote. This requires that the cost is such that twice of it is between the high and low values. In the next two sections, we will see that insight gained from this example carries over to more general models.

# 3 Aggregate Supporter Certainty

## 3.1 Model

Here we model a committee making a binary decision such as which of two districts, $A$ or $B$, to build a casino. There are an equal number of representatives from each district, and each representative has a private value for it being built in his district and a common cost for showing up to vote.

Formally, there are two types of voters and $n$ voters of each type (overall there are $2n$ voters). Each voter has cost $c \geq 0$. Assume that each voter $i$ has value $v_i \geq 0$ that is randomly drawn according to the non-atomic cumulative distribution $F$ which has support $[0, \overline{v}]$, where $\overline{v} > 2c$. If $1 \leq i \leq n$, voter $i$ is a type $A$ voter who values a win by $A$ at $v_i$ and a win by $B$ at 0. If $n+1 \leq i \leq 2n$, voter $i$ is a type $B$ voter who values a win by $B$ at $v_i$ and a win by $A$ at 0. Choice $A$ wins if the number of votes it receives, denoted by $\#_A$, is strictly greater the number of votes choice $B$ receives, denoted by $\#_B$. Choice $B$ wins if $\#_B > \#_A$. If there is a tie, $\#_B = \#_A$, then the winner is determined randomly with equal probability of each winning. Note that the voter's preferences for platform $A$ or $B$ is modelled in the style of Palfrey and Rosenthal (1983, 1985).

## 3.2 Equilibrium and social surplus

Denote $v^*(c)$ as a cutoff strategy such that a voter $i$ votes if his value is above $v^*(c)$ and doesn't vote if his value is below $v^*(c)$. We denote $\Pr_{-i}(event|v^*)$ as the probability that *event* occurs given that all voters except voter $i$ follows cutoff strategy $v^*(c)$ and voter $i$ does not vote.

A voter $i$ where $1 \leq i \leq n$ (a type $A$) and with value $v_i$ will vote if

$$v_i \left[ \frac{1}{2} \Pr_{-i}(\#_A = \#_B | v^*) + \frac{1}{2} \Pr_{-i}(\#_A = \#_B - 1 | v^*) \right] > c.$$

The expression $\Pr_{-i}(\#_A = \#_B|v^*)$ represents the case when all other votes are tied. Hence, voter $i$ is pivotal since by $i$ voting, the outcome will change the vote from a tie to a win by voting. The expression $\Pr_{-i}(\#_A = \#_B - 1|v^*)$ represents the case when voter $i$ will change the outcome of a vote from losing to a tie. The gains in either instance is half the value, $\frac{v_i}{2}$.

**Lemma 1** *A cutoff $v^*(c)$ forms a Bayes-Nash equilibrium if*

$$v^* \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i+1} F\left(v^*\right)^{2(n-i-1)} \left(1 - F\left(v^*\right)\right)^{2i} \left[1 + \frac{2i-n+1}{n-i} \cdot F(v^*)\right] = 2c. \tag{1}$$

**Proof.** The cutoff will be such that the value for voting equals the cost.

$$v^*(c) \cdot \left[\frac{1}{2} \Pr_{-i}(\#_A = \#_B | v^*) + \frac{1}{2} \Pr_{-i}(\#_A = \#_B - 1 | v^*)\right] = c.$$

When the voter $i$ prefers $A$ ($i \leq n$), we can rewrite the probabilities in this equation as follows:

$$\begin{aligned}
&\Pr_{-i}(\#_A = \#_B | v^*) \\
&\quad = \Pr_{-i}(\#_A = \#_B = 0 | v^*) + \Pr_{-i}(\#_A = \#_B = 1 | v^*) + ... + \Pr_{-i}(\#_A = \#_B = n-1 | v^*) \\
&\quad = \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i} \left(1 - F\left(v^*\right)\right)^{2i} F\left(v^*\right)^{2(n-i)-1}
\end{aligned}$$

and

$$\begin{aligned}
&\Pr_{-i}(\#_A = \#_B - 1 | v^*) \\
&\quad = \Pr_{-i}(\#_A = 0, \#_B = 1 | v^*) + \Pr_{-i}(\#_A = 1, \#_B = 2 | v^*) + ... + \Pr_{-i}(\#_A = n-1, \#_B = n | v^*) \\
&\quad = \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i+1} \left(1 - F\left(v^*\right)\right)^{2i+1} F\left(v^*\right)^{2(n-i-1)}.
\end{aligned}$$

Substituting these expressions into the cutoff value equation yields:

$$v^* \sum_{i=0}^{n-1} \left[\binom{n-1}{i} \binom{n}{i} \left(1 - F\left(v^*\right)\right)^{2i} F\left(v^*\right)^{2(n-i)-1} + \binom{n-1}{i} \binom{n}{i+1} \left(1 - F\left(v^*\right)\right)^{2i+1} F\left(v^*\right)^{2(n-i-1)}\right] = 2c. \tag{2}$$

Since the equilibrium entails the probability of being pivotal equalling twice costs over benefits, the above equation resembles the equilibrium condition in Palfrey and Rosenthal (1983, 1985). This equation can then be simplified as

$$v^* \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i+1} F\left(v^*\right)^{2(n-i-1)} \left(1 - F\left(v^*\right)\right)^{2i} \left[1 + \frac{2i-n+1}{n-i} \cdot F(v^*)\right] = 2c$$

The same equation holds for voters preferring $B$ ($i > n$). ■

**Lemma 2** *When there is a zero cost of voting, everyone votes,* $v^*(0) = 0$.

**Proof.** This follows directly from equation (1). ■

**Lemma 3** *If* $n = 1$ *or* $\lim_{v \to 0} vF'(v) = 0$, *then* $\lim_{c \to 0} v_c^*(c) = 2$.

**Proof.** Equation (1) must hold for all $c$, so we can take the derivative w.r.t. $c$ and take the limit as $c \to 0$. As $c \to 0$, we have $v \to 0$ (from Lemma 2), so $\lim_{v \to 0} (1 - F) = 1$, $\lim_{v \to 0} F = 0$. Notice that from this we need only worry about the term when $i = n - 1$. (For $n > 1$, the rest vanish.) This yields:

$$\left[1 - (n-1) \lim_{v* \to 0} v^* \cdot F'(v^*)\right] = \frac{2}{v_c^*(0)}.$$

Hence, $v_c^*(0) = 2$ when $n = 1$ or $\lim_{v \to 0} vF'(v) = 0$. ■

**Lemma 4** *The social surplus to voting is the expected value of the winner minus the costs of voting:*

$$\begin{aligned} SSV(c) &= \sum_{a=0}^{n} \sum_{b=0}^{n} \binom{n}{a} \binom{n}{b} F\left(v^*(c)\right)^{2n-a-b} \left(1 - F\left(v^*(c)\right)\right)^{a+b} \left[ \begin{array}{c} (n - \max\{a,b\}) E[v | v < v^*(c)] + \\ \max\{a,b\} E[v | v > v^*(c)] \end{array} \right] \\ &\quad -2(1 - F\left(v^*(c)\right)) n \cdot c. \end{aligned} \quad (3)$$

The expected value of the winner is computed by going through the possible number of voters for each candidate where $a$ is the votes for candidate $A$ and $b$ is the votes for candidate $B$. The probability of each case is calculated and multiplied by the expected value of the winner, which is calculated by the expected value of those that voted for the winner plus the expected value of those that wanted the winner to win but nonetheless didn't vote for him. The expression $2n(1 - F(v^*(c)))c$ is the expected cost of the voters voting since $1 - F(v^*(c))$ is the probability of each voter voting and there are $n$ voters of each type.

### 3.3 When is it optimal to have a voting cost?

In the following proposition, we derive the conditions when it is optimal to have a cost.

**Proposition 1** *If* $\lim_{v \to 0} F'(v)v = 0$, $\lim_{v \to 0} F'(v)F(v) = 0$ *and* $E[v] \cdot F'(0) > 1$, *then it is optimal to have* $c > 0$.

**Proof.** To show that it is optimal to have $c > 0$, it is sufficient to show that $\lim_{c\to 0} SSV'(c) > 0$. We will prove this by showing that the derivative of the expected value of the voters that prefer the winning candidate is higher than the derivative of the expected costs.

Given $\lim_{v\to 0} F'(v)v = 0$, $\lim_{v\to 0} F'(v)F(v) = 0$ and $E[v] \cdot F'(0) > 1$, the derivative of $SSV$ as $c$ goes to 0 can be determined as follows. Using the product rule, the derivative is equal to the sum of the values times the derivative of the probabilities plus the sum of the probabilities times the derivative of the values. The probability of $a$ voters voting for $A$ and $b$ voters voting for $B$ is $\binom{n}{a}\binom{n}{b} F\left(v^*(c)\right)^{2n-a-b}\left(1 - F\left(v^*(c)\right)\right)^{a+b}$. The limit of this term as $c \to 0$ is zero if $a + b < 2n$ and 1 if $a + b = 2n$. If $a + b = 0$, then the derivative of this probability term is $2nF^{2n-1}F'v_c$ which goes to 0 since $\lim_{v\to 0} F'(v)F(v) = 0$ and $\lim_{c\to 0} v^*_c = 2$ by Lemma 3. If $a + b = 2n$, the derivative is $-2n\left(1 - F\right)^{2n-1} F'v_c$ which goes to $-2nF'v_c$. Otherwise, the derivative of this term is $\binom{n}{a}\binom{n}{b}\left[(2n - a - b) F\left(v^*(c)\right)^{2n-a-b-1}\left(1 - F\left(v^*(c)\right)\right)^{a+b} - (a + b)F\left(v^*(c)\right)^{2n-a-b}\left(1 - F\left(v^*(c)\right)\right)^{a+b-1}\right] F'v_c$ which goes to zero unless $a+b = 2n-1$ in which case it goes to $\binom{n}{a}\binom{n}{b}F'v_c$. Note that $a+b = 2n-1$ when either $a = n$ and $b = n - 1$ or vice-versa. In each case, $\binom{n}{a}\binom{n}{b} = n$.

Let us now look at the sum of the values times the derivative of the probabilities. When $a+b = n$, we have $(n - \max\{a, b\})E[v_i|v_i < v^*(c)] + \max\{a, b\}E[v_i|v_i > v^*(c)] = nE[v_i|v_i > v^*(c)]$. When $a + b = 2n - 1$, we also have $(n - \max\{a, b\})E[v_i|v_i < v^*(c)] + \max\{a, b\}E[v_i|v_i > v^*(c)] = nE[v_i|v_i > v^*(c)]$. Thus, when multiplied by the derivative of the probabilities and summed over the possible values for $a$ and $b$, the limit goes to 0.

We are left with the sum of the probabilities times the derivative of the values. As we saw above, as $c$ goes to 0 the only time the probabilities are non-zero is when $a = b = n$. Thus,

$$\lim_{c\to 0} SSV'(c) = \frac{ndE[v_i|v_i > v^*(c)]}{dc} - 2n + \lim_{c\to 0} 4n \cdot c \cdot F'(v^*(c)).$$

Since $E[v|v > v^*(c)] = \frac{\int_{v^*(c)}^{\infty} vdF(v)}{1-F(v^*(c))}$, we have

$$\frac{dE[v|v > v^*(c)]}{dc} = \frac{-v^*_c(c)v^*(c)F'(v^*(c))}{(1 - F(v^*(c))} + \frac{F'(v^*(c))v^*_c(c)\int_{v^*(c)}^{\infty} vdF(v)}{(1 - F(v^*(c))^2}.$$

Hence, $\lim_{c\to 0} \frac{dE[v|v>v^*(c)]}{dc} = \lim_{c\to 0} 2F'(v^*(c))\int_0^{\infty} vdF(v) = \lim_{c\to 0} 2F'(v^*(c))E[v]$. Thus, if $F'(0)E[v] > 1$, $SSV'(0) > 0$ and it is optimal to increase costs.

Note that even though there is a unique equilibrium at $c = 0$, when $c > 0$, there is a possibility of multiple equilibria (see Palfrey and Rosenthal, 1983, 1985). However, even if this is the case, under these conditions we have shown that all of the equilibria must yield higher social surplus than the equilibrium when $c = 0$. ■

Of the three conditions of Proposition 1, the first two, $\lim_{v\to 0} F'(v)v = 0$, $\lim_{v\to 0} F'(v)F(v) = 0$, are fairly innocuous. Anytime $F'(0)$ is finite they are satisfied. The last condition, $E[v] \cdot F'(0) > 1$, is the one of interest and has two components that depend upon the distribution of $v$: the density at zero and the expected value. The combination of these two components must be large enough. Too low a value of the density at zero would mean that increasing cost does not eliminate enough low value votes. Too low an expected value would mean that the benefit to eliminating these low-value voters is not large enough. This condition is equivalent to $\lim_{c\to 0} \frac{dE[v|v>v^*(c)]}{dc} > 2$. This implies that the expected value of those voting is increasing in cost by a sufficient amount, namely 2. In other words, if one increases cost marginally by a dollar (at zero), then the expected value of those voting should go up by 2 in order for costly voting to be beneficial. This condition is also equivalent to $\lim_{v^*\to 0} \frac{dE[v|v>v^*]}{dv^*} > 1$. This states that the mean-residual-lifetime function (MRL) of $F$ is strictly increasing at zero. It is satisfied by all strict log-convexity distributions (see Proposition 2 of Heckman and Honore, 1990). Similar conditions (such as having a monotone MRL) are used in a variety of economic applications (see Bagnoli and Bergstrom, 2005). These include McAfee and Miller (2012) who show that an increasing MRL at zero implies allocating appointments (or objects) by reservations is inefficient for low transportation costs.

The following example illustrates Proposition 1.

**Example 1** $F(v) = v^\alpha$ *where* $\alpha > 0$, $c < 1/2$, $n = 1$.

We have $\lim_{v\to 0} vF'(v) = \lim_{v\to 0} \alpha v^\alpha = 0$. From (1), $v^*(c) = 2c$. We can then write equation (3) as

$$\begin{aligned}(2c)^{2\alpha} E[v|v &< 2c] + (1-(2c)^{2\alpha})E[v|v>2c] - 2(1-(2c)^\alpha)c \\ &= \frac{\alpha - 2c(1+\alpha) + (2c)^\alpha(\alpha+2c)}{1+\alpha}.\end{aligned}$$

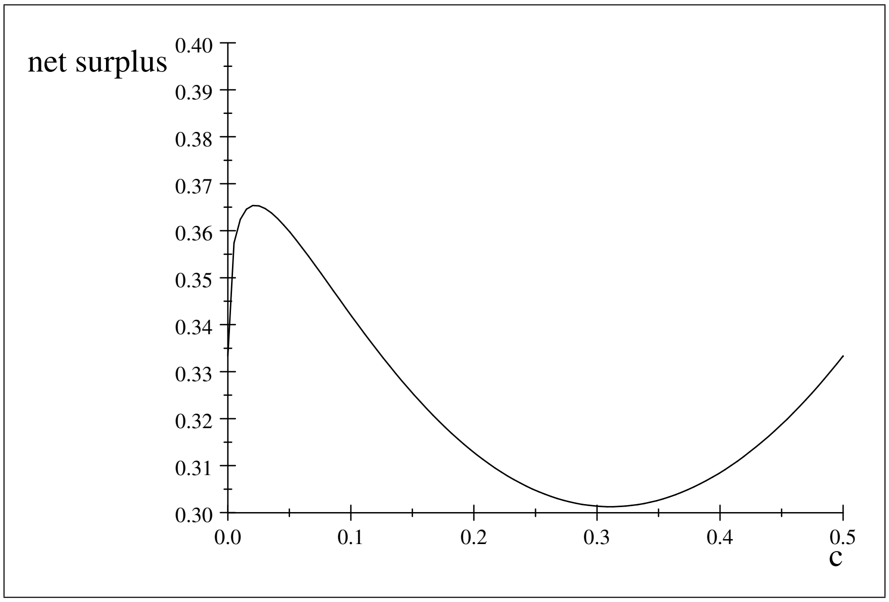


**Figure 2.** The social surplus net of voting costs versus the cost of voting $c$ when $F(v) = v^{0.5}$ and $n = 1$.

If $\alpha = 0.5$, the net surplus is plotted in Figure 2. As we see here, the ideal $c$ is strictly positive. It reaches a maximum at $c \approx 0.0223$. We can also examine the probability density function of $v$. This density is $0.5v^{-.5}$ and shown in Figure 3.

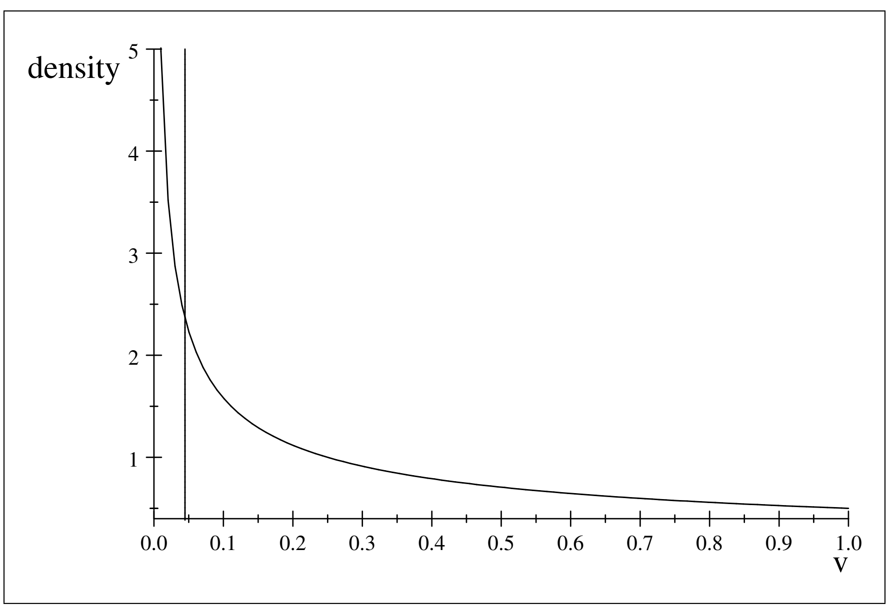


**Figure 3.** The graph of the density function of $f(v) = 0.5v^{-0.5}$ and a vertical line at $v = 2c$ at $c \approx 0.0223$ (the optimal $c$).

In Figure 3, the voters to the left of the vertical line do not vote when $c$ is at the optimal level. We now ask: in our example, for which $\alpha$ is there a gain in surplus to increasing the cost of voting? For $n = 1$, the slope of the surplus w.r.t. $c$ is

$$SSV'(c) = -2 + \frac{2^\alpha c^{\alpha-1}(\alpha^2 + 2(1+\alpha)c)}{1+\alpha}.$$

For $\alpha > 1$, $\lim_{c\to 0} SSV'(c) = -2$. For $\alpha = 1$, $\lim_{c\to 0} SSV'(c) = -1$. For $0 < \alpha < 1$, $\lim_{c\to 0} SSV'(c) = \infty$. Hence, when $0 < \alpha < 1$, the surplus improves by increasing the cost. It also turns out that for $\alpha \geq 1$, the surplus is at the highest when cost is zero. (When $\alpha \geq 1$, $SSV'(c)$ is strictly increasing in $c$ for all $c > 0$, hence $SSV'(c)$ can equal zero only once. Since $SSV(0) = SSV(1/2)$, no one votes in either case, and $SSV'(0) < 0$, that point at which $SSV'(c) = 0$ must be a minimum.)

We also see that the conditions of Proposition 1 are satisfied in Example 1 for $1/2 \leq \alpha < 1$. We have $E[v] = \frac{\alpha}{\alpha+1}$ and for $0 < \alpha < 1$, $F'(0) = \lim_{\alpha\to 0} \alpha v^{\alpha-1} = \infty$. Hence, $F'(0)E[v] > 1$. Also, $\lim_{v\to 0} F'(v)v = \alpha v^\alpha = 0$. If $\alpha \geq 1/2$, we further have $\lim_{v\to 0} F'(v)F(v) = \alpha v^{2\alpha-1} = 0$. We also demonstrate with this example that Proposition 1's conditions are sufficient but not necessary.

# 4 Aggregate supporter uncertainty.

## 4.1 Model and Initial Results

In the previous section, we assumed that there were an equal number of supporters for either A or B. Here we assume there are $n$ voters and each voter has an equal and independent chance of desiring each outcome. This leads to aggregate supporter uncertainty (ASU). Again, each voter has a level of support for his desired outcome drawn according to the non-atomic cumulative distribution function $F$ with support $[0, \overline{v}]$, where $\overline{v} > 2c$. The voter preferences over platforms is similar in style to that in Börgers (2000, 2004), we will discuss the differences later.

In the following lemma, we show there is a unique equilibrium and determine its cutoff condition.

**Lemma 5** *There is a unique Bayes-Nash equilibrium with cutoff $v^*(c)$ that satisfies*

$$v^* \sum_{a=0}^{n-1} \binom{n-1}{a} F(v^*)^{n-1-a} (1 - F(v^*))^a \begin{cases} \binom{a}{a/2} \left(\frac{1}{2}\right)^a & \textit{if } a \textit{ is even,} \\ \binom{a}{(a-1)/2} \left(\frac{1}{2}\right)^a & \textit{if } a \textit{ is odd.} \end{cases} = 2c. \quad (4)$$

**Proof.** There are $n$ voters overall. Take the decision of an individual voter. Consider each case where exactly $a$ other voters vote. This occurs with probability $\binom{n-1}{a} F(v^*)^{n-1-a}(1-F(v^*))^a$. If $a$ is even, this voter is pivotal only if there is a tie. This happens if exactly $a/2$ vote for each outcome, which occurs with probability $\binom{a}{a/2}\left(\frac{1}{2}\right)^a$. If $a$ is odd then the voter is pivotal if there is exactly one less voter that votes for his preferred outcome. This has $(a-1)/2$ voting for his outcome and $(a+1)/2$ voting for the other outcome. This occurs with probability $\binom{a}{(a-1)/2}\left(\frac{1}{2}\right)^a$. Using the above forms equation (4). The LHS of (4) is 0 when $v^* = 0$. The LHS equals $\overline{v}$ when $v^* = \overline{v}$ which is larger than 2c. Hence, since the LHS is continuous, there exists an interior solution to the equation. Finally, we want to show uniqueness. The probability of being pivotal given that $a$ other voters vote is decreasing in $a$. As $v^*$ increases the distribution of the number of other voters stochastically shifts downwards. Hence, as $v^*$ increases, the overall probability of being pivotal increases. Thus, the LHS of (4) is strictly increasing in $v^*$ and we have a unique solution. ■

**Lemma 6** *When there is a zero cost of voting, everyone votes,* $v^*(0) = 0$.

**Proof.** This follows directly from equation (4). ■

**Lemma 7** *If* $\lim_{v\to 0} F'(v)v = 0$, *then*

$$\lim_{c\to 0} v_c(c) = \begin{cases} \frac{2^n}{\binom{n-1}{n/2-1}} & \text{if } n \text{ is even,} \\ \frac{2^n}{\binom{n-1}{(n-1)/2}} & \text{if } n \text{ is odd.} \end{cases}$$

**Proof.** We can then take the total derivative w.r.t. $c$ of the cutoff equation (4).

If $\lim_{v\to 0} F'(v)v = 0$, then the only term remaining on the LHS is when $a = n-1$. Thus,

$$\begin{aligned} \lim_{c\to 0} v_c^*(c) &= 2/\begin{cases} \binom{n-1}{(n-1)/2}\left(\frac{1}{2}\right)^{n-1} & \text{if } n-1 \text{ is even,} \\ \binom{n-1}{(n-2)/2}\left(\frac{1}{2}\right)^{n-1} & \text{if } n-1 \text{ is odd.} \end{cases} \\ &= 2^n/\begin{cases} \binom{n-1}{(n-1)/2} & \text{if } n \text{ is odd,} \\ \binom{n-1}{n/2-1} & \text{if } n \text{ is even.} \end{cases} . \end{aligned}$$

■

**Proposition 2** *Under ASU, if* $n$ *is even,* $\lim_{v\to 0} F'(v)v = 0$, $\lim_{v\to 0} F'(v)F(v) = 0$, *and* $E[v] \cdot F'(0) > \frac{1}{2}$, *then it is optimal to have* $c > 0$.

**Proof.** The social surplus of the equilibrium (above random allocation) is the expected benefits minus the costs of voting. The expected benefits depends only upon the number of voters that vote (in expectation all those that don't vote balance each other out). If two vote for an option and two vote against it, the surplus is zero. If three vote for an option and two vote against it, the surplus is $E[V_i|V_i > v^*(c)]$. In general, the social surplus is the number that voter for the winning option minus the number that vote for the losing option times $E[V_i|V_i > v^*(c)]$. Given there is an equal chance that a voter that chooses to vote votes for either option, the number of votes for option A minus those for option B follows a one-dimensional random walk as voters vote. The social surplus given that a certain number of voters vote is the expected absolute value of this number times $E[V_i|V_i > v^*(c)]$. If $a$ voters vote, then this expectation is $\begin{cases} \frac{(a-1)!!}{(a-2)!!} & \text{if } a \text{ is even,} \\ \frac{a!!}{(a-1)!!} & \text{if } a \text{ is odd.} \end{cases}$ (see Weisstein, 2010). Note that the double factorial, $n!!$, is either all strictly positive even numbers up to $n$ multiplied together or all strictly positive odd numbers up to $n$ multiplied together depending upon whether $n$ is even or odd.

$$\begin{aligned} SSV(c) &= \sum_{a=0}^{n} \binom{n}{a} F\left(v^*(c)\right)^{n-a} \left(1 - F\left(v^*(c)\right)\right)^a \left[ E[v|v > v^*(c)] \cdot \begin{cases} \frac{(a-1)!!}{(a-2)!!} & \text{if } a \text{ is even,} \\ \frac{a!!}{(a-1)!!} & \text{if } a \text{ is odd.} \end{cases} \right] \\ &\quad -(1 - F\left(v^*(c)\right))n \cdot c. \end{aligned}$$

Taking the limit as $c \to 0$ of the derivative yields:

$$\begin{aligned} \lim_{c \to 0} SSV'(c) &= \lim_{c \to 0} v_c \left( \begin{array}{c} \left( \frac{dE[v|v>v^*(c)]}{dv^*} - nE[v|v > v^*(c)]F'(v^*(c)) \right) \cdot \begin{cases} \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even,} \\ \frac{n!!}{(n-1)!!} & \text{if } n \text{ is odd.} \end{cases} \\ +nE[v|v > v^*(c)] \cdot F'(v^*(c)) \begin{cases} \frac{(n-2)!!}{(n-3)!!} & \text{if } n \text{ is odd,} \\ \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even.} \end{cases} \end{array} \right) \\ &\quad + \lim_{c \to 0} F'((v^*(c)))n \cdot v_c \cdot c - n. \end{aligned} \tag{5}$$

Note that $v_c \frac{dE[v|v>v^*(c)]}{dv^*} = \frac{dE[v|v>v^*(c)]}{dc^*}$ and $\lim_{c \to 0} \frac{dE[v|v>v^*(c)]}{dc^*} = F'(0)E[v] \lim_{c \to 0} v_c$, thus $\lim_{c \to 0} \frac{dE[v|v>v^*(c)]}{dv^*} = E[v] \cdot F'(0)$. Also note that $\lim_{c \to 0} F'((v^*(c))) \cdot v_c \cdot c = 0$ if $\lim_{v \to 0} F'(v)v = 0$ and $v_c$ is finite. Hence, we can simplify (5) to yield:

$$\lim_{c\to 0} SSV'(c) = \left(\lim_{c\to 0} v_c\right) E[v]\cdot F'(0) \left( n \begin{cases} \frac{(n-2)!!}{(n-3)!!} & \text{if } n \text{ is odd,} \\ \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even.} \end{cases} - (n-1) \begin{cases} \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even,} \\ \frac{n!!}{(n-1)!!} & \text{if } n \text{ is odd.} \end{cases} \right)$$
$$-n.$$

Look at the case when $n$ is even. We have: $\lim_{c\to 0} SSV'(c) = 2^n/\binom{n-1}{n/2-1}\frac{(n-1)!!}{(n-2)!!}\cdot E[v]F'(0) - n = 2^n \cdot \frac{(n/2-1)!(n/2)!}{(n-1)!}\cdot\frac{(n-1)!!}{(n-2)!!}\cdot E[v]F'(0) - n$. Since $\frac{2^{n/2}}{n!} = \frac{1}{(n/2)!(n-1)!!}$ and $2^{n/2-1}(n/2-1)! = (n-2)!!$, we have $\lim_{c\to 0} SSV'(c) = (2E[v]F'(0) - 1)\cdot n$. Now this is strictly greater than zero if and only if $E[v]\cdot F'(0) > \frac{1}{2}$.

Note that if we try the same method when $n$ is odd, we have: $\lim_{c\to 0} SSV'(c) = \frac{2^n}{\binom{n-1}{(n-1)/2}} E[v]\cdot F'(0)\left(n\frac{(n-2)!!}{(n-3)!!} - \frac{n!!}{(n-3)!!}\right) - n = -n$. Now this is never greater than zero. Hence, in that case, using this method we are unable to determine when it is optimal to have a positive cost when $n$ is odd.
■

Notice that the condition with aggregate supporter uncertainty $E[v]\cdot F'(0) > \frac{1}{2}$ and an even number of voters is weaker than that when there is certainty $E[v]\cdot F'(0) > 1$ (which by our assumptions also has an even number of total voters). Thus, for all distributions where it would be worthwhile to have voting costs without uncertainty in number of supporters for each outcome, it would also be worthwhile to have positive voting costs with such supporter uncertainty.

## 4.2 What is the optimal level of voting?

We saw in the previous subsection that voting, even if it is costly, can have benefits to social surplus. However, in the previous subsections, who voted was determined by equilibrium conditions. In this subsection, we wish to ask what should be the correct level of voting for society. This is the equivalent of asking if a social planner can decide a critical level of value only above which people should vote, what should it be?

Finding this level allows us to compare the optimal level to the equilibrium level. We can then ask what policy recommendations we can give to induce this level – having penalties for not voting or adding a poll tax. These penalties and taxes are just transfers and thus do not affect overall welfare.[7] Remember, in contrast, a cost of voting yields no direct benefit to anyone and given

[7]Another method would be to employ some criteria for voting that reflects values. For instance, Jefferson felt

the same voting outcome is a waste to society. In the following proposition, we can compare the equilibrium with the optimal level of voting.

**Proposition 3** *Under ASU, (i) there is overvoting (ii) there should be no fines to encourage voting (no mandatory voting) (iii) there should be a poll tax to discourage voting.*

**Proof.** A voter's vote will be pivotal in two instances. Case (A): when there is a tie in votes without his vote. Case (B): when the other candidate leads by 1 without his vote. In case (A), there will be no externality imposed since the other voters balance each other out. For case (B), the net externality imposed by a voter on others by voting is $E[v|v > v^*]$. This externality is negative and a voter doesn't take this into account. Thus, in an optimal there should be less voting. Formalizing this logic, the optimal cutoff (if interior) should then solve:

$$\sum_{a=0}^{n-1} \binom{n-1}{a} F(v^*)^{n-1-a} (1 - F(v^*))^a \begin{cases} \binom{a}{a/2} \left(\frac{1}{2}\right)^a v^* & \text{if } a \text{ is even,} \\ \binom{a}{(a-1)/2} \left(\frac{1}{2}\right)^a (v^* - E[v|v > v^*]) & \text{if } a \text{ is odd.} \end{cases} = 2c. \tag{6}$$

From this equation, the equilibrium cutoff will then be lower than the optimal cutoff. To see this the LHS is similar to the LHS of equation (4) but $(v^* - E[v|v > v^*])$ multiplied by one of the probabilities instead of $v^*$, which is then smaller. Since the LHS of (4) is increasing in $v^*$. The LHS of (4) will be smaller than $2c$ for all $v^*$ less than the equilibrium cutoff. Consequently, the LHS of (6) is smaller than $2c$ for all $v^*$ less than the equilibrium cutoff. Hence, the solution to (6) must be higher than the equilibrium cutoff. Since the optimal level of voting is lower than the equilibrium level, a government could charge for voting in order to implement the optimal cutoff. This would be change the $c$ in equation (4) such that the $v^*$ that solves that the solution of equation (4) matches the solution to the equation (6). Note that while (6) may have more than one solution, all solutions would be at a higher $v^*$, then the equilibrium. Also, the only possible non-interior optimal cutoff for $c > 0$ is where no one votes (it can never be socially optimal for a zero-valued voter to vote). Thus again, the optimal cutoff would be higher than the equilibrium. ■

Börgers (2000, 2004) develops a model with costly voting and shows (in Börgers, 2004) that

only the educated should vote (Padover 1952, page 43). In our model, they would have better information and hence higher values for certain candidates. There have also been literacy and property ownership as requirements. While literacy might have been used to disenfranchise certain minority groups, a property ownership requirement for the most part was to restrict voting to groups that had a stake in the country (high values).

the unique equilibrium is superior to both mandatory voting still with voting costs (or equivalently bribing people to vote) and random choice (with no one voting). He also graphically shows (in Börgers, 2000) that the unique equilibrium may be superior to one with lower voting costs, but importantly did not show that it is superior to one with zero voting costs. With the technique used in the above proposition, it becomes apparent that Börgers (2004) also has overvoting.[8] The main difference of our model to that of Börgers (2000, 2004) is that we allow for different intensities of preferences while Börgers (2000, 2004) has different voting costs. In both models, it is always optimal to charge a poll tax when there is a positive cost of voting; however, only in our model is it worthwhile to have a poll tax when the cost of voting is zero. This is because when the marginal voter has a value of 0 and when pivotal (moving from a loss to a tie) is replacing a voter with a higher value. In Börgers (2000, 2004), it is optimal to have all voters voting when there is zero voting costs, since everyone has the same valuation (in absolute terms) and thus this will maximize the information aggregated.

There are no restrictions on $F$ for a poll tax to be optimal. The tax is a transfer unlike a wasteful cost of voting so is a superior alternative. It is not clear that a poll tax is politically viable. This then leads to our previous sub-section where it may be worthwhile to maintain (or even induce) a cost of voting. Doing so eliminates voters with little intensity for their preferred candidate.

## 5 Conclusion

Since the nineteenth century, political scientists have been in agreement that increasing the franchise will be beneficial to the society (Lakeman and Lambert, 1959, page 19). So over the last century in democracies, the right to vote has been given to most of the adult society and the requirements of registration to vote such as property qualifications have been removed. Social scientists have further asked the question whether or not it makes sense to require people to vote. In addition this requirement to vote also gets around the paradox that since each individual may find his vote negligible will choose not to vote if there is a cost to voting. We show that not only should one not require people to vote but there is a distinct benefit to having some people not vote and increasing

[8] The externality imposed on others by voting is absent in case (A) and equal to the value in case (B). For the same reason as the proof of Proposition 2, there is overvoting in the Börgers model as well.

the (wasteful) cost to voting may paradoxically be beneficial to society. (Note increasing a wasteful cost of voting may be politically more viable than imposing a poll tax.) For instance, if a committee has an important vote, scheduling the meeting at an inconvenient time may improve the outcome. This also shows that allowing absentee ballots or internet voting can be damaging.[9]

Since this is the first paper to show that it may be beneficial having costly voting (over costless voting), there are many directions of future research where one can expand the result. One direction is to increase the number of alternatives on the ballot to more than two. With committee voting this seems quite logical. Furthermore, once this is done, one can introduce approval voting to ameliorate strategic voting (see Brams and Fishburn, 1978). Another direction is to introduce a common value element in addition to a private value as in Osborne and Turner (2010).

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[9] In Israel, absentee ballots are not allowed. Voters fly back to Israel specifically for the election (Los Angeles Times, May 9, 1999). Our results show that this waste may actually be socially efficient.

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