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ABSTRACT. The problem of decomposing a cash flow has been treated in recent years by Gronchi (1986, 1987), Peccati (1987, 1991, 1992), Stewart (1991), Pressacco and Stucchi (1997). After showing that the *Economic Value Added* introduced by Stewart bears a strong resemblance to (and in some conditions coincides with) the periodic Net Present (or Final) Value in Peccati's model and that Pressacco-Stucchi's model can be seen as a formal generalization of Stewart's model, this paper proposes a different decomposition model introducing the *Systemic Value Added*, which lends itself to a disaggregation in periodic shares whose uncompounded sum coincides with Peccati's and Pressacco-Stucchi's Net Final Value. The model proposed offers the opportunity to dwell on the notion of *residual income*, showing that the interpretation given by the three previous models fails to explain the correct evolution of the investor's financial system. The evaluation process is then reshaped by introducing the concept of *shadow project*, by means of which Peccati's and Stewart's model can be retrieved. Pressacco-Stucchi's model can also be retrieved and generalized and some of its assumptions are relaxed. The formal results in the last section provide sufficient and necessary conditions for integrating all models in the systemic framework here adopted. Finally, some hints shows that the results Pressacco and Stucchi obtain can be proved by using the *systemic* approach here suggested.

JEL Classification: G11, G31

Introduction

Evaluation of investments and decomposition of a cash flow stream under certainty has gained in recent years a renewed interest thanks to Gronchi (1986, 1987), who generalizes the concept of internal rate of return, Peccati (1987, 1991, 1992), who decomposes a certain cash flow stream, Pressacco and Stucchi (1997), who generalize Peccati's model. In the same years, Stewart (1991) introduces a new profitability index, called Economic Value

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Added (EVA). It has attracted increasing attention among academics and professionals, and is considered a reliable index for assessing the value of firms (and projects) as well as a helpful tool for rewarding management. It is presented in any recent finance text-book (e.g. Brealey and Myers (2000)) and even its critics recognize that "the introduction of EVA [...] can rightly be regarded as one of the most significant management innovations of the past decade" (Biddle *et al.* (1999), p.78) It is well-known that EVA translates, formally, the classical economic concept of *residual income* (also known as *Goodwill* in the accounting literature) and that the aggregation of the periodic EVAs leads to the Net Present (or Final) Value. This paper briefly summarizes the aforementioned models, which turn out to be akin in that they are grounded on the same assumptions, and then proposes a different decomposition model. The goal of this work is twofold: 1) It will be shown that the first three models rely on an implicit notion of *residual income* which seems to be somehow fallacious; 2) it will be shown that the index here introduced, the *Systemic Value Added*, lends itself to a safe economic interpretation and that it is possible to integrate the latter with the other decomposition models. The paper is organized as follows. The first three section briefly present, respectively, Peccati's model, Pressacco and Stucchi's model and Stewart's model, showing the formal analogies between the three. Sec.4 introduces the *Systemic Value Added* (SVA) model. Sec.5 shows the inconsistencies of the first three models, revealing the way they interpret the economic concept of *residual income*. Sec.6 reshapes the evaluation process so as to integrate the previous models in the systemic approach. The concept of *shadow project* is here introduced. Sec.7 present formal results providing sufficient and necessary conditions to accomplish the integration of all models. Some remarks conclude the paper.

In the sequel I shall rest (among others) on the following assumptions, unless otherwise specified: A decision maker aims at evaluating a project P with initial outlay $-a_0<0$ at time 0 and equidistant cash flows are $a_s \in \mathbb{R}$ at time s. All flows are certain. The term net worth is to be intended as a synonym of wealth. I assume that maximization of net worth at a fixed terminal horizon T is the goal of the evaluator. No generality is loss assuming $T=n$. I shall more frequently refer to the Net Final Value (NFV) rather than the Net Present Value (NPV). As we will see, the NFV rule is nothing but the NPV rule expressed in terms of a terminal fixed horizon time n. I shall furhter assume, as it is natural, that P is a nonzero project.

1. Peccati's model

The NFV rule assumes that cash flows released by a project P are withdrawn from (if negative) and invested in (if positive) an account, say C, whose value C_s at time s evolves according to the following recurrence equation:

$$C_s = C_{s-1}(1+i) + a_s \qquad s = 1, \ldots, n$$

where i is the so-called *opportunity cost of capital*. Following Peccati (1991) I shall assume $C_0{=}E_0$, where E_0 denotes the value of the evaluator's net worth at time 0, $E_0 \in \mathbb{R}$. The decision maker faces two alternative courses of action: (i) to invest in project P, (ii) to keep her wealth in account C. Let us denote with E_s and E^s, $s \geq 1$, the net worth at time s for case (i) and (ii) respectively. We define the Net Final Value of P as the difference

$$E_n - E^n,$$

i.e.

$$(E_0 - a_0)(1+i)^n + \sum_{s=1}^{n} a_s (1+i)^{n-s} - E_0(1+i)^n = -a_0(1+i)^n + \sum_{s=1}^{n} a_s (1+i)^{n-s}. \quad (1)$$

The NPV is obtained by discounting (1) at present time. We aim at decomposing (1) in n shares G_s so that $G_1 + \ldots G_n$=NFV. Assuming that P belongs to the class of Soper (1959), the *outstanding capital* (or *project balance*) of P at time s is given by

$$\begin{aligned} w_0 &= a_0 \\ w_s &= w_{s\text{–}1}(1+x) - a_s \qquad s = 1, 2, \ldots, n. \end{aligned} \quad (2)$$

where x is the internal rate of return. Following Peccat's argument, we focus on a generic period s: The investor invests the sum $w_{s\text{–}1}$ at the beginning of the period and receives $w_s{+}a_s$ at the end of the period. The gain is $xw_{s\text{–}1}$. So doing, she gives up the opportunity of investing $w_{s\text{–}1}$ at the opportunity cost of capital i, that is she foregoes the gain $iw_{s\text{–}1}$. The sum $w_{s\text{–}1}(x-i)$ is the *residual income* in period s, that is the difference between what the investor earns by choosing P and what she would earn should she decide to keep funds in C. As every such share is money referred to time s, we must compound to time n before we can sum all shares. We have then

$$G_s = w_{s\text{–}1}(x-i)(1+i)^{n-s} \quad (3a)$$

In such a way, the model meets both the requirements of finding periodic values for project P being significant from an economic point of view (they measure the residual income of period s) and aggregating these values so as to obtain the NFV (which is the overall residual income).

It will be useful to note that if the project is partly financed with debt G_s becomes

$$G_s = \big(w_{s\text{–}1}(x-i) + D_{s\text{–}1}(i-\delta)\big)(1+i)^{n-s} \quad (3b)$$

where δ is the debt rate and

$$\begin{aligned} D_0 &= f_0 \\ D_s &= D_{s\text{–}1}(1+\delta) - f_s \end{aligned}$$

is the outstanding debt, with f_s denoting the debt's cash flows (where, for convenience, we pick $f_0{>}0$ and $-f_s{<}0$ for $s{\geq}1$).

2. Pressacco-Stucchi's model

Pressacco and Stucchi (*op.cit.*, henceforth P&S) aim at generalizing Peccati's model allowing account C to evolve according to the following recurrence equation:

$$C_s = C_{s-1}(1 + i(C_{s-1})) + a_s \tag{4}$$

where

$$i(C_{s-1}) = i_P \qquad \text{if} \quad C_{s-1} > 0,$$

$$i(C_{s-1}) = i_N \qquad \text{if} \quad C_{s-1} < 0,$$

with $i_P \neq i_N$,[2] and allowing for non-Soper projects. P&S introduce account C picking $C_0 = -a_0$. This has (at least) two different economic interpretations:

(a) account C is strictly connected with the project, so that it exists only as long as the project exists;
(b) the initial net worth is zero: If so, prior to the undertaking of the project we have E_0=0=C_0 which is the value of account C. At the beginning of period 1 a_0 is then withdrawn from account C and invested in P, leading to $C_0 = -a_0$, which is the starting point for P&S.

Interpretation (a) seems the most plausible one, as the authors present C as a "project account". However (a) and (b), while formally extending the scope of application, limit Peccati's model from an economic point of view, as they represent very particular assumptions about possible ways of investing funds (case (a)) or about the value of the investor's wealth (case (b)). Actually, Pressacco and Stucchi do not deal with the case in which E_0 is nonzero and is invested in account C.[3] According to these definitions and due to the assumption $C_0 = -a_0$, the NFV for P is

$$\text{NFV} = -a_0(1 + i(C))^n + \sum_{s=1}^{n} a_s(1 + i(C))^{n-s} \tag{5}$$

where

$$(1 + i(C))^{n-s} := \prod_{k=s+1}^{n} (1 + i(C_{k-1}))$$

[2] P stands for "positive", N for "negative".
[3] The need of thoroughly inquiring implicit assumptions will turn to be essential.

Note that in P&S model $i(C_{s-1})$ cannot be given the economic interpretation of an opportunity cost of capital as it represents a genuine rate (of cost or return depending on the sign of C_{s-1}). The outstanding capital w_s at the internal rate $x(w_{s-1})$is given by

$$\begin{aligned} w_0 &= a_0 \\ w_s &= w_{s-1}(1 + x(w_{s-1})) - a_s \end{aligned} \tag{6}$$

where

$$\begin{aligned} x(w_{s-1}) &= x_P \qquad \text{if} \qquad w_{s-1} > 0, \\ x(w_{s-1}) &= x_N \qquad \text{if} \qquad w_{s-1} < 0. \end{aligned}$$

I shall henceforth assume $x_P \neq x_N$. Further, all interest rates we will be dealing with are assumed to be nonzero.[4]

$x(w_{s-1})$ is such that

$$w_n = -a_0(1 + x(w))^n + \sum_{s=1}^{n} a_s(1 + x(w))^{n-s} = 0 \tag{7}$$

where

$$(1 + x(w))^{n-s} := \prod_{k=s+1}^{n} (1 + x(w_{k-1})),$$

in analogy with $i(C)$.[5] The main result of P&S (Theorem 6.2) is here summarized:

P&S Theorem. *Peccati's model can be generalized in*

$$\text{NFV} = \sum_{s:w_{s-1}>0}^{n} w_{s-1}(x_P - i_N)(1+i(C))^{n-s} + \sum_{s:w_{s-1}<0}^{n} w_{s-1}(x_N - i_P)(1+i(C))^{n-s} \tag{8a}$$

if and only if

$$x(w_{s-1}) = x_P \qquad \textit{iff} \qquad i(C_{s-1}) = i_N. \tag{8b}$$

It is worthwhile noting that P&S generalize Peccati's model only under a particular perspective. As we noted, P&S model can be seen as assuming E_0=0, whereas Peccati allows

[4] I shall never define the value of a two-valued rate when its argument is zero. In this case, we can pick whatever value we want.

[5] Obviously, we have infinite pairs (x_P, x_N) satisfying (7).

for all values of E_0. But more important is that they take account of external financing by means of the two-valued rate of account C. Peccati's account C has a one-valued rate but he generalizes his model by allowing for one or more debts (whose values cannot obviously be negative), as we have briefly seen at the end of the previous section.

3. Stewart's model

The Economic Value Added is a profitability index introduced by Stewart in order to provide a tool for evaluating (projects and) firms as well as for evaluating and compensating managers. The basic objective of EVA is to create a measure of periodic performance based on the concept of residual income.

> "*Recognized by economists since the 1770's, residual income is based on the premise that, in order for a firm to create wealth for its owners, it must earn more on its total capital invested than the cost of that capital*" (Biddle *et al.* (1999), p.70)

To compute it, we calculate the firm's (or project's) total cost of capital, given by the product of the Weighted Cost of Capital (WACC) and the total capital invested (TC). Then the total cost of capital is subtracted from the Net Operating Profit After Taxes (NOPAT). Notationally, we have, for period s,

$$\text{EVA}_s = \text{NOPAT} - \text{WACC} * \text{TC}. \tag{9a}$$

Summing for s and discounting at time 0 (or compounding at time n) at a rate i' we obtain the overall residual income, which Stewart calls Market Value Added (MVA)

$$\text{MVA} = \sum_{s=1}^{n} \frac{\text{EVA}_s}{(1+i')^s}.$$

It is easy to show that (9a) is equivalent to (3b). In fact, (9a) can be rewritten as

$$\text{EVA}_s = \text{ROA}{*}\text{TC} - \frac{(\text{ROD}{*}\text{Debt}{+}\text{i}{*}\text{Equity})}{\text{Debt}{+}\text{Equity}} * \text{TC} \tag{9b}$$

whence

$$\begin{aligned} \text{EVA}_s &= \text{ROA}{*}\text{TC}{-}\text{ROD}{*}\text{Debt-i}{*}(\text{TC}{-}\text{Debt}) \\ &= \text{TC}{*}(\text{ROA}{-}i){+}\text{Debt}{*}(i{-}\text{ROD}) \end{aligned} \tag{9c}$$

where ROA is the Return on Assets, ROD is the Return on Debt, and i is the opportunity cost of capital. All values in (9) obviously refer to period s. Applying this argument to project P, we have TC=w_{s-1}, ROA=x, Debt=D_{s-1}, ROD=δ. and the relation between (3b) and (9) is straightforward:

$$G_s = \text{EVA}_s(1+i)^{n-s}.$$

If i'=i the overall residual income in Stewart's model, denoted by the acronym MVA (Market Value Added), coincides with Peccati's NPV:

$$\text{NPV} = \frac{\text{NFV}}{(1+i)^n} = \frac{1}{(1+i)^n}\sum_{s=1}^{n} G_s = \sum_{s=1}^{n} \text{EVA}_s(1+i)^{-s} = \text{MVA}$$

The equivalence vanishes only in discounting each EVA_s: Stewart uses the Weighted Average Cost of Capital (i'=WACC), whereas Peccati uses the opportunity cost of capital itself (i'=i). I shall not dwell on this issue (the reader can refer to Peccati (1996) about the use of the WACC for discounting), and adopt a zero debt assumption (i.e. D_s=0 for all s), so that i'=WACC=i. However, the arguments presented hold regardless of whatever assumption on external debt.

It should be also clear that P&S generalize the EVA model from a formal point of view, in the same sense in which they generalize Peccati's model.

4. Systemic Value Added

In this section a decomposition model is offered differing in various aspects from the previous ones. Let us focus on Peccati's assumptions and on a generic period s. At time 0 the decision maker compares two lines of action:

(i) undertaking the project
(ii) investing funds at the opportunity cost of capital i.

As for (i) at time s the decision maker's net worth E_s can be seen as a financial system consisting of the sum of C_s and the outstanding capital w_s; if (ii) is instead chosen, the decision maker's wealth E^s will be given by E_0 plus the interest yielded by account C. We have then that the following recurrence equations hold:

$$\begin{aligned}
&C_0 = E_0 - w_0 \\
&w_0 = a_0 \\
&C_s = C_{s-1}(1+i) + a_s && \text{for } s \geq 1 \\
&w_s = w_{s-1}(1+x) - a_s && \text{for } s \geq 1
\end{aligned} \tag{10i}$$

for (i) and

$$\begin{aligned} &C^0 = E_0 \\ &C^s = C^{s-1}(1+i) &\text{for } s \geq 1 \end{aligned} \tag{10ii}$$

for (ii). I define *Systemic Value Added* for period s (SVA_s) the difference between net earnings for (i) and net earning for (ii), that is

$$\mathrm{SVA}_s = \left[(E_s - E_{s-1}) - (E^s - E^{s-1})\right] = (C_s + w_s - C_{s-1} - w_{s-1}) - \left(C^s - C^{s-1}\right).$$

This represents the differential net profit of (i) over (ii) to be ascribed to period s, that is the residual income generated by project P in period s. Summing for s we obtain the (overall) *Systemic Value Added* (SVA)

$$\mathrm{SVA} = \sum_{s=1}^{n} \left((E_s - E_{s-1}) - (E^s - E^{s-1})\right) = E_n - E^n. \tag{11}$$

Since the right-hand side is, by definition, the NFV of P, we have SVA=NFV.

This result is consistent with the NFV rule in that it states that the total residual income SVA coincides with the Net Final Value, and the decision maker will accept the project if and only if

$$\mathrm{SVA} = \mathrm{NFV} = \mathrm{NPV}(1+i)^n > 0.$$

But while coinciding in overall terms, they give rise to different partitions: We have, from (10i) and (10ii)

$$\mathrm{SVA}_s = xw_{s-1} - i\left(C^{s-1} - C_{s-1}\right) \tag{12}$$

which represents the periodical residual income, that is the s-the sahre of the SVA of P. Conversely, the s-th share of the NFV is the compound amount G_s, where

$$G_s = \mathrm{EVA}_s(1+i)^{n-s} \neq xw_{s-1} - i\left(C^{s-1} - C_{s-1}\right).$$

The SVA model is grounded on a systemic way of reasoning: the net worth is a system structured in accounts whose value evolves in time following different laws. The sum of the accounts constitutes the value of the whole net worth. This enables us to avoid compounding, whereas Peccati-EVA model rests on the concept of Net Final (Present) Values and on capitalization processes. In a sense, by using a systemic perspective we are able to sum cash regardless of its maturity. This result, far from being illicit, suggests that we can create a cognitive outlook where there is no need of capitalization: time dimension is considered by means of the system's time evolution.

To shed more light on this issue, we can investigate it thouroughly. Using (2) we have $\left(C^{s-1} - C_{s-1}\right) = w_{s-1} - \mathrm{EVA}_1(1+i)^{s-2} - \mathrm{EVA}_2(1+i)^{s-3} - \cdots - \mathrm{EVA}_{s-2}(1+i) - \mathrm{EVA}_{s-1}$. Substituting in (12) we obtain

$$\mathrm{SVA}_s = \mathrm{EVA}_s + \sum_{k=1}^{s-1} i\mathrm{EVA}_k(1+i)^{s-k-1}. \tag{13}$$

By induction,

$$\sum_{k=1}^{s} \mathrm{SVA}_k = \sum_{k=1}^{s} \mathrm{EVA}_k(1+i)^{s-k} \tag{14a}$$

for every $s \geq 1$. Hence,

$$\mathrm{SVA} = \sum_{k=1}^{n} \mathrm{SVA}_k = \sum_{k=1}^{n} \mathrm{EVA}_k(1+i)^{n-k} = \sum_{k=1}^{n} G_k = \mathrm{NFV} \tag{14b}$$

as we expected. Note that, due to (13) and (14a),

$$\mathrm{SVA}_s = \mathrm{EVA}_s + i\left(\sum_{k=1}^{s-1} \mathrm{SVA}_k\right). \tag{15a}$$

Using (15a) and choosing, for the sake of convenience, n=3, let us decompose project P by means of G_s and SVA.

$$\begin{aligned}
G_1 &= \mathrm{EVA}_1(1+i)^2 & \mathrm{SVA}_1 &= \mathrm{EVA}_1\\
G_2 &= \mathrm{EVA}_2(1+i) & \mathrm{SVA}_2 &= \mathrm{EVA}_2 + i\mathrm{SVA}_1\\
G_3 &= \mathrm{EVA}_3 & \mathrm{SVA}_3 &= \mathrm{EVA}_3 + i\mathrm{SVA}_1 + i\mathrm{SVA}_2
\end{aligned} \tag{15b}$$

or

$$\begin{aligned}
G_1 &= \mathrm{EVA}_1 + (i\mathrm{EVA}_1) + (i\mathrm{EVA}_1 + i^2\mathrm{EVA}_1) & \mathrm{SVA}_1 &= \mathrm{EVA}_1\\
G_2 &= \mathrm{EVA}_2 + (i\mathrm{EVA}_2) & \mathrm{SVA}_2 &= \mathrm{EVA}_2 + (i\mathrm{EVA}_1)\\
G_3 &= \mathrm{EVA}_3 & \mathrm{SVA}_3 &= \mathrm{EVA}_3 + (i\mathrm{EVA}_1 + i^2\mathrm{EVA}_1)\\
& & &\quad + (i\mathrm{EVA}_2)
\end{aligned} \tag{15c}$$

For Peccati-EVA model the idea is the following: G_1, G_2, G_3 are the three shares for period 1, 2, 3 respectively. As this is money referred to the dates 1, 2, 3, respectively, the basic principles of financial calculus force the evaluator to compound (or discount) flows to take time into consideration. After capitalization (and only after) the evaluator may sum the three shares. Conversely, in the light of our systemic perspective the decision maker can construct, in a gradual way, the three shares of the SVA. The first share is EVA_1, which exactly represents the difference between what the investor receives in the first period and what she would receive should she decide to forego the project opportunity and invest her funds at the opportunity cost of capital i. In the second period the difference between what she receives and what she would have received must take into account that, in addition to EVA_2, the first share yields interest equal to $i\text{EVA}_1$ ($=i\text{SVA}_1$). Iterating the argument, the third share must consider the return on the two first shares EVA_1 and EVA_2, as well as the returnt gained on $i\text{EVA}_1$, which are produced just in the third period. Financially speaking, we can interpret every SVA_s as a capital invested at time s, yielding linear interest at the rate i until n, for a total interest of $(i(n-s)\text{SVA}_s)$ each. In fact, we can easily check that

$$\begin{aligned}\text{NFV} = \text{SVA} &= \sum_{s=1}^{n} \text{SVA}_s \\ &= \sum_{s=1}^{n} \text{EVA}_s + \sum_{s=1}^{n} i \left(\sum_{h=1}^{s-1} \text{SVA}_h \right) \\ &= \sum_{s=1}^{n} \text{EVA}_s + \sum_{s=1}^{n-1} i(n-s)\text{SVA}_s\end{aligned}$$

On the contrary, in Peccati-EVA model G_1 embodies the term $i\text{EVA}_1$ which is instead generated in the second period, and comprehends $i\text{EVA}_1 + i^2\text{EVA}_1$ which in turn is related to the third period. Further, G_2 includes $i\text{EVA}_2$, which relates to period 3, but lacks the term $i\text{EVA}_1$ (previously embodied in G_1). Finally, the third share G_3 forgets the return on previous periods' shares. Therefore capitalization of the EVAs is unwarranted, because it means attaching to each EVA interest which is not pertinent: The factor $(1+i)^{n-s}$, used to compound the residual income until time n, gather future interest which is just generated in periods successive to period s.

Now we can extend our model allowing for two-valued rates i and x depending on the sign of C_{s-1} and w_{s-1} respectively, as in P&S model. Let $i(C^{s-1})$ be such that $i(C^{s-1}){=}i_P$ if $C^{s-1}{>}\ 0$, $i(C^{s-1}){=}i_N$ if $C^{s-1}{<}\ 0$; let $i(C_{s-1})$ and $x(w_{s-1})$ be defined as in P&S model; (12) can be written as

$$\text{SVA}_s = x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1}. \tag{16}$$

The Net Final Value of P is given by

$$\text{NFV} = E_n - E^n = E_0\Big(\big(1+i(C)\big)^n - \big(1+i(E_0)\big)^n\Big) - a_0\big(1+i(C)\big)^n + \sum_{s=1}^{n} a_s\big(1+i(C)\big)^{n-s}$$

since

$$\begin{aligned} E_n &= (E_0 - a_0)(1+i(C))^n + \sum_{s=1}^{n} a_s(1+i(C))^{n-s} \\ E^n &= E_0(1+i(C^0))^n = E_0(1+i(E_0))^n \end{aligned}$$

and the NFV is, by definition, the difference between the two final net worths. Note also that picking E_0=0 (and therefore C_0=$-a_0$) we get to (5) as in Pressacco and Stucchi's model. It is worthwhile anticipating that the latter differs from SVA model: *In primis*, the model here offered is more general in that account C is allowed to take on whatsoever value at time 0; *in secundis* the decomposition of the NFV is different: even if we assume C_0=$-a_0$ the periodic shares do not coincide.[6]

The decomposition we have arrived to is different from EVA model since relies on a well-specified notion of periodic residual income: The latter is based on drawing up two sequences of n double-entry sheets, the sequence for alternative (i) and the sequence for alternative (ii). We calculate all periodic net profits from both sequences and take the the difference between alternative net profits: This gives us the periodic residual income. Such an interpretation limit, at the very best, the use of EVA model. Actually, the residual income Stewart and Peccati (as well as Pressacco and Stucchi) refer to is not the difference between alternative net profits (remember also (15)). But then, what notion of residual income do all these authors implicitly assume?

5. Different notions of residual income

This section is devoted to discussing the economic interpretation underlying the NFV-based models, as opposed to the systemic approach. The latter focuses on the periodic evolution of the evaluator's financial system and periodically records changes of wealth for both alternatives (i) and (ii). The NFV-based approaches exposued in the first three sections overlook this diachronic evolution. So doing they seem to be self-inconsistent. I shall discuss Peccati-EVA model and P&S model separately.

5.1 Peccati-EVA model (henceforth, PE model)

[6]Note also that if i_P=i_N we get back to Peccati's NFV.

While the SVA model is based on alternatives (i) and (ii), in PE model a sequence of n pairs of periodic implicit assumptions (I_s, II_s) are introduced such that:

(I_s) at time $(s-1)$ the sum w_{s-1} is invested for one period at the rate x
(II_s) at time $(s-1)$ project P is displaced and the sum w_{s-1} is invested in account C

for *every period* s. Fix s^* such that $1 \leq s^* \leq n$. (I_{s^*}) implies that the periodic net profit is

$$xw_{s^*-1} + iC_{s^*-1}.$$

(II_{s^*}) implies that the value of the net worth at time (s^*-1) is given by

$$E_{s^*-1} = C_{s^*-1} + w_{s^*-1} \tag{17a}$$

and, at time s^*, we have

$$E_{s^*} = C_{s^*} = (C_{s^*-1} + w_{s^*-1})(1+i). \tag{17b}$$

The periodic net profit is therefore

$$i\,(C_{s^*-1} + w_{s^*-1})\,.$$

The residual income RI is the difference between net profit for (I_{s^*}) and net profit for (II_{s^*}):

$$\mathrm{RI} = xw_{s^*-1} + iC_{s^*-1} - i\,(C_{s^*-1} + w_{s^*-1}) = w_{s^*-1}(x-i) = \mathrm{EVA}_s$$

as we expected.

But as (17a) holds for *every* s, we have, at time s^*,

$$E_{s^*} = C_{s^*} + w_{s^*} = [\text{for (10i)}] = C_{s^*-1}(1+i) + a_{s^*} + w_{s^*} \tag{17c}$$

(17b) and (17c) are incompatible since, in general,

$$w_{s^*-1}(1+i) \neq w_{s^*} + a_{s^*}$$

(the two amounts coincides if $i{=}x$, but this makes the evaluation problem rather trivial and uninteresting).

In PE model the notion of residual income is inconsistent with the time evolution of the decision maker's financial system. If (II_s) holds for an s, then project P has been displaced and we cannot recover it by assuming (I_k) nor (II_k) for $k{>}s$: The game is over, so to say.

5.2 P&S model

P&S model is somewhat less simple to deal with. Let us focus on (II_s). If interpretation (a) is adopted account C disappears as well as project P, since the former exists as long as the latter exists; if interpretation (b) is adopted project P disappears and the financial system collapses into a single account C bearing interest at the rate $i(C_{s-1})$ (which will be constant from that moment on).

Let us begin with (b), that is E_0=0.

5.2.1 Interpretation (b)

Fix s^* such that $1\leq s^*\leq n$. (I_{s^*}) implies that the periodic net profit is

$$x(w_{s^*-1})w_{s^*-1} + i(C_{s^*-1})C_{s^*-1}.$$

(II_{s^*}) may not mean that w_{s^*-1} is invested *in account* C. In fact, if this was the case we would have

$$C_{s^*} = (C'_{s^*-1})(1 + i(C'_{s^*-1}))$$

where C'_{s^*-1}:=C_{s^*-1}+w_{s^*-1}. The periodic net profit would be

$$i(C'_{s^*-1})C'_{s^*-1}.$$

The residual income (RI) would therefore be

$$\begin{aligned}\mathrm{RI} &= x(w_{s^*-1})w_{s^*-1} + i(C_{s^*-1})C_{s^*-1} - i(C'_{s^*-1})C'_{s^*-1}\\ &= x(w_{s^*-1})w_{s^*-1} + i(C_{s^*-1})C_{s^*-1} - i(C'_{s^*-1})C_{s^*-1} - i(C'_{s^*-1})w_{s^*-1}.\end{aligned}$$

We have

$$\mathrm{RI} = w_{s^*-1}\left(x(w_{s^*-1}) - i(C_{s^*-1})\right)$$

if and only if

$$i(C'_{s^*-1}) = i(C_{s^*-1}). \tag{18}$$

If then (8b) holds too, then we obtain

$$\mathrm{RI} = w_{s^*-1}\left(x_P - i_N\right)$$

or

$$\mathrm{RI} = w_{s^*-1}\left(x_N - i_P\right)$$

as we would expect from P&S Theorem. But to reach this result we have added, as you see, assumption (18) to (8b). This implies that C_{s^*-1} and C'_{s^*-1} have the same sign (both positive or both negative). But this is not necessarily the case. As P&S themselves admit in their Proposition 7.3, we can think of the case for which w_{s^*-1}>0 (and therefore C_{s^*-1}<0) but $C'_{s^*-1}>0$. This means that P&S implicitly assume that w_{s^*-1} is invested *at the same rate of account* C*, but not in account* C. So, if we are to salvage the argument, (II_s) must be slightly modified:

(II_s) at time $(s-1)$ project P is displaced and the sum w_{s-1} is invested *at the same rate of account* C*, but not in* C.

But we cannot accept it, because it means that a new account, say K, is added, yielding interest at the same rate as account C.[7] This alternative has never been stated and cannot represent an opportunity cost for the decision maker. The opportunity cost is a course of action alternative to the one undertaken. The activation of account K is not an available opportunity, for it has not been mentioned at the outset (if it was, things would change). Moreover (and more important), why should the rate of interest for K be tied to the value of account C rather than the value of K itself?

Nevertheless, let us assume that our modified (II_s) holds for *every* s. Since

$$E_{s^*-1} = C'_{s^*-1} = C_{s^*-1} + w_{s^*-1}$$

(II_{s^*}) entails

$$E_{s^*} = C'_{s^*-1}(1 + i(C_{s^*-1})). \tag{19a}$$

But since our modified (II_{s^*+1}) must hold too, we also have

$$E_{s^*} = C'_{s^*} = C_{s^*} + w_{s^*} = C_{s^*-1}(1 + i(C_{s^*-1})) + a_{s^*} + w_{s^*} \tag{19b}$$

(19a) and (19b) are incompatible, since, in general,

$$w_{s^*-1}(1 + i(C_{s^*-1})) \neq w_{s^*} + a_{s^*}.$$

5.2.2 Interpretation (a)

If (a) is adopted, then what does (II_s) mean? If project P and account C are being generated at the same time and share their lives, then, whenever the project is displaced account C dissolves too. We can imagine a situation of the following kind: Assuming for simplicity that $w_{s-1}>0$, then in P&S model C_{s-1} is negative, as we have seen. When the investor removes the project, account C is being paid off. The investor receives then the net sum $w_{s-1}+C_{s-1}=C'_{s-1}$. If the latter is positive, it will be invested somewhere, if it is negative it will be withdrawn from somewhere. This implies the activation of an account, say F. Again, regardless of the fact that we cannot make this assumption, as it is not part

[7] Note that we would have the same rate playing the role of rate of cost for account C and rate of return for account K, since the two accounts' values would be different in sign (due to (8b)).

of the decision process, it is easy to show that we get to another inconsistency. I will not dwell on it (the reader may think that if P along with C have dissolved at time s, then we cannot assume that they dissolve again at a later time, which means that is (II_s) is incompatible with (II_k), $k>s$).

6. The shadow project

We now introduce the concept of *shadow project*. We say that project $\overline{P}$ is the *shadow project* of project P (or that $\overline{P}$ is the *shadow* of P) if its cash flows are such that

$$\overline{P} = (-\overline{a}_0, \overline{a}_1, \ldots, \overline{a}_n)$$

where

$$\overline{a}_s = a_s + \mathrm{SVA}_s \qquad s = 0, 1, \ldots, n, \qquad\qquad \mathrm{SVA}_0 := 0$$

and where obviously SVA_s refers to project P. We now apply PE model to the shadow project $\overline{P}$. Letting

$$\overline{w}_s := C^s - C_s \qquad\qquad s = 0, 1, \ldots, n$$

and using (10), we obtain

$$\overline{w}_0 = \overline{a}_0 \qquad\qquad \overline{w}_s = \overline{w}_{s-1}(1 + \overline{x}) - \overline{a}_s \qquad\qquad s = 0, 1, \ldots, n \tag{20}$$

where

$$\overline{x} := x\,\frac{w_{s-1}}{\overline{w}_{s-1}}.$$

We can then interpret $\overline{w}_s$ as the *project balance* of $\overline{P}$ at the rate $\overline{x}$, and the $\overline{a}_s$ are withdrawn from (if positive) or invested in (if negative) an account yielding interest at the rate $\overline{x}$. At the beginning of period s, the investor invests $\overline{w}_{s-1}$ and receives the sum $\overline{w}_s + \overline{a}_s$ at the end of that period. So doing she renounces to the opportunity of investing that sum at the rate of interest i. She therefore foregoes the receipt $-\overline{w}_{s-1}(1+i)$. The Economic Value Added of $\overline{P}$ is

$$\begin{aligned}
\overline{\mathrm{EVA}}_s &= -\overline{w}_{s-1}(1 + i) + \overline{w}_s + \overline{a}_s \\
&= -\overline{w}_{s-1}(1 + i) + (\overline{w}_{s-1}(1 + \overline{x}) - \overline{a}_s) + \overline{a}_s \\
&= \overline{w}_{s-1}(\overline{x} - i).
\end{aligned} \tag{21}$$

The Economic Value Added of $\overline{P}$ coincides with the Systemic Value Added of P: In fact

$$\begin{aligned}
\overline{\mathrm{EVA}}_s &= \overline{w}_{s-1}(\overline{x} - i) \\
&= x w_{s-1} - i\overline{w}_{s-1} \\
&= x w_{s-1} - i\left(C^{s-1} - C_{s-1}\right) \\
&= \mathrm{SVA}_s.
\end{aligned}$$

As one can note we have been able to retrieve PE model and adjust for a *systemic* partition of the Net Final Value of P. We discover an interesting result: If we are to partition the NFV of P we can indeed use the concept of Economic Value Added as it is introduced by Stewart and Peccati, provided that we apply it to the shadow project $\overline{P}$ and do not capitalize the Economic Value Added so obtained. Actually, PE model's partition is such that the sum of the periodic EVA_s of P turns out to differ from the Net Final Value of P. We are then forced to capitalize the shares to a common date. Conversely, in our systemic approach, which focuses on the financial system's evolution, the sum of the periodic $\overline{\mathrm{EVA}}_s$ of the shadow project $\overline{P}$ coincides with the overall SVA of P, which is but P's Net Final Value.

In reframing the decision/evaluation process we have then applied Peccati's argument to project $\overline{P}$. In this way, the contradiction found in 5.1 relates to project $\overline{P}$, not to project P. So doing, we *shift* the contradiction, moving it from P to $\overline{P}$. Peccati's argument can be now safely applied (without capitalization) because its contradictory assumptions invalidate the decomposition of $\overline{P}$, while recovering at the same time the decomposition of P. The latter coincides with the decomposition accomplished by the SVA. To say it in Stewart's terms: to decompose a project P take $\overline{\mathrm{EVA}}_s$ not EVA_s (and forget capitalization!).

The following section keeps on analyzing the relations among all models presented. I shall generalize and adopt a more formal approach in order to obtain some results which will enable us to integrate all models presented via shadow project.

In particular, we will make use of a project P with external pair (i_P, i_N) depending on the sign of account C (i_P if positive, i_N if negative, as usual) and internal pair (x_P, x_N) depending on the sign of the outstanding capital (x_P if positive, x_N if negative, as usual). The shadow project $\overline{P}$ will therefore consist of the initial outlay $\overline{a}_0{=}a_0$ and subsequent cash flows

$$\overline{a}_s = a_s + \mathrm{SVA}_s = a_s + x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1}.$$

We will also make use of the rate $\overline{x}(\overline{w}_{s-1})$ which is defined as follows:

$$\overline{x}(\overline{w}_{s-1}) = \overline{x}_P \qquad \text{if} \quad \overline{w}_{s-1} > 0$$

$$\overline{x}(\overline{w}_{s-1}) = \overline{x}_N \qquad \text{if} \quad \overline{w}_{s-1} < 0$$

where $\overline{x}_N{:=}x_N\frac{w_{s-1}}{\overline{w}_{s-1}}$ and $\overline{x}_P{:=}x_P\frac{w_{s-1}}{\overline{w}_{s-1}}$.

7. The SVA Theorems

Definition 1: *A pair (i_P, i_N) is said to be a twin-pair if for all s, $i(C^s){=}i(C_s)$*

Definition 2: *A pair* (i_P, i_N) *is said to be an* i_P*-twin-pair if it is a twin-pair and* $i(C_s)$=i_P. *A pair* (i_P, i_N) *is said to be an* i_N*-twin-pair if it is a twin-pair and* $i(C_s)$=i_N.

Definition 3: $\overline{P}$ *is said to be a Soper project if for all* s $\overline{x}(\overline{w}_{s-1})$ =$\overline{x}_P$. P *is said to be a Soper project if for all* s $x(w_{s-1})$=x_P

Definition 4: *The shadow pair* $(\overline{x}_P, \overline{x}_N)$ *and the internal pair* (x_P, x_N) *are said to be parallel if, for all* s,

$$x(w_{s-1}) = x_P \qquad \textit{iff} \qquad \overline{x}(w_{s-1}) = \overline{x}_P.$$

Proposition 1. *If for all* s C_s *and* C^s *are both nonnegative or both nonpositive, then* (i_P, i_N) *is a twin-pair.*

Proof: From Definition 1 (and pointing out that $i(0)$ can be defined *ad libitum*).

Proposition 2. *If* (i_P, i_N) *is a twin-pair and there exists some* s *such that* C_s *and* C^s *do not have the same sign, then* (i_P, i_N) *is both* i_P*-twin and* i_N*-twin.*

Proof: The assumptions imply i_P=i_N.

Remark 1: In Peccati's model (i_P, i_N) is both i_P-twin and i_N-twin.

Proposition 3. *If* E_0=0*, then* (i_P, i_N) *is a twin-pair and* C_s=$-\overline{w}_s$ *for all* s.

Proof: We have C^s=0 for all s and $-C_s$=$C^s - C_s$=$\overline{w}_s$ for all s. Further, we have that C^s=0 for all s implies that, for all s, they are both nonnegative or both nonpositive, whence (i_P, i_N) is a twin-pair (Proposition 1).

Proposition 4. *If* (i_P, i_N) *is an* i_P*-twin-pair, then* $E_0 \neq 0$.

Proof: If, for absurd, E_0=0, then C_0=$-a_0$<0, which contradicts the assumption.

Proposition 5. *Suppose* E_0=0. *Then* $\overline{P}$ *is a Soper project if and only if* (i_P, i_N) *is an* i_N*-twin-pair.*

Proof: If $E_0{=}0$ then $C_s{=}-\overline{w}_s$ for all s and (i_P, i_N) is twin (Proposition 3). Then, if $\overline{P}$ is a Soper project, $C_s{\le}0$ and $i(C_s){=}i_N$ for all s. Conversely, if (i_P, i_N) is i_N-twin then $C_s{\le}0$ for all s and therefore $\overline{w}_s{\ge}0$ for all s. Hence $\overline{x}(\overline{w}_{s-1}){=}\overline{x}_P$.

Proposition 6. *If both P and $\overline{P}$ are Soper project, then the internal pair and the shadow pair are parallel. In particular, $x(w_{s-1}){=}x_P$ and $\overline{x}(\overline{w}_{s-1}){=}\overline{x}_P$.*

Proof: From Definitions 3 and 4.

Proposition 7. *Suppose the shadow pair and the internal pair are parallel. Then P is a Soper project if and only if $\overline{P}$ is a Soper project.*

Proof: From Definitions 3 and 4.

Theorem (SVA1). *If (i_P, i_N) is a twin-pair and the shadow pair and the internal pair are parallel, then*

$$\begin{aligned}\mathrm{SVA}_s &= \overline{w}_{s-1}(\overline{x}(\overline{w}_{s-1}) - i(C_{s-1})C_{s-1}) \\ &= \overline{w}_{s-1}(\overline{x}_P - i_N)^{s_\sigma(1-s_\tau)}(\overline{x}_N - i_P)^{(1-s_\sigma)s_\tau}(\overline{x}_P - i_P)^{s_\sigma s_\tau}(\overline{x}_N - i_N)^{(1-s_\sigma)(1-s_\tau)}\end{aligned}$$

$$\textit{for every } s, \qquad (22a)$$

where $s_\tau{=}1$ if C_{s-1} is positive, $s_\tau{=}0$ if C_{s-1} is negative, $s_\sigma{=}1$ if $\overline{w}_{s-1}$ is positive, $s_\sigma{=}0$ if $\overline{w}_{s-1}$ is negative. Summing for s we have

$$\mathrm{SVA} = \mathrm{NFV} \qquad (22b)$$

or, more explicitly,

$$\begin{aligned}\mathrm{SVA} = &\sum_{s:\overline{w}_{s-1}>0, C_{s-1}<0}^{n} \overline{w}_{s-1}(\overline{x}_P - i_N) + \sum_{s:\overline{w}_{s-1}<0, C_{s-1}>0}^{n} \overline{w}_{s-1}(\overline{x}_N - i_P) \\ &+ \sum_{s:\overline{w}_{s-1}>0, C_{s-1}>0}^{n} \overline{w}_{s-1}(\overline{x}_P - i_P) + \sum_{s:\overline{w}_{s-1}<0, C_{s-1}<0}^{n} \overline{w}_{s-1}(\overline{x}_N - i_N)\end{aligned}$$

Further

$$\mathrm{SVA}_s = \overline{\mathrm{EVA}}_s \qquad \textit{for every } s \qquad (22c)$$

Proof: For the sake of convenience I shall label some propositions with conventional notations:

A_1: (i_P, i_N) is a twin-pair

A_2: the shadow pair $(\overline{x}_P, \overline{x}_N)$ and the internal pair (x_P, x_N) are parallel

A_3: for all s, $i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1} = -i(C_{s-1})\overline{w}_{s-1}$

A_4: $x(w_{s-1})w_{s-1} = \overline{x}(\overline{w}_{s-1})\overline{w}_{s-1}$

A_5: $\text{SVA}_s = \overline{w}_{s-1}(\overline{x}(\overline{w}_{s-1}) - i(C_{s-1}))$

A_6: $\overline{\text{EVA}}_s = -\overline{w}_{s-1}(1 + i(C_{s-1})) + \overline{w}_s + \overline{a}_s = -\overline{w}_{s-1}(1 + i(C_{s-1})) + \overline{w}_{s-1}(1 + \overline{x}(\overline{w}_{s-1}))$.

A_1 implies A_3, A_2 implies A_4. A_3, A_4 and (16) imply A_5, which in turn implies (22a).

Consider now that the Net Final Value of P (NFV) is, by definition, the difference between the terminal net worths relative to the alternative courses of action (i) and (ii): $\text{NFV}=E_n - E^n$. But $E_n - E^n$ coincides, for (11), with SVA, which is the sum of all SVA_s. Hence, (22b) holds.

Let us now calculate the shadow project's Economic Value Added ($\overline{\text{EVA}}_s$). It is easy to see that

$$\overline{w}_s = \overline{w}_{s-1}(1 + \overline{x}(\overline{w}_{s-1})) - \overline{a}_s$$

since the shadow pair and the internal pair are parallel. We can then interpret $\overline{w}_s$ as $\overline{P}$'s project balance at time s at the rate $\overline{x}(\overline{w}_{s-1})$. Applying Peccati's argument we get to A_6. The latter coincides with A_5, so that (22c) holds. (Q.E.D.)

Note that (22) tells us that for all s, one of the following holds:

(*) $\overline{w}_{s-1}(\overline{x}_P - i_N) = \overline{\text{EVA}}_s$
(**) $\overline{w}_{s-1}(\overline{x}_N - i_P) = \overline{\text{EVA}}_s$
(***) $\overline{w}_{s-1}(\overline{x}_P - i_P) = \overline{\text{EVA}}_s$
(****) $\overline{w}_{s-1}(\overline{x}_N - i_N) = \overline{\text{EVA}}_s$

in the following cases, respectively:

(*) $\overline{w}_{s-1} > 0$ and $C_{s-1} < 0$
(**) $\overline{w}_{s-1} < 0$ and $C_{s-1} > 0$
(***) $\overline{w}_{s-1} > 0$ and $C_{s-1} > 0$
(****) $\overline{w}_{s-1} < 0$ and $C_{s-1} < 0$

Theorem (SVA2). *If E_0=0 and the shadow pair and the internal pair are parallel, then (22) holds with*

$$s_\tau = 1 \qquad \textit{iff} \qquad s_\sigma = 0.$$

Proof: As before let us make use of the following conventions:

B_1: E_0=0

B_2: the shadow pair $(\overline{x}_P, \overline{x}_N)$ and the internal pair (x_P, x_N) are parallel

B_3: (i_P, i_N) is a twin-pair

B_4: for all s, C_s=$-\overline{w}_s$

B_5: $\overline{x}(\overline{w}_{s-1})$=$\overline{x}_P$ if and only if $i(C_{s-1})$=i_N

B_6: $\overline{x}_{s-1}(\overline{w}_{s-1})-i(C_{s-1})=(\overline{x}_P - i_N)$ or $\overline{x}_{s-1}(\overline{w}_{s-1})-i(C_{s-1})=(\overline{x}_N - i_P)$

B_7: s_τ=1 if and only if s_σ=0

B_1 implies B_3 and B_4 (Proposition 3). B_2 and B_3 imply (22) (SVA1). B_4 implies B_5. B_5 implies B_6. B_6 and (22a) imply B_7.

(Q.E.D.)

Proposition 8. *(22a) holds if and only if*

$$x(w_{s-1})w_{s-1} - \overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} = C^{s-1}\left[i(C^{s-1}) - i(C_{s-1})\right] \qquad \textit{for every } s \qquad (23)$$

Proof: (22a) holds if and only if

$$\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} - i(C_{s-1})\overline{w}_{s-1} = x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1}$$

whence

$$\begin{aligned} x(w_{s-1})w_{s-1} - \overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} &= i(C^{s-1})C^{s-1} - i(C_{s-1})\left[\overline{w}_{s-1} + C_{s-1}\right] \\ &= C^{s-1}\left[i(C^{s-1}) - i(C_{s-1})\right]. \end{aligned}$$

I now prove that the assumptions of SVA1 and SVA 2 are not necessary for (22) to hold, by providing a counterexample. Choose E_0=-30, n=2, a_0=700,a_1=850, x_N=0.35, x_P=0.3, i_N=0.15, i_P=0.0630434782608. We have then

$$\begin{aligned}
C^0 &= -30 < 0 & C^1 &= -30(1.15) = -34.5 < 0\\
C_0 &= -730 < 0 & C_1 &= -730(1.15) + 850 = 10.5 > 0\\
w_0 &= a_0 = 700 > 0 & w_1 &= 700(1.3) - 850 = 60 > 0\\
\overline{w}_0 &= C^0 - C_0 = 700 > 0 & \overline{w}_1 &= C^1 - C_1 = -45 < 0\\
i(C^0) &= i_N & i(C^1) &= i_N\\
i(C_0) &= i_N & i(C_1) &= i_P\\
x(w_0) &= x_P & x(w_1) &= x_P\\
\overline{x}(\overline{w}_0) &= \overline{x}_P = \frac{x_P w_0}{\overline{w}_0} & \overline{x}(\overline{w}_1) &= \overline{x}_N = \frac{x_N w_1}{\overline{w}_1}
\end{aligned}$$

and a_2 is univocally determined (=78). (23) holds, since

$$0.3 * 700 - 0.3 * 700 = -30\,[0.15 - 0.15]$$

for period 1, and

$$0.3 * 60 - (-45) * \frac{0.35 * 60}{-45} = -34.5\,[0.15 - 0.0630434782608]$$

for period 2. Further,

$$\mathrm{SVA}_1 = \overline{w}_0(\overline{x}_P - i_N)$$

$$\mathrm{SVA}_2 = \overline{w}_1(\overline{x}_P - i_N).$$

Therefore, (22a) holds, where s_τ=1 if and only if s_σ=0, as required in SVA2.

I have provided a counterexample which proves that the assumptions of both SVA1 and SVA2 are not necessary, since (22) holds, whereas neither of their assumptions holds: We have, in fact,

(#1) $E_0 \neq 0$
(#2) (i_P, i_N) is non-twin
(#3) the shadow pair and the internal pair are not parallel.

Remark 2: Note that (#2) implies (#1) (Proposition 3, *modus tollens*).

Proposition 9. *If (22) holds then (i_P, i_N) is twin if and only if the shadow pair and the internal pair are parallel.*

Proof: (23) holds (Proposition 8). Assume (i_P, i_N) is twin. Then the right-hand side of (23) must be zero for all s, which implies the same for the left-hand side, that is the the shadow pair and the internal pair are parallel. Assume now the latter. Then the left-hand side of (23) is zero for all s, which implies the same for the right-hand side. Therefore, E_0 is zero or (i_P, i_N) is twin. Should the former of these two hold, then the latter is implied (Proposition 3).

Proposition 9 enables us to prove that if (22) holds we cannot have (#2) without (#3) and vice versa. Thus, if we want to prove that the assumptions of SVA1 are not necessary we cannot invalidate only one of them.

Proposition 10. *Suppose that (22) holds alongside (#2) or (#3). Then the other one also holds.*

Proof: Proposition 9 and Proposition 10 are tautological equivalent.

For convenience of the reader I restate here P&S Theorem, making explicit the implicit assumption E_0=0:

P&S Theorem. *Assume E_0=0. Then the* NFV *of P can be written as*

$$\text{NFV} = \sum_{s:w_{s-1}>0}^{n} w_{s-1}(x_P - i_N)(1+i(C))^{n-s} + \sum_{s:w_{s-1}<0}^{n} w_{s-1}(x_N - i_P)(1+i(C))^{n-s}$$

if and only if the shadow pair and the internal pair are parallel.[8]

We are now ready to state the systemic counterpart of P&S Theorem.

Theorem (SVA3). *Assume E_0=0. Then (22) holds with*

$$s_\tau = 1 \qquad \text{iff} \qquad s_\sigma = 0$$

if and only if the shadow pair and the internal pair are parallel.

In particular, the NFV *of P can be written as*

$$\text{NFV} = \sum_{s:\overline{w}_{s-1}>0}^{n} \overline{w}_{s-1}(\overline{x}_P - i_N) + \sum_{s:\overline{w}_{s-1}<0}^{n} \overline{w}_{s-1}(\overline{x}_N - i_P)$$

[8]Strictly speaking, P&S assume (at the outset of their paper) C_0=$-a_0$, which has (at least) the two interpretations previously seen. I focus on interpretation (b), because it is the more natural one in our approach, but nothing would change should we adopt (a).

Proof: Assume that, in addition to E_0=0, the shadow pair and the internal pair are parallel: then (22) holds, with

$$s_\tau = 1 \qquad \text{if and only if} \qquad s_\sigma = 0$$

(SVA2). Conversely, assume that, in addition to E_0=0, (22) holds with

$$s_\tau = 1 \qquad \text{if and only if} \qquad s_\sigma = 0.$$

Then (i_P, i_N) is a twin-pair (Proposition 3) and the shadow pair and the internal pair are parallel (Proposition 9). (Q.E.D.)

Theorem (SVA4). *If both P and $\overline{P}$ are Soper projects and E_0=0, then (22) holds with s_σ=1 and s_τ=0 for all s.*

Proof: Let

C_1: E_0=0

C_2: P is a Soper project

C_3: $\overline{P}$ is a Soper project

C_4: the shadow pair $(\overline{x}_P, \overline{x}_N)$ and the internal pair (x_P, x_N) are parallel

C_5: (i_P, i_N) is a twin-pair

C_6: (i_P, i_N) is an i_N-twin-pair

C_7: s_σ=1 and s_τ=0 for all s.

C_1 implies C_5 (Proposition 3). C_2 and C_3 imply C_4 (Proposition 6). C_4 and C_5 imply (22) (SVA1). C_1 and C_3 imply C_6 (Proposition 5). C_1 and C_4 imply that, for all s, one of the following holds:

$$\text{SVA}_s = \overline{w}_{s\text{-}1}(\overline{x}_P - i_N) \tag{24a}$$

$$\text{SVA}_s = \overline{w}_{s\text{-}1}(\overline{x}_N - i_P) \tag{24b}$$

(SVA3). As C_6 holds, (24b) must be ruled out, and (24a) coincides with C_7. (Q.E.D.)

I restate here Proposition 6.1 of P&S (*op.cit.*, p.179) in our systemic parlance:

Proposition 11.1. *If E_0=0, (i_P, i_N) is an i_N-twin-pair, P is a Soper project, then*

$$\text{NFV} = \sum_{s=1}^{n} w_{s\text{-}1}(x_P - i_N)(1 + i_N)^{n-s}.$$

I now prove the systemic counterpart of Proposition 11.1

Proposition 11.2. *If E_0=0, (i_P, i_N) is an i_N-twin-pair, P is a Soper project, then the conclusion of* SVA4 *holds.*

In particular, we have

$$\text{NFV} = \text{SVA} = \sum_{s=1}^{n} \overline{w}_{s\text{-}1}(\overline{x}_P - i_N).$$

Proof: The first two hypotheses imply that $\overline{P}$ is a Soper project (Proposition 5). The latter, the first hypothesis and the third hypothesis are just SVA4's assumptions, so that (24a) holds.

Remark 3: The two Propositions get back to a particular case of Peccati's model, in which E_0 is zero, P is assumed to be a Soper project and the value of account C is always negative. Even though, strictly speaking, they are not inconsistent each other in overall terms, it is clear that the periodic NFV's shares differ and that different perspectives are at work. My systemic decomposition is "accounting-flavored", Peccati's, Stewart's and P&S's decompositions are "NFV-flavored", so to say. This is true for all the results here obtained.

Remark 4: A striking result is that Proposition 11.1 of P&S can be easily proved if we make use of our systemic approach. The proof is straightforward, due to Proposition 11.2, (14b) and the following equalities:

$$\begin{aligned}
&\overline{w}_{s\text{-}1}(\overline{x}_P - i_N) = \overline{\text{EVA}}_s = \text{SVA}_s \\
&w_{s\text{-}1}(x_P - i_N) = \text{EVA}_s.
\end{aligned}$$

Note also that the first two hypotheses in Propositions 11.1 and 11.2 imply that $\overline{P}$ is a Soper project. This suggests us that we can relax the first hypothesis:

Proposition 11.2.1. *If (i_P, i_N) is an i_N-twin-pair and both P and $\overline{P}$ are Soper projects, then the conclusion of* SVA4 *holds.*

In particular, we have

$$\text{NFV} = \text{SVA} = \sum_{s=1}^{n} \overline{w}_{s\text{-}1}(\overline{x}_P - i_N).$$

Proof: The first hypothesis implies (i_P, i_N) is a twin-pair, with $i(C_{s-1})=i(C^{s-1})=i_N$. The second and the third hypotheses imply that the shadow pair and the internal pair are parallel, with $x(w_{s-1}) = x_P$ and $\overline{x}(\overline{w}_{s-1}) = \overline{x}_P$ (Proposition 6). Hence, (22) holds with s_σ=1 and s_τ=0 for all s.

As for P&S model, we have the following

Proposition 11.1.1. *If* (i_P, i_N) *is an* i_N*-twin-pair and both* P *and* $\overline{P}$ *are Soper projects, then*

$$\text{NFV} = \sum_{s=1}^{n} w_{s-1}(x_P - i_N)(1 + i_N)^{n-s}.$$

Proof: We just have to make use of the systemic approach. The proof mirrors the argument in *Remark 4*, relying on Proposition 11.2.1, (14b) and the equalities shown.

Remark 5: We could further generalize Proposition 11.1.1 by removing the third assumption on $\overline{P}$ being a Soper project. The latter is essential only if we want to prove the Proposition via Proposition 11.2.1. The first two hypotheses are actually sufficient to get to the conclusion, because (14b) holds regardless of being $\overline{P}$ a Soper project or not.[9]

Remark 6: On the basis of the latter Proposition's proof and *Remark 4* one may wonder whether we can use the systemic approach to prove all the results P&S have reached. The answer is yes but I will not dwell on it, leaving a thourough investigation for a next paper. I just give some hints for the the proof of P&S Theorem. The proof is easy: We just have to use SVA3, (14b) and remember that

$$\overline{\text{EVA}}_s = \text{SVA}_s = \overline{w}_{s-1}(\overline{x}_P - i_N) \quad \text{whenever} \quad \text{EVA}_s = w_{s-1}(x_P - i_N)$$

$$\overline{\text{EVA}}_s = \text{SVA}_s = \overline{w}_{s-1}(\overline{x}_N - i_P) \quad \text{whenever} \quad \text{EVA}_s = w_{s-1}(x_N - i_P).$$

Conclusive Remarks

This paper deals with different aspects of the decomposition of a cash flow stream. For the sake of clarity I briefly summarize the main results as follows:

[9]However, if P is a Soper project but $\overline{P}$ is not, the shadow pair and the internal pair are not parallel, as Proposition 7 indirectly suggests. This means we are not sure that

$$\overline{w}_{s-1}(\overline{x}_P - i_N) = \text{SVA}_s$$

for every s, so that Proposition 11.2.1 needs the "Soper condition" for both P and $\overline{P}$ to ensure its conclusion.

(A) Peccati's model coincides, formally and to a certain extent, with Stewart's EVA model
(B) P&S generalize, but only at a certain extent, PE model
(C) the notion of Systemic Value Added is introduced and used to reach a decomposition of a project's NFV different from the aforementioned models
(D) some relations between SVA and EVA are studied. In particular, SVA model enables us to avoid capitalization processes. The sum of the periodic SVA_s coincide with the NFV, whereas the sum of the periodic EVA_s do not (we have to compound them before we can sum)
(E) each of the models presupposes a particular notion of residual income. PE decomposition and P&S decomposition are NFV-flavored, SVA model is accounting-flavored. The former ones show some flaws, in that the implicit assumptions on the financial system's evolution are self-contradictory
(F) Peccati's and Stewart's arguments can be formally retrieved by introducing the concept of shadow project. In order to obtain the periodic Systemic Value Added of P we just have to compute the Economic Value Added of the shadow project. The NFV of P is thus obtained by the sum of all periodic Economic Value Added so obtained, with no need of compounding.
(G) the concept of shadow project is essential. It is project P's *alter ego*, and enables us to incorporate the NFV-based models into the systemic framework. Applying the NFV-based argument to $\overline{P}$ we can formally write the NFV of P as the sum of n addends, each of which is given by the difference between the rate of return of $\overline{P}$ and the rate of account C, times the total capital invested in $\overline{P}$. Capitalization turns out to be, in a systemic perspective, inessential and unnatural, as we have seen in Sec.4 (see (15b) and (15c)). The shadow project is therefore that project that allows us to adopt a notion of residual income consistent with the financial system's evolution, while expressing the residual income in terms of Economic Value Added.
(H) some formal results are provided shedding lights on the relations among the concepts of twin-pair, parallel pairs, Soper project, as well as between some results obtained by P&S and the results here obtained. The results of Sec.4 are generalized providing sufficient conditions (e.g. SVA1, SVA2, SVA3, SVA4, Proposition 8) and necessary conditions (e.g. SVA3, Proposition 8, Proposition 9) for the models to be embodied in a systemic approach. In particular, SVA3 and P&S Theorem play the same role in the two approaches, and Proposition 11.1 is shown to be the counterpart of Proposition 11.2, and the assumptions for both are finally relaxed.
(I) strictly speaking, either PE model or P&S model can be seen as the generalization of the other. P&S generalize PE model in that they allow for a two-valued rate for account C, PE model generalizes P&S model in that it is not confined to the assumption $C_0 = -a_0$. As seen, SVA model generalizes all these models, in that it can cope with a two-valued rate for C and allow for whatsoever C_0. This generalization is

based on a particular interpretation of the concept of residual income which does not imply, contrary to the other models, any contradictions or ambiguities (the ambiguity of the concept of EVA is studied in Magni (2000b)).

(L) at the end of the paper we have discovered that the systemic perspective is capable of proving some of the results P&S have reached. Proposition 11.1.1 can be proved by using the results we have obtained by applying the new concept of Systemic Value Added. P&S Theorem can also be proved via systemic approach: To this end, (14b) plays a central role, as it shows us the relation between EVA and SVA, so that the proof (which I have just sketched) can be said to rely on transparent financial arguments.

(M) the striking result according to which the Systemic Value Added of P is just the Economic Value Added of $\overline{P}$ may suggest that a financial calculus could be developed where the notion of discounting and compounding is, in some cases, superfluous. Further, in problems where disaggregation of cash flows is under consideration, the capitalization process seems to be only a device to adjust the periodic shares so as to obtain a correct overall evaluation

(N) accounting is often thought of as misleading if used in evaluating projects. "Accountancy deals with accounting values, not with cash values" is often said. Yet (not accounting itself but) the systemic perspective accounting relies on can be quite useful in apprasing investment opportunities. In this sense accounting and finance can be strictly linked and are far from being incompatible. In this light, our SVA model is maybe a second step toward that integration between the two disciplines, whose first step is represented by the pioneering contributions of Peccati and Stewart (see also Magni (1999) on this issue).

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