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Job-Searching and Job-Matching in a Two-Sector General Equilibrium Model

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Abstract: We extend the benchmark model of DMP in a two-sector general equilibrium framework by introducing a frictionless segment of the labour market. We also examine the effects of trade reforms and labour market reforms on equilibrium rate of unemployment and wage inequality in our stylitzed economy. We find that both these reforms reduce equilibrium rate of unemployment. However, trade reforms raise wage inequality but labour market reforms reduce it. These results provide a strong theoretical basis for labour market reform in a small open economy characterized by frictional labour market.

Jel Classification: F 16

Key Words: Job-searching, Job-matching, General equilibrium, Trade liberalization.

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1. Introduction:

Job-searching and job-matching are the two striking features of the labour market. In the labour market, we find flows of workers, flows of jobs offered, new vacancies are created, old jobs are destroyed, workers search good jobs, firms search good workers. All these facts have been included in the matching models of the labour market. The job-matching models generally explain the existence of equilibrium rate of unemployment and give insights to the planners to pursue various economic policies in the fractional labour market.

The benchmark model on the job-matching is the Diamond-Mortensen-Pissarides (called DMP hereafter) model. Other notable works include the models of Diamond (1982a, 1982b, 1984), Pissarides (1979, 1984, 1985a, 1985b, 1986, 2000), Mortensen (1987), Mortensen and Pissarides (1994, 1998, 1999), Sheng and Xu (2007) etc. All these models emphasized the role of matching in the labour market. In reality, both the workers and the firms search to meet each others. Production starts only when they are matched. But matching is not an instantaneous process; rather it takes time and is costly. Matching generates surplus which is distributed between workers and firm. The most commonly used surplus-sharing rule is the Nash-bargaining solution.[1]

The matching function has been used first in the models of Hall (1979), Pissarides (1979), Diamond and Maskin (1979), Bowden (1980). Matching function is described as black-box where vacancies and unemployed workers jointly produce output.[2] Empirically, it has been found that matching is a function of unemployment rate and vacancy rate and is subject to CRS.[3]

In this paper, we develop a two-sector matching model in general equilibrium framework along the line of DMP. Actually, we extend the DMP model in a two-sector general equilibrium framework by incorporating a frictionless segment of the labour market. Here, one sector has frictionless labour market where job-searching and job-matching are

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[1] In the standard literature, we also find wage-posting as a tool for the split of match surplus.

[2] See Pissarides (2000).

[3] The Cobb-Douglas type matching functions have been used by Blanchard et al. (1989), Pissarides (2000) and Stevens (2007).

instantaneous but in the other sector labour market is frictional and work like the DMP model. The comparative static exercises of our analysis shows that both the trade reforms and the labour market reforms soften the problem of unemployment and labour market reform lowers wage inequality while trade reforms raise it in a small open economy having frictional labour market. We also show that under the Hosios (1990) efficiency condition, the equilibrium rate of unemployment would be lower if the elasticity of the matching function is higher.

2. The Model:

We consider a two-sector small open economy. The two sectors are sector 1 which is the export sector and produces commodity, X_1 and the other sector is the import-competing sector 2 which produces commodity, X_2. The prices of the two commodities P_1, P_2 are given due to the small country assumption. The price of commodity 1 is chosen as numeraire. The two sectors use both labour and capital in production. The production functions are subject to CRS and diminishing marginal productivity. Capital is mobile between the two sectors and this gives a unique rate of return on capital.

Labour is also mobile across the sectors but labour market is segmented. Both workers and the firms search in both sectors.. But in one sector (sector 1), job-search and job-match are instantaneous and so this sector is frictionless. Here, workers are paid according to their marginal products. However, the other sector is frictional where matchings are time consuming and costly. In this sector (sector 2) the two-way searching gets fruitful after incurring some costs.

In the frictional labour market job-search is an ongoing process. Jobs are offered to the workers and the workers arrive at the jobs offered. So, there exists job-matching between workers and firm in sector 2. Following DMP we may consider the matching function as $m = m(u, v)$, where m stands for matching, u is the rate of unemployment and v is the vacancy rate and $m_1, m_2 > 0, m_{11}, m_{22} < 0, m_{12} = m_{21} = 0$. Total flow of matches is

$m = au$ and total flow of jobs is $m = vq$. So, $\frac{m}{u} \equiv a$ is the job arrival rate and $\frac{m}{v} = q$ is the job offer rate. Matching function is assumed to possess CRS property and so we may

write $q = q(\theta), a \equiv \frac{m}{u} = \frac{m}{v}\frac{v}{u} = \theta q(\theta)$ where $\theta = \frac{v}{u}$ is the labour market tightness and $q'(\theta) < 0, \left|e^{\theta}_{q}\right| < 1$.[4]

2.1 Value Equations:

The Bellman equations for the values of unemployment (U), employment (W), vacancy (V) and jobs filled in (J) are

$$rU = \theta q(\theta)(W - U) \tag{1}$$
$$rW = w_2 - \lambda(W - U) \tag{2}$$
$$rV = -C + q(\theta)(J - V) \tag{3}$$
$$rJ = P_2^* t - w_2 - rk_2 - \lambda J \tag{4}$$

Equation (1) states that unemployment gives option of a discrete change in the valuation from U to W.[5] This equation holds at steady state where discount rate, transaction rate and income flows are constant. Equation (2) embraces that the asset value of employment is the wage rate in sector $2(w_2)$ less employment gain when negative shock arises, where λ the job destruction rate which is given exogenously. Equation (3) shows that the asset value of vacancy yields, at the rate $q(\theta)$, a discrete change in its valuation from V to J less a given flow cost C to maintain vacancy**.** Finally, Equation (4) states that the

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[4] Note that in steady state , $\frac{1}{q(\theta)}$ is the expected duration of vacancy and $\frac{1}{\theta q(\theta)}$ is the expected duration of unemployment (Pissarides, 2000).

[5] We assume that there is no unemployment benefit.

value of a job filled in is the flow of profit ($P_2^* t - w_2 - rk_2$) to the firm less the jobs destroyed where t is the match productivity.[6]

2.2 Job-Creation Condition:

A firm creates jobs up to the point where $V = 0$. Putting this condition into Equation (3) one gets

$$J = \frac{C}{q(\theta)} \tag{5}$$

Substituting (5) into (4) we can write

$$P_2^* t = w_2 + rk_2 + \frac{(r+\lambda)C}{q(\theta)} \tag{6}$$

This is the job-creation condition at steady state. This shows that at steady state, value of the product is equal to the wage cost plus rental cost plus recruitment cost of labour. [7]

2.3 Wage function in the frictional Sector:

In the search and matching model, production begins when firm and workers are matched. If the match is broken both of them again search and can produce after new match. But the search is expensive which can be saved by staying together. So, match generates surplus. This surplus can be shared by both the matched workers and firms. The most commonly used surplus sharing rule is the Nash-bargaining solution. The Nash-bargaining solution allocates surplus according to the returns from search on both sides.
The Nash-bargaining solution can be obtained from the following exercise:

$$\underset{w_2}{Max}\ \Omega = (W-U)^{\beta}\, (J-V)^{(1-\beta)} \tag{7}$$

---

[6] We may assume full productivity of match i.e. $t = 1$.
[7] Since each match produces output, value of the match coincides with the value of the product.

Where β is the bargaining strength of the workers and $1 > \beta > 0$.
Assuming interior solutions exist, the first order condition is

$$(W - U) = \beta (W - U + J - V) \tag{8}$$

This is the surplus sharing rule in search equilibrium. This rule states that at steady state the net gain to the workers $(W - U)$ is equal to the fixed proportion, β of the total surplus, $(W - U + J - V)$.

Using Equations (1), (2), (4) and the zero-profit condition for the firm, $V = 0$, from (8) we can get[8]

$$w_2 = \beta \left(P_2^* t + C\theta - rk_2 \right) \tag{9}$$

This is the wage equation for the frictional sector. This shows that wage in frictional sector depends positively on the productivity of the sector and on the market tightness, given the discount rate, r.[9]

Now solving the two basic Equations (6) and (9) we can get the equilibrium values of w_2, θ.

2.4 Unemployment rate:

The conventional labour force is assumed to be fixed. Following Pissarides (2000) we may derive the rate of unemployment in the following way:

Suppose, at time t unemployment is u_t and employment is $\left(1 - u_t\right)$. In short time interval αt, $\theta_t q\left(\theta_t\right) u_t \alpha t$ workers are matched and $\lambda\left(1 - u_t\right)\alpha t$ lose their jobs. So, unemployment in this interval is

$$u_{t+\alpha t} = u_t - \theta_t q\left(\theta_t\right) u_t \alpha t + \lambda\left(1 - u_t\right)\alpha t \tag{10}$$

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8 See Appendix A.
9 At steady state, k is constant.

$$\therefore u_{t+\alpha t} - u_t = -\theta_t q\left(\theta_t\right) u_t \alpha t + \lambda\left(1-u_t\right)\alpha t$$

$$\therefore \underset{\alpha t \to 0}{Lt}\left(\frac{u_{t+\alpha t} - u_t}{\alpha t}\right) = -\theta_t q\left(\theta_t\right) u_t + \lambda\left(1-u_t\right)$$

$$\therefore \dot{u} = -\theta q(\theta) u + \lambda(1-u) \tag{11}$$

At steady state, $\dot{u} = 0$.

$$\therefore u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{12}$$

This is the equilibrium rate of unemployment. This is also known as the Beveridge curve which shows an inverse relation between u,θ, given λ. Putting the equilibrium value of θ into (12) we can get equilibrium u

2.5 **The General Equilibrium Structure of the Model**

The structure of the two-sector general equilibrium model consists of the following equations:

The price equations of the two sectors are

$$w_1 a_{L1} + r a_{K1} = 1 \tag{13}$$

$$w_2 a_{L2} + r a_{K2} = P_2^* \tag{14}$$

Where P_2^* is the tariff-inclusive price of the commodity 2. All a_{ij} are functions of $w_j, r \forall j = 1,2$.

The wage equation for the frictional sector is given by

$$w_2 = rU + \frac{\beta}{(1-\beta)}(r+\lambda)\frac{C}{q(\theta)} \tag{15}$$

An unemployed worker in sector 2 either searches job in this sector or get employed in sector 1. As job-seeker he gets unemployment income rU and as worker in sector 1 he gets wage, w_1. The no-arbitrage condition implies that in equilibrium,

$$rU = w_1 \tag{16}$$

The equilibrium rate of unemployment is

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{17}$$

Labour is not fully employed but capital is fully employed. Thus, the two factor endowment equations are

$$a_{L1}X_1 + a_{L2}X_2 = (1-u)L \tag{18}$$

$$a_{K1}X_1 + a_{K2}X_2 = K \tag{19}$$

Where L, K are the fixed supply of labour and capital respectively.

Using (1) and (16) into (13) and (14) one gets[10]

$$\xi q(\theta)\theta a_{L1} + ra_{K1} = 1 \tag{13.1}$$

$$\xi\left[r + \lambda + \theta q(\theta)\right]a_{L2} + ra_{K2} = P_2^* \tag{14.1}$$

Where $\xi = \frac{\beta}{(1-\beta)}\frac{C}{q(\theta)}$ is the frictional cost of labour in sector 2. We may write $\xi = \xi(\beta, \theta)$ with $\xi_\beta, \xi_\theta > 0$ (ξ_i is the elasticity of frictional cost of labour with respect to i where $i = \beta, \theta$.

[10] See Appendix A.

Now we can determine the equilibrium values of seven endogenous variables: $w_1, w_2, r, \theta, u, X_1, X_2$ from seven Equations (13.1), (14.1), (15)-(19). Solving (13.1) and (14.1) we get equilibrium values of θ, r. Then, from (15), (16) and (17) we get w_1, w_2, u. Finally, Equations (18) and (19) yield X_1, X_2.

3. Comparative Static Exercises:

Taking total differentials of Equations (13.1), (14.1), (15), (16) and after simplification the following results can be obtained:[11]

$$\left(\frac{\hat{w}_2 - \hat{w}_1}{\hat{P}_2^*}\right) < 0, \quad \left(\frac{\hat{w}_2 - \hat{w}_1}{\hat{\beta}}\right) > 0 \tag{20}$$

These results lead to the following proposition:

Proposition 1: In the presence of search friction in the labour market a fall in the tariff-inclusive price of the commodity produced in the import-competing sector raises wage inequality and a fall in the bargaining strength of the labour reduces it in a small open economy.

We may give an intuitive explanation of proposition 1. Trade liberalization reduces P_2^*. It can be verified from Equations (13.1) and (14.1) that a fall in P_2^* leads to an increase in θ and a decrease in r. When θ rises the average recruitment cost, $C\theta$ rises. As a result value of unemployment, rU rises, given β. This, under the no-arbitrage condition, implies that w_1 also rises. From (14.1) it can be observed that w_2 also rises. Under the capital-intensity condition, $\left(\theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2}\right) > 0$ w_2 rises more than w_1. Therefore, trade liberalization raises wage inequality in our small open economy where labour

[11] See Appendix B Appendix C.

market is frictional. On the contrary, a fall in the bargaining power of the labour raises both θ, r. From (16) it can be verified that rU falls and so also w_1. From (14.1) it can be observed that w_2 also falls. Here also the capital-intensity condition implies that w_2 falls more than w_1. So, wage inequality decreases.

Taking total differentials of (17) and using (13.1), (14.1) and after simplification one gets[12]

$$\left(\frac{\hat{u}}{\hat{P}_2^*}\right) > 0, \left(\frac{\hat{u}}{\hat{\beta}}\right) > 0 \tag{21}$$

These results give the following proposition:

Proposition2: Both trade reform and labour market reform lower the equilibrium rate of unemployment in a small open economy having frictional labour market.

Proposition 2 can be explained as follows. From (13.1) and (14.1) it can be verified that a fall in P_2^* and/ β raises θ. From the Beveridge curve (Equation, 17) it is evident that u must fall when θ rises.

4. Efficiency and Matching Function:

One of the most important aspects of the DMP model is to determine the equilibrium rate of unemployment in the frictional labour market. The social planner may question the efficiency of this unemployment rate at the steady state equilibrium. Hosios (1990) derived the condition under which the unemployment rate would be the efficient one. The condition of efficiency is that the firm’s share to the match surplus is equal to the elasticity of the matching function with respect to vacancy rate i.e. $\beta = 1 - e_m^v$.

[12] See Appendix B, Appendix C.

It is generally agreed that the matching function is subject to the CRS. However, Mortensen (2011) uses two types of matching functions: one is linear, $m = a_1 u + a_2 v$ and the other is quadratic, $m = \left(a_1 + a_2\right) uv$ where a_1, a_2 are constant terms. In the linear case, $e_m^v < 1$. This implies that $1 > \beta > 0$ and this is compatible to our model. On the other hand, in the quadratic form of the matching function as considered by Mortensen (2011) $e_m^v = 1$. In this case $\beta = 0$ and in our set-up this implies that $w_2 = \text{rU} = \text{w}_1 = 0, \xi = 0$.[13] This shows that labour is like a free gift of nature which is purely utopian. Further, as the value of e_m^v rises the value of β falls under the Hosios (1990) Efficiency condition. In our set-up, low value of β implies high value of θ which in turn gives low equilibrium u and this leads to the following proposition:

Proposition 3: Under the Hosios (1990) efficiency condition, we get an inverse relation between the elasticity of the matching function and the equilibrium rate of unemployment.

5. Concluding Remarks:

In this paper we extend the DMP model in general equilibrium framework. Like the DMP model we also assume determination of wage rate in the frictional sector through the Nash-bargaining solution. However, the marginal productivity rule is applied to determine wage rate in the frictionless sector and the unique discount rate.

We introduce no-arbitrage condition in the labour market.[14] Our theoretical analysis shows that trade reform softens the problem of unemployment but raises wage inequality in a small open economy characterized by search friction in the labour market. However,

[13] In this situation wage posting may be considered as an alternative to the Nash-bargaining solution and this may lead to a single wage equal to the reservation wage if only one wage is offered (Pissarides, 2000).

[14] See Sheng and Xu (2007) in this context.

labour market reform lowers both wage inequality and equilibrium rate of unemployment. Thus, our theoretical results provide a strong ground for labour market reform and weak ground for trade reform in a small open economy where labour market is frictional.

Finally, we again establish the role of the matching function in the determination of the equilibrium rate of unemployment. We find that under the Hosios (1990) efficiency condition, the greater the elasticity of the matching function the smaller the equilibrium rate of unemployment in the frictional sector.

Appendix A Derivation of Some Useful Expressions:

The Nash –bargaining problem is

$$\underset{w_2}{Max}\ \Omega = (W-U)^{\beta}\,(J-V)^{(1-\beta)} \tag{7}$$

The first order condition for maximization is

$$\beta\,(J-V)\frac{\partial}{\partial w_2}(W-U)+(1-\beta)(W-U)\frac{\partial}{\partial w_2}(J-V)=0 \tag{A.1}$$

Using (2), (4) and the zero-profit condition $V=0$ into (A.1) one gets

$$(1-\beta)\left(w_2-rU\right)=\beta\left(P_2^{*}t-w_2-rk_2\right)$$

Or

$$w_2=(1-\beta)\,rU+\beta\left(P_2^{*}t-rk_2\right) \tag{A.2}$$

Using (2), (5), (A.1) and V=0 from (1) we can write

$$rU=\frac{\beta C\theta}{(1-\beta)} \tag{1.1}$$

Using (1.1) into (A.2) one gets

$$w_2 = \beta\left(P_2^* t + C\theta - rk_2\right) \tag{9}$$

Appendix B Effects of a Change in P_2^*, β on θ, r, w_1, w_2, u:

Using (1.1) from (16) we get

$$w_1 = \frac{\beta C\theta}{(1-\beta)} = \xi q(\theta)\theta \tag{A.3}$$

Using (1.1) into (15) we may get

$$w_2 = \frac{\beta}{(1-\beta)}\frac{C}{q(\theta)}\left(r+\lambda+\theta q(\theta)\right)$$
$$= \xi\left(r+\lambda+\theta q(\theta)\right) \tag{A.4}$$

Taking total differentials of Equations (13.1) and (14.1) and after simplifications we get

$$\left(1+e_q^\theta+e_\xi^\theta\right)\theta_{L1}\hat{\theta}+\theta_{K1}\hat{r} = -\theta_{L1}e_\xi^\beta\hat{\beta} \tag{A.5}$$

$$\left[e_\xi^\theta+\frac{\xi q(\theta)\theta}{w_2}\left(1+e_q^\theta\right)\right]\theta_{L2}\hat{\theta}+\left(\frac{\xi r}{w_2}\theta_{L2}+\theta_{K2}\right)\hat{r} = \hat{P}_2^* - e_\xi^\beta\theta_{L2}\hat{\beta} \tag{A.6}$$

Solving (A.5) and (A.6) we get

$$\hat{\theta} = \frac{1}{\Delta}\left[-\theta_{K1}\hat{P}_2^* - \left\{\underset{(+)}{\left(\theta_{L1}\theta_{K2}-\theta_{K1}\theta_{L2}\right)}+\frac{\xi r}{w_2}\theta_{L1}\theta_{L2}\right\}\underset{(+)}{e_\xi^\beta}\hat{\beta}\right] \tag{A.7}$$

$$\hat{r} = \frac{1}{\Delta}\left[\underset{(+)}{\left(1+e_q^\theta+e_\xi^\theta\right)}\theta_{L1}\hat{P}_2^* + \underset{(+)}{\left(1+e_q^\theta\right)}\underset{(-)}{\left(\frac{w_1}{w_2}-1\right)}\theta_{L1}\theta_{L2}\underset{(+)}{e_\xi^\beta}\hat{\beta}\right] \tag{A.8}$$

where

$$
\begin{aligned}
\Delta &= \left(1+e_q^{\theta}+\xi_q^{\theta}\right)\left(\frac{\xi r}{w_2}\theta_{L2}+\theta_{K2}\right)\theta_{L1}-\left[\xi_q^{\theta}+\frac{\xi q(\theta)\theta}{w_2}\left(1+e_q^{\theta}\right)\right]\theta_{L2}\theta_{K1} \\
&= \left(1+e_q^{\theta}+\xi_q^{\theta}\right)\frac{\xi r}{w_2}\theta_{L1}\theta_{L2}+\left[\begin{array}{l}\left(1+e_q^{\theta}+\xi_q^{\theta}\right)\theta_{L1}\theta_{K2}- \\ \qquad\left\{\xi_q^{\theta}+\frac{\xi q(\theta)\theta}{w_2}\left(1+e_q^{\theta}\right)\right\}\theta_{L2}\theta_{K1}\end{array}\right]>0 \\
&\qquad \because\left(\theta_{L1}\theta_{K2}>\theta_{L2}\theta_{K1}\right) \text{ and} \left(1+e_q^{\theta}+\xi_q^{\theta}\right)>\left\{\xi_q^{\theta}+\frac{\xi q(\theta)\theta}{w_2}\left(1+e_q^{\theta}\right)\right\}
\end{aligned}
\tag{A.9}
$$

From (A.7)-(A.9) we get

$$
\frac{\hat{\theta}}{\hat{P}_2^*}=-\frac{1}{\underset{(+)}{\Delta}}\theta_{K1}<0,\quad \frac{\hat{\theta}}{\hat{\beta}}=-\frac{1}{\underset{(+)}{\Delta}}\left\{\underset{(+)}{\left(\theta_{L1}\theta_{K2}-\theta_{K1}\theta_{L2}\right)}+\frac{\xi r}{w_2}\theta_{L1}\theta_{L2}\right\}\underset{(+)}{e_{\xi}^{\beta}}<0 \tag{A.7.1}
$$

$$
\frac{\hat{r}}{\hat{P}_2^*}=\frac{1}{\underset{(+)}{\Delta}}\underset{(+)}{\left(1+e_q^{\theta}+e_{\xi}^{\theta}\right)}\theta_{L1}>0,\quad \frac{\hat{r}}{\hat{\beta}}=\frac{1}{\underset{(+)}{\Delta}}\underset{(+)}{\left(1+e_q^{\theta}\right)}\underset{(-)}{\left(\frac{w_1}{w_2}-1\right)}\theta_{L1}\theta_{L2}\underset{(+)}{e_{\xi}^{\beta}}<0 \tag{A.8.1}
$$

Again from (13) and (14) we get

$$
\hat{w}_1=-\frac{\theta_{K1}}{\theta_{L1}}.\hat{r} \tag{A.10}
$$

and

$$
\hat{w}_2=-\frac{\theta_{K2}}{\theta_{L2}}.\hat{r} \tag{A.11}
$$

Using (A.10) and (A.11) we get

$$\left(\hat{w}_2-\hat{w}_1\right)=-\left(\frac{\theta_{K2}}{\theta_{L2}}-\frac{\theta_{K1}}{\theta_{L1}}\right).\hat{r} \tag{A.12}$$

$$(+)$$

Using (A.7.1), (A.8.1) from (A.12 one gets

$$\frac{\left(\hat{w}_2-\hat{w}_1\right)}{P_2^*}=-\left(\frac{\theta_{K2}}{\theta_{L2}}-\frac{\theta_{K1}}{\theta_{L1}}\right).\frac{\hat{r}}{P_2^*}<0 \tag{A.12.1}$$

$$(+) \qquad (+)$$

$$\frac{\left(\hat{w}_2-\hat{w}_1\right)}{\hat{\beta}}=-\left(\frac{\theta_{K2}}{\theta_{L2}}-\frac{\theta_{K1}}{\theta_{L1}}\right).\frac{\hat{r}}{\hat{\beta}}>0 \tag{A.12.2}$$

$$(+) \qquad (-)$$

Taking total differentials of (17) and after simplifying one gets

$$\hat{u}=-(1-u)\left(1+e_q^{\theta}\right)\hat{\theta} \tag{A.13}$$

Using (A.7.1) from (A.13) we get

$$\frac{\hat{u}}{\hat{P}_2^*}=-(1-u)\left(1+e_q^{\theta}\right)\frac{\hat{\theta}}{\hat{P}_2^*}>0 \tag{A.13.1}$$

$$(+) \quad (-)$$

$$\frac{\hat{u}}{\hat{\beta}}=-(1-u)\left(1+e_q^{\theta}\right)\frac{\hat{\theta}}{\hat{\beta}}>0 \tag{A.13.2}$$

$$(+) \quad (-)$$

Appendix C: Effects of Changes in p_2^*, β on Sectoral Output

Taking total differentials of Equations (18) and (19) and using the definitions of the elasticity of factor substitutions and after simplifications we get

$$\lambda_{L1}\hat{X}_1+\lambda_{L2}\hat{X}_2=\lambda_{L1}\sigma_1\theta_{K1}\hat{w}_1+\lambda_{L2}\sigma_2\theta_{K2}\hat{w}_2-\left(\lambda_{L1}\sigma_1\theta_{K1}+\lambda_{L2}\sigma_2\theta_{K2}\right)\hat{r}-(1-u)\hat{L}-u\hat{u} \tag{A.14}$$

$$\lambda_{K1}\hat{X}_1+\lambda_{K2}\hat{X}_2=-\lambda_{K1}\sigma_1\theta_{L1}\hat{w}_1-\lambda_{K2}\sigma_2\theta_{L2}\hat{w}_2+\left(\lambda_{K1}\sigma_1\theta_{L1}+\lambda_{K2}\sigma_2\theta_{L2}\right)\hat{r}+\hat{K} \tag{A.15}$$

Solving (A.14) and (A.15) by Cramer's rule one gets

$$\hat{X}_1=\frac{1}{|\lambda|}\begin{bmatrix}\left(\lambda_{K2}\lambda_{L1}\theta_{K1}+\lambda_{L2}\lambda_{K1}\theta_{L1}\right)\sigma_1\hat{w}_1+\lambda_{K2}\lambda_{L2}\sigma_2\hat{w}_2- \\ \left\{\left(\lambda_{K2}\lambda_{L1}\theta_{K1}+\lambda_{L2}\lambda_{K1}\theta_{L1}\right)\sigma_1+\lambda_{K2}\lambda_{L2}\sigma_2\right\}\hat{r}+ \\ \lambda_{\mathrm{K}2}(1-u)\hat{L}-\lambda_{L2}\hat{K}-\lambda_{K2}u\hat{u}\end{bmatrix} \tag{A.16}$$

$$\hat{X}_2=\frac{1}{|\lambda|}\begin{bmatrix}\lambda_{L1}\hat{K}-\lambda_{L1}\lambda_{K1}\sigma_1\hat{w}_1-\left(\lambda_{L1}\lambda_{K2}\theta_{L2}+\lambda_{K1}\lambda_{L2}\theta_{K2}\right)\sigma_2\hat{w}_2+ \\ \left\{\lambda_{L1}\lambda_{K1}\sigma_1+\left(\lambda_{L1}\lambda_{K2}\theta_{L2}+\lambda_{K1}\lambda_{L2}\theta_{K2}\right)\sigma_2\right\}\hat{r}- \\ \lambda_{K1}(1-u)\hat{L}+\lambda_{K1}u\hat{u}\end{bmatrix} \tag{A.17}$$

where

$|\lambda|=\left(\lambda_{L1}\lambda_{K2}-\lambda_{K1}\lambda_{L2}\right)>0$ (Since sector 1is assumed to be labour-intensive vis-à-vis sector 2).

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