Choice of strategic variables under relative profit maximization in asymmetric oligopoly: non-equivalence of price strategy and quantity strategy

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Choice of strategic variables under relative profit maximization in asymmetric oligopoly: non-equivalence of price strategy and quantity strategy

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Abstract

We consider a simple model of the choice of strategic variables under relative profit maximization by firms in an asymmetric oligopoly with differentiated substitutable goods such that there are three firms, Firm 1, 2 and 3, demand functions are linear and symmetric, marginal costs are constant, there is no fixed cost, Firm 2 and 3 have the same cost function, but Firm 1 has a different cost function. In such a model we show that there are two pure strategy sub-game perfect equilibria. One is such that all firms choose the outputs as their strategic variables, and the other is such that Firm 2 and 3 choose the outputs as their strategic variables, and Firm 1 chooses the price as its strategic variable.

Keywords. relative profit maximization; asymmetric oligopoly; choice of strategic variables

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1 Introduction

In a symmetric duopoly with differentiated goods Tanaka (2013) has shown the following results.

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> If firms maximize their relative profits, the choice of strategic variables, price or output, is irrelevant to the outcome of the game in the sense that the equilibrium outputs, prices and profits of the firms are the same in all situations. Thus, any combination of strategy choice by the firms constitutes a sub-game perfect equilibrium in a two stage game such that in the first stage the firms choose their strategic variables and in the second stage they determine the values of their strategic variables.

Symmetry means that demand functions are symmetric and firms have the same cost function. This conclusion can be extended to a symmetric oligopoly and an asymmetric duopoly. But it can not be extended to an asymmetric oligopoly. Asymmetry means that demand functions may be asymmetric or firms may have different cost functions.

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. Please see Gibbons and Murphy (1990), Lu (2011), Matsumura, Matsushima and Cato (2013), Miller and Pazgal (2001), Vega-Redondo (1997) and Schaffer (1989).

In Vega-Redondo (1997), it is argued that, in a homogeneous good case, if firms maximize their relative profits, a competitive equilibrium can be induced. But in the case of differentiated goods, the result under relative profit maximization is different from the competitive result.

We think that seeking for relative profit or utility is based on the nature of human. Even if a person earns a big money, if his brother/sister or close friend earns a bigger money than him, he is not sufficiently happy and may be disappointed. On the other hand, even if he is very poor, if his neighbor is more poor, he may be consoled by that fact.

In this paper we consider a simple model of the choice of strategic variables under relative profit maximization by firms in an asymmetric oligopoly with differentiated substitutable goods such that there are three firms, Firm 1, 2 and 3, demand functions are linear and symmetric, marginal costs of the firms are constant, there is no fixed cost, Firm 2 and 3 have the same cost function, but Firm 1 has a different cost function. In such a model we show the following result.

> There are two pure strategy sub-game perfect equilibria. One is such that all firms choose the outputs as their strategic variables, and the other is such that Firm 2 and 3 choose the outputs as their strategic variables, and Firm 1 chooses the price as its strategic variable.

In the next section we present a model of this paper. In Section 3 we analyze equilibria in the second stage of the game. In this stage (the managers of) firms determine the values of their strategic variables so as to maximize their relative profits. In Section 4 we investigate equilibria in the first stage of the game. In this stage the owners of firms choose the strategic variables so as to maximize the relative profits of the firms. In Section 5 we briefly mention the results when the owners of firms seek to maximize the absolute profits of the firms. Miller and Pazgal (2001) presented a similar study. They showed the equivalence of price strategy and quantity strategy in a delegation game of a duopoly when owners of firms control managers of firms seek to maximize an appropriate combination of absolute and relative profits. In their model the owner of each firm determines the weight on the rival firm's profit in the objective function of its firm. We do not consider, however, such a delegation game. In our analysis owners of firms choose the strategic variables.

2 The model

There are three firms. Firm 1, 2 and 3. They produce differentiated substitutable goods. The outputs and prices of the goods of the firms are denoted by x_1, x_2, x_3, p_1, p_2 and p_3. The inverse demand functions are

$$p_1 = a - x_1 - bx_2 - bx_3, \tag{1}$$

$$p_2 = a - x_2 - bx_1 - bx_3, \tag{2}$$

$$p_3 = a - x_3 - bx_1 - bx_2. \tag{3}$$

We assume $a > 0$ and $0 < b < 1$.

From (2) and (3), from (1) and (3) and from (1) and (2) we have

$$p_2 + p_3 = 2a - (1+b)(x_2 + x_3) - 2bx_1,$$

$$p_1 + p_3 = 2a - (1+b)(x_1 + x_3) - 2bx_2,$$

$$p_1 + p_2 = 2a - (1+b)(x_1 + x_2) - 2bx_3.$$

Substituting them into (1), (2) and (3), we obtain the following ordinary demand functions.

$$x_1 = \frac{1}{(1-b)(1+2b)}[(1-b)a - (1+b)p_1 + bp_2 + bp_3], \tag{4}$$

$$x_2 = \frac{1}{(1-b)(1+2b)}[(1-b)a - (1+b)p_2 + bp_1 + bp_3], \tag{5}$$

$$x_3 = \frac{1}{(1-b)(1+2b)}[(1-b)a - (1+b)p_3 + bp_1 + bp_2]. \tag{6}$$

The inverse and ordinary demand functions are symmetric.

The constant marginal costs of Firm 1, 2 and 3 are c_1, c_2 and c_3. There is no fixed cost. In Section 4 we assume $c_2 = c_3$ and $c_1 \neq c_2$.

The relative profit of a firm is defined as the difference between its (absolute) profit and the average of the (absolute) profits of the rival firms.

We consider a two stage game. In the first stage (owners of) the firms choose their strategic variables, price or quantity (output), and in the second stage (managers of) the firms determine the values of their strategic variables. In Section 4 we consider a case where owners of firms choose the strategic variables to maximize the relative profits, and in Section 5 we briefly mention a case where owners of firms choose the strategic variables to maximize the absolute profits.

3 The second stage of the game

3.1 One firm is a price setting firm

In this subsection we assume that one of the firms is a price setting firm, that is, it chooses the price as its strategic variable, and other two firms are quantity setting firms, that is, they choose the outputs as their strategic variables. The price setting firm is Firm 2, and the quantity setting firms are Firm 1 and 3. The output and the price of the good of Firm 1 are denoted by x_1 and p_1, and so on. The ordinary demand function for Firm 2 is

$$x_2 = a - p_2 - bx_1 - bx_3. \tag{7}$$

From this and (1), (2) and (3), the inverse demand functions for Firm 1 and 3 are derived as follows.

$$p_1 = (1-b)a + bp_2 - (1-b^2)x_1 - b(1-b)x_3, \tag{8}$$

and

$$p_3 = (1-b)a + bp_2 - (1-b^2)x_3 - b(1-b)x_1. \tag{9}$$

Denote the relative profits of Firm 1, 2 and 3 by Π_1, Π_2 and Π_3. Then,

$$\begin{aligned}\Pi_1 =&[(1-b)a + bp_2 - (1-b^2)x_1 - b(1-b)x_3]x_1 - c_1x_1 - \frac{1}{2}\{(a-p_2-bx_1-bx_3)(p_2-c_2)\\&+[(1-b)a + bp_2 - (1-b^2)x_3 - b(1-b)x_1]x_3 - c_3x_3\},\end{aligned}$$

$$\begin{aligned}\Pi_2 =&(a-p_2-bx_1-bx_3)(p_2-c_2) - \frac{1}{2}\{[(1-b)a + bp_2 - (1-b^2)x_1 - b(1-b)x_3]x_1\\&-c_1x_1 + [(1-b)a + bp_2 - (1-b^2)x_3 - b(1-b)x_1]x_3 - c_3x_3\},\end{aligned}$$

and

$$\begin{aligned}\Pi_3 =&[(1-b)a + bp_2 - (1-b^2)x_3 - b(1-b)x_1]x_3 - c_3x_3 - \frac{1}{2}\{(a-p_2-bx_1-bx_3)(p_2-c_2)\\&+[(1-b)a + bp_2 - (1-b^2)x_1 - b(1-b)x_3]x_1 - c_1x_1\}.\end{aligned}$$

The conditions of relative profit maximization for Firm 1, 2 and 3 are

$$(1-b)a + bp_2 - 2(1-b^2)x_1 - b(1-b)x_3 - c_1 - \frac{1}{2}[-bp_2 - b(1-b)x_3 + bc_2] = 0,$$

$$a - 2p_2 - bx_1 - bx_3 + c_2 - \frac{1}{2}(bx_1 + bx_3) = 0,$$

and

$$(1-b)a + bp_2 - 2(1-b^2)x_3 - b(1-b)x_1 - c_3 - \frac{1}{2}[-bp_2 - b(1-b)x_1 + bc_2] = 0.$$

Arranging the terms,

$$-3bp_2 + 4(1-b^2)x_1 + b(1-b)x_3 = 2(1-b)a - 2c_1 - bc_2, \tag{10}$$
$$4p_2 + 3bx_1 + 3bx_3 = 2a + 2c_2, \tag{11}$$
$$-3bp_2 + b(1-b)x_1 + 4(1-b^2)x_3 = 2(1-b)a - 2c_3 - bc_2 \tag{12}$$

are derived. From (10), (11), (12) we get the equilibrium price of Firm 2 and the equilibrium outputs of Firm 1 and 3 as follows.

$$p_2^{1p} = \frac{(1-b)(4-b)a + (4+b-2b^2)c_2 + 3bc_1 + 3bc_3}{(2+b)(4-b)},$$

$$x_1^{1q} = \frac{(1-b)(4-b)(4+3b)a + b(1-b)(4+3b)c_2 - (16-7b^2)c_1 + b(4+5b)c_3}{(1-b)(2+b)(4-b)(4+3b)},$$

and

$$x_3^{1q} = \frac{(1-b)(4-b)(4+3b)a + b(1-b)(4+3b)c_2 + b(4+5b)c_1 - (16-7b^2)c_3}{(1-b)(2+b)(4-b)(4+3b)}.$$

A superscript p indicates *price setting firm*, a superscript q indicates *quantity setting firm*, and 1 indicates the case of this subsection. Assuming that $c_2 = c_3$ and $c_1 \neq c_2$, the relative profits of Firm 1, 2 and 3 are written as

$$\begin{aligned}
\Pi_1^{1q} =& \frac{c_2 - c_1}{(1-b)(2+b)(4-b)^2(4+3b)^2}[(256 - 288b^2 - 16b^3 + 57b^4 - 9b^5)a \\
&- (128 + 64b - 104b^2 - 28b^3 + 21b^4)c_1 - (128 - 64b - 184b^2 + 12b^3 + 36b^4 - 9b^5)c_2], \\
\Pi_2^{1p} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4-b)^2(4+3b)^2}[(256 - 288b^2 - 16b^3 + 57b^4 - 9b^5)a \\
&- (128 + 64b - 104b^2 + 8b^3 + 39b^4)c_1 - (128 - 64b - 184b^2 - 24b^3 + 18b^4 - 9b^5)c_2], \\
\Pi_3^{1q} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4-b)^2(4+3b)^2}[(256 - 288b^2 - 16b^3 + 57b^4 - 9b^5)a \\
&- (128 + 64b - 104b^2 - 64b^3 + 3b^4)c_1 - (128 - 64b - 184b^2 + 48b^3 + 54b^4 - 9b^5)c_2].
\end{aligned}$$

If Firm 3 is a price setting firm, the relative profits of the firms are written as follows.

$$\begin{aligned}
\Pi_1^{1q} =& \frac{c_2 - c_1}{(1-b)(2+b)(4-b)^2(4+3b)^2}[(256 - 288b^2 - 16b^3 + 57b^4 - 9b^5)a \\
&- (128 + 64b - 104b^2 - 28b^3 + 21b^4)c_1 - (128 - 64b - 184b^2 + 12b^3 + 36b^4 - 9b^5)c_2], \\
\Pi_2^{1q} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4-b)^2(4+3b)^2}[(256 - 288b^2 - 16b^3 + 57b^4 - 9b^5)a \\
&- (128 + 64b - 104b^2 - 64b^3 + 3b^4)c_1 - (128 - 64b - 184b^2 + 48b^3 + 54b^4 - 9b^5)c_2], \\
\Pi_3^{1p} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4-b)^2(4+3b)^2}[(256 - 288b^2 - 16b^3 + 57b^4 - 9b^5)a \\
&- (128 + 64b - 104b^2 + 8b^3 + 39b^4)c_1 - (128 - 64b - 184b^2 - 24b^3 + 18b^4 - 9b^5)c_2].
\end{aligned}$$

If Firm 1 is a price setting firm, the relative profits of the firms are written as follows.

$$\Pi_1^{1p} = \frac{(c_2 - c_1)[(16 - 8b + b^2)a - (8 + b^2)c_1 - 8(1 - b)c_2]}{(2 + b)(4 - b)^2},$$

$$\Pi_2^{1q} = -\frac{(c_2 - c_1)[(16 - 8b + b^2)a - (8 + b^2)c_1 - 8(1 - b)c_2]}{2(2 + b)(4 - b)^2},$$

$$\Pi_3^{1q} = -\frac{(c_2 - c_1)[(16 - 8b + b^2)a - (8 + b^2)c_1 - 8(1 - b)c_2]}{2(2 + b)(4 - b)^2}.$$

3.2 Two firms are price setting firms

In this subsection we assume that one of the firms is a quantity setting firm, and other two firms are price setting firms. The quantity setting firm is Firm 3 and the price setting firms are Firm 1 and 2. The inverse demand function for Firm 3 is

$$p_3 = \frac{1}{1 + b}[(1 - b)a - (1 - b)(1 + 2b)x_3 + bp_2 + bp_1] \tag{13}$$

From this and (4), (5) and (6), the ordinary demand functions for Firm 1 and 2 are derived as follows.

$$\begin{aligned} x_1 =& \frac{1}{(1 - b)(1 + 2b)}\{(1 - b)a - (1 + b)p_1 \\ &+ \frac{b}{1 + b}[(1 - b)a - (1 - b)(1 + 2b)x_3 + bp_2 + bp_1] + bp_2\} \\ =& \frac{1}{1 - b^2}\{(1 - b)a - p_1 + bp_2 - b(1 - b)x_3\}, \end{aligned} \tag{14}$$

and

$$\begin{aligned} x_2 =& \frac{1}{(1 - b)(1 + 2b)}\{(1 - b)a - (1 + b)p_2 \\ &+ \frac{b}{1 + b}[(1 - b)a - (1 - b)(1 + 2b)x_3 + bp_2 + bp_1] + bp_1\} \\ =& \frac{1}{1 - b^2}\{(1 - b)a - p_2 + bp_1 - b(1 - b)x_3\}. \end{aligned} \tag{15}$$

The relative profits of Firm 1, 2 and 3 are written as follows.

$$\begin{aligned} \Pi_1 =& \frac{1}{1 - b^2}[(1 - b)a - p_1 + bp_2 - b(1 - b)x_3](p_1 - c_1) \\ &- \frac{1}{2}\left\{\frac{1}{1 + b}[(1 - b)a - (1 - b)(1 + 2b)x_3 + bp_2 + bp_1]x_3 - c_3x_3 \right.\\ &\left. + \frac{1}{1 - b^2}[(1 - b)a - p_2 + bp_1 - b(1 - b)x_3](p_2 - c_2)\right\}, \end{aligned}$$

$$\begin{aligned}\Pi_2 =& \frac{1}{1-b^2}\left[(1-b)a - p_2 + bp_1 - b(1-b)x_3\right](p_2 - c_2) \\ &- \frac{1}{2}\left\{\frac{1}{1+b}[(1-b)a - (1-b)(1+2b)x_3 + bp_2 + bp_1]x_3 - c_3x_3 \right. \\ &\left. + \frac{1}{1-b^2}\left[(1-b)a - p_1 + bp_2 - b(1-b)x_3\right](p_1 - c_1)\right\},\end{aligned}$$

and

$$\begin{aligned}\Pi_3 =& \frac{1}{1+b}[(1-b)a - (1-b)(1+2b)x_3 + bp_2 + bp_1]x_3 - c_3x_3 \\ &- \frac{1}{2}\left\{\frac{1}{1-b^2}\left[(1-b)a - p_2 + bp_1 - b(1-b)x_3\right](p_2 - c_2) \right. \\ &\left. + \frac{1}{1-b^2}\left[(1-b)a - p_1 + bp_2 - b(1-b)x_3\right](p_1 - c_1)\right\}.\end{aligned}$$

The conditions of relative profit maximization for Firm 1, 2 and 3 are

$$\frac{1}{1-b^2}\left[(1-b)a - 2p_1 + bp_2 - b(1-b)x_3 + c_1\right] - \frac{1}{2}\left[\frac{b}{1+b}x_3 + \frac{b}{1-b^2}(p_2 - c_2)\right] = 0,$$

$$\frac{1}{1-b^2}\left[(1-b)a - 2p_2 + bp_1 - b(1-b)x_3 + c_2\right] - \frac{1}{2}\left[\frac{b}{1+b}x_3 + \frac{b}{1-b^2}(p_1 - c_1)\right] = 0,$$

and

$$\begin{aligned}&\frac{1}{1+b}[(1-b)a - 2(1-b)(1+2b)x_3 + bp_2 + bp_1] - c_3 \\ &- \frac{1}{2(1-b^2)}[-b(1-b)(p_2 - c_2) - b(1-b)(p_1 - c_1)] = 0.\end{aligned}$$

Arranging the terms,

$$2(1-b)a + bp_2 - 4p_1 - 3b(1-b)x_3 + bc_2 + 2c_1 = 0, \tag{16}$$

$$2(1-b)a - 4p_2 + bp_1 - 3b(1-b)x_3 + 2c_2 + bc_1 = 0, \tag{17}$$

$$\begin{aligned}&2(1-b)^2a - 4(1-b)^2(1+2b)x_3 + 3b(1-b)p_2 + 3b(1-b)p_1 \\ &- 2(1-b)(1+b)c_3 - b(1-b)c_2 - b(1-b)c_1 = 0\end{aligned} \tag{18}$$

are derived. From (16), (17), (18) we get the equilibrium output of Firm 3 and the equilibrium prices of Firm 1 and 2 as follows.

$$x_3^{2q} = \frac{(1-b)(4+5b)a - (1+b)(4-b)c_3 + b(1+2b)c_2 + b(1+2b)c_1}{(1-b)(2+b)(4+5b)},$$

$$\begin{aligned}p_1^{2p} =& \frac{1}{(2+b)(4+b)(4+5b)}[(1-b)(4+b)(4+5b)a + 3b(1+b)(4+b)c_3 \\ &+ 3b(4+7b+2b^2)c_2 + (1+b)(16+16b+b^2)c_1] + c_1,\end{aligned}$$

and

$$
\begin{aligned}
p_2^{2p} =& \frac{1}{(2+b)(4+b)(4+5b)}[(1-b)(4+b)(4+5b)a + 3b(1+b)(4+b)c_3 \\
&+ (1+b)(16+16b+b^2)c_2 + 3b(4+7b+2b^2)c_1] + c_2.
\end{aligned}
$$

A superscript 2 indicates the case of this subsection. Assuming that $c_2 = c_3$ and $c_1 \neq c_2$, the relative profits of Firm 1, 2 and 3 are written as

$$
\begin{aligned}
\Pi_1^{2p} =& \frac{c_2 - c_1}{(1-b)(2+b)(4+b)^2(4+5b)^2}[(256+512b-21b^2-496b^3-215b^4-25b^5)a \\
&-(128+320b+152b^2-124b^3-91b^4-16b^5)c_1 \\
&-(128+192b-184b^2-372b^3-124b^4-9b^5)c_2], \\
\Pi_2^{2p} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4+b)^2(4+5b)^2}[(256+512b-21b^2-496b^3-215b^4-25b^5)a \\
&-(128+320b+152b^2-88b^3-37b^4+2b^5)c_1 \\
&-(128+192b-184b^2-408b^3-178b^4-27b^5)c_2], \\
\Pi_3^{2q} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4+b)^2(4+5b)^2}[(256+512b-21b^2-496b^3-215b^4-25b^5)a \\
&-(128+320b+152b^2-160b^3-145b^4-34b^5)c_1 \\
&-(128+192b-184b^2-336b^3-70b^4-9b^5)c_2].
\end{aligned}
$$

If Firm 2 is a quantity setting firm, the relative profits of the firms are written as follows.

$$
\begin{aligned}
\Pi_1^{2p} =& \frac{c_2 - c_1}{(1-b)(2+b)(4+b)^2(4+5b)^2}[(256+512b-21b^2-496b^3-215b^4-25b^5)a \\
&-(128+320b+152b^2-124b^3-91b^4-16b^5)c_1 \\
&-(128+192b-184b^2-372b^3-124b^4-9b^5)c_2], \\
\Pi_2^{2q} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4+b)^2(4+5b)^2}[(256+512b-21b^2-496b^3-215b^4-25b^5)a \\
&-(128+320b+152b^2-160b^3-145b^4-34b^5)c_1 \\
&-(128+192b-184b^2-336b^3-70b^4-9b^5)c_2], \\
\Pi_3^{2p} =& -\frac{c_2 - c_1}{2(1-b)(2+b)(4+b)^2(4+5b)^2}[(256+512b-21b^2-496b^3-215b^4-25b^5)a \\
&-(128+320b+152b^2-88b^3-37b^4+2b^5)c_1 \\
&-(128+192b-184b^2-408b^3-178b^4-27b^5)c_2].
\end{aligned}
$$

If Firm 1 is a quantity setting firm, the relative profits of the firms are written as follows.

$$\begin{aligned}
\Pi_1^{2q} =& \frac{c_2-c_1}{2(1-b)(2+b)(4+b)^2(4+5b)^2}[(16+24b-15b^2-25b^3)a \\
&-(8+16b+b^2-7b^3)c_1-2(4+4b-8b^2-9b^3)c_2], \\
\Pi_2^{2p} =& -\frac{c_2-c_1}{2(1-b)(2+b)(4+b)^2(4+5b)^2}[(16+24b-15b^2-25b^3)a \\
&-(8+16b+b^2-7b^3)c_1-2(4+4b-8b^2-9b^3)c_2], \\
\Pi_3^{2p} =& -\frac{c_2-c_1}{2(1-b)(2+b)(4+b)^2(4+5b)^2}[(16+24b-15b^2-25b^3)a \\
&-(8+16b+b^2-7b^3)c_1-2(4+4b-8b^2-9b^3)c_2].
\end{aligned}$$

3.3 Cournot and Bertrand equilibria

Consider the Cournot equilibrium in which all firms are quantity setting firms and the Bertrand equilibrium in which all firms are price setting firms. The equilibrium outputs at the Cournot equilibrium are obtained as follows.

$$x_1^C = \frac{(4-b)a-(4+b)c_1+bc_2+bc_3}{(2+b)(4-b)},$$

$$x_2^C = \frac{(4-b)a-(4+b)c_2+bc_1+bc_3}{(2+b)(4-b)},$$

and

$$x_3^C = \frac{(4-b)a-(4+b)c_3+bc_1+bc_2}{(2+b)(4-b)}.$$

C indicates *Cournot*. Assuming $c_3 = c_2$ and $c_1 \neq c_2$, the relative profits of Firm 1, 2 and 3 at the Cournot equilibrium are written as

$$\begin{aligned}
\Pi_1^C &= \frac{(c_2-c_1)[(4-b)^2a-(8+b^2)c_1-8(1-b)c_2]}{(2+b)(4-b)^2}, \\
\Pi_2^C &= -\frac{(c_2-c_1)[(4-b)^2a-(8+b^2)c_1-8(1-b)c_2]}{2(2+b)(4-b)^2}, \\
\Pi_3^C &= -\frac{(c_2-c_1)[(4-b)^2a-(8+b^2)c_1-8(1-b)c_2]}{2(2+b)(4-b)^2}.
\end{aligned}$$

The equilibrium prices at the Bertrand equilibrium are

$$p_1^B = \frac{(1-b)(4+5b)a+(4+7b+4b^2)c_1+3b(1+b)c_2+3b(1+b)c_3}{(2+b)(4+5b)},$$

$$p_2^B = \frac{(1-b)(4+5b)a+(4+7b+4b^2)c_2+3b(1+b)c_1+3b(1+b)c_3}{(2+b)(4+5b)},$$

and

$$p_3^B = \frac{(1-b)(4+5b)a + (4+7b+4b^2)c_3 + 3b(1+b)c_1 + 3b(1+b)c_2}{(2+b)(4+5b)}.$$

B indicates *Bertrand*. Assuming that $c_2 = c_3$ and $c_1 \neq c_2$, the relative profits of Firm 1, 2 and 3 at the Bertrand equilibrium are

$$\Pi_1^B = \frac{(c_2-c_1)[(16+24b-15b^2-25b^3)a-(8+16b+b^2-7b^3)c_1-2(4+4b-8b^2-9b^3)c_2]}{(1-b)(2+b)(4+5b)^2},$$

$$\Pi_2^B = -\frac{(c_2-c_1)[(16+24b-15b^2-25b^3)a-(8+16b+b^2-7b^3)c_1-2(4+4b-8b^2-9b^3)c_2]}{2(1-b)(2+b)(4+5b)^2},$$

$$\Pi_3^B = -\frac{(c_2-c_1)[(16+24b-15b^2-25b^3)a-(8+16b+b^2-7b^3)c_1-2(4+4b-8b^2-9b^3)c_2]}{2(1-b)(2+b)(4+5b)^2}.$$

4 The first stage of the game

The owner of each firm maximizes the relative profit of its firm.

4.1 The best responses of Firm 1

1. Assume that Firm 2 and 3 are quantity setting firms. Compare the relative profit of Firm 1 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\Pi_1^{1p} - \Pi_1^C = -\frac{9b^3(4+b)(c_2-c_3)^2}{2(1-b)(4-b)^2(4+3b)^2} = 0. \quad (19)$$

   Thus, price and output are indifferent, and both are best responses.

2. Assume that Firm 2 and 3 are price setting firms. Compare the relative profit of Firm 1 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\Pi_1^B - \Pi_1^{2q} = -\frac{9b^3(4+3b)(c_2-c_3)^2}{2(1-b)(4+b)^2(4+5b)^2} = 0. \quad (20)$$

   Thus, price and output are indifferent, and both are best responses.

3. Assume that one of Firm 2 and 3 is a quantity setting firm and the other is a price setting firm. Compare the relative profit of Firm 1 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\Pi_1^{2p} - \Pi_1^{1q} = -\frac{144b^7(2+b)(c_1-c_2)^2}{(1-b)(4-b)^2(4+b)^2(4+3b)^2(4+5b)^2} < 0. \quad (21)$$

   Thus, the output is the best response of Firm 1.

4.2 The best responses of Firm 2 (or 3)

The situation of Firm 2 and that of Firm 3 are symmetric.

1. Assume that Firm 1 and 3 are quantity setting firms. Compare the relative profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, since $c_1 \neq c_2$ and $c_2 = c_3$ we have

$$\Pi_2^{1p} - \Pi_2^C = -\frac{9b^3(4+b)(c_1-c_2)^2}{2(1-b)(4-b)^2(4+3b)^2} < 0. \tag{22}$$

   Thus, the output is the best response of Firm 2.

2. Assume that Firm 1 and 3 are price setting firms. Compare the relative profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\Pi_2^B - \Pi_2^{2q} = -\frac{9b^3(4+3b)(c_1-c_2)^2}{2(1-b)(4+b)^2(4+5b)^2} < 0. \tag{23}$$

   Thus, the output is the best response of Firm 2.

3. Assume that Firm 1 is a price setting firm and Firm 3 is a quantity setting firm. Compare the relative profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\Pi_2^{2p} - \Pi_2^{1q} = -\frac{9b^3(3b^3+12b^2+80b+64)(c_1-c_2)^2}{2(1-b)(4-b)^2(4+b)^2(4+5b)^2} < 0. \tag{24}$$

   Thus, the output is the best response of Firm 2.

4. Assume that Firm 3 is a price setting firm and Firm 1 is a quantity setting firm. Compare the relative profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\Pi_2^{2p} - \Pi_2^{1q} = \frac{9b^3(7b^3-44b^2-112b-64)(c_1-c_2)^2}{2(1-b)(4-b)^2(4+3b)^2(4+5b)^2} < 0. \tag{25}$$

   Thus, the output is the best response of Firm 2.

4.3 Sub-game perfect equilibria

From these results we obtain the following conclusion.

Proposition 1 *There are two pure strategy sub-game perfect equilibria as follows.*

1. *All firms choose the outputs as their strategic variables.*
2. *Firm 2 and 3 choose the outputs as their strategic variables, and Firm 1 chooses the price as its strategic variable.*

4.4 A note on a symmetric case

If the oligopoly is symmetric, that is, $c_1 = c_2 = c_3$, all of (21), (22), (23), (24) and (25) are zero. Then, output and price are indifferent for all firms in all situations, and so any combination of strategies of the firms constitutes a sub-game perfect equilibrium.

5 Absolute profit maximizing owners

It may be natural that the owners of firms seek to maximize the absolute profits of their firms. In this section we briefly mention the results of that case. The equilibrium outputs and prices are the same as those in the previous section. We consider the first stage of the game.

Denote the absolute profit of Firm 1 when it is a price setting firm and other firms are quantity setting firms by π_1^{1p}, and so on.

5.1 The best responses of (the owner of) Firm 1

1. Assume that Firm 2 and 3 are quantity setting firms. Compare the absolute profit of Firm 1 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\pi_1^{1p} - \pi_1^C = 0. \tag{26}$$

Thus, price and output are indifferent, and both are best responses.

2. Assume that Firm 2 and 3 are price setting firms. Compare the absolute profit of Firm 1 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\pi_1^B - \pi_1^{2q} = 0. \tag{27}$$

Thus, price and output are indifferent, and both are best responses.

3. Assume that one of Firm 2 and 3 is a quantity setting firm and the other is a price setting firm. Compare the absolute profit of Firm 1 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\pi_1^{2p} - \pi_1^{1q} = -\frac{48b^5(3b^3 - 10b^2 - 48b - 32)(c_1 - c_2)^2}{(1-b)(4-b)^2(4+b)^2(4+3b)^2(4+5b)^2} > 0. \tag{28}$$

Thus, the price is the best response of Firm 1.

5.2 The best responses of (the owner of) Firm 2 (or 3)

The situation of Firm 2 and that of Firm 3 are symmetric.

1. Assume that Firm 1 and 3 are quantity setting firms. Compare the absolute profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\pi_2^{1p} - \pi_2^C = 0. \tag{29}$$

Thus, price and output are indifferent, and both are best responses.

2. Assume that Firm 1 and 3 are price setting firms. Compare the absolute profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\pi_2^B - \pi_2^{2q} = 0. \tag{30}$$

Thus, price and output are indifferent, and both are best responses.

3. Assume that Firm 1 is a price setting firm and Firm 3 is a quantity setting firm. Compare the absolute profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\pi_2^{2p} - \pi_2^{1q} = -\frac{6b^4(b^2 - 24b - 16)(c_1 - c_2)^2}{(1-b)(4-b)^2(4+b)^2(4+5b)^2} > 0.$$

Thus, the price is the best response of Firm 2.

4. Assume that Firm 3 is a price setting firm and Firm 1 is a quantity setting firm. Compare the absolute profit of Firm 2 when it chooses the price as a strategic variable and that when it chooses the output as a strategic variable. Then, we have

$$\pi_2^{2p} - \pi_2^{1q} = \frac{6b^4(9b^2 - 8b - 16)(c_1 - c_2)^2}{(1-b)(4-b)^2(4+3b)^2(4+5b)^2} < 0. \tag{31}$$

Thus, the output is the best response of Firm 2.

From these results about the model of this paper we obtain the following conclusion.

Proposition 2 *There are the following four pure strategy sub-game perfect equilibria.*

1. *All firms choose the outputs as their strategic variables.*

2. *All firms choose the prices as their strategic variables.*

3. *Firm 1 and 2 choose the prices as their strategic variables, and Firm 3 chooses the output as its strategic variable.*

4. *Firm 1 and 3 choose the prices as their strategic variables, and Firm 2 chooses the output as its strategic variable.*

6 Conclusion

We have studied the choice of strategic variables under relative profit maximization in an asymmetric oligopoly with differentiated substitutable goods. We considered a case with three firms such that two firms have the same cost functions and the other one firm has a different cost function, and have shown that there are two pure strategy sub-game perfect equilibria. In duopoly and symmetric oligopoly the equivalence of price and output strategies under relative profit maximization have been proved. In an asymmetric oligopoly, however, they are not equivalent.

On the other hand in the last section we have shown that if the owners of firms seek to maximize the absolute profits of their firms even though (the managers of) firms maximize their relative profits, there are four sub-game perfect equilibria.

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