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Abstract

This paper examines the endogenous choice of competition mode with strategic export policies in vertically related markets. We show that (i) regardless of the nature of goods, choosing Bertrand competition is the dominant strategy for downstream firms, which leads downstream firms to face a prisoners' dilemma; (ii) the optimal export intervention can be a subsidy under Bertrand competition; and (iii) when the choice of competition mode is delegated to upstream firms or to the upstream firm on country and the downstream firm in the other country, multiple equilibria (quantity-price and price-quantity competitions) can be sustained except those for which goods are sufficiently close complements. With the exception of such a case, Bertrand competition can be sustained with this delegation of competition mode choice. Thus, a conflict of interest between downstream and upstream firms may or may not occur, as social welfare depends on who chooses the competition mode and the degree of imperfect complementarity. This contrasts with the result under free trade, which shows that there is no conflict of interests between upstream and downstream firms with Cournot (Bertrand) competition when the goods are substitutes (complements) in equilibrium.

JEL Classification: F12, F13, L13.
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1 Introduction

The theory of a strategic trade policy has progressed remarkably in terms of the game theoretical approach in the 1980s. Adopting a two-stage game, a pioneering work by Brander and Spencer (1985) revealed that strategic export subsidization may enhance the exporting country's welfare under Cournot competition in the third market; the rent-shifting effect of the strategic subsidy can be explained by the firms' distorted objective functions[1]. On the other hand, Eaton and Grossman (1986) showed that strategic export taxation is optimal under Bertrand competition.

Based on both Brander and Spencer (1985) and Eaton and Grossman (1986), Spencer and Jones (1992) investigated whether an import tariff on an intermediate input may reduce the input price under a vertically integrated structure model. Recently, many papers attempt to explain the vertical structure with the strategic trade policy. For example, Bernhofen (1997) showed that a downstream country's optimal policy calls for a tax or a subsidy depending on whether or not the upstream firm charges a discriminating or uniform price. Moreover, this approach has been refined by Ishikawa and Lee (1997), Ziss (1997), Ishikawa and Spencer (1999) and Kawabata (2010), among others, to examine the impacts of vertically related markets on strategic trade policies[2]. The existing literature, however, has been paid little attention to discuss the endogenous choice of strategic variables for prices or

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[1]For more detailed discussion of subsidy policy, see Cooper and Riezman (1989), Brainard and Martimort (1997), Hwang and Mai (2007), and Brander (1995) and references therein.

[2]The vertical structure under the international trade in intermediate goods is the focus of many recent theoretical

quantities on strategic trade policies in the presence of trade in intermediate goods. This study is a step to fills this gap. For the purpose, we combine the model of both Brander and Spencer (1985) and Eaton and Grossman (1986) and discuss the endogenous choice of strategic variables for prices or quantities with either subsidy or tax regime.

Since our issue was addressed in the industrial organization context, the potential impact of government subsidy or tax policy was not theoretically incorporated. Key paper in this area includes Singh and Vives (1984) with the endogenous choice of strategic variables. They were the first to analyze this issue and to show, from the standpoint of consumer surplus and social welfare, that Bertrand competition is more efficient than Cournot competition regardless of the nature of goods. They also show that when goods are substitutes, Cournot equilibrium profits are higher than Bertrand equilibrium profits, and vice versa, when goods are complements. In the industrial organization context, many strands of the literature have produced an array of extensions and generalizations of the analysis in Singh and Vives (1984)[3]. However, none of the previous works have considered a case in which both a domestic firm and a foreign firm choose to set prices or quantities in the vertically related markets under what conditions government needs to help the strategic export policy[4].

In contrast to the choice of strategic variables described by Singh and Vives (1984), we consider the issue of the choice of strategic variables with strategic delegation in vertically related markets. That is, the main issue is who should make the endogenous choice of prices or quantities under a vertically related-international duopoly when each upstream firm sells its intermediate good to its own country's downstream firm. In this paper, we assume that the upstream firm in each country prohibits its country's downstream firm from transacting and distributing the product produced by the rival upstream firm, and that only one downstream firm serves a given upstream firm. Thus, we examine three cases of the delegation problem with respect to the choice of competition mode: (i) the choice of strategic variables for prices or quantities is delegated to the downstream firm in each country; (ii) the choice of these variables is delegated to the upstream firm in each country; (iii) the choice of these variables is delegated to the upstream firm in one country and to the downstream firm in the other country. Using this setting with the vertical structure, we investigate how the preference of each firm (or government) affects social welfare. The firms' profit can change the choice of competition mode when the governments impose export subsidies or taxes on the final good.

The main result of our paper is that regardless of the nature of goods, even though each downstream firm can earn higher profits under Cournot competition than under Bertrand competition, choosing Bertrand competition is the dominant strategy for downstream firms when the choice of competition mode is delegated to the downstream firm in each country with strategic trade policies in a third-country market. The intuition is as follows. The downstream firms' profits depend on the subsidy granted by each government and the cost charged by each upstream firm. Therefore, regardless of the nature of goods, downstream firms produce a higher output with a lower price when choosing

and empirical studies, such as Spencer and Jones (1991), Feenstra and Hanson (1996), McLaren (2000), Grossman and Helpman (2005), Chen *et al.* (2004) and Yi (2003).

[3]For example, Qiu (1997), Lambertini (1997), Hackner (2000), and Zanchettin (2006) among other reveal counter-results based on the original framework by allowing for a wider range of cost and demand asymmetries.

[4]In the literature of strategic trade policies, some works with comparisons of Bertrand and Cournot competition are Cheng (1988), Bagwell and Staiger (1994), Maggi (1996) and among others, where the endogenous choice of strategic variables is not provided, and does not consider the vertical structure.

the price variable than when choosing the quantity variable. The upstream firms charge a higher input price when choosing the quantity variable than when choosing the price variable. Therefore, each downstream firm's optimal strategy leads to a higher output and lower price under the choice of the price variable regardless of the competition mode chosen by the rival firm. This forces both downstream firms to be aggressive in determining their output under Bertrand competition. This leads each firm to face a prisoners' dilemma in equilibrium regardless of the nature of goods (except for the case of a sufficiently high degree of imperfect complementarity). Moreover, we find that the optimal export intervention can be a subsidy under Bertrand competition[5]. This contrasts with the result in Eaton and Grossman (1986) and Clarke and Collie (2006) that, in the absence of vertical structures, the optimal export policy is a tax under Bertrand competition.

However, when the choice of strategic variables is delegated to upstream firms or in the mixed case (i.e., the upstream firm in one country and the downstream firm in the other country), the multiple equilibria of the quantity-price and price-quantity competitions can be sustained unless the degree of imperfect complementarity is sufficiently high. Each downstream firm chooses a lower quantity under Cournot competition and a higher quantity under Bertrand competition. Understanding this fact, the upstream firm uses its input price to increase its profit. Accordingly, the best response of the upstream firm is to charge a higher input price in accordance with the given subsidy and the choice of strategic variables of its country's downstream firm. However, the upstream firm charges a lower input price when the downstream firm chooses the quantity variable than when it chooses the price variable, which means that selling more intermediate goods with a lower input price is the dominant strategy than by selling fewer goods with a higher input price. However, choosing Bertrand competition is the dominant strategy for upstream firms and for the mixed case when the degree of imperfect complementarity is sufficiently large. As a result, a conflict of interest between downstream and upstream firms may or may not occur depending on who chooses the competition mode and the degree of imperfect complementarity.

These two main results imply that social welfare depends on who chooses the competition mode. That is, social welfare when the choice of competition mode is delegated to each downstream firm becomes the same as when there is Bertrand competition in each country. Moreover, the social welfare gap becomes greater if the goods are substitutes when the choice is delegated to both upstream firms or in the mixed case than when the choice is delegated to both downstream firms. In the case of complementary goods, the largest social welfare, which the government prefers, is obtained if the choice is delegated to both downstream firms, while a level of social welfare that is disadvantageous for the government is obtained when the choice of competition mode is delegated to both upstream firms or in the mixed case, with the exception of when goods are sufficiently close complements. Accordingly, for social welfare, governments necessarily determine their policy for the optimal delegation of the choice of the strategic variables according to such circumstances as the nature of goods and product differentiation[6]. This result is in stark contrast to the result under freer trade, which shows that

[5]The intuition is as follows. An export subsidy on the final good increases the demand for the intermediate good, which reduces the input price. This causes a decrease in the marginal cost faced by each downstream firm, which implies that for our setting, the effect of an export tax that increases the final-good price is dominated by the effect of an export subsidy.

[6]In our companion paper, Lim *et al.* (2014) considered analogous analysis under strategic trade policies in a third market in the absence of vertical structures, which exists the conflict of interest between the downstream firms and gov-

there is no conflict of interest between upstream and downstream firms and that choosing Cournot (Bertrand) competition is the dominant strategy for downstream and upstream firms when the goods are substitutes (complements).

2 The Model

We consider two vertically related activities in the home and foreign countries in where the upstream and downstream sectors are modeled as in Brander and Spencer (1985) and a home and foreign final-good firms export all of their output to a third country final-good market. Let 1 and 2 also represent two countries, upstream and downstream firms $i, i = 1, 2$ belonging to country i. The inverse and direct demands are:

$$p_i = 1 - q_i - bq_j, \text{ and } q_i = \frac{1 - b - p_i + bp_j}{1 - b^2}; i, j = 1, 2, i \neq j, \tag{1}$$

where p_i is the price, q_i is the quantity, and $b \in (-1, 1)$. If $b > (<)0$, the products are substitutes (complements). Throughout the main part of the paper, assuming $b \neq 0$, we postulate that firms are engaged in the case of substitutes. In each proposition in our paper, we show that the main results also carry over to the case of complements.

The technology of the final-good production is simplified by assuming that one unit of the intermediate good is required to produce one unit of the final good. Denote the price of the intermediate good in each country by w_i and w_j, respectively. Given an input price w_i for each downstream firm, the two exporting downstream firms' profit are as shown in the following function:

$$\pi_i = (p_i + s_i - w_i)q_i; i = 1, 2, \tag{2}$$

where the home and foreign governments impose export subsidies or taxes s_i, on the final good.

On the other hand, we assume that both downstream firms purchase the intermediate goods from its country's supplier (i.e., upstream firm), which is located in the foreign and home countries. That is, each upstream firm located in each country who is each supplier of the intermediate good offers an input price w_i to each downstream firm. The upstream firm's maximization problem is as follows[7]:

$$u_i = (w_i - c)q_i. \tag{3}$$

Thus, social welfare SW_i is given by

$$SW_i = u_i + \pi_i - s_i q_i = (p_i - c)q_i, \tag{4}$$

This study considers that each firm can make two types of binding contracts with third-market consumers, as described by Singh and Vives (1984). Three cases of delegation are distinguished as introduced in Introduction[8]. Specifically, a four-stage game model is used. In the first stage, either

ernments. That is, there is no delegation problem for the choice of competition mode between upstream and downstream firms.

[7] We exclude the case in where two competing downstream firms through two-part tariffs (i.e., input price and fixed fee) since we focus on the delegation problem with respect to the choice of competition mode between the upstream and downstream firms.

[8] We do not extend to a bargaining problem between upstream and downstream firms in each country. More clear extensions for the bargaining model are left to future research to develop the analysis more generally.

upstream firm or downstream firm i simultaneously commit to choosing strategic variable, i.e., either price or quantity, to set in the vertical structure. In the second stage, the exporting countries decide on the optimal subsidy or tax s_i to maximize its welfare[9]. In third stage, each upstream firm located in each country simultaneously offers input price, w_i to each downstream (home and foreign) firm. In the fourth stage, each downstream firm chooses its quantity or price simultaneously, in order to maximize its objective knowledge of the strategic variable.

3 The Choice of Competition Mode under Free Trade

We briefly present the choice of endogenous strategic variables under free trade. As stated in the Introduction, the model and results differ from those of industrial organization since we consider two vertically related activities in the home and foreign countries in which the downstream and upstream sectors are modeled.

First, since we are considering the case of the free trade regime, we set $s_1 = s_2 = 0$. The profits for downstream firm i are given by $\pi_i^C = (1 - bq_j - q_i - w_i)q_i$, and $\pi_i^B = [(p_i - w_i)(1 - b - w_i - p_i + bp_j)]/(1 - b^2), i = 1, 2$, where superscript C (B) denotes Cournot (Bertrand) competition. At stage four, downstream firm i's best response functions under Cournot and Bertrand competition are given by $BR_i^C(q_j, w_i, s_i = s_j = 0) = (1-bq_j-w_i)/2$ and $BR_i^B(p_j, w_i, s_i = s_j = 0) = (1-b+bp_j+w_i)/2$. Thus, it is straightforward that the equilibrium downstream firm i's profit is derived under each competition mode: $\pi_i^C = \frac{(2-b-2w_i+bw_j)^2}{(4-b^2)^2}$ and $\pi_i^B = [2-b-b^2-w_i(2-b^2)+bw_j]^2/(1-b^2)(4-b^2)^2$. As third stage, the upstream firm simultaneously offers input price w_i to downstream firm. The profit for upstream firm i under Bertrand competition is given by $u_i^C = (w_i - c)q_i$ and $u_i^B = \frac{(w_i-c)(1-b-p_i+bp_j)}{1-b^2}$, which yields that upstream firm i's best response function is given by $BR_i^C(w_j, s_i = s_j = 0) = (2 - b + 2c + bw_j)/4$ and $BR_i^B(w_j, s_i = s_j = 0) = [2 - b - b^2 + c(2 - b^2) + bw_j]/2(2 - b^2)$. Note that we distinguish the ultimate equilibrium values by using notation "$\wedge$" to signify the equilibrium outcomes under free trade, and we use a notation $1 - c \equiv \mu$. From the maximization problem, we obtain the equilibrium values as follows:

Lemma 1: *Suppose that the goods under Cournot and Bertrand competition for free trade, are substitutes. Then, if the upstream firm offers an input price to its country's downstream firm, the equilibrium output, the input price, the price of the final product, the upstream and downstream firms' profits, and*

[9] In the games considered so far, exporting firms are assumed to choose a strategic variable before exporting countries (i.e., governments). Such moving firms first before governments in the literature of strategic trade policy is Brander and Spencer (1987), Blonigen and Ohno (1998), Konishi *et al.* (1999), among others. If we switch stage one and stage two, governments have incentive to lead firms to choose the strategic variable in the sense from the welfare viewpoint. That is, governments do not necessarily set the optimal tax or subsidy in some case. For example, when a government wants to induce Cournot competition, it may commit providing the optimal subsidy if the firm chooses quantity as a strategic variable but impose an extremely high tax if the firm chooses price as a strategic variable.

social welfare are given, respectively, by

$$\hat{q}_i^C = \frac{2\mu}{(4-b)(2+b)}, \quad \hat{w}_i^C = c + \frac{(2-b)\mu}{4-b}, \quad \hat{p}_i^C = c + \frac{(6-b^2)\mu}{(2+b)(4-b)}, \quad \hat{u}_i^C = \frac{2(2-b)\mu^2}{(2+b)(4-b)^2},$$
$$\hat{\pi}_i^C = \frac{4\mu^2}{(2+b)^2(4-b)^2}, \quad S\hat{W}_i^C = \frac{2(6-b^2)\mu^2}{(2+b)^2(4-b)^2}, \quad \hat{q}_i^B = \frac{(2-b^2)\mu}{(1+b)(2-b)(4-b-2b^2)},$$
$$\hat{w}_i^B = c + \frac{(2-b-b^2)\mu}{4-b-2b^2}, \quad \hat{p}_i^B = c + \frac{2(1-b)(3-b^2)\mu}{(2-b)(4-b-2b^2)}, \quad \hat{u}_i^B = \frac{(1-b)(2+b)(2-b^2)\mu^2}{(1+b)(2-b)(4-b-2b^2)^2},$$
$$\hat{\pi}_i^B = \frac{(1-b)(2-b^2)^2\mu^2}{(1+b)(2-b)^2(4-b-2b^2)^2}, \quad S\hat{W}_i^B = \frac{2(1-b)(2-b^2)(3-b^2)\mu^2}{(1+b)(2-b)^2(4-b-2b^2)^2}.$$

Lemma 1 suggests that since input prices under Cournot and Bertrand competition are strategic complements from $BR_i^C(w_j, s_i = s_j = 0)$ and $BR_i^B(w_j, s_i = s_j = 0)$, each upstream firm i sets the input price to be above its marginal cost, which stems from the so called double-marginalization problem. Thus, each downstream firm i sets the final-good price to be above its marginal cost, which enhancing each upstream firm's profit. Moreover, note that under free trade, there are no subsidies under Cournot and Bertrand competition, so government i's payoff is the same as the combined profits of the upstream and downstream firms.

To find endogenous choice of strategic variables for prices or quantities, we need to consider the case of which implies that the firm i optimally choose its quantity as a best response to any price chosen by firm j, and the firm j optimally choose its price as a best response to any quantity chosen by firm i (hereafter we call this "asymmetric competition"). Both demand functions that firms i and j face are given by $p_i = 1 - b + bp_j - (1-b^2)q_i$ and $q_j = 1 - bq_i - p_j$, respectively. Let us denote firm i's (j's) equilibrium values with superscript "$Q(P)$" when firm i (j) sets quantity (price) as a best response to any price (quantity) chosen by firm $j(i)$. By using same process as before, straightforward computation yields the following equilibrium values (straightforward calculations are in Appendix A):

Lemma 2: *Suppose that the goods under asymmetric competition for free trade, are substitutes. Then, if the upstream firm offers an input price to its country's downstream firm, the equilibrium output, the input price, the price of the final product, the upstream and downstream firms' profits, and social welfare are given, respectively, by*

$$\hat{q}_i^Q = \frac{2\mu(8-2b-5b^2+b^3)}{(16-9b^2)(4-3b^2)}, \quad \hat{w}_i^Q = c + \frac{(8-2b-5b^2+b^3)\mu}{16-9b^2}, \quad \hat{u}_i^Q = \frac{2(8-2b-5b^2+b^3)^2\mu^2}{(16-9b^2)^2(4-3b^2)^2},$$
$$\hat{p}_i^Q = c + \frac{(48-12b-70b^2+16b^3+25b^4-5b^5)\mu}{(16-9b^2)(4-3b^2)}, \quad \hat{\pi}_i^Q = \frac{4(1-b^2)(8-2b-5b^2+b^3)^2\mu^2}{(16-9b^2)^2(4-3b^2)^2},$$
$$S\hat{W}_i^Q = \frac{2(6-5b^2)(8-2b-5b^2+b^3)^2\mu^2}{(16-9b^2)^2(4-3b^2)^2}, \quad \hat{p}_j^P = c + \frac{(48-12b-62b^2+8b^3+20b^4)\mu}{(16-9b^2)(4-3b^2)},$$
$$\hat{w}_j^P = c + \frac{(8-2b-5b^2)\mu}{16-9b^2}, \quad \hat{q}_j^P = \frac{(2-b^2)(8-2b-5b^2)\mu}{(16-9b^2)(4-3b^2)} \quad \hat{u}_j^P = \frac{(2-b^2)(8-2b-5b^2)\mu^2}{(16-9b^2)^2(4-3b^2)^2},$$
$$\hat{\pi}_j^P = \frac{(2-b^2)^2(8-2b-5b^2)^2\mu^2}{(16-9b^2)^2(4-3b^2)^2}, \quad S\hat{W}_j^P = \frac{2(2-b^2)(3-2b^2)(8-2b-5b^2)^2\mu^2}{(16-9b^2)^2(4-3b^2)^2}.$$

It is straightforward to verify that under free trade, choosing Cournot (Bertrand) competition is the best downstream and upstream firms can do, which is government i's payoff is the same, when the goods are substitutes (complements). The following proposition can be stated (straightforward calculations are in Appendix A).

Proposition 1: *Suppose that under free trade in a vertically related market, a home and a foreign firm both export to a third-country market. Then, if the choice of competition mode is delegated to either upstream firms or downstream firms, choosing Cournot (Bertrand) competition is the dominant strategy for both downstream and upstream firms when the goods are substitutes (complements). Thus, regardless of the nature of goods except for the case of* $b \in [0.81, 1)$*, the interests of the downstream and upstream firms always coincide with the aspects of social welfare.*

Proof: See Appendix A. ■

Under free trade, there are no subsidies or taxes, so government i's payoff is the same as the combined profits of the upstream and downstream firms. Therefore, it is straightforward to verify that under free trade, welfare is larger in Bertrand competition than in Cournot competition regardless of the nature of goods in the case of $b \in (-1, 0.8]$ and $b \neq 0$. On the other hand, if b falls into range of $b \in [0.81, 1)$, welfare is larger in Cournot competition than in Bertrand competition. That is, the upstream firm recognizes the strategic variable of the downstream firm as a means by which to increase its profit. When the downstream firms compete with each other under Bertrand competition, each downstream firm chooses a higher quantity, but a lower market price, which results in a lower input price. From the viewpoint of social welfare, it is desirable to force the downstream firm to pursue more profit under Bertrand competition than to force the upstream firm to pursue more profit in the case of $b \in (-1, 0.8]$ and $b \neq 0$, while this effect is reversed when b falls into range of $b \in [0.81, 1)$ under Cournot competition. In the end, social welfare faces a trade-off between Bertrand competition and Cournot competition when product differentiation increases in the range of $b \in [0.81, 1)$.

4 Equilibrium Outcomes under the Subsidy Regime

Before the type of contract is applied under subsidy regime in the international model to identify the point of equilibrium, four different cases of contract games are explained. In Bertrand competition, firms set prices, whereas in Cournot competition, firms set quantities. In asymmetric cases, firm i sets the quantity and firm j sets the price and vice versa. Such a game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium (SPNE).

[**Cournot Competition**]: At stage four, taking arbitrary subsidy rates (s_1, s_2) and using inverse demand functions, $p_i = 1 - q_i - bq_j$, we obtain that the downstream firm i's best response function under Cournot competition is given by $BR_i(q_j, s_i, w_i) = (1 - bq_j + s_i - w_i)/2$, which is downward-sloping. Thus, it is straightforward that the equilibrium downstream firm i's profit is derived at stage three: $\pi_i^C = \frac{(2-b+2s_i-bs_j-2w_i+bw_j)^2}{(4-b^2)^2}$. At the third stage, each upstream firm offers the input price w_i to each downstream firm. The upstream firm's maximization problem is as follows: $\max_{w_i} u_i^C = (w_i - c)q_i$. Hence, the upstream firm i's best response function is given by $w_i = (2 - b + 2c + 2s_i - bs_j + bw_j)/4$. Solving the system of response functions, we obtain input price, price and quantity under Cournot competition:

$$w_i^C = c + \frac{(8-2b-b^2)\mu + s_i(8-b^2) - 2b^3 s_j}{16-b^2}, \quad q_i^C = \frac{2(8-2b-b^2)\mu + s_i(8-b^2) - 2bs_j}{64-20b^2+b^4},$$
$$p_i^C = c + \frac{(48-12b-14b^2+2b^3+b^4)\mu - 2s_i(8-3b^2) - 2bs_j(6-b^2)}{64-20b^2+b^4}.$$

Given the output and price at third stage, each government simultaneously chooses subsidy in order to maximize social welfare at the second stage:

$$\max_{s_i} SW_i^C = \frac{2[\alpha][(8-2b-b^2)\mu + (8-b^2)s_i - 2bs_j]}{(4-b^2)(2-b)^2(2+b)^2(4+b)^2},$$
$$\text{where } \alpha = (48-12b-14b^2+2b^3+b^4)\mu - 2s_i(8-3b^2) - 2bs_j(6-b^2).$$

Differentiating SW_i^C with respect to s_i, invoking symmetry ($s_1^C = s_2^C$) and solving yields

$$s_i^C = \frac{(32-8b^2+b^4)\mu}{2(16+8b-4b^2-b^3)} > 0,$$

where $\mu = 1 - c$. These optimal subsidies lead to the following expression for the equilibrium values, which summarize these results in Lemma 3.

Lemma 3: *Suppose that the goods under Cournot competition for subsidy regime, are substitutes. Then, if the upstream firm offers an input price to its country's downstream firm, the equilibrium output, the input price, the price of the final product, the upstream and downstream firms' profits, and social welfare are given, respectively, by*

$$q_i^C = \frac{(8-b^2)\mu}{16+8b-4b^2-b^3}, \quad w_i^C = c + \frac{(4-b^2)(8-b^2)\mu}{2(16+8b-4b^2-b^3)}, \quad p_i^C = c + \frac{(8-3b^2)\mu}{16+8b-4b^2-b^3},$$
$$u_i^C = \frac{(4-b^2)(8-b^2)^2\mu^2}{2(16+8b-4b^2-b^3)^2}, \quad \pi_i^C = \frac{(8-b^2)\mu^2}{(16+8b-4b^2-b^3)^2}, \quad SW_i^C = \frac{(8-b^2)(8-3b^2)\mu^2}{(16+8b-4b^2-b^3)^2}.$$

[**Bertrand Competition**]: Consider that firm i faces the direct demand function as in equation (1) with arbitrary subsidy rates (s_i). At stage four, firm i's best response function under Bertrand competition with arbitrary subsidy rates (s_1, s_2) is given by $\pi_i^B = (1 - b - p_i + bp_j)(p_i + s_i - w_i)/(1 - b^2), i = 1, 2$. The downstream firm i's best response function under Bertrand competition is given by $BR_i(p_j, s_i, w_i) = (1 - b + bp_j - s_i + w_i)/2$. Thus, it is straightforward that the equilibrium downstream firm i's profit is derived at stage four: $\pi_i^B = \frac{[(2+b)(1-b)+(2-b^2)(s_i-w_i)-b(s_j-w_j)]^2}{(1-b^2)(4-b^2)^2}$.

As regards the third stage, each upstream firm offers an input price w_i to each downstream firm. The upstream firm's maximization problem is as follows: $\max_{w_i} u_i^B = (w_i - c)q_i$. Hence, the upstream firm i's best response function under Bertrand competition is given by $w_i = [2 - b - b^2 + c(2 - b^2) +$

$s_i(2-b^2) - bs_j + bw_j]/2(2-b^2)$. Solving the system of response function, we obtain price and quantity under Bertrand competition:

$$w_i = c + \frac{\mathcal{A}\mu + s_i\mathcal{B} - bs_j(2-b^2)}{16 - 17b^2 + 4b^4},\ p_i = c + \frac{\mathcal{C}\mu - s_i(16 - 14b^2 + 3b^4) - s_j(12 - 10b^2 + 2b^4)}{64 - 84b^2 + 33b^4 - 4b^6},$$

$$q_i = \frac{(2-b^2)[\mathcal{A}\mu + s_i\mathcal{B} - bs_j(2-b^2)]}{64 - 148b^2 + 117b^4 - 37b^6 + 4b^8},\ \text{where } \mathcal{A} = (8 - 2b - 9b^2 + b^3 + 2b^4),\ \mathcal{B} = (8 - 9b^2 + 2b^4),$$

and $\mathcal{C} = (48 - 12b - 70b^2 + 10b^3 + 30b^4 - 2b^5 - 4b^6)$.

Given the output and the price at the third stage, each government simultaneously chooses to subsidy in order to maximize social welfare at the second stage

$$\max_{s_i} SW_i^B = \frac{(2-b^2)[\beta][(8 - 2b - 9b^2 + b^3 + 2b^4)\mu + s_i(8 - 9b^2 + 2b^4) - bs_j(2-b^2)]}{(1-b^2)(64 - 84b^2 + 33b^4 - 4b^6)^2},$$

where $\beta = (48 - 12b - 70b^2 + 10b^3 + 30b^4 - 2b^5 - 4b^6)\mu - s_i(16 - 14b^2 + 3b^4) - bs_j(12 - 10b^2 + 2b^4)$.

Differentiating SW_i^B with respect to s_i, invoking symmetry ($s_1^B = s_2^B$) and solving yields

$$s_i^B = \frac{(1-b)(32 - 56b^2 + 27b^4 - 4b^6)\mu}{(2-b^2)(16 - 8b - 12b^2 + 3b^3 + 2b^4)}.$$

Note that if $b \in (0.97, 1)$ or $b \in (-1, -0.97)$[10], then s_i^B becomes a negative value that is related to the export tax, as analyzed in Eaton and Grossman (1986). However, in contrast to the result of Eaton and Grossman (1986), around the range of b, the optimal subsidy is positive instead of the optimal tax. The negative value s_i^B when $b \in (0.97, 1)$ or $b \in (-1, -0.97)$ implies that governments want to tax under Bertrand competition if and only if each competition is sufficiently severe or mild in the product market between downstream firms. It is desirable to force the downstream firm to set a higher price by imposing an export tax when $b \in (0.97, 1)$ or $b \in (-1, -0.97)$, while it is desirable to force the downstream firm to increase more its output by imposing an export subsidy even under Bertrand competition as long as b falls into the range $b \in (-0.97, 0.97)$. The intuition for this result can be explained as follows. An export subsidy of the final good increases the demand for the intermediate good, which reduces the input price. This causes a decrease in the marginal cost faced by each downstream firm, which leads the best response function to be inward under Bertrand competition. These optimal subsidies lead to the following expression for the equilibrium values in Lemma 4.

Lemma 4: *Suppose that the goods under Bertrand competition for the subsidy regime are substitutes. Then, if the upstream firm offers input price w_i to its country's downstream firm, the equilibrium output, the input price, the price of the final product, the upstream and downstream firms' profits, and social welfare are given, respectively, by*

$$q_i^B = \frac{(8 - 9b^2 + 2b^4)\mu}{(1+b)(16 - 8b - 12b^2 + 3b^3 + 2b^4)},\quad w_i^B = c + \frac{(1-b)(32 - 44b^2 + 17b^4 - 2b^6)\mu}{(2-b^2)(16 - 8b - 12b^2 + 3b^3 + 2b^4)},$$

$$p_i^B = c + \frac{(1-b)(8 - 3b^2)\mu}{16 - 8b - 12b^2 + 3b^3 + 2b^4},\quad u_i^B = \frac{(1-b)(4-b^2)(8 - 9b^2 + 2b^4)^2\mu^2}{(1+b)(2-b^2)(16 - 8b - 12b^2 + 3b^3 + 2b^4)^2},$$

$$\pi_i^B = \frac{(1-b)(8 - 9b^2 + 2b^4)^2\mu^2}{(1+b)(16 - 8b - 12b^2 + 3b^3 + 2b^4)^2},\quad SW_i^B = \frac{(1-b)(8 - 3b^2)(8 - 9b^2 + 2b^4)\mu^2}{(1+b)(16 - 8b - 12b^2 + 3b^3 + 2b^4)^2}.$$

[10] See Tables A-2 and A-3 in Appendix B for numerical examples.

[**Asymmetric Competition**]: Let firm i optimally choose its quantity as a best response to any price chosen by firm j, and let the firm j optimally choose its price as a best response to any quantity chosen by firm i. Both demand functions that firms i and j face are given by $p_i = 1-b+bp_j-(1-b^2)q_i$ and $q_j = 1-bq_i-p_j$, respectively. Let us denote firm i's (j's) equilibrium values with superscript "$Q(P)$" when firm i (j) sets quantity (price) as a best response to any price (quantity) chosen by firm $j(i)$.

At stage four, taking arbitrary subsidy rates, (s_1, s_2) and input price (w_1, w_2), we obtain that the each firm's best response function under asymmetric competition is given by $BR_i(p_j, s_i, w_i) = (1-b+bp_j+s_i-w_i)/2(1-b^2)$ and $BR_j(q_i, s_i, s_j) = (1-bq_i-s_j+w_j)/2$, which are upward- and downward-sloping, respectively. As regards the third stage, each upstream firm offers an input price w_i to each downstream firm. The upstream firm's maximization problem is as follows: $\max_{w_i} u_i^Q = (w_i-c)q_i$ and $\max_{w_j} u_j^P = (w_j-c)q_j$. Hence, the upstream firm i's and j's best response functions under asymmetric competition are given by $w_i = (2-b+2c+2s_i-bs_j+bw_j)/4$, and $w_j = (2-b-b^2+2c-b^2c+2s_j-b^2s_j-bs_i+bw_i)/2(2-b^2)$. Solving the system of response function, we obtain price and quantity under asymmetric competition:

$$w_i = c+\frac{(8-2b-5b^2+b^3)\mu+s_i(8-5b^2)-bs_j(2-b^2)}{16-9b^2},\ w_j = c+\frac{(8-2b-5b^2)\mu-2bs_i+s_j(8-5b^2)}{16-9b^2},$$

$$p_i = c+\frac{(48-12b-70b^2+16b^3+25b^4-5b^5)\mu-s_i(16-14b^2+2b^4)-bs_j(12-16b^2+5b^4)}{(16-9b^2)(4-3b^2)},$$

$$p_j = c+\frac{(48-12b-62b^2+8b^3+20b^4)\mu-s_j(16-22b^2+7b^4)-bs_i(12-8b^2)}{(16-9b^2)(4-3b^2)},$$

$$q_i = \frac{2[\mu(8-2b-5b^2+b^3)+s_i(8-5b^2)-bs_j(2-b^2)]}{(16-9b^2)(4-3b^2)},\ q_j = \frac{(2-b^2)[\mu(8-2b-5b^2)+s_j(8-5b^2)-2bs_i]}{(16-9b^2)(4-3b^2)}.$$

Given the output and the price at the third stage, each government simultaneously chooses to subsidy in order to maximize social welfare at the second stage

$$\max_{s_i} SW_i^Q = \frac{2[\gamma][\mu(8-2b-5b^2+b^3)+s_i(8-5b^2)-bs_j(2-b^2)]}{(16-9b^2)^2(4-3b^2)^2},$$

where $\gamma = (48-12b-70b^2+16b^3+25b^4-5b^5)\mu-s_i(16-14b^2+2b^4)-bs_j(12-16b^2+5b^4)$,

$$\max_{s_j} SW_j^P = \frac{(2-b^2)[\delta][\mu(8-2b-5b^2)+s_j(8-5b^2)-2bs_i]}{(16-9b^2)^2(4-3b^2)^2},$$

where $\delta = (48-12b-62b^2+8b^3+20b^4)\mu-s_j(16-22b^2+7b^4)-bs_i(12-8b^2)$.

The first-order conditions for each government given by

$$s_i = \frac{(32-56b^2+23b^4)[\mu(8-2b-5b^2+b^3)-bs_j(2-b^2)]}{4(8-5b^2)(8-7b^2+b^4)},$$

$$s_j = \frac{(32-40b^2+13b^4)[\mu(8-2b-5b^2)-2bs_i]}{2(2-b^2)(8-5b^2)(8-7b^2)}.$$

Hence, straightforward calculation yields

$$s_i^Q = \frac{(16-8b-12b^2+5b^3)(32-56b^2+23b^4)\mu}{2(256-448b^2+240b^4-37b^6)},$$

$$s_j^P = \frac{(16-8b-12b^2+5b^3+b^4)(32-40b^2+13b^4)\mu}{(2-b^2)(256-448b^2+240b^4-37b^6)} > 0.$$

Note that if $b \in (0.96, 1)$ or $b \in (-1, -0.96)$, then s_i^Q has a negative value (see Tables A-2 and A-3 in Appendix B for numerical examples). The negative value s_i^Q when $b \in (0.96, 1)$ or $b \in (-1, -0.96)$ implies that governments want to tax when the downstream firms use quantity strategies if and only if the competition between downstream firms is sufficiently severe or not severe in the product market. However, governments want to subsidize when the downstream firms use price strategies. It is desirable to force the downstream firm to set a lower price by imposing an export subsidy when downstream firm j optimally chooses its price as a best response to any quantity chosen by downstream firm i as long as b falls into the range $b \in (-0.96, 0.96)$, while it is desirable to force the downstream firm to produce less output by imposing an export tax if $b \in (0.96, 1)$ or $b \in (-1, -0.96)$. The intuition is as follows. An export subsidy of the final good increases the demand for the intermediate good when the rival downstream firm chooses the price strategy, which reduces the input price, while an export tax has the reverse effect when the rival downstream firm chooses the price strategy. This implies that depending on the degree of imperfect complementarity or substitutability, the effect of an export tax that increases the input price is dominated by the effect of the export subsidy, and vice versa. Thus, these optimal subsidies lead to the following expression for the equilibrium values in Lemma 5.

Lemma 5: *Suppose that the goods under asymmetric competition for the subsidy regime are substitutes. Then, if the upstream firm offers input price w_i to its country's downstream firm, the equilibrium output, the input price, the price of the final product, the upstream and downstream firms' profits, and social welfare are given, respectively, by*

$$q_i^Q = \frac{(8-5b^2)\zeta\mu}{256-448b^2+240b^4-37b^6}, \quad u_i^Q = \frac{(4-3b^2)(8-5b^2)^2\zeta^2\mu^2}{2(256-448b^2+240b^4-37b^6)^2},$$

$$\pi_i^Q = \frac{(1-b^2)(8-5b^2)^2\zeta^2\mu^2}{(256-448b^2+240b^4-37b^6)^2}, \quad SW_i^Q = \frac{(8-5b^2)(8-7b^2+b^4)\zeta^2\mu^2}{(256-448b^2+240b^4-37b^6)^2},$$

$$w_i^Q = c + \frac{(512-256b-1088b^2+512b^3+768b^4-340b^5-180b^6+75b^7)\mu}{2(256-448b^2+240b^4-37b^6)},$$

$$p_i^Q = c + \frac{(128-64b-208b^2+96b^3+100b^4-43b^5-12b^6+5b^7)\mu}{256-448b^2+240b^4-37b^6}, \ \textit{where } \zeta = (16-8b-12b^2+5b^3),$$

$$q_j^P = \frac{(8-5b^2)\eta\mu}{256-448b^2+240b^4-37b^6}, \quad u_j^P = \frac{(4-3b^2)(8-5b^2)^2\eta^2\mu^2}{(2-b^2)(256-448b^2+240b^4-37b^6)^2},$$

$$\pi_j^P = \frac{(8-5b^2)^2\eta^2\mu^2}{(256-448b^2+240b^4-37b^6)^2}, \quad SW_j^P = \frac{(8-5b^2)(8-7b^2)\eta^2\mu^2}{(256-448b^2+240b^4-37b^6)^2},$$

$$w_j^P = c + \frac{(512-256b-1088b^2+512b^3+800b^4-340b^5-224b^6+75b^7+15b^8)\mu}{(2-b^2)(256-448b^2+240b^4-37b^6)},$$

$$p_j^P = c + \frac{(128-64b-208b^2+96b^3+92b^4-35b^5-7b^6)\mu}{256-448b^2+240b^4-37b^6}, \ \textit{where } \eta = (16-8b-12b^2+5b^3+b^4).$$

5 The Choice of Competition Mode under the Subsidy Regime

Once the equilibria for the four fixed types of contract and social-welfare levels are derived as discussed in the preceding section, the type of contract can be determined endogenously by taking each social welfare level and firm's profit as given. Therefore, we will consider the cases of substitutes and complements simultaneously.

To employ the four-stage game, let "C" and "B" represent, respectively, Cournot and Bertrand competition with regard to each firm's choice. In this section of firm's choice of competition mode under subsidy regime, the SPNE will be found in the first stage for any given pair of competition types. Suppose that the choice of competition mode is delegated to downstream firm in each country. Thus, the payoff matrix for the competition mode between downstream firms can be represented by the following Table 1.

Table 1: The Downstream Firm's Choice of Competition Mode under Subsidy Regime

$i \setminus j$	C	B
C	π_i^C, π_j^C	π_i^Q, π_j^P
B	π_i^P, π_j^Q	π_i^B, π_j^B

Comparing each downstream firm's profit shows that

$$\pi_i^C - \pi_i^P = -b^4(A)(128 - 96b^2 - 8b^3 + 8b^4 + 5b^5)\mu^2 < 0,$$
$$\text{where } A = 4096 - 7680b^2 + 4864b^4 - 1168b^6 - 8b^8 + 5b^9,$$
$$\pi_i^Q - \pi_i^B = -b^4(B)(1-b)(128 - 224b^2 - 8b^3 + 128b^4 + 5b^5 - 24b^6)\mu^2 < 0,$$
$$\text{where } B = 4096 - 11776b^2 + 12800b^4 - 6480b^6 + 8b^7 + 1498b^8 - 5b^9 - 124b^{10}.$$

Hence, choosing Bertrand competition is the firm's best option regardless of whether the goods are substitutes or complements. The following proposition can then be stated.

Proposition 2: *Suppose that a home firm and a foreign firm both export to a third-country market under either the subsidy or tax regime. Then, if the upstream firm offers an input price to its country's downstream firm, choosing Bertrand competition is the dominant strategy for both downstream firms regardless of the nature of the goods.*

Noting that s_i^B and s_i^Q can have positive values depending on the degree of product differentiation, Proposition 1 suggests that an export subsidy can be used under Bertrand competition and that choosing Bertrand competition is the dominant strategy for both downstream firms regardless of the nature of goods. The intuition is as follows. Regardless of the nature of goods, downstream firms produce a higher output with a lower price when they choose the price variable than when they choose the quantity variable (i.e., $q_i^P > q_i^B > q_i^C > q_i^Q$ and $p_i^C > p_i^Q > p_i^P > p_i^B$). Therefore, downstream firms' profits depend on how much subsidy is granted by each government and how much cost is charged by each upstream firm. Each upstream firm charges a higher input price when it chooses the quantity variable than when it chooses the price variable (i.e., $w_i^C > w_i^P$ and $w_i^Q > w_i^B$). Therefore, downstream firms can predict their own optimal strategy to maximize their profit when they choose the competition mode. The downstream firm receives a smaller subsidy when it chooses the quantity variable than when it chooses the price variable, $s_i^C < s_i^P$, which leads to a higher output and a lower price (i.e., $q_i^C < q_i^P$ and $p_i^P < p_i^C$). A higher export subsidy forces both downstream firms to be aggressive in determining the output with $w_i^C > w_i^P$. This is because the aggressive effect in determining the output with a larger subsidy dominates the defensive effect in determining the output with a smaller subsidy. On the other hand, when comparing π_i^B and π_i^Q, if $b \in (0, 0.93)$, then $s_i^B < s_i^Q$ with $q_i^Q < q_i^B$. The effect of a higher subsidy for the downstream firm causes a higher price for intermediate goods, which gives rise to a high cost (i.e., $w_i^Q > w_i^B$). That is, the downstream firm has a lower final output price, but a higher output. Therefore, the downstream firm gains higher profits with a smaller subsidy and a lower cost when it chooses the price variable than when it chooses the quantity variable. In other words, even if $s_i^Q < s_i^B$ when $b \in (0.93, 1)$, this effect also leads to a higher output and a lower price, so the downstream firm gains higher profits with a higher subsidy or tax and a lower cost when it chooses the price variable than it chooses the quantity variable. Thus, each firm prefers price variable to the quantity variable regardless of the nature of goods. Hence, our result differs from that of Singh and Vives (1984), who showed that a dominant strategy exists for both firms that choose Cournot (Bertrand) competition if the goods are substitutes (complements).

Moreover, from the relationships, $\pi_i^Q < \pi_i^B < \pi_i^C < \pi_i^P$ (except for the case of $b \in (-1, -0.99]$)[11] under the subsidy regime, we understand that even though each downstream firm can earn higher profits under Cournot competition than under Bertrand competition, choosing Bertrand competition is a dominant strategy for both downstream firms. Consequently, for both downstream firms, the endogenous choice of a contract might be Pareto inferior regardless of the nature of goods except for the case of $b \in (-1, -0.99]$. Hence, each downstream firm faces a prisoners' dilemma regardless of the nature of goods under the subsidy regime in the vertical structure. This observation leads to the following proposition.

Proposition 3: *Suppose that a home firm and a foreign firm both export to a third-country market under either the subsidy or tax regime. Then, if the upstream firm offers an input price to its country's downstream firm, except in case of* $b \in (-1, -0.99]$*, each downstream firm faces a prisoners' dilemma regardless of the nature of goods in the vertical structure.*

Proposition 3 suggests that even though each firm could obtain higher profit by choosing Cournot competition, the endogenous choice of the strategic variable is Bertrand competition regardless of the nature of goods. The intuition behind Proposition 3 is as follows. By straightforward comparisons, we obtain $q_i^C < q_i^B \Leftrightarrow p_i^B < p_i^C \Leftrightarrow \pi_i^B < \pi_i^C$ except for the case of $b \in (-1, -0.99]$. This implies that the effect on a higher price with a lower output under Cournot competition dominates the effect on a lower price with a higher output under Bertrand competition. Note that in the range $b \in (-1, -0.99]$, the dominant strategy equilibrium is Pareto superior to the others for downstream firms since (with complements) Bertrand profits are larger than Cournot profits.

Next, we investigate the subgame perfect Nash equilibrium (SPNE) of the other case, namely, when the choice of competition mode is delegated to the upstream firm in each country. Before the type of contract is applied in the model to identify the point of equilibrium, we provide the relationships between the upstream firms in the four cases as follows:

Lemma 6: *Suppose that a home and a foreign firm both export to a third-country market under either*

[11] See Table A-4 in Appendix B for numerical examples.

the subsidy or tax regime. Then, if the upstream firm offers an input price to its country's downstream firm,

$$u_i^P > u_i^C > u_i^Q > u_i^B (u_i^P > u_i^C > u_i^B > u_i^Q) \text{ if } b \in (0, -0.74)(b \in (-0.75, -0.97)),$$
$$u_i^P > u_i^B > u_i^C > u_i^Q (u_i^B > u_i^P > u_i^C > u_i^Q) \text{ if } b = -0.98(b \in (-0.99, -1)),$$

when the goods are complements. On the other hand, we obtain that $u_i^P > u_i^C > u_i^Q > u_i^B$ *when the goods are substitutes.*

Proof: See the Appendix B for numerical examples. ■

Similar to the choice of strategic variables among downstream firms, the choice of strategic variables among upstream firms can be represented by the following Table 2.

Table 2: The Upstream Firm's Choice of Competition Mode under Subsidy Regime

$i \setminus j$	C	B
C	u_i^C, u_j^C	u_i^Q, u_j^P
B	u_i^P, u_j^Q	u_i^B, u_j^B

Using Lemma 6 and Tables A-5 and A-6 in Appendix B, which show numerical examples, we summarize these results in Proposition 4.

Proposition 4: *Suppose that a home firm and a foreign firm both export to a third-country market under either the subsidy or tax regime. Then, if the choice of competition mode is delegated to upstream firms, multiple SPNEs with the range* $b \in (-0.75, 1)$ *can be sustained regardless of the nature of goods, whether (B, C) or (C, B). On the other hand, a unique SPNE under the subsidy regime can be sustained with (B, B) when the goods are complements with the range* $b \in (-1, -0.75]$.

The intuition behind Proposition 4 is as follows. The output level of the downstream firm is determined by its strategic variable and the subsidy granted by its government. That is, the downstream firms choose a lower quantity when competing with each other under Cournot competition, and but choose a higher quantity under Bertrand competition. Knowing this fact, each upstream firm uses its input price to increase its profit. Accordingly, the best strategy for the upstream firm is to charge a higher input price in accordance with the given subsidy and the choice of strategic variables of its downstream firm. That is, when each downstream firm receives a higher subsidy from its government, the corresponding upstream firm charges a higher input price, namely, $s_i^P > s_i^C > s_i^Q > s_i^B$ if $b \in (0, 0.93)$, $s_i^P > s_i^C > s_i^B > s_i^Q$ if $b \in (0.93, 1)$, and $w_i^C > w_i^P > w_i^Q > w_i^B$. However, the upstream charges a lower input price when the downstream firm chooses the quantity variable than when it chooses the price variable, which means that profit increases by selling more intermediate goods with a lower input price is the dominant strategy rather than selling a lower level of output with a higher input price. This results in $u_i^P > u_i^C > u_i^Q > u_i^B$.

A conflict of interest between downstream and upstream firms can occur regardless of the nature of goods, while the degree of imperfect complementarity falls into the range $b \in (-1, -0.75]$ when the goods are complements under the subsidy regime. The conflict of interest between them is then resolved by choosing (B, B). Thus, the greater the degree of imperfect complementarity is, the higher the upstream firms' profit becomes when they choose the price strategy[12]. That is, if the degree of imperfect complementarity falls into the range $b \in (-1, -0.75]$, the greater the degree of imperfect complementarity is, the higher the upstream firm's profit becomes under the subsidy regime in Bertrand competition.

Given Propositions 2, 3, and 4, we show in Table 3 the results when the choice of the strategic variables (prices or quantities) is delegated to an upstream firm in one country and to a downstream firm in other country (hereafter, we call this the mixed delegation case).

Table 3: The Mixed Choice of Competition Mode under Subsidy Regime

$i \setminus j$	C	B
C	u_i^C, π_j^C	u_i^Q, π_j^P
B	u_i^P, π_j^Q	u_i^B, π_j^B

Thus, we obtain Proposition 5.

Proposition 5: *Suppose that a home firm and a foreign firm both export to a third-country market under either the subsidy or tax regime. Then, if the choice of strategic variables for prices or quantities is delegated to the upstream firm in country $i(j)$, and to the downstream firm in country $j(i)$, the equilibrium involves (C, B)[(B,C)] regardless of the nature of goods except for the case of $b \in (-1, -0.75]$. On the other hand, a unique SPNE under the subsidy regime can be sustained with (B, B) in this range $b \in (-1, -0.75]$.*

Proposition 5 suggests that when the choice of strategic variables for prices or quantities has a mixed delegation case, the government's welfare when the choice of strategic variables is delegated to each downstream firm is greater than when the choice of strategic variables is delegated to each upstream firm, since $SW_i^P > SW_i^Q$ when the goods are substitutes, and vice versa when the goods are complements (see Proposition 6 below).

Finally, with the endogenous choice of strategic variables and the equilibrium of subsidies or tax levels, we are ready to assess the impacts on social welfare. By comparing the social welfare obtained under the subsidy or tax regime in the vertical structure, we can determine the governments' preference orderings for the subsidy and tax regimes as follows (all calculations and numerical examples are in Appendix B):

Proposition 6: *Suppose that a home firm and a foreign firm both export to a third-country market under either the subsidy or tax regime. Then, the government's preference orderings for the roles are as follows:*

$$SW_i^P > SW_i^C > SW_i^B > SW_i^Q (SW_i^P > SW_i^C > SW_i^Q > SW_i^B)$$

[12]Straightforward calculations and numerical examples are in the Appendix B.

if the goods are substitutes and $b \in (0, 0.81)[[0.82, 1)]$,

$$SW_i^B > SW_i^Q > SW_i^P > SW_i^C,$$

if the goods are complements.

Proposition 6 shows that social welfare depends on who chooses the competition mode. When the choice of competition mode is delegated to each downstream firm, social welfare becomes the same in each country with Bertrand competition. Moreover, if the goods are substitutes, the social welfare gap widens when the choice is delegated to the upstream firms in both countries or to the mixed delegation case compared to when the choice is delegated to both downstream firms. This implies that when the goods are substitutes, the delegation over two upstream firms can obtain the highest welfare level if and only if the upstream firm i sets price as a best response to any quantity chosen by upstream firm j. Otherwise, lower or the lowest welfare level can be obtained as long as either the upstream firm or downstream firm chooses price or quantity strategy when other firms choose the price strategy in the three cases of the delegation problem with respect to the choice of competition mode.

In the case of complementary goods, the highest social welfare, which the government prefers, is obtained if the choice is delegated to the downstream firms, while a disadvantageous level of social welfare is obtained when the choice delegated to either both upstream firms or in the mixed delegation case, except for the case in which the goods are sufficiently close complements (i.e., $b \in (-1, -0.75)$, as in Proposition 5). When the goods are complements (except for $b \in (-1, -0.75)$), this is detrimental to welfare if the endogenous choice of the strategic variable is delegated to both upstream firms or there is a mixed delegation case. However, with complements, it is beneficial for welfare point if the choice of strategic variables is delegated to the downstream firms.

Consequently, Proposition 6 suggests that the downstream firms' prisoners' dilemma is not resolved when the choice of competition mode is delegated to each downstream firm in the case of substitutes, while its delegation in the case of complements is Pareto superior to the others in terms of social welfare since the Bertrand welfare is the largest. In sum, the welfare consequences in the three delegation scenarios differ substantially. Accordingly, for social welfare, governments necessarily set their policy for optimally delegating the choice of the strategic variable according to such circumstances as the nature of goods and product differentiation. This result is in stark contrast to the result under freer trade, under which, if the choice of competition mode is delegated to either upstream firms or downstream firms (or the mixed delegation case), choosing Cournot (Bertrand) competition is the dominant strategy for downstream firms, upstream firms, and the mixed delegation case when the goods are substitutes (complements). Thus, the interests of downstream and upstream firms always coincide with aspects of social welfare regardless of the nature of goods.

6 Concluding Remarks

In the present study, we extended the analysis of Singh and Vives (1984) by incorporating the third-market model into strategic export policy and the delegation of the choice of strategic variables under the vertically related-market for free trade versus the subsidy regime. Unlike the industrial organization context, in which the choice of strategic variables is delegated to each downstream firm, we have suggested that regardless of the nature of goods, choosing Bertrand competition can be the dominant strategy for downstream firms, which face a prisoners' dilemma. However, if the choice of competition mode is delegated to both upstream firms or if there is a mixed delegation case, a conflict of interest between downstream and upstream firms may or may not occur depending on the degree of imperfect complementarity. From the perspective of the government, which faces the problem of delegating the choice of competition mode, when the goods are substitutes, the best strategy to obtain the highest welfare level is for the two upstream firms to choose the mode if and only if one upstream firm sets price as a best response to any quantity chosen by other upstream firm. Otherwise, lower or the lowest welfare level can be obtained as long as either the upstream firm or downstream firm chooses price or quantity strategy when other firms choose the price strategy in the three delegation scenarios. However, this is not necessarily so in the case of complements.

We have used the simplifying assumption that the one home firm and the one foreign firm are symmetric under the vertical structure. By making this assumption, we did not take into account any cost or demand difference that may arise from the subsidy regime. Moreover, we did not analyze strategic outsourcing with trade liberalization in the intermediate-product market. International trade may exist that is related to the determinants of the location of upstream activity, vertical mergers, and the optimal domestic response of countervailing duties. Thus, we need to re-examine such relationships from a more realistic perspective. Finally, we did not consider nonlinear demand structures. The extension of our model in these directions is left for future research.

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Appendix A (Free Trade)

Proof of Lemma 2

As in the main text, at the third stage, taking arbitrary input price (w_1, w_2), we obtain that the each downstream firm’s best response function under asymmetry competition is given by $BR_i^Q(p_j, w_i) = (1-b+bp_j-w_i)/2(1-b^2)$ and $BR_j^P(q_i, w_j) = (1-bq_i-w_j)/2$, which are upward- and downward-sloping, respectively. With these $BR_i^Q(p_j, w_i)$ and $BR_j^P(q_i, w_j)$, the maximization problem of the upstream firm yields each upstream firm’s best response function. Solving the system of response function, straightforward computation yields Lemma 2 as in main text. ■

Proof of Proposition 1

To find subgame perfect Nash equilibrium, we compare $\hat{\pi}_i^C$ and $\hat{u}_i^C$ with $\hat{\pi}_i^P$ and $\hat{u}_i^P$, and compare $\hat{\pi}_i^B$ and $\hat{u}_i^B$ with $\hat{\pi}_i^Q$ and $\hat{u}_i^Q$;

$$\hat{\pi}_i^C - \hat{\pi}_i^P = b^3(16 - 8b - 8b^2 + 5b^3)(256 - 336b^2 - 16b^3 + 116b^4 + 8b^5 - 5b^6)\mu^2,$$
$$\hat{\pi}_i^Q - \hat{\pi}_i^B = -b^3(1-b)(16 - 8b - 16b^2 + 5b^3 + 4b^4)(256 - 464b^2 + 16b^3 + 268b^4 - 16b^5 - 49b^6 + 4b^7)\mu^2,$$
$$\hat{u}_i^C - \hat{u}_i^P = b^3(1024 - 512b - 1088b^2 + 512b^3 + 240b^4 - 130b^5 + 25b^6)\mu^2,$$
$$\hat{u}_i^Q - \hat{u}_i^B = -b^3(1024 - 512b - 2112b^2 + 896b^3 + 1648b^4 - 546b^5 - 583b^6 + 129b^7 + 80b^8 - 8b^9)\mu^2.$$

Hence, Table A-1 provides the game of choice of competition mode, noting that let "C" and "B" represent Cournot and Bertrand competition with regard to each downstream and upstream firm's choice.

Table A-1: The Choice of Competition Mode under Free Trade

$i \setminus j$	C	B
C	$\hat{\pi}_i^C, \hat{\pi}_j^C$	$\hat{\pi}_i^Q, \hat{\pi}_j^P$
B	$\hat{\pi}_i^P, \hat{\pi}_j^Q$	$\hat{\pi}_i^B, \hat{\pi}_j^B$

$i \setminus j$	C	B
C	$\hat{u}_i^C, \hat{u}_j^C$	$\hat{u}_i^Q, \hat{u}_j^P$
B	$\hat{u}_i^P, \hat{u}_j^Q$	$\hat{u}_i^B, \hat{u}_j^B$

$i \setminus j$	C	B
C	$\hat{u}_i^C, \hat{\pi}_j^C$	$\hat{u}_i^Q, \hat{\pi}_j^P$
B	$\hat{u}_i^P, \hat{\pi}_j^Q$	$\hat{u}_i^B, \hat{\pi}_j^B$

Moreover, we find that

$$S\hat{W}_i^B - S\hat{W}_i^C = b^2(8 - 3b^2)(32 - 40b - 16b^2 + 20b^3 + 2b^4 - 2b^5).$$

Thus, when $b \in (0, 0.81)[b \in (0.81, 1)]$, $S\hat{W}_i^B > S\hat{W}_i^C (S\hat{W}_i^B < S\hat{W}_i^C)$. For simplicity, we omit numerical examples in comparing $S\hat{W}_i^B$ and $S\hat{W}_i^C$. ■

Appendix B (Subsidy Regime)

Numerical Examples

Table A-2: Numerical Examples in Subsidy s_i^B and s_i^Q with $b \in (0, 1)$

Substitutes			
b	s_i^B	s_i^Q	$s_i^B - s_i^Q$
0.01	0.99492531032335	0.99492531032336	-1.54321E-14
0.1	0.94279163399473	0.94279164930704	-1.53123E-08
0.3	0.78937572627941	0.78938676515098	-1.10389E-05
⋮	⋮	⋮	⋮
0.93	0.03466519657146	0.03520207063108	-0.000536874
0.94	0.02336677361365	0.02193886748370	0.001427906
0.95	0.01315194543537	0.00900592186176	0.004146024
0.96	0.00440694809566	-0.00300349113843	0.007898087
0.97	-0.0023196601377	-0.01541279648572	0.013093136
0.98	-0.0062256575414	-0.03670727915761	0.020345494
0.99	-0.0060950194195	-0.04545454545455	0.03061226

Table A-3: Numerical Examples in Subsidy s_i^B and s_i^Q with $b \in (-1, 0)$

Complements			
b	s_i^B	s_i^Q	$s_i^B - s_i^Q$
-0.01	1.00492468530147	0.99492531032336	0.009999375
-0.1	1.04216443549385	0.94279164930704	0.099372786
-0.3	1.071943930582940	0.78938676515098	0.282557165
⋮	⋮	⋮	⋮
-0.93	0.164515587009284	0.03520207063108	0.129313516
-0.94	0.121653250929093	0.02193886748370	0.099714383
-0.95	0.076813299068174	0.00900592186176	0.067807377
-0.96	0.029871825096847	-0.00349113843443	0.033362964
-0.97	-0.019306524564097	-0.01541279648572	-0.003893728
-0.98	-0.070869916495128	-0.02657115137601	-0.044298765
-0.99	-0.124980895219703	-0.03670727915761	-0.088273616

Table A-4: Comparison of Downstream Firm's Profit under Subsidy Regime with $b \in (-1, 1)$

Substitutes		Complements	
b	$\pi_i^C - \pi_i^B$	b	$\pi_i^C - \pi_i^B$
0.01	2.47522E-05	-0.01	2.52522E-05
0.1	0.002270283	-0.1	0.002773742
0.3	0.017185541	-0.3	0.031568513
0.5	0.041107564	-0.5	0.116223426
0.7	0.071037037	-0.7	0.323261479
0.9	0.107429154	-0.9	0.818064542
0.98	0.12787454	-0.98	0.679991622
0.99	0.131493861	-0.99	-0.058629134

Table A-5: Comparison of Upstream Firm's Profit under Subsidy Regime with $b \in (0, 1)$

Substitutes				
b	u_i^P	u_i^C	u_i^Q	u_i^B
0.01	0.49503719005722	0.49503719005720	0.49501243757580	0.49501243757579
0.1	0.45346135	0.453461329	0.451188062	0.451188054
0.3	0.377002	0.376988	0.359576	0.359571
0.5	0.319132	0.316105	0.273295	0.273175
0.7	0.268624	0.265814	0.188049	0.186945
0.9	0.246130	0.222954	0.095126	0.086730
0.99	0.284429	0.205484	0.043749	0.013600

Table A-6: Comparison of Upstream Firm's Profit under Subsidy Regime with $b \in (-1, 0)$

Complements				
b	u_i^P	u_i^C	u_i^Q	u_i^B
-0.01	0.50503781509941	0.50503781509939	0.50501256258049	0.50501256258048
-0.1	0.554090602	0.554090575	0.551313537	0.551313528
-0.5	0.910949	0.909923	0.791031	0.790862
⋮	⋮	⋮	⋮	⋮
-0.74	1.422464	1.393378	0.988016	0.987834
b	u_i^P	u_i^C	u_i^B	u_i^Q
-0.75	1.457006	1.423716	0.998660	0.998556
-0.8	1.661585	1.595662	1.061217	1.057440
-0.9	2.369686	2.084756	1.310829	1.239123
-0.97	3.587310	2.624425	2.432307	1.536322
b	u_i^P	u_i^B	u_i^C	u_i^Q
-0.98	3.899701	3.249546	2.722565	1.612458
b	u_i^B	u_i^P	u_i^C	u_i^Q
-0.99	5.718314	4.281946	2.827527	1.705661

■

Proof of Proposition 4

To find endogenous choice of strategic variables for prices or quantities with Tables A-5 and A-6, we need to consider the case of comparing each government's social welfare as follows:

$u_i^C - u_i^P = -b^6\mu^2[D]$, where $D = 1572864 - 262144b - 5046272b^2 + 851968b^3 + 6602752b^4 - 1118208b^5$
$- 4515840b^6 + 750592b^7 + 1716224b^8 - 266752b^9 - 355008b^{10} + 45840b^{11} + 36408b^{12} - 2700b^{13} - 1519b^{14}$,
$u_i^Q - u_i^B = b^6\mu^2[E]$, where $E = 524288 - 262144b - 2818048b^2 + 1572864b^3 + 6275072b^4 - 3747840b^5$,
$- 7604224b^6 + 4750336b^7 + 5492736b^8 - 3539968b^9 - 2417344b^{10} + 1593520b^{11} + 629832b^{12}$,
$- 422612b^{13} - 87981b^{14} + 60031b^{15} + 4952b^{16} - 3452b^{17}$.

As in numerical examples, we obtain that $u_i^Q < u_i^B$ when the goods are complements with the range $b \in (-1, -0.75]$. ■

Proof of Proposition 6

Comparing each government's social welfare shows that

$$\begin{aligned} SW_i^C - SW_i^P &= -b^6\mu^2[F] < 0, \quad \text{where } F = 65536 - 32768b - 180224b^2 + 90112b^3 + 186368b^4 \\ &\quad - 93696b^5 - 88832b^6 + 45056b^7 + 18816b^8 - 9568b^9 - 1368b^{10} + 630b^{11} + 35b^{12}, \\ SW_i^Q - SW_i^B &= -b^6\mu^2[G] < (>)0, \text{ where } G = 65536 - 98304b - 212992b^2 + 335872b^3 + 296960b^4 \\ &\quad - 484864b^5 - 233728b^6 + 384256b^7 + 113536b^8 - 180960b^9 \\ &\quad - 34024b^{10} + 50670b^{11} + 5711b^{12} - 7781b^{13} - 400b^{14} + 500b^{15}, \end{aligned}$$

when $b \in (0, 0.81][b \in (0.81, 1)]$ with substitutes. Moreover, when the goods are complements, we obtain that $SW_i^Q < SW_i^B$. Finally, we obtain that

$$SW_i^B - SW_i^C = -2b^5(8 - 3b^2)(64 - 16b - 64b^2 + 8b^3 + 17b^4 - b^5 - b^7)\mu^2.$$

The numerical analysis of Tables A-7 and A-8 shows that depending on the degree of imperfect substitutability or complementarity, social welfare ranking is determined as follows:

Table A-7: Comparison of Social Welfare under Subsidy Regime with $b \in (0, 1)$

Substitutes				
b	SW_i^P	SW_i^C	SW_i^B	SW_i^Q
0.01	0.2475185949513	0.2475185949512	0.247518594949704	0.247518594949700
0.1	0.226729957	0.226729953	0.226729813	0.226729810
0.3	0.188446727	0.186756934	0.188414585	0.188412570
0.5	0.157752744	0.157712381	0.157317636	0.157281029
0.7	0.132015121	0.131696285	0.129027381	0.128837277
0.8	0.120560048	0.199786413	0.113072259	0.112984393
0.81	0.119464592	0.118622011	0.111229702	0.111201296
b	SW_i^P	SW_i^C	SW_i^Q	SW_i^B
0.82	0.118378643	0.117461493	0.109363033	0.109312044
0.83	0.117302350	0.116304617	0.107462776	0.107307191
0.9	0.110045146	0.108288004	0.091608611	0.089115995
0.99	0.100622299	0.098086677	0.055939960	0.021120917

Table A-8: Comparison of Social Welfare under Subsidy Regime with $b \in (-1, 0)$

Complements				
b	SW_i^B	SW_i^Q	SW_i^P	SW_i^C
-0.01	0.252518907472362	0.252518907472358	0.2525189074707840	0.2525189074707800
-0.1	0.277044599	0.277044594	0.277044423	0.277044419
-0.3	0.347449045	0.347443865	0.347389672	0.347384566
-0.5	0.455446647	0.455236927	0.454176881	0.453983083
-0.7	0.654359980	0.650032167	0.638942419	0.635722164
-0.9	1.346894007	1.193306524	1.059490125	1.012560301
-0.99	8.880753909	2.180951065	1.514824418	1.349702536

■