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R & D Sector Outsourcing, Human Capital Formation and Growth in the Context of Developed versus Developing Economies

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Abstract

An advanced economy relies on innovation activity for its further technology improvement. On the other hand a backward economy depends on both imitation from the world technology frontier and innovation activities – innovation being more skilled-intensive than imitation. In this paper I theoretically examine the impact of R & D outsourcing from an economy which is in the innovation-only regime to an imitation-innovation regime. I show that dependence on imitation activities rises and as a consequence of which share of skilled human capital falls and both skilled and unskilled human capital shifts from innovation to imitation activities in the backward economy. As a result proportion of outsourcing from advanced economy to backward economy falls. Thus, growth rate of the backward economy declines as time progresses. In the long run backward economy will get into a trap and gap from the world technology frontier rises, even if it falls in the initial period.

*Journal of Economic Literature Classifications:* I24, J20, O30, O31, O33, O40.

Key Words: R & D activity, Outsourcing, Economic Growth, Endogenous Labor Composition, Imitation-Innovation, Convergence.

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1 Introduction

The World Investment Report (2005) pointed out that due to the improvement of telecommunication services, the outsourcing of R & D activities to the developing countries, specifically to the Asian economies, has risen over time. Their finding reveals that around 800 leading global transnational corporations are operating their R & D centers and R & D based firms in India and China, mainly in the information communication technology, telecommunication, pharmaceutical and automobiles sectors. According to the ranking of R & D spending in 2002, China holds the sixth position in the world. Among the developing economies, South-East Europe and Common Wealth of Independent States, China, Korea, Russian Federation, India and Singapore respectively hold the first, second, fifth, sixth and eighth positions. In view of the growing importance of R & D outsourcing, I would like to study the impact of this phenomena on the growth rate and the convergence process in both advanced and backward countries.

Let us first discuss the consequences of R & D outsourcing on growth in the destination economy. Being a backward economy (that is, in the imitation-innovation regime) it would direct a proportion of their skilled and unskilled human capital in the R & D activity of the advanced economy (this one being in the innovation-only regime) instead of its own technology improvement thus lowering its own economic growth rate. On the one hand, this might raise the income of skilled and unskilled human capital in the short run. In the long run this may increase the technology gap between the destination (backward) economy and the originating (advanced) economy. Specifically, seemingly, that in the short run outsourcing may increase the income level of the destination economy but in the long run it may be harmful for it. So, there is an intertemporal trade off in the benefit of different generations. Along with these, I are also considering that as an advanced economy outsources their R & D activity, there is a possibility of technology diffusion. But with the assumption that the possibility of diffusion is limited I would like to study the impact of these two opposite effects on the growth rate and the convergence condition of the destination economy.

I next discuss the consequences of R & D activity in the originating economy. There
also two effects play out. On one hand, attracted by cheap human capital in the relatively backward economy, an intermediate input producer of the advanced economy would like to outsource its innovation activity hence spring technological change and economic growth. But, innovation is skilled human capital intensive and the relative composition of skilled human capital is low in the backward economy, thus lowering the technological change and therefore the growth in the short run. So, by considering these two opposing effects I would like to endogenize the optimal R & D outsourcing by the intermediate input producer in the advanced economy. Additionally, due to outsourcing, the demand for human capital decreases in the advanced economy, which may reduce wages and, consequently, income of the individuals of the advanced economy. Thus, it is possible that in the originating economy R & D outsourcing may increase the short run benefit of the intermediate input producer but may reduce the increment to technology level in the long run. In this context, I would like to study the combined impact of these opposing effects on the long run growth rate of the advanced economy. Also, I would want to study the convergence prospects of both the advanced and the backward economy, along with the effects on the income, consumption and inequality paths of skilled and unskilled human capital in these two countries. To the best of our knowledge, this would be the first theoretical analysis of the economy wide decision of outsourcing of R & D activity and the impact of that on growth, inequality and convergence condition for relatively advanced and backward economies.

The earlier literatures mainly concentrate their studies on outsourcing of basic production which is mostly unskilled labor intensive. These studies largely tend to focus on the rising inequality debate within these countries. The phenomenon of rising inequality is attributed to the skilled biased technical change\textsuperscript{1} or else to the increase in import competition from the low-wage countries.\textsuperscript{2} Supporting the second view Feenstra (1996a) shows that any relative increment in capital stock of the South or neutral technological progress in South would

\textsuperscript{1}See Davis and Haltiwanger (1991), Lawrence and Slaughter (1993) and Berman et al. (1994).

\textsuperscript{2}See Leamer (1993 and 1994), Borjas and Ramey (1995), Wood (1994) and Hijzen (2007) show that both the factors have significant impact.
raise the wage inequality in both the northern and southern economies.\(^3\) But, on contrary, by introducing the higher productivity improvement due to outsourcing,\(^4\) Kamal and Saggi (2001) theoretically shows that outsourcing of the production activity, lowers the wage gap between developing and developed economies. By allowing that basic production can shift to less developed economy through imitation, Sayek and Sener (2006) shows that outsourcing raises wage inequality in technologically advanced economy and under certain condition a backward economy might face rising or declining trend in wage inequality.\(^5\) empirically show that in Mexico, FDI (Foreign Direct Investment) raises the demand for skilled human capital and consequently relative wage rate of them. But none of the studies consider that an advanced economy can also outsource skill-intensive activity and may be a backward economy can also perform some R & D activity independently. So, I want to shed light on those issues in the distance to frontier framework.

Instead of taking outsourcing as given, another strand of literature considers the firm level decision of choosing alternative production strategies of vertical integration at home, vertical integration at abroad, outsourcing, foreign sourcing, FDI or export. Using a general equilibrium framework they show that in the absence of firm level heterogeneity or increasing returns to matching between firms all the firms will opt for the same production activity.\(^6\) Another strand of literature shows that when firms have significant market power, in an oligopolistic setting they show that even when firms are ex-ante symmetric, their make or buy decision can vary depending upon their cost structure and the strategic policy.\(^7\) All the above discussed papers are talking about production share. But L.C.Lai et al. (2009)

\(^3\)Also See Feenstra (1996b).
\(^4\)Instead of the assumption of Feenstra (1996a), that difference in capital stock is the driving fact behind outsourcing, Glass and Saggi (2001) assume that technology difference in both the economies will play the role.
\(^5\)Empirically supported by Feenstra and Hanson (1997) and Geishecker and Gorg (2008 and 2005)
\(^7\)Chen (2004), Shy and Stenbacka (2003), Leahy and Montagna (2007) and Nickerson and Vanden Bergh (1999)
considers only cost reducing R & D and shows that revenue sharing contract raises R & D outsourcing.

1.1 Definition, Rationale and Scope of the Study

Aghion et al. (2009), Vandenbussche et al. (2006) and Basu and Mehra (2014) bring out the contribution of human capital to economy-wide technological improvements through the two channels of imitation and innovation. Technological progress is a dual phenomenon which makes a different use of human capital inputs at different levels of economic growth. When an economy is far away from the technological frontier, imitation of technologies is the main engine of total factor productivity growth. The first two mentioned papers also use this theoretical insight to revisit the empirical relationship between schooling level and growth in rich countries for developed economy, which the previous research had found to be slightly negative. Our work aims to extend this line of research to address the following specific aspects.

In this paper I would like to introduce the concept of R & D outsourcing. Since wage rate is lower in an imitation-innovation regime, an R & D producer in the innovation-only regime would like to outsource their R & D activity to another country that is in the former regime. However, the skill composition is lower in an imitation-innovation regime. As a consequence of these two opposing effects, R & D producer would optimally choose the proportion of R & D that he/ she wants to outsource. On the other hand, due to outsourcing the resources (that is, skilled and unskilled human capital) of the backward economy are diverted to the technology improvement of the advanced economy. In one hand, outsourcing reduces the technology improvement of the backward economy. As well as outsourcing changes the composition of human capital in a backward economy. So, the interaction of all these factors determines the short run and long run dynamics of the originating economy. In this case I want to study how R & D outsourcing would change the dynamics of an economy in both the originating and destination economies. I mainly focus on the impact of R & D outsourcing on the labor market equilibrium condition, aggregate growth rate, income and consumption
levels under both the regimes and as well as on the convergence condition of the economy.

Now, I address the research question under this paper

1. Under what economic conditions will an R & D intermediate producer of innovation-only regime outsource his/ her R & D activity to an imitation-innovation regime? What proportion of their R & D activity will be outsourced? How does this depend on the distance of the destination economy from the frontier? How would the R & D decision of an intermediate producer of innovation-only regime impact the labor market equilibrium condition in both the economies? Does this alter the growth trajectory of both the economies?

For this, the research hypothesis is – outsourcing from innovation-only regime to imitation-innovation regime raises the growth rate of an economy in innovation-only regime but reduces it for an economy in an imitation-innovation regime.

2. How does outsourcing affect the convergence condition of the two – originating and destination-economies?

The research hypothesis for this part is that – the convergence to the world technology frontier becomes more difficult for a relatively technologically backward economy than the earlier scenario when there was no outsourcing.

2 Economic Environment

2.1 Production

Production of final output requires land which is normalized to one and continuum of mass one unit of intermediate inputs. To produce one unit of intermediate input I need one unit of final output. Intermediate input producer of each sector is a monopolist who possess the highest available technology in that particular sector $i$ in period $t$. Let us assume discrete
time interval. I consider Cobb-Douglas production function of the form

\[ Y_t = l_t^{1-\alpha} \int_0^1 A_{t}^{1-\alpha} x_{it}^\alpha di, \quad 0 < \alpha < 1, \]

where, \( Y_t \) is the final output in period \( t \), \( l_t \) is the total supply of land, \( A_{t} \) is the level of technology in sector \( i \) in period \( t \), and \( x_{i,t} \) is the amount of intermediate input used in sector \( i \) in period \( t \).

With the assumption that final good sector is perfectly competitive, the price of each of the intermediate input is equal to its marginal product. That is,

\[ p_{it} = \frac{\partial Y_t}{\partial x_{it}} = \alpha A_t^{1-\alpha} x_{it}^{\alpha-1}, \]

where \( p_{it} \) denotes the price of the intermediate input in sector \( i \) in period \( t \). The monopolist chooses \( x_{it} \) by solving

\[ \max_{x_{it}} (p_{it} x_{it} - x_{it}). \]

The monopolist produces the following amount of the intermediate good in sector \( i \) in period \( t \)

\[ x_{it} = \alpha^{\frac{2}{1-\alpha}} A_t \]

Accordingly, he/she would earn the following amount of profit in sector \( i \) in period \( t \)

\[ \pi_{it} = (p_{it} - 1)x_{it} = \left( \frac{1}{\alpha} - 1 \right) \alpha^{\frac{2}{1-\alpha}} A_t = \delta_t A_t, \quad (1) \]

where \( \delta = \left( \frac{1}{\alpha} - 1 \right) \alpha^{\frac{2}{1-\alpha}}. \)

Note that the technology adjusted intermediate inputs and the profit are same for all the sectors in every period.

2.2 Dynamics of Productivity

R & D sector is different between home and foreign countries. Technology improvement depends both on imitation and innovation activities in the home country whereas it depends only on innovation activity in the foreign country. That is, home country is in the diversified
regime while foreign country is in the innovation-only regime. Both skilled and unskilled human capital are required for both imitation and innovation activities. Technology improvement specification for the home country is

\[
A_t = A_{t-1} + \lambda \left[ u_{nit}^{1-\sigma} \left( \frac{A_{t-1} - A_{t-1}}{\lambda} \right) + \gamma u_{nit}^{1-\phi} \phi \left( A_{t-1} - A_{t-1} \right) \right] \quad \lambda > 0, \gamma > 0, 0 < \phi < \sigma < 1 \quad (2)
\]

where, \( \sigma \) (resp. \( \phi \)) is the elasticity of the unskilled human capital in the imitation (resp. innovation) activity, \( \lambda \) measures the efficiency of the overall process of technological progress and \( \gamma \) measures the relative efficiency of innovation compared to imitation. \( u_{nit} \) (resp. \( u_{nit} \)) represents the level of unskilled human capital engaged in the imitation (resp. innovation) activity and \( s_{mit} \) (resp. \( s_{mit} \)) represents the level of skilled human capital engaged in the imitation (resp. innovation) activity in the home country. \( (\overline{A}_{t-1} - A_{t-1}) \) captures the scope of imitation. Along with the advantage of backwardness, there is also a disadvantage of backwardness, as mentioned by Gerschekron (1952). I also divide the scope of imitation by the targeted world technology level, that is, \( \overline{A}_t \). That is more advanced the world leader, more difficult it is to imitate for a backward economy.

**A1. I assume that imitation is more unskilled human capital intensive and innovation is more skilled human capital intensive. That is, \( \sigma > \phi \).**

Now, I discuss the technology improvement specification of the foreign country. I assume that foreign country is the technology leader. Technology improvement of the world leader depends how efficiently skilled and unskilled human capital innovate upon its own technology level.

\[
A_t = A_{t-1} + \lambda \gamma U_{mit}^{\phi} S_{mit}^{1-\phi} \overline{A}_{t-1}, \quad \lambda > 0, \gamma > 0. \quad (3)
\]

where \( U_{mit} \) (resp. \( S_{mit} \)) denotes the level of unskilled (resp. skilled) human capital in the foreign country.

**A2. Innovation is more skilled human capital intensive. That is, \( \phi < \frac{1}{2} \).**

**A3. World technology frontier is growing at a constant exogenous rate \( \bar{g} \).**
2.3 Consumption

I assume a one period overlapping generation model. In the first period an individual decides whether he/ she goes for education. If the decision is yes, then he/ she spends $\theta$ fraction of his/ her first period life in education and works as a skilled worker for the remaining time. Otherwise, he/ she works as an unskilled worker. I also assume that cognitive ability differs among individuals. It is uniformly distributed over the interval $[0, 1]$. Moreover, cost of education has one-one inverse mapping with the cognitive ability of an individual. That is, cost of education rises as cognitive ability of an individual falls. I assume $(1 - \theta)$ is the cost of education for an individual with $\theta$ cognitive ability. I assume log-linear utility of an individual. It depends on his/ her consumption. Let utility function of $k^{th}$ individual be

$$W_{kt} = c_{kt},$$

(4)

where $W_{kt}$ and $c_{kt}$ respectively denote life time utility and consumption of $k^{th}$ individual in period $t$. Let budget constraint of skilled worker be

$$c_{kt} = \theta w_{st},$$

(5)

Budget constraint of unskilled worker be

$$c_{kt} = w_{ut},$$

(6)

where $w_{st}$ (resp. $w_{ut}$) denotes wage rate of skilled (resp. unskilled) worker.

I assume individuals have perfect foresight. There is no population growth. Each parent has one child. I also assume perfectly competitive labor market. At the end of $(t - 1)^{th}$ generation, next generation appears.

3 Analytical Result

In this section we derive our main findings.
### 3.1 Condition for Outsourcing

First, I find out the condition which determine when a R & D intermediate input producers of foreign country decide whether to engage in the outsourcing activity. They will outsource only if the profit of intermediate input producers is higher from outsourcing. Technology leader relies only on innovation activity for its further technology improvement. By A2, innovation is skilled intensive. Now, wage rate of skilled worker is lower in the home country which entails a reduction in the cost of innovation if foreign country outsources. Also proportion of skilled worker is lower in the home country. So, there is a trade off between cost and increment of technology. Also, I assume that there is a fixed cost of outsourcing. An intermediate input producer of foreign country outsources if

\[
(N_{OH} - w_{OH}) + (N_{OF} - w_{OF}) - F \geq (N_{WOF} - w_{WOF})
\]

where \(N_{OH}\) is the amount of innovation that occurs in the backward economy if advanced economy outsources, \(w_{OH}\) is the cost of innovation in the backward economy if advanced economy outsources, \(N_{OF}\) is the amount of innovation that occurs in the advanced economy if advanced economy outsources, \(w_{OF}\) is the cost of innovation in the advanced economy if advanced economy outsources, \(N_{WOF}\) is the amount of innovation that occurs in the advanced economy if advanced economy does not outsource, \(w_{WOF}\) is the cost of innovation in the advanced economy if advanced economy does not outsource and \(F\) is the fixed cost of outsourcing.\(^8\)

\[
N_{OH} - w_{OH} = \left[ \lambda \delta \gamma \bar{u}_{nit}^\phi \bar{s}_{nit}^{1-\phi} \bar{A}_{t-1} - (w_{nit} \bar{a}_{nit} + w_{nit} \bar{s}_{nit}) \right] \mu(a_{t-1})
\]

\[
N_{OF} - w_{OF} = \left[ \lambda \delta \gamma U_{Fit} S_{Fit} \bar{A}_{t-1}^{-1} - (w_{Fit} U_{Fit} + w_{Fit} S_{Fit}) \right] (1 - \mu(a_{t-1}))
\]

\[
N_{WOF} - w_{WOF} = \lambda \delta \gamma U_{Fit} S_{Fit} \bar{A}_{t-1}^{-1} - (w_{Fit} U_{Fit} + w_{Fit} S_{Fit})
\]

\(^8\)We are assuming that if intermediate input producer of the home country engage in the R & D activity of the foreign country then they will not perform innovation activity for their own country. This assumption is also true in equilibrium. For detailed proof please contact author.
That is, \((N_{OH} - w_{OH})\) measures the profit of the intermediate input producer of the foreign country that he/she gains from the home country after outsourcing. \((N_{OF} - w_{OF})\) measures the profit of the intermediate input producer of the foreign country that he/she gains from the foreign country after outsourcing. That is, \((N_{OH} - w_{OH}) + (N_{OF} - w_{OF})\) measures the total gross profit of the intermediate input producer of the foreign country after outsourcing. After deducting the fixed cost of outsourcing (that is, \(F\)), I get the net profit of the intermediate input producer of the foreign country after outsourcing. Whereas \((N_{WOF} - w_{WOF})\) measures the net profit of the intermediate input producer of the foreign country without outsourcing.

Substituting eq. (8) in eq. (7), I get the following condition for outsourcing\(^9\)

\[
\mu(a_{t-1}) \geq \frac{F}{\left\{ \lambda \delta \gamma \left[ \tilde{u}_{nit}^{\phi} \tilde{s}_{nit}^{1-\phi} \tilde{A}_{t-1} - (w_{at} \tilde{u}_{nit} + w_{st} \tilde{s}_{nit}) \right] - \left\{ \lambda \delta \gamma \left[ U_{Fit}^{\phi} S_{Fit}^{1-\phi} \tilde{A}_{t-1} - (w_{Fat} U_{Fit} + w_{Fst} S_{Fit}) \right] \right\}}
\]

(9)

It implies that proportion of outsourcing depends on the profit gap of the originating and destination economy and the fixed cost of outsourcing. If the above condition is satisfied then foreign country will engage in outsourcing of the R & D activity. Once they are engaged in it they will outsource at least as much as mentioned in eq. (9). If the condition in eq. (9) comes with strict inequality then it implies that profit from outsourcing is strictly higher than profit without outsourcing. In that case, intermediate input producer of foreign country will outsource its full R & D activity to the backward economy. That is, \(\mu(a_{t-1}) = 1\). The reverse of this condition with strict inequality implies \(\mu(a_{t-1}) = 0\). Our focus of analysis is concentrated with the strict equality of eq. (9).

### 3.2 Maximization Exercise of the Intermediate Input Producer of the Foreign Country

In this subsection I derive the maximization exercise of the foreign country after outsourcing. This would enable us to figure out the wage rate of skilled and unskilled human capital in

\(^9\)Detail Mathetical Derivations are provided in eq. A1 in the Appendix A.
that case. Let maximization exercise of the foreign country be
\[
\max_{U_{Fit}^{\phi}, S_{Fit}^{\phi}} \lambda \delta \gamma U_{Fit}^{\phi} S_{Fit}^{1-\phi} A_{t-1} (1 - \mu(a_{t-1})) - (w_{Fit} U_{Fit} + w_{Fit} S_{Fit});
\]
where \(w_{Fit}\) (resp. \(w_{Fit}\)) represents wage rate of unskilled (resp. skilled) human capital in the foreign country in period \(t\).

Given the outsourcing decision of an intermediate producer of foreign country wage rate of skilled and unskilled human capital be\(^{10}\)
\[
\begin{align*}
    w_{Fit} &= \lambda \delta \gamma \phi U_{Fit}^{\phi-1} S_{Fit}^{1-\phi} A_{t-1} (1 - \mu(a_{t-1})) \\
    w_{Fit} &= \lambda \delta \gamma (1 - \phi) U_{Fit}^{\phi} S_{Fit}^{1-\phi} A_{t-1} (1 - \mu(a_{t-1}))
\end{align*}
\]
As proportion of outsourcing increases wage rate of both skilled and unskilled human capital falls in the foreign country.

Given that all intermediate producers face the same maximization problem, in equilibrium I have
\[
U_{Fit} = U_{Ft}, \quad S_{Fit} = S_{Ft}.
\]
Substituting eq. (11) and eq. (12) in eq. (9), I get\(^{11}\)
\[
\lambda \delta \gamma U_{Ft}^{\phi} S_{Ft}^{1-\phi} A_{t-1} \mu^2(a_{t-1}) + [(w_{ut} \bar{u}_{nit} + w_{st} \bar{s}_{nit}) - \lambda \delta \gamma \bar{u}_{nit} \bar{s}_{nit} A_{t-1}] \mu(a_{t-1}) + F = 0
\]
Now, \(A = \lambda \delta \gamma U_{Ft}^{\phi} S_{Ft}^{1-\phi} A_{t-1} > 0\)
\[
B = [(w_{ut} \bar{u}_{nit} + w_{st} \bar{s}_{nit}) - \lambda \delta \gamma \bar{u}_{nit} \bar{s}_{nit} A_{t-1}]
\]
\[
C = F > 0
\]
Therefore, \(\mu(a_{t-1}) = \frac{-B \pm \sqrt{B^2 - 4AF}}{2A}\). (13)
Given that \(A > 0\) and \(C > 0\), \(\mu(a_{t-1}) > 0 \Rightarrow B < 0\).

\(^{10}\)Detailed Mathematical Derivations are provided in eq. (A2) in the Appendix A.

\(^{11}\)Detailed Mathematical Derivations are Provided in eq. (A4) in the Appendix A.
3.3 Maximization Exercise of the Intermediate Input Producer of the Home Country

Now, I look at the maximization exercise of the R & D producer of the home country. 

\[
\max_{u_{mit}, s_{mit}, \tilde{u}_{nit}, \tilde{s}_{nit}} A_{it-1} + \lambda \delta \left[ u_{mit}^{\sigma} s_{mit}^{1-\sigma} \frac{(A_{t-1} - A_{t-1})}{A_t} + \gamma \tilde{u}_{nit}^{\phi} \tilde{s}_{nit}^{1-\phi} A_{t-1} \mu(a_{t-1}) \right]
\]

\[
- w_{ut} (u_{mit} + \tilde{u}_{nit}) - w_{st} (s_{mit} + \tilde{s}_{nit})
\]

(14)

where \(\tilde{u}_{nit}\) (resp. \(\tilde{s}_{nit}\)) represents the amount of unskilled (resp. skilled) human capital of the home country which is engaged in the R & D activity of the foreign country what they had outsourced.

First order condition of the maximization exercise be

\[ L_{H} = A_{it-1} + \lambda \delta \left[ u_{mit}^{\sigma} s_{mit}^{1-\sigma} \frac{(A_{t-1} - A_{t-1})}{A_t} + \gamma \tilde{u}_{nit}^{\phi} \tilde{s}_{nit}^{1-\phi} A_{t-1} \mu(a_{t-1}) \right]
\]

\[- w_{ut} (u_{mit} + \tilde{u}_{nit}) - w_{st} (s_{mit} + \tilde{s}_{nit})
\]

(14)

\[
\frac{\partial L_{H}}{\partial u_{mit}} = \lambda \delta \sigma u_{mit}^{\sigma-1} s_{mit}^{1-\sigma} \frac{(A_{t-1} - A_{t-1})}{A_t} - w_{ut} = 0
\]

\[
\frac{\partial L_{H}}{\partial s_{mit}} = \lambda \delta (1-\sigma) u_{mit}^{\sigma} s_{mit}^{1-\sigma} \frac{(A_{t-1} - A_{t-1})}{A_t} - w_{st} = 0
\]

\[
\frac{\partial L_{H}}{\partial \tilde{u}_{nit}} = \lambda \delta \gamma \tilde{u}_{nit}^{\phi-1} \tilde{s}_{nit}^{1-\phi} A_{t-1} \mu(a_{t-1}) + \lambda \delta \gamma \tilde{u}_{nit}^{\phi} \tilde{s}_{nit}^{1-\phi} A_{t-1} \frac{\partial \mu(a_{t-1})}{\partial \tilde{u}_{nit}} - w_{ut} = 0
\]

\[
\frac{\partial L_{H}}{\partial \tilde{s}_{nit}} = \lambda \delta \gamma (1-\phi) \tilde{u}_{nit}^{\phi} \tilde{s}_{nit}^{1-\phi} A_{t-1} \mu(a_{t-1}) + \lambda \delta \gamma \tilde{u}_{nit}^{\phi} \tilde{s}_{nit}^{1-\phi} A_{t-1} \frac{\partial \mu(a_{t-1})}{\partial \tilde{s}_{nit}} - w_{st} = 0
\]

(15)

Given that all intermediate producers face the same maximization problem, in equilibrium I have

\[ u_{mit} = u_{mt}, \quad \tilde{u}_{nit} = \tilde{u}_{nt}, \quad s_{mit} = s_{mt}, \quad \tilde{s}_{nit} = \tilde{s}_{nt}. \]

(16)

There is mass 1 of intermediate firms, so that labor market equilibrium condition is

\[ S_t = s_{mt} + \tilde{s}_{nt}, \quad U_t = u_{mt} + \tilde{u}_{nt}. \]

(17)
Let us define the relative wage rate of skilled and unskilled human capital in the imitation activity. From eq. (15), I get,

\[ \frac{w_{st}}{w_{ut}} = \frac{(1 - \sigma)}{\sigma} \frac{u_{ut}}{s_{st}} \]  

(18)

Now, I try to figure out the relative wage of unskilled and skilled human capital in period \( t \). To find out that first I need to reexpress the proportion of outsourcing in a different way. I use this expression further in our analysis. Rewriting eq. (13) I get,\(^1\)

\[ \pm \left[ \left( w_{ut} \tilde{u}_{nt} + w_{st} \tilde{s}_{nt} \right) - \lambda \delta \gamma \tilde{u}_{nt}^\phi \tilde{s}_{nt}^{1-\phi} \bar{A}_{t-1} \right]^2 - 4AF \right]^{\frac{1}{2}}

= 2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \tilde{u}_{nt}^\phi \tilde{s}_{nt}^{1-\phi} \bar{A}_{t-1} - (w_{ut} \tilde{u}_{nt} + w_{st} \tilde{s}_{nt}) \right] \]  

(19)

To understand the first order condition of the maximization exercise of the intermediate input producer of the home country, I need to find out the change in the proportion of the outsourcing due to a change in allocation of unskilled human capital in the innovation activity (that is, \( \frac{\partial \mu(a_{t-1})}{\partial \tilde{u}_{nt}} \)). Also I need to know the expression of the change in the proportion of the outsourcing due to a change in allocation of skilled human capital in the innovation activity (that is, \( \frac{\partial \mu(a_{t-1})}{\partial \tilde{s}_{nt}} \)). Now, first I define the change in the proportion of outsourcing due to a change in allocation of unskilled human capital in the innovation activity. Differentiating eq. (13) w.r.t \( \tilde{u}_{nt} \) and using eq. (16), I get,\(^2\)

\[ \frac{\partial \mu(a_{t-1})}{\partial \tilde{u}_{nt}} = \frac{\mu(a_{t-1}) \left[ \lambda \delta \gamma \phi \tilde{u}_{nt}^{\phi-1} \tilde{s}_{nt}^{1-\phi} \bar{A}_{t-1} - w_{st} \right]}{2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \tilde{u}_{nt}^\phi \tilde{s}_{nt}^{1-\phi} \bar{A}_{t-1} - (w_{ut} \tilde{u}_{nt} + w_{st} \tilde{s}_{nt}) \right]} \]  

(20)

Now, I try to work out the change in the proportion of the outsourcing (that is, \( \mu(a_{t-1}) \)) due to a change in the allocation of skilled human capital in the innovation activity (that is, \( \tilde{s}_{nt} \)). Differentiating eq. (13) w.r.t \( \tilde{s}_{nt} \) and using eq. (16), I get,\(^3\)

\[ \frac{\partial \mu(a_{t-1})}{\partial \tilde{s}_{nt}} = \frac{\mu(a_{t-1}) \left[ \lambda \delta \gamma (1-\phi) \tilde{u}_{nt}^\phi \tilde{s}_{nt}^{-\phi} \bar{A}_{t-1} - w_{st} \right]}{2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \tilde{u}_{nt}^\phi \tilde{s}_{nt}^{1-\phi} \bar{A}_{t-1} - (w_{ut} \tilde{u}_{nt} + w_{st} \tilde{s}_{nt}) \right]} \]  

(21)

\(^1\)Detailed Mathematical Derivations are Provided in eq. (A5) in the Appendix A.

\(^2\)Detailed Mathematical Derivations are Provided in eq. (A6) in the Appendix A.

\(^3\)Detailed Mathematical Derivations are Provided in eq. (A7) in the Appendix A.
Now substituting eq. (20) and eq. (21), in eq. (15), I get the wage rate of skilled and unskilled workers in the innovation activity of the home country. Rearranging those conditions I get,\(^\text{15}\)

\[
\frac{w_{ut} - \lambda \delta \gamma \phi \bar{u}_{\text{ut}}^{\phi-1} \bar{s}_{\text{ut}}^{1-\phi} A_{t-1} \mu(a_{t-1})}{\lambda \delta \gamma \phi \bar{u}_{\text{ut}}^{\phi-1} \bar{s}_{\text{ut}}^{1-\phi} A_{t-1} - w_{ut}} = \frac{\lambda \delta \gamma \bar{u}_{\text{ut}}^{\phi} \bar{s}_{\text{ut}}^{1-\phi} A_{t-1} \mu(a_{t-1})}{2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \bar{u}_{\text{ut}}^{\phi} \bar{s}_{\text{ut}}^{1-\phi} A_{t-1} - (w_{ut} \bar{u}_{\text{ut}} + w_{st} \bar{s}_{\text{st}}) \right]}
\]

\[
\frac{w_{ut}}{w_{st}} = \frac{\phi}{(1 - \phi)} \bar{u}_{\text{ut}} \tag{22}
\]

Now, equating relative wage rate of skilled and unskilled human capital from imitation and innovation activities, I get the relative demand of skilled and unskilled human capital in the innovation activity. That is, from eq. (18) and eq. (22), I get,\(^\text{16}\)

\[
\Rightarrow \bar{s}_{\text{st}} \frac{\bar{u}_{\text{st}}}{S_{t}} = \psi S_{t} - (\psi - 1) \bar{s}_{\text{st}} \tag{23}
\]

where \(\psi = \frac{\sigma(1 - \phi)}{\phi(1 - \sigma)} > 1\). Now, I try to figure out the implicit function of the demand of skilled human capital in the innovation activity. By equating the wage rate of skilled and unskilled human capital in imitation and innovation activities, I get the implicit solution of skilled human capital in the innovation activity. Substituting eq. (21) in eq. (15), I get,\(^\text{17}\)

\[
\left[ (1 - \sigma) u_{\text{ut}}^{\sigma} s_{\text{st}}^{\sigma} \frac{(A_{t-1} - A_{t-1})}{A_{t}} - \gamma (1 - \phi) \bar{u}_{\text{ut}}^{\phi} \bar{s}_{\text{ut}}^{1-\phi} A_{t-1} \mu(a_{t-1}) \right]
\]

\[
\left[ 2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \bar{u}_{\text{ut}}^{\phi} \bar{s}_{\text{ut}}^{1-\phi} A_{t-1} - (w_{ut} \bar{u}_{\text{ut}} + w_{st} \bar{s}_{\text{st}}) \right] \right]
\]

\[
\Rightarrow 2A \mu(a_{t-1}) \left[ \frac{z(a_{t-1}) U_{t}^{\sigma-\phi}}{[\psi S_{t} - (\psi - 1) \bar{s}_{\text{st}}]^{\sigma-\phi} - \mu(a_{t-1})} \right]
\]

\[
- \lambda \delta \gamma z(a_{t-1}) U_{t}^{\sigma} \bar{s}_{\text{st}} A_{t-1} \left[ 1 - \frac{z(a_{t-1}) U_{t}^{\sigma-\phi}}{[\psi S_{t} - (\psi - 1) \bar{s}_{\text{st}}]^{\sigma-\phi}} \right] = 0; \tag{24}
\]

\(^{15}\)Detailed Mathematical Derivations are Provided in eqs. (A8), (A9) and (A10) in the Appendix A.

\(^{16}\)Detailed Mathematical Derivations are provided in eq. (A11) in the Appendix A.

\(^{17}\)Detailed Mathematical Derivations are provided in eqs. (A12-A16) in the Appendix A.
where $z(a_{t-1}) = \frac{(1-\sigma)\psi}{\gamma(1-\phi)} \frac{(A_{t-1}-A_t)}{A_t}$.

As mentioned in eq. (A14), $B < 0 \Rightarrow \frac{z(a_{t-1}) U_t^{\sigma-\phi}}{[\psi S_t - (\psi - 1)\bar{s}_n]^{\sigma-\phi}} < 1.

Therefore, from eq. (24), I get,

$\Rightarrow \frac{z(a_{t-1}) U_t^{\sigma-\phi}}{[\psi S_t - (\psi - 1)\bar{s}_n]^{\sigma-\phi}} - \mu(a_{t-1}) > 0$

$\Rightarrow \mu(a_{t-1}) < \frac{z(a_{t-1}) U_t^{\sigma-\phi}}{[\psi S_t - (\psi - 1)\bar{s}_n]^{\sigma-\phi}} < 1$

$\Rightarrow \mu(a_{t-1}) < 1.$

(25)

3.4 Labor Supply

In this section I derive the supply curve of skilled and unskilled human capital in period $t$. I already know that individuals have different cognitive ability and cost of education also varies accordingly. Now, an individual goes for education if his/ her lifetime income as skilled is higher than unskilled. Therefore the cut off level of cognitive ability above which an individual goes for education is

$\theta w_{st} = w_{ut}$

$U_t = \frac{\psi S_t - (\psi - 1)\bar{s}_n}{U_t}$

$\Rightarrow \phi \frac{\psi S_t - (\psi - 1)\bar{s}_n}{U_t}$

From eq. (18) and eq. (23)]

(26)

3.5 Labor Market in Equilibrium

Now, the interaction of the demand and supply curve of skilled and unskilled human capital in imitation and innovation activities determine the equilibrium level of skilled and unskilled human capital in period $t$ in the home country. This in turn determines the equilibrium allocation of skilled and unskilled human capital in imitation and innovation activities. Now I want to show that there exist an equilibrium level of skilled and unskilled human capital so that participation in both the activities are positive. That is,
our focus is to show that $0 < s_{at} < S_t$ and $0 < u_{at} < U_t$. Analytically it is difficult to find. With the help of simulation I would like to show our findings. I show it for two distinct cases. In the first (resp. second) case, overall technological progress (that is, $\lambda$) and efficiency of innovation compare to imitation (that is, $\gamma$) is less (resp. more) efficient. For the first case arbitrary parameter values are $[\lambda, \delta, \gamma, \sigma, \phi, \bar{g}, F, U_F, a(1), \bar{A}(1)] = [0.3, 1, 0.5, 0.7, 0.2, 0.02, 0.001, 0.4, 0.6, 5]$. For the second case arbitrary parameter values are $[\lambda, \delta, \gamma, \sigma, \phi, \bar{g}, F, U_F, a(1), \bar{A}(1)] = [0.8, 1, 1, 0.7, 0.2, 0.02, 0.001, 0.4, 0.6, 5]$. One can easily observe that in the two cases all other parameter values are same except $\lambda$ and $\gamma$.

First I would like to study the composition of human capital as time progresses and the change in it due to a change in the relative gap of an economy’s technology level from the frontier. With the help of simulation, simultaneously solving eq. (24) and eq. (26), I get that when technological progress is less efficient (that is, case 1), skilled human capital falls and unskilled human capital rises as gap from the world technology frontier increases. I illustrate it in figure (1a). As an economy regresses its dependence on imitation rises and innovation falls. By A1, imitation is unskilled human capital intensive. Thus, as an economy moves away from the frontier stock of unskilled human capital rises and skilled human capital falls. However, in case 2, where technological progress is more efficient, as time progresses level of skilled (resp. unskilled) human capital falls (resp. rises) irrespective of whether economy progresses or regresses (as shown in figure (1b) (resp. (1c))). Due to outsourcing the home country performs innovation activity for the foreign country and carries out imitation for the home country. Therefore, as time progresses dependence on imitation increases (resp. decreases) and as well as unskilled (resp. skilled) human capital rises (falls) irrespective of whether economy progresses or regresses. This finding is not in line with Basu and Mehra (2014). They show that without outsourcing skilled human capital rises as an economy progresses.

**Lemma 1**

**Under A1,**

*The stock of skilled human capital decreases and unskilled human capital increases as a*
country moves away from or moves to the world technology frontier, even as the aggregate stock of human capital remains unchanged, for the country which is in the imitation-innovation regime and which is performing R & D activity for the foreign country.

Now, I would focus on the equilibrium allocation of skilled and unskilled human capital in the imitation and innovation activities as the distance from the frontier increases or decreases. In case 1, as an economy regresses both skilled and unskilled human capital rises (resp. falls) in the imitation (resp. innovation) activity. Whereas, in case 2, as time progresses allocation of both skilled and unskilled human capital increases (resp. decreases) in the imitation (resp. innovation) activity irrespective of whether the gap from the world technology frontier improves or worsens. FromLemma 1and by A1, as an economy regresses or progresses, unskilled human capital rises and since imitation is unskilled intensive, imitation attracts more unskilled human capital. Due to complementarity, imitation also attracts skilled human capital and so on. As a result of which both skilled and unskilled human capital rises (resp. falls) in the imitation (resp. innovation) activity. This finding is not similar with Basu
and Mehra (2014). They show that without outsourcing as an economy progresses (resp. regresses) both skilled and unskilled human capital shifts from imitation (resp. innovation) to the innovation (resp. imitation) activity.

**Lemma 2**

*Under A1,*

*The equilibrium amount of skilled and unskilled human capital employment decreases in imitation and increases in innovation activity, as a country moves to or moves away from the world technology frontier.*

### 3.6 Wage Rate of the Home Country

In this subsection, I define the wage rate of skilled and unskilled human capital as time progresses. By **Lemma 1**, skilled human capital falls and unskilled human capital rises irrespective of whether economy progresses or regresses. As an outcome of which marginal productivity of both skilled and unskilled human capital rises and so does the wage rate.
This is not in line with Basu and Mehra (2014). They show that without outsourcing as economy progresses (resp. regresses), wage rate of skilled human capital rises (resp. falls) and unskilled human capital falls (resp. rises).

\textit{Proposition 1}

\textit{Under A1,}

\textit{Due to outsourcing, in the imitation-innovation regime, wage rate of both skilled and unskilled human capital increases irrespective of whether an economy moves to or moves away from the world technology frontier.}

\section{3.7 Proportion of Outsourcing}

In this subsection I look at the equilibrium level of the proportion of outsourcing as time progresses. Attracted by the lower wage rate of the home country intermediate input producers of the foreign country outsource their R & D activity to the home country. Skill composition of human capital is also lower in the home country. By \textit{A2}, R & D activity of the foreign country is skilled intensive. Whereas by \textit{Lemma 1}, as time progresses proportion of skilled in the home country falls. Moreover, by \textit{Proposition 1}, wage rate of it rises. As a consequence of which profit of the intermediate input producer of the foreign country falls irrespective of whether economy improves or worsens. This leads to a reduction in the proportion of outsourcing as time progresses.
Proposition 2

Under A2, 

Equilibrium level of R & D outsourcing falls as time progresses irrespective of whether gap from the world technology frontier improves or worsens.

3.8 Growth Rate

In this subsection I characterize the growth rate of an economy in period $t$ depending on its distance to frontier. I also characterize the growth enhancing education policy of an economy depending on its distance to the frontier.

Rearranging the technological progress eq. (2) and summing over all $i$, one can define the growth rate of a decentralized economy in period $t$ as

$$g_t = \int_0^1 \frac{A_{it} - A_{i, t-1}}{A_{t-1}} di$$

$$= \lambda u^\sigma s^{1-\sigma} \frac{(A_{i, t-1} - A_{i, t})}{A_{t-1} A_t}$$

$$= \lambda u^\sigma s^{1-\sigma} \frac{(1 - a_{i, t-1})}{(1 + \bar{g})a_{i, t-1} A_{i, t-1}}$$

I am assuming that there is no technology diffusion. Even with small amount of diffusion findings remain unchanged. In case 1 as an economy regresses growth rate falls. While in case 2 growth rate reduces irrespective of whether economy progresses or regresses. As time progresses, by Lemma 1, both skilled and unskilled human capital shifts away from
innovation to imitation activity. But due to the disadvantage of backwardness, scope of imitation falls as time progresses and as well as growth rate of the home country. This finding is not similar with Basu and Mehra (2014).

**Proposition 3**

*Under A2 and A2,*

*Growth rate of home country falls irrespective of whether distance from the world technology frontier improves or worsens.*

### 3.9 Dynamics of Distance to Frontier

In case 1, as time progresses distance of an economy from the world technology frontier rises. Whereas in case 2, initially distance to frontier falls and it seems that even after outsourcing economy will progress. But after a certain time point the situation gets reverse. That is, due to outsourcing in the long run home country will get into a trap. For case 1, from **Proposition 3**, growth rate is falling as time progresses. As a consequence of which distance of an economy from the world frontier decreases. However in case 2, as time progresses, growth rate is also falling. But due the high efficiency of overall technology level (that is, \( \lambda \)) and high efficiency of innovation compared to imitation (that is, \( \gamma \)) growth rate of home country is initially higher than the world technology leader. As a outcome of which initially distance of an economy form the world technology frontier falls. But as time progresses, due to a significant reduction of skilled human capital as mentioned in
Lemma 1, growth rate of the home country falls below the growth rate of world leader. As a consequence of which distance to frontier frontier increases and in the long run economy will get stuck. This finding is not in line with Basu and Mehra (2014). They show that without outsourcing in the long run all the economies will converge to the world technology frontier.

**Proposition 4**

*Under A1, A2 and (A3),*

*Due to outsourcing, in the long run the home country will get into a trap.*

4 Conclusion

Technological progress is a dual phenomenon. A backward country can improve its technology level by imitating from the frontier or by innovating new knowledge. Advanced economy relies only on innovation activities for technology improvement. I assume that different types of human capital are efficient in different activities. I consider a situation where technology leader is outsourcing a proportion of their R & D activity to a backward economy. We show that share of unskilled human capital rises as time progresses. As an outcome of which both skilled and unskilled human capital shifts from innovation to imitation activity. This leads to a reduction in the proportion of outsourcing from the advanced economy to a backward economy. In the long run the economy will get stuck and the gap of the country’s technology...
level from the world leader rises.

Our work can be extended in several directions. First, in this entire work, we assume that new knowledge is freely available to all the economies. Instead, one can characterize the growth path and the convergence condition of the economy by ruling out the assumption that world technology level is freely accessible. Second, till now all the work in this area has been abstracted from international trade in commodities. One can develop a dynamic Ricardian model of international trade around the core idea of our work and can study cross-sectoral allocation of skilled and unskilled human capital in the context of international specialization in goods production and trade. Third, one can analyze the consequences of heterogeneous cost of education depending on his/her parental education level and can study the impact of that on growth rate, inequality and intergenerational mobility of an economy depending on its distance to frontier. This would certainly yield further insights on the relationship between distance to frontier and composition of human capital and economic growth.

**Appendix A**

**Condition for Outsourcing**

\[
\left[ \lambda \delta \gamma \ U_{fit}^{\phi} \ S_{fit}^{1-\phi} \ A_{t-1} - (w_{fit} U_{fit} + w_{fst} S_{fit}) \right] (1 - \mu(a_{t-1})) \\
+ \left[ \lambda \delta \gamma \ \tilde{u}_{nit}^{\phi} \ A_{t-1} - (w_{nit} \tilde{u}_{nit} + w_{st} \tilde{s}_{nit}) \right] \mu(a_{t-1}) - F \\
\geq \lambda \delta \gamma \ U_{fit}^{\phi} \ S_{fit}^{1-\phi} \ A_{t-1} - (w_{fit} U_{fit} + w_{fst} S_{fit}) \\
\Rightarrow \mu(a_{t-1}) \geq \frac{F}{\left\{ \lambda \delta \gamma \ \tilde{u}_{nit}^{\phi} \ A_{t-1} - (w_{nit} \tilde{u}_{nit} + w_{st} \tilde{s}_{nit}) \right\} - \left\{ \lambda \delta \gamma \ U_{fit}^{\phi} \ S_{fit}^{1-\phi} \ A_{t-1} - (w_{fit} U_{fit} + w_{fst} S_{fit}) \right\}} \\
\] (A1)
Maximization Exercise of the Intermediate Input Producer of the Foreign Country

\[ \mathbb{L} = \lambda \delta \gamma U_{Fit}^{1-\phi} S_{Fit}^{1-\phi} \bar{A}_{t-1} (1 - \mu(a_{t-1})) - (w_{Fit} U_{Fit} + w_{Fat} S_{Fit}) \]

\[ \frac{\partial \mathbb{L}}{\partial U_{Fit}} = \lambda \delta \gamma \phi U_{Fit}^{\phi - 1} S_{Fit}^{1-\phi} \bar{A}_{t-1} (1 - \mu(a_{t-1})) - w_{Fit} = 0 \]

\[ \frac{\partial \mathbb{L}}{\partial S_{Fit}} = \lambda \delta \gamma (1 - \phi) U_{Fit}^{\phi} S_{Fit}^{1-\phi} \bar{A}_{t-1} (1 - \mu(a_{t-1})) - w_{Fat} = 0 \] (A2)

\[ w_{Fit} U_{Fit} + w_{Fat} S_{Fit} = \lambda \delta \gamma U_{Fit}^{\phi} S_{Fit}^{1-\phi} \bar{A}_{t-1} (1 - \mu(a_{t-1})). \] (A3)

Substituting eq. (A3) in eq. (A1), I get

\[ \mu(a_{t-1}) = \frac{F}{\left\{ (w_{Fit} U_{Fit} + w_{Fat} S_{Fit}) - (w_{ut} \bar{u}_{nit} + w_{st} \bar{s}_{nit}) \right\} - \left\{ \lambda \delta \gamma U_{Fit}^{\phi} S_{Fit}^{1-\phi} \bar{A}_{t-1} - \lambda \delta \gamma \bar{u}_{nit} \bar{s}_{nit}^{1-\phi} \bar{A}_{t-1} \right\}} \]

\[ \Rightarrow \lambda \delta \gamma U_{Fit}^{\phi} S_{Fit}^{1-\phi} \bar{A}_{t-1} \mu^2(a_{t-1}) + \left\{ (w_{ut} \bar{u}_{nit} + w_{st} \bar{s}_{nit}) - \lambda \delta \gamma \bar{u}_{nit} \bar{s}_{nit}^{1-\phi} \bar{A}_{t-1} \right\} \mu(a_{t-1}) + F = 0 \] (A4)

Maximization Exercise of the Intermediate Input Producer of the Home Country

In this section I would try to figure out the relative wage rate of unskilled and skilled human capital. To find out this, first I would like to write the proportion of outsourcing into a different fashion. For mathematical tractability I need this following expression.

\[ \mu(a_{t-1}) = \frac{\[ \lambda \delta \gamma \bar{u}_{nit} \bar{s}_{nit}^{1-\phi} \bar{A}_{t-1} - (w_{ut} \bar{u}_{nit} + w_{st} \bar{s}_{nit}) \] \pm \left\{ (w_{ut} \bar{u}_{nit} + w_{st} \bar{s}_{nit}) - \lambda \delta \gamma \bar{u}_{nit} \bar{s}_{nit}^{1-\phi} \bar{A}_{t-1} \right\}^2 - 4AF}{2A} \]

\[ \Rightarrow \pm \left\{ (w_{ut} \bar{u}_{nit} + w_{st} \bar{s}_{nit}) - \lambda \delta \gamma \bar{u}_{nit} \bar{s}_{nit}^{1-\phi} \bar{A}_{t-1} \right\} \mu(a_{t-1}) + F = 0 \] (A5)
Now, to solve the first order maximization exercise of the intermediate input producer of the home country first I characterize the change in the proportion of the outsourcing due to a change in allocation of unskilled human capital in the innovation activity. Differentiating eq. (13) w.r.t \( \tilde{u}_{it} \) and using eq. (16), I get,

\[
2A \frac{\partial \mu(a_{t-1})}{\partial \tilde{u}_{it}} = \left[ \lambda \delta \gamma \phi \tilde{u}_{it}^{\phi-1} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} - w_{it} \right]
+ \left[ (w_{it} \tilde{u}_{it} + w_{it} \tilde{s}_{it}) - \lambda \delta \gamma \tilde{u}_{it}^{\phi} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} \right] \left[ w_{it} - \lambda \delta \gamma \phi \tilde{u}_{it}^{\phi-1} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} \right] \]

\[
\frac{\partial \mu(a_{t-1})}{\partial \tilde{u}_{it}} = \frac{\mu(a_{t-1}) \left[ \lambda \delta \gamma (1 - \phi) \tilde{u}_{it}^{\phi-1} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} - w_{it} \right]}{2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \tilde{u}_{it}^{\phi} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} - (w_{it} \tilde{u}_{it} + w_{it} \tilde{s}_{it}) \right]} \tag{A6}
\]

Now, I derive the demand for wage rate of skilled and unskilled human capital in the home country. Now, substituting eq. (A6) in eq. (15), I get

\[
w_{it} = \lambda \delta \gamma \phi \tilde{u}_{it}^{\phi-1} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} \mu(a_{t-1}) + \lambda \delta \gamma \tilde{u}_{it}^{\phi} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} \frac{\partial \mu(a_{t-1})}{\partial \tilde{u}_{it}}
\]

\[
\Rightarrow \quad \frac{w_{it} - \lambda \delta \gamma \phi \tilde{u}_{it}^{\phi-1} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} \mu(a_{t-1})}{\lambda \delta \gamma \tilde{u}_{it}^{\phi} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} \mu(a_{t-1})} = \frac{2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \tilde{u}_{it}^{\phi} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1} - (w_{it} \tilde{u}_{it} + w_{it} \tilde{s}_{it}) \right]}{\lambda \delta \gamma \phi \tilde{u}_{it}^{\phi-1} \tilde{s}_{it}^{1-\phi} \bar{A}_{t-1}} \tag{A8}
\]
Now substituting eq. (A7) in eq. (15), I get
\[
 w_{st} = \lambda \delta \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1}) + \lambda \delta \gamma \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \frac{\partial \mu(a_{t-1})}{\partial \tilde{u}_{st}} \\
\Rightarrow \frac{w_{st} - \lambda \delta \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1})}{\lambda \delta \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} - w_{st}} = \frac{\lambda \delta \gamma \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1})}{2A \mu(a_{t-1}) - \left[ \lambda \delta \gamma \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} - (w_{st} \tilde{u}_{st} + w_{st} \tilde{s}_{st}) \right]} 
\]
(A9)

Now, equating eqs. (A8) and (A9), I get the relative wage rate of unskilled and skilled human capital of the innovation activity in the home country.
\[
\frac{w_{st} - \lambda \delta \gamma \phi \tilde{u}_{st}^{\phi - 1} \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1})}{\lambda \delta \gamma \phi \tilde{u}_{st}^{\phi - 1} \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} - w_{st}} = \frac{w_{st} - \lambda \delta \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1})}{\lambda \delta \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} - w_{st}} \\
\Rightarrow \frac{w_{st}}{w_{st}} = \frac{\phi}{(1 - \phi)} \frac{\tilde{s}_{st}}{\tilde{u}_{st}} 
\]
(A10)

From eq. (18) and eq. (22), I get,
\[
\frac{\sigma}{(1 - \sigma)} u_{st} = \frac{\phi}{(1 - \phi)} \tilde{s}_{st} \\
\Rightarrow \frac{\tilde{u}_{st}}{\tilde{s}_{st}} = \frac{U_t}{\psi S_t - (\psi - 1)\tilde{s}_{st}} 
\]
(A11)

Now, I try to figure out the implicit function of the equilibrium level of skilled human capital in the innovation activity. By equating the wage rate of skilled and unskilled human capital in imitation and innovation activities, I get the implicit solution of skilled human capital in the innovation activity. Substituting eq. (A7) in eq. (15), I get,
\[
\lambda \delta (1 - \sigma) u_{st}^\sigma s_{st}^{1 - \sigma} \left( \frac{\bar{A}_{t-1} - \bar{A}_t}{\bar{A}_t} \right) = \lambda \delta \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1}) \\
+ \lambda \delta \gamma \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \frac{\partial \mu(a_{t-1})}{\partial \tilde{s}_{st}} \\
\Rightarrow \left[ (1 - \sigma) u_{st}^\sigma s_{st}^{1 - \sigma} \left( \frac{\bar{A}_{t-1} - \bar{A}_t}{\bar{A}_t} \right) - \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1}) \right] \\
= \gamma \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} \mu(a_{t-1}) \left[ \lambda \delta \gamma (1 - \phi) \tilde{u}_{st}^\phi \tilde{s}_{st}^{1 - \phi} \bar{A}_{t-1} - w_{st} \right] 
\]
(A12)
Now, I define these above three expressions separately. First, I determine the first expression of eq. (A12).

\[
(1 - \sigma)u^\sigma_{nt} s^{-\sigma}_{nt} \frac{(\bar{A}_{t-1} - A_{t-1})}{\bar{A}_t} - \gamma(1 - \phi) \bar{u}^\phi_{nt} \bar{s}^{-\phi}_{nt} \bar{A}_{t-1} \mu(a_{t-1})
\]

\[
= \frac{\gamma(1 - \phi) U^\phi_t \bar{A}_{t-1}}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^\phi} \left[ \frac{z(a_{t-1}) U^{\sigma - \phi}_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^{\sigma - \phi}} - \mu(a_{t-1}) \right]; \tag{A13}
\]

where \( z(a_{t-1}) = \frac{(1 - \sigma)^{\psi \sigma}}{\gamma(1 - \phi)} \frac{(\bar{A}_{t-1} - A_{t-1})}{\bar{A}_t A_{t-1}} = \frac{(1 - \sigma)^{\psi \sigma}}{\gamma(1 - \phi)} \frac{(1 - a_{t-1})}{(1 + \bar{g}) A_{t-1}} \). Now, I determine the second expression of eq. (A12).

\[
\lambda \delta \gamma \bar{u}^\phi_{nt} \bar{s}^{-\phi}_{nt} \bar{A}_{t-1} - (w_{ut} \bar{u}_{nt} + w_{st} \bar{s}_{nt})
\]

\[
= \lambda \delta \gamma \frac{U^\phi_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^\phi} \bar{s}_{nt} \bar{A}_{t-1} \left[ 1 - \frac{z(a_{t-1}) U^{\sigma - \phi}_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^{\sigma - \phi}} \right] > 0, \quad \text{[since } B < 0 \text{]} \tag{A14}
\]

Therefore, \( B < 0 \) \( \Rightarrow \) \( \frac{z(a_{t-1}) U^{\sigma - \phi}_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^{\sigma - \phi}} < 1 \).

Next, I determine the third expression of eq. (A12).

\[
\lambda \delta \gamma (1 - \phi) \bar{u}^\phi_{nt} \bar{s}^{-\phi}_{nt} \bar{A}_{t-1} - w_{st}
\]

\[
= \lambda \delta \gamma (1 - \phi) \frac{U^\phi_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^\phi} \bar{A}_{t-1} \left[ 1 - \frac{z(a_{t-1}) U^{\sigma - \phi}_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^{\sigma - \phi}} \right] > 0. \tag{A15}
\]

Substituting eq. (A13), eq. (A14), eq. (A15) and eq. (A11) in eq. (A12), I get

\[
\left[ (1 - \sigma)u^\sigma_{nt} s^{-\sigma}_{nt} \frac{(\bar{A}_{t-1} - A_{t-1})}{\bar{A}_t} - \gamma(1 - \phi) \bar{u}^\phi_{nt} \bar{s}^{-\phi}_{nt} \bar{A}_{t-1} \mu(a_{t-1}) \right]
\]

\[
= \gamma \bar{u}^\phi_{nt} \bar{s}^{-\phi}_{nt} \bar{A}_{t-1} \mu(a_{t-1}) \left[ \lambda \delta \gamma (1 - \phi) \bar{u}^\phi_{nt} \bar{s}^{-\phi}_{nt} \bar{A}_{t-1} - w_{st} \right]
\]

\[
\Rightarrow 2A \mu(a_{t-1}) \left[ \frac{z(a_{t-1}) U^{\sigma - \phi}_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^{\sigma - \phi}} - \mu(a_{t-1}) \right]
\]

\[
- \lambda \delta \gamma z(a_{t-1}) U^{\sigma}_t \bar{s}_{nt} \bar{A}_{t-1} \left[ 1 - \frac{z(a_{t-1}) U^{\sigma - \phi}_t}{[\psi S_t - (\psi - 1)\bar{s}_{nt}]^{\sigma - \phi}} \right] = 0 \tag{A16}
\]
Bibliography


