Intergenerational mobility, composition of human capital and distance to frontier

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Abstract

An economy can improve its technology level through two channels – imitating from the world technology frontier and innovating on its own technology level – innovation being more skilled-intensive than imitation. I develop a growth model based on the endogenous ability-driven skill acquisition decision of an individual in an imperfect credit market. It is shown that there exists a constant level of skilled and unskilled human capital in the imitation-only and innovation-only regimes. In the imitation-innovation regime stock of skilled human capital rises whereas that of unskilled human capital falls in the imitation-innovation regime. Also, both skilled and unskilled human capital shift from imitation to the innovation activity as an economy progresses. Moreover, growth rate falls in the imitation-only regime. But in the diversified regime growth rate rises even if it falls initially and there exists constant level of growth rate in the innovation-only regime. In the long run all the economies will converge to the world technology frontier and grow at a same rate. In the imitation-only and innovation-only regimes, there exists constant level of upward and downward mobility. However, in the diversified regime both upward and downward mobility falls as an economy progresses to the frontier. Along with that, I show that wage rate and average income of both skilled and unskilled human capital falls in the imitation-only regime and the same rises in the innovation-only regime. However, wage rate and average income of skilled human capital rises and unskilled human capital falls in the diversified regime. Also, there exists constant level of between group income inequality in the imitation-only and innovation-only regimes. However, wage and income inequality between skilled and unskilled human capital rises as an economy bridges its gap from the world technology frontier. There exists constant level of income inequality within skilled and unskilled human capital due to parental income differences and due to difference in cognitive ability, in the imitation-only and innovation-only regime. On the other hand, income inequality within skilled and unskilled human capital rises due to parental income differences and due to difference in cognitive ability, in the imitation-innovation regime.

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1 Introduction

For quite a long time economists have been struggling to understand the possible causes and mechanisms driving economic growth and cross-country income differences. In 2010, the purchasing power parity (PPP) adjusted GDP per capita for Argentina relative to the United States of America (US) was 31.16, for Mexico 28.84, for India 8.58 and for Nigeria it was only 3.5.[1] In the same year, the PPP adjusted GDP per capita in current prices in US was US $46568.57, whereas in Mexico it was only US $13430.03. In India it was around US $3995.61 and in Nigeria it was $1629.50. The question that surfaces is: why different economies have different income levels and economic growth rates. Is there any possibility of convergence or over time will this gap increase?

The first thing that strikes is why are people so concerned about high growth rate and higher levels of income? Is it in any way related to better well-being of individuals? Acemoglu (2008, Ch:1) points toward several development indicators that corroborate this. It is found that the richest countries are not only producing 30 times more than the poorest countries but also consuming 30 times more than them. Further, life expectancy at birth is almost 80 years in the richest countries whereas the same for Sub-Saharan Africa is only between 40-50 years. According to Acemoglu et al. (2001, 2002), Banerjee and Iyer (2005) and Guha and Guha (2008) the divergence among countries have started around 200 years ago, when industrial revolution happened in the European countries and they began to trade with the entire world. Institutions evolved during that time still persist today.

Looking at the literature on institutions, composition of human capital and economic growth I find that when technological progress involves both imitation and innovation, Vandenbussche et al. (2006), Aghion and Howitt (2006), Aghion et al. (2009) have shown that skilled human capital is growth enhancing in a relatively advanced economy whereas un-

[1]Data Sources are Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 7.1, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, July 2012.

skilled human capital is growth enhancing in a relatively backward economy.[2] Thus, to be on a higher growth trajectory, a relatively backward economy should invest more on primary and secondary education and as it becomes relatively advanced it should change its policy and concentrate more on tertiary education. In Basu and Mehra (2014), I endogenize the composition of human capital and show that skilled human capital is growth enhancing in the diversified regime and not the unskilled human capital. Moreover, they show that skilled human capital is growth enhancing in the innovation-only regime whereas unskilled human capital is growth enhancing in the imitation-only regime. However, none of the above studies have considered the situation when capital markets are imperfect – they ignore the fact that acquiring education is more difficult for the poor. So, in this paper, on the consumption side I incorporate the possibility of capital market imperfection by assuming that an individual cares about his/ her child's income and thus leaves bequest for him/ her. That is, one can connect the intergenerational income of individuals by allowing the possibility of bequests. An individual who has acquired higher education level, and consequently has higher income, leaves more bequest for his/her child. That is, the child of a richer parent is more privileged and therefore has a higher probability of becoming more educated. To be precise, under capital market imperfection, the probability of a family remaining on a higher income trajectory becomes higher. On the production side, I consider knowledge-based Aghion and Howitt (1992) type of model, except that here I consider a more specific form of technology evolution. That is, both imitation and innovation contribute toward technology improvement depending on the distance of the economy to the frontier. In this case I assume that both skilled and unskilled human capital are used as inputs in the R & D sector.[3] The interactions between production and consumption activities determine the optimal composition of

[2]Here unskilled human capital implies those individuals who have technical skill but do not acquire the general purpose technology. In this entire synopsis I have used low skilled, technical skilled and unskilled human capital as synonyms while on the other side skilled, high skilled and general skill human capital refer to the same skill-set.

[3]Like Vandenbussche et al. (2006), Aghion and Howitt (2006), Aghion et al. (2009), Di Maria and Stryszowski (2009) and Basu and Mehra (2014).

skilled-unskilled human capital as well as economic growth, average income and consumption of skilled and unskilled human capital of the aggregate economy, all of which are different for different economies depending on their own technology level. Additionally, I also analyze whether, in the long run, all the economies converge to the same steady state. Furthermore, I characterize what happens to intergenerational upward or downward mobility of different economies depending on their distance to the frontier? Moreover, I study the wage and income inequality between skilled and unskilled human capital depending on its distance to frontier. Along with that I look at the within skilled and unskilled group income inequality due to parental income differences and due to differnece in the cognitive ability depeding on their gap from world technology frontier.

In addition to the literature discussed so far, our work is also related to the earlier literature in several respects. I first discuss the existing literature on inequality and intergenerational mobility. Without considering that return to education changes over time, under imperfect capital market Becker and Tomes (1997, 1986) and Loury (1981) show that a more equal society leads to higher mobility and economic development. Becker and Tomes (1986) also shows that intergenerational mobility is smaller when endowments are transferred from parents to children. By considering macroeconomic dynamic, Borjas (1992) empirically shows that the skill of today's generation depends not only on parental income but also on the average skills of the ethnic group in the parent's generation. With perfect capital market, Galor and Tsiddon (1997) shows that a major technological invention initially raises inequality and consequently mobility. Over time, as technology becomes more accessible, both mobility and inequality decrease. That is, there exists a positive relation between inequality and intergenerational mobility in the short run. But, with the assumption of capital market imperfection, Owen and Weil (1998) and Moav and Moaz (1999) show that inequality and mobility move in opposite directions.[4] While Owen and Weil (1998) characterizes only the steady state condition, Moav and Moaz (1999) charts out the transitional dynamics of inequality, mobility and allocation of education along the growth path and shows that both

[4]For empirical support see Andrew and Leigh (2009).

downward and upward mobility increases as an economy progresses. But none of these studies addresses the issue of technological progress over time and interaction of it with inequality and mobility. Hassler et al. (2007) shows that difference in skill-biased technology or wage compression exhibits a positive relation between inequality and mobility. This holds under the assumption that poor parents not only have less ability to spend on children's education but also have a lower willingness to pay. Das (2007) shows that initial income differences persist even under convex technology and convex preferences. Empirical findings of Solon (1992) and Chadwick and Solon (2002) show that intergenerational correlation in the long run income is relatively high for both son's and daughter's income.[5] However, none of these studies have considered the possibility of endogenous R & D based approach in the analysis of intergenerational mobility and inequality. By incorporating this endogeneity along with the concept of distance to frontier I propose to study the pattern of upward and downward mobility of different generations and also the relationship between mobility and inequality depending on the economy's distance to frontier.

Our work also interfaces with the existing literature on convergence theory. Using the classical approach Barro and Sala-i-Martin (1991, 1992, 1995) and Sala-i-Martin (1994, 1996) show that there exists cross-sectional conditional β-convergence even if σ-convergence is not valid. But by using cointegration, Durlauf (1995) shows very little evidence of the convergence of output. By criticizing the earlier methodology, Quah (1996a, 1996b, 1996c, 1996d, 1997), Howitt and Mayer-Foulkes (2004) and Maasoumia et al. (2007) empirically show that income distribution is polarizing into twin peaks of rich and poor; that is, poor are becoming poorer, rich are getting richer and middle class is diminishing. With the assumption that an economy improves it technology level only through innovation, Howitt (2000) theoretically demonstrates the possibility of club convergence. With nonparametric analysis Mayer-Foulkes (2002) shows that, in the long run, the world income converges in three steady states – semi-stagnation, semi-development and development, depending on whether countries have overcome barriers to human capital and technological innovation.

[5] Also see Lee, solon (2009).

By making the assumption that as technology level increases, innovation becomes more difficult, Howitt and Mayer-Foulkes (2005) theoretically shows club convergence. Di-Maria and Stryszowski (2009) and Basu and Mehra (2014) show that without any distortion, such as migration, technology transfer from advanced economy leads to absolute convergence in the long run.[6]

2 Economic Environment

2.1 Production

There are finite numbers of small open economies. In each economy there is an entrepreneur, who is engaged in the production of a final output, and there are continuum of mass one of intermediate input producers. Each intermediate input producer produces the monopoly output, and invest their monopoly profit in the R & D activity. I assume free entry and exit in the R & D sector.

I consider discrete time interval. Final output is produced competitively by using land and a continuum of mass one of intermediate inputs. For simplicity I normalize the total supply of land to one. I consider Cobb-Douglas production function of the form

$$Y_{\mathrm{t+1}} = l_{\mathrm{t+1}}^{1-\alpha} \int_0^1 A_{\mathrm{i,t+1}}^{1-\alpha} x_{\mathrm{i,t+1}}^{\alpha} di, \quad 0 < \alpha < 1,$$

where, $Y_{\mathrm{t+1}}$ is the final output in period $t+1$, $l_{\mathrm{t+1}}$ is the total amount of land, $A_{\mathrm{i,t+1}}$ is the technology level in sector i in period $t+1$ and $x_{\mathrm{i,t+1}}$ is the amount of intermediate input used in sector i in period $t+1$.

Since the final good sector produces under perfect competition, price of each of the intermediate input i is equal to its marginal product. That is,

$$p_{\mathrm{i,t+1}} = \frac{\partial Y_{\mathrm{t+1}}}{\partial x_{\mathrm{i,t+1}}} = \alpha A_{\mathrm{i,t+1}}^{1-\alpha} x_{\mathrm{i,t+1}}^{\alpha-1},$$

where $p_{i,t+1}$ denotes the price of the intermediate input in sector i in period $t+1$. One unit of the final good is required to produce one unit of intermediate input. The monopolist

[6] Aghion and Howitt (2006).

chooses $x_{i,t+1}$ by solving

$$\max_{x_{\mathrm{i,t+1}}} (p_{\mathrm{i,t+1}} x_{\mathrm{i,t+1}} - x_{\mathrm{i,t+1}}).$$

The monopolist produces the following amount of the intermediate good in sector i in period $t+1$

$$x_{\mathrm{i,t+1}} = \alpha^{\frac{2}{1-\alpha}} A_{\mathrm{i,t+1}}$$

Profit of the intermediate input producer is

$$\pi_{\mathrm{i,t+1}} = (p_{\mathrm{i,t+1}} - 1) x_{\mathrm{i,t+1}} = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} A_{\mathrm{i,t+1}} = \delta_1 A_{\mathrm{i,t+1}}, \tag{1}$$

where $\delta_1 = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}}$.

Note that both the equilibrium level of production and the profit of the intermediate input producers in sector i in period $t+1$ is linearly dependent on the local/ national technology level in sector i in that period. That is, both the technology adjusted intermediate inputs and the profit are same for all the sectors in every period.

2.2 Dynamics of Productivity

Technological progress depends not only on the innovation upon local/ national technology level but also on the imitation of technology from the world technology frontier. This is in similar line with Benhabib and Speigel (1994) and Acemoglu Aghion and Zilibott (2006). However, in both of these two work technology improvement depends only on the total stock of human capital and not on the composition of it. That is, whether skilled or unskilled human capital are engaged in imitation or innovation activities that does not have any impact on it. This is a rather restrictive assumption. So, I improve upon this by considering a specification similar to Vandenbussche, Aghion, Meghir (2006) and Aghion, Boustan, Hoxby, Vandenbussche (2009) where imitation and innovation activities require both skilled and unskilled human capital, but with differing intensity of use. I assume that innovation is relatively skilled human capital intensive and imitation is unskilled human capital intensive. A technologically backward (resp. advance) economy specializes in imitation (resp. innovation) activity. The in between economies will perform both the activities.[7] Now, I look at the technology improvement specification of an economy which is in the imitation-only regime.

2.2.1 Imitation-Only Regime

$$A_{i,t+1} = A_{i,t} + \lambda \left[\widetilde{u}^{\sigma}_{\mathrm{i,t+1}} \widetilde{s}^{1-\sigma}_{\mathrm{i,t+1}} \frac{1}{\overline{A}_{t+1}} (\overline{A}_t - A_t) \right], \quad \lambda > 0, \tag{2}$$

where, $\widetilde{u}_{\mathrm{i,t+1}}$ (resp. $\widetilde{s}_{\mathrm{i,t+1}}$) measures the level of unskilled (resp. skilled) human capital engaged in the imitation-only regime, σ is the elasticity of unskilled human capital in the imitation activity and λ measures the efficiency of the overall technology improvement. $(\overline{A}_{\mathrm{t}} - A_{\mathrm{t}})$ captures the scope of imitation. Along with the advantage of backwardness, there is also a disadvantage of backwardness, as mentioned by Gerschekron (1952). I divide the scope of imitation also by its targeted world technology level, that is, $\overline{A}_{t+1}$. That is, more advanced the world leader, it is more difficult to imitate for a backward economy.

2.2.2 Imitation-Innovation Regime

Now, I discuss the technology improvement pattern of an economy which is in the imitation-innovation regime. The specific technology improvement function is postulated as

$$A_{\mathrm{i,t+1}} = A_{\mathrm{i,t}} + \lambda \left[u^{\sigma}_{\mathrm{mi,t+1}} s^{1-\sigma}_{\mathrm{mi,t+1}} \frac{1}{\overline{A}_{\mathrm{t+1}}} (\overline{A}_{\mathrm{t}} - A_{\mathrm{t}}) + \gamma \, u^{\phi}_{\mathrm{ni,t+1}} s^{1-\phi}_{\mathrm{ni,t+1}} A_{\mathrm{t}} \right], \quad \lambda > 0, \ \gamma > 0, \tag{3}$$

where, $u_{\mathrm{mi,t+1}}$ (resp. $s_{\mathrm{mi,t+1}}$) represents the amount of unskilled (resp. skilled) human capital in the imitation activity, $u_{\mathrm{ni,t+1}}$ (resp. $s_{\mathrm{ni,t+1}}$) represents the amount of unskilled (resp. skilled) human capital in the innovation activity, ϕ is the elasticity of unskilled human capital in the innovation activity and γ measures the relative efficiency of innovation compare to imitation. That is, in the diversified regime, an economy can improve its technology level through two channels – imitation (that is, $(\overline{A}_{\mathrm{t}} - A_{\mathrm{t}})$) and innovation (that is, A_{t}).

[7] Later on in this work I show that this is true in equilibrium.

2.2.3 Innovation-Only Regime

$$A_{\mathrm{i,t+1}} = A_{\mathrm{i,t}} + \lambda\gamma \widehat{U}^{\phi}_{\mathrm{i,t+1}} \widehat{S}^{1-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}}, \quad \lambda > 0, \gamma > 0. \tag{4}$$

In the innovation-only regime, an economy is so advanced that it increase its technology through innovation only – how efficiently skilled and unskilled human capital innovates that determine the next period technology level.[8]

To satisfy our basic assumption that innovation (resp. imitation) is relatively skilled (resp. unskilled) human capital intensive than imitation (resp. innovation), I make the following assumption:

A1. *The elasticity of skilled human capital is higher in the innovation activity than in the imitation activity, that is,* $\sigma > \phi$.

In the same vein, under the imitation-only regime, imitation is unskilled human capital intensive implies $\sigma > \frac{1}{2}$ *and in the innovation-only regime, innovation is skilled human capital intensive, that is,* $\phi < \frac{1}{2}$.[9]

A2. *World technology frontier is growing at a constant exogenous rate* $\bar{g}$.

2.3 Consumption Side

I assume that an individual lives for two periods in an overlapping generation model. He/ she has a log-linear utility function. Utility depends on his/ her consumption in both the periods and the level of bequest that he/ she leaves for his/ her child. In the first period of his/ her life, an individual takes a decision on whether to opt for education or not. In the second period, depending on his/ her education decision, he/ she works as a skilled/ unskilled worker. Like Moaz and Moav (1999) I assume complete absence of capital market. So that individuals can not borrow or lend. That is, income and expenditure in the two

[8]Here efficiency or productivity of skilled and unskilled human capital is measured in terms of the elasticity of skilled and unskilled human capital in the imitation and innovation activities.

[9]In the diversified regime our work does not require any assumption on the absolute intensity of skilled or unskilled human capital in the imitation or innovation activities. hence, these parametric restrictions pertain only to the specialized economies.

periods are independent. An individual spends bequest on the first period consumption and education (if he/ she opts for it) and allocates the second period income on his/ her own consumption and leaves a bequest. I assume that individuals have different cognitive ability (captured by parameter θ), which is uniformly distributed over the interval $[0, 1]$. The cost of education is negatively related to the individual's cognitive ability and positively with the wage rate of unskilled human capital. That is,

$$E(\theta, A_{t-1}) = \frac{Hw_{\text{ut}}}{\theta}, \tag{5}$$

where H is any positive constant. Both skilled and unskilled worker maximize his/ her lifetime utility subject to the budget constraint. Each individual maximizes the following lifetime utility function

$$\mathbb{W}_k = \; c_{\text{k,t,t}}\sqrt{c_{\text{k,t,t+1}} \; x_{\text{k,t,t+1}}}, \quad \text{where k = s, u} \tag{6}$$

where $\mathbb{W}_k$ measures the lifetime utility of k^{th} individual, $c_{\text{k,t,t}}$ is the consumption level of k^{th} individual in period t who born at period t, $c_{\text{k,t,t+1}}$ is the consumption level of k^{th} individual in period $(t+1)$ who born at period t and $x_{\text{k,t,t+1}}$ is level of bequest that k^{th} individual who born at period t leaves for his/ her child at period $(t+1)$. Budget constraint of skilled worker who born at period t is

$$c_{\text{s,t,t}} + \frac{Hw_{\text{ut}}}{\theta} = x_{\text{t,t}}$$

$$c_{\text{s,t,t+1}} + x_{\text{s,t,t+1}} = w_{\text{s,t+1}}$$

Budget constraint of unskilled worker who born at period t is

$$c_{\text{u,t,t}} = x_{\text{t,t}}$$

$$c_{\text{u,t,t+1}} + x_{\text{u,t,t+1}} = w_{\text{u,t+1}}$$

where $w_{s,t+1}$ and $w_{u_{t+1}}$ are respectively wage rate of skilled and unskilled worker at period $(t+1)$, $x_{t,t}$ be the level of bequest that an individual received from his/ her parent. It depends on whether his/ her parent was skilled or unskilled worker.

I assume perfectly competitive labor markets. Individuals have perfect foresight. There is no population growth. That is, each parent has one child. At the end of t^{th} generation, a new $(t+1)^{th}$ generation appears. This ensures mathematical tractability.

The interaction of production and consumption side determine the equilibrium composition of human capital. This in turn determines the allocation of skilled and unskilled human capital between the imitation and innovation activities, which determines the overall technology improvement. Consequently, this ascertains the growth rate, convergence condition, income, consumption, inequality and intergenerational mobility path of the economy as time progresses.

3 Analytical Results

In this section I derive the main findings of our result.

3.1 Labour Supply

First, I look at the supply curve of an economy. As already mentioned income and consumption in two periods are not interrelated. So, first I maximize second period utility function w.r.t second period budget constraint.

Log-liner utility function ensures that an individual spends his/ her income equally on second period consumption and bequest.[10] Individual goes for education if his/ her lifetime income as skilled worker is greater than unskilled worker. That is,[11]

$$\begin{aligned} & \mathbb{W}_s \geq \mathbb{W}_u \\ \Rightarrow \quad & \theta_{\mathrm{t+1}} \geq \frac{H w_{\mathrm{ut}}}{x_{\mathrm{t,t}}\left[1-\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}\right]} \end{aligned} \tag{7}$$

That is, an individual goes for education if his/ her cognitive ability is higher than the as mentioned in eq. (7). It depends on the future wage gap between skilled and unskilled

[10]Detailed Mathematical derivations are provided in eq. (A1) in the Appendix A.

[11]Detailed Mathematical derivations are provided in eq. (A2) in the Appendix A.

human capital, level of bequest that an individual received from his/ her parent and also on the cost of education. If an individual's parent worked as skilled (resp. unskilled), he/she leaves a higher (resp. lower) bequest, that is, $\frac{w_{st}}{2}$ (resp. $\frac{w_{ut}}{2}$). Therefore, the cutoff level of cognitive ability above which an individual goes for education depends on whether his/ her parent was educated or not. Therefore, the total number of unskilled human capital in period $(t+1)$ is an weighted average of proportion of educated individuals in period $(t+1)$ whose parent was educated and also uneducated in period t.

$$\theta^{\mathrm{u}}_{\mathrm{t+1}} = \frac{2H}{1 - \frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}} \quad \text{and} \quad \theta^{\mathrm{s}}_{\mathrm{t+1}} = \frac{2H \ \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}}{1 - \frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}} \tag{8}$$

where θ^{u}_{t+1} (resp. θ^{s}_{t+1}) measures the cut off cognitive ability above which an individual goes for education if his/ her parent was unskilled (resp. skilled). Note that $\theta^{\mathrm{s}}_{\mathrm{t+1}} < \theta^{\mathrm{u}}_{\mathrm{t+1}}$. It implies that child of an educated parent has higher opportunity of acquiring education than child of an uneducated parent. So, education decision is not only correlated with the cognitive ability of an individual but is also associated with the parental education decision and income. This finding is in line with Galor and Moav (1999).

$$U_{\mathrm{t+1}} = \theta^{\mathrm{u}}_{\mathrm{,t+1}} U_{\mathrm{t}} + \theta^{\mathrm{s}}_{\mathrm{t+1}} S_{\mathrm{t}} = \frac{2H \ \left[U_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}} S_{\mathrm{t}}\right]}{1 - \frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}} \tag{9}$$

That is, total number of unskilled (resp. skilled) human capital in period $(t+1)$ depends on the composition of human capital in period t and also on the expected future wage gap of skilled and unskilled human capital. So, there is a trade off between history vs. expectation, as mentioned in Krugman (1991).

3.2 Imitation-Only Regime

Now, I try to characterize the equilibrium findings of the economies which are in the imitation-only regime. First, I illustrate the demand curve of skilled and unskilled human capital in the imitation-only regime. After that I figure out the equilibrium level of skilled and unskilled human capital. Subsequently I find the growth rate of an economy and the wage path of skilled and unskilled human capital in the imitation-only regime.

3.2.1 Demand for Skilled and Unskilled Human Capital

Now, let me look at the demand curve of skilled and unskilled human capital in the imitation-only regime. Since an intermediate input producer operates the production process for one period only, he/ she maximizes current profit net of labor costs. Profit maximizing exercise of the intermediate input producer is

$$\max_{u_{i,t+1}, s_{i,t+1},} \lambda\delta_1 \left[\widetilde{U}^{\sigma}_{\mathrm{i,t+1}} \widetilde{S}^{1-\sigma}_{\mathrm{i,t+1}} \frac{1}{\overline{A}_{\mathrm{t+1}}} (\overline{A}_{\mathrm{t}} - A_{\mathrm{t}})\right] - \left[w_{\mathrm{u,t+1}} \widetilde{U}_{\mathrm{i,t+1}} + w_{\mathrm{s,t+1}} \widetilde{S}_{\mathrm{i,t+1}}\right], \tag{10}$$

where $\widetilde{w}_{\mathrm{i,t+1}} = \left[w_{\mathrm{u,t+1}} \widetilde{U}_{\mathrm{i,t+1}} + w_{\mathrm{s,t+1}} \widetilde{S}_{\mathrm{i,t+1}}\right]$ measures the total labor cost of R & D activity in the imitation-only regime.

From eq. (10), I derive the first order condition of the maximization exercise of R & D activity in the imitation-only regime

$$\begin{aligned} \frac{\partial \mathbb{L}^{\mathrm{M}}_{1,\mathrm{t+1}}}{\partial \widetilde{U}_{\mathrm{i,t+1}}} &= \lambda\delta_1 \sigma \widetilde{U}^{\sigma-1}_{\mathrm{i,t+1}} \widetilde{S}^{1-\sigma}_{\mathrm{i,t+1}} \frac{1}{\overline{A}_{\mathrm{t+1}}} (\overline{A}_{\mathrm{t}} - A_{\mathrm{t}}) - w_{\mathrm{u,t+1}} = 0; \\ \frac{\partial \mathbb{L}^{\mathrm{M}}_{1,\mathrm{t+1}}}{\partial \widetilde{S}_{\mathrm{i,t+1}}} &= \lambda\delta_1 (1-\sigma) \widetilde{U}^{\sigma}_{\mathrm{i,t+1}} \widetilde{S}^{-\sigma}_{\mathrm{i,t+1}} \frac{1}{\overline{A}_{\mathrm{t+1}}} (\overline{A}_{\mathrm{t}} - A_{\mathrm{t}}) - w_{\mathrm{s,t+1}} = 0. \end{aligned} \tag{11}$$

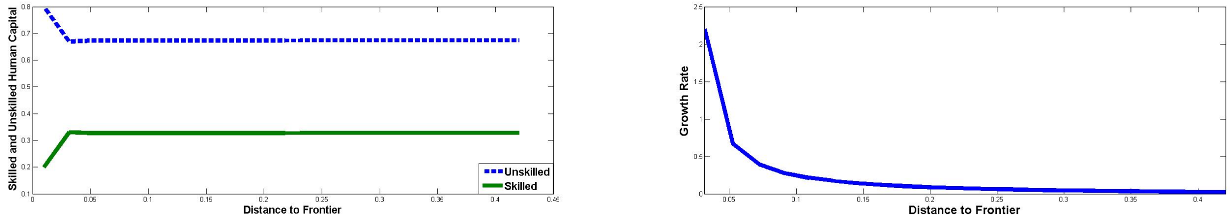
Now, let me look at the relative demand curve for skilled and unskilled human capital in the imitation-only regime. From eq. (11), I get,

$$\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}} = \frac{\sigma}{(1-\sigma)} \frac{\widetilde{S}_{\mathrm{i,t+1}}}{\widetilde{U}_{\mathrm{i,t+1}}} \tag{12}$$

3.2.2 Equilibrium

Now, let me look at the equilibrium level of skilled and unskilled human capital in the the imitation-only regime. Perfectly competitive labor market ensures that at a competitive wage rate labor demand equates labor supply. Now, from eq. (9) and eq. (12), I get the total number of unskilled human capital in the imitation-only regime:

$$\widetilde{U}_{\mathrm{t+1}} = \sigma + 2H\ (1-\sigma) \left[\widetilde{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}} \widetilde{S}_{\mathrm{t}}\right] \tag{13}$$



(a) Skilled and Unskilled Human Capital (b) Growth Rate

Figure 1: Imitation Only – Composition of Skilled and Unskilled Human Capital and Growth Rate

Now, from eq. (13), I get the total number of skilled human capital in the imitation-only regime:

$$\widetilde{S}_{\mathrm{t+1}} = 1 - \widetilde{U}_{\mathrm{t+1}} = (1-\sigma)\left[1 - 2H\,\left[\widetilde{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widetilde{S}_{\mathrm{t}}\right]\right] \tag{14}$$

To ensure the essentiality of inputs I need the following condition:

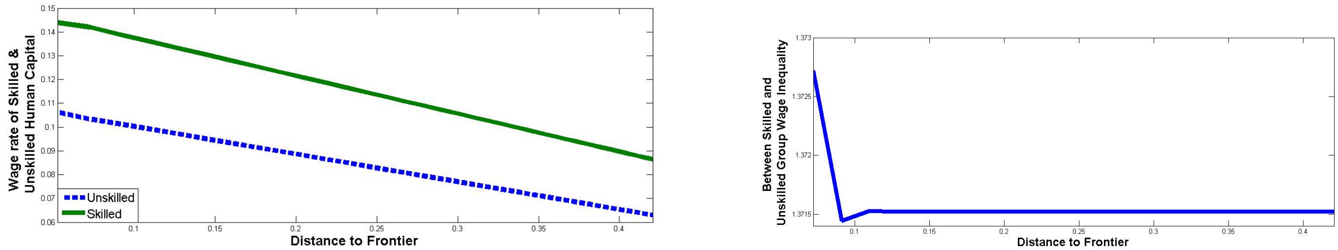
$$\widetilde{S}_{\mathrm{t+1}} > 0 \quad \Rightarrow \quad H < \frac{1}{2\left[\widetilde{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widetilde{S}_{\mathrm{t}}\right]}$$

To do the comparative static analysis, I use simulation. The arbitrary parametric values are [λ, δ, γ, σ, ϕ, H, $\bar{g}$, $\overline{A}_1$]= [0.4, 0.6, 0.1, 0.6, 0.15, 0.1, 0.02, 10]. Now, I give the specific arbitrary parametric values of the imitation-only regime. That is, [U(1), a(1)]=[0.8, 0.01] From fig. (1a), one can see that equilibrium level of unskilled human capital is higher than skilled human capital in the imitation-only regime. After initial adjustment there exists a constant composition of human capital.

3.2.3 Growth Rate

Now, I characterize the growth rate of an economy in the imitation-only regime. From eq. (2), I get,

$$\widetilde{g}_{\mathrm{t+1}} = \int_0^1 \frac{A_{\mathrm{i,t+1}} - A_{\mathrm{i,t}}}{A_{\mathrm{it}}}\mathrm{di} = \frac{\lambda}{(1+\bar{g})\overline{A}_{\mathrm{t}}}\widetilde{U}^{\sigma}_{\mathrm{t+1}}\widetilde{S}^{(1-\sigma)}_{\mathrm{t+1}}\frac{(1-a_{\mathrm{t}})}{a_{\mathrm{t}}}, \tag{15}$$



(a) Wage Rate

(b) Between Group Wage Inequality

Figure 2: Imitation-Only Regime – Wage Rate and Between Group Wage Inequality of Skilled-Unskilled Human Capital

where, $\widetilde{g}_{\mathrm{t+1}}$ measures the growth rate of an economy in period $t+1$. Now, I do the comparative static analysis. First, I would like to study the transition path of an economy. As relative gap of an economy from the world technology frontier decreases, scope of imitation decreases. As a consequence of which increment of technology is lower in the imitation-only regime and that leads to a lower growth rate, as observed in fig. (1b).

3.2.4 Wage Rate

Now I want to study the wage path of skilled and unskilled human capital in the imitation-only regime as an economy progresses. Also I would like to study the relative wage gap between skilled and unskilled human capital. From eq. (11) and eq. (13), I get,

$$\widetilde{w}_{\mathrm{u,t+1}} = \lambda\delta_1 \frac{\sigma(1-\sigma)^{(1-\sigma)}}{(1+\bar{g})} \frac{\left[1 - 2H\ \left[\widetilde{S}_{\mathrm{t}} + \frac{x_{\mathrm{s,t}}}{x_{\mathrm{u,t}}}\widetilde{U}_{\mathrm{t}}\right]\right]^{(1-\sigma)}}{\left[\sigma + 2H\ (1-\sigma)\left[\widetilde{S}_{\mathrm{t}} + \frac{x_{\mathrm{s,t}}}{x_{\mathrm{u,t}}}\widetilde{U}_{\mathrm{t}}\right]\right]^{(1-\sigma)}}(1-a_{\mathrm{t}})$$

$$\widetilde{w}_{\mathrm{s,t+1}} = \lambda\delta_1 \frac{(1-\sigma)^{(1-\sigma)}}{(1+\bar{g})} \frac{\left[1 - 2H\ \left[\widetilde{S}_{\mathrm{t}} + \frac{x_{\mathrm{s,t}}}{x_{\mathrm{u,t}}}\widetilde{U}_{\mathrm{t}}\right]\right]^{-\sigma}}{\left[\sigma + 2H\ (1-\sigma)\left[\widetilde{S}_{\mathrm{t}} + \frac{x_{\mathrm{s,t}}}{x_{\mathrm{u,t}}}\widetilde{U}_{\mathrm{t}}\right]\right]^{-\sigma}}(1-a_{\mathrm{t}})$$

Form fig. (2a), as an economy progresses, scope of imitation falls, consequently marginal productivity of skilled and unskilled human capital falls and so does the wage rate falls. Now, the question is, what happens to the relative wage gap between skilled and unskilled human capital in the imitation-only regime. From fig. (2b), it is clear that, after initial adjustment, there exists a constant level of wage inequality between skilled and unskilled human capital in the imitation-only regime.

3.3 Innovation-Only Regime

Now, our focus of study is the innovation-only regime. In this subsection, I derive the equilibrium outcome of skilled and unskilled human capital in the innovation-only regime. Consequently that decides the growth rate of the economy. Furthermore, I study wage path of an economy which is in the innovation-only regime.

3.3.1 Demand for Skilled and Unskilled Human Capital

First, I examine the demand curve of skilled and unskilled human capital. Let me look at the maximization exercise of the R & D activity of the intermediate input producers in the innovation-only regime.

$$\max_{\widehat{U}_{i,t+1},\widehat{S}_{i,t+1},} \lambda\gamma\delta_1 \widehat{U}^{\phi}_{\mathrm{i,t+1}} \widehat{S}^{1-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}} - \left[w_{\mathrm{u,t+1}} \widehat{U}_{\mathrm{i,t+1}} + w_{\mathrm{s,t+1}} \widehat{S}_{\mathrm{i,t+1}} \right],$$

where $\widehat{w}_{\mathrm{i,t+1}} = \left[w_{\mathrm{u,t+1}} \widehat{U}_{\mathrm{i,t+1}} + w_{\mathrm{s,t+1}} \widehat{S}_{\mathrm{i,t+1}} \right]$ measures the cost associated with the R & D activity of an intermediate input producer who is in the innovation-only regime.

The first order condition of maximization exercise of the innovation-only regime is

$$\frac{\partial \mathbb{L}^{\mathrm{N}}_{1,\mathrm{t+1}}}{\partial \widehat{U}_{\mathrm{i,t+1}}} = \lambda\delta_1\gamma\phi \widehat{U}^{\phi-1}_{\mathrm{i,t+1}} \widehat{S}^{1-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}} - w_{\mathrm{u,t+1}} = 0;$$

$$\frac{\partial \mathbb{L}_{\mathrm{N}_{1,\mathrm{t+1}}}}{\partial \widehat{S}_{\mathrm{i,t+1}}} = \lambda\gamma\delta_1(1-\phi) \widehat{U}^{\sigma}_{\mathrm{i,t+1}} \widehat{S}^{-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}} - w_{\mathrm{s,t+1}} = 0. \tag{16}$$

From eq. (16), I derive the relative demand for skilled-unskilled human capital in the innovation-only regime is

$$\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}} = \frac{\phi}{(1-\phi)} \frac{\widehat{S}_{\mathrm{i,t+1}}}{\widehat{U}_{\mathrm{i,t+1}}} \tag{17}$$

3.3.2 Equilibrium

Now, I look at the equilibrium level of skilled and unskilled human capital in the innovation-only regime. From eq. (9) and eq. (17), I get,

$$\widehat{U}_{\mathrm{t+1}} = \phi + 2H\ (1-\phi)\left[\widehat{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widehat{S}_{\mathrm{t}}\right];$$

$$\widehat{S}_{\mathrm{t+1}} = (1-\phi)\left[1 - 2H\ \left(\widehat{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widehat{S}_{\mathrm{t}}\right)\right]. \tag{18}$$

Similar to the imitation-only regime, here also stock of skilled and unskilled human capital in period $(t+1)$ is history dependent. From fig. (3a), total stock of skilled human capital is higher than the unskilled human capital in the innovation-only regime. The specific parametric values for the innovation-only regime are $[U(1),\, a(1)]$=[0.3, 0.6].

Now, regularity condition for positive stock of skilled and unskilled human capital is the following

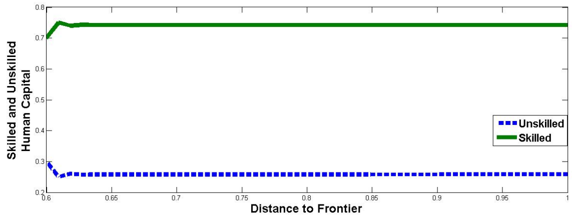
$$\text{Now,} \quad \widehat{S}_{\mathrm{t+1}} > 0 \quad \Rightarrow \quad H < \frac{1}{2\left[\widehat{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widehat{S}_{\mathrm{t}}\right]}$$

Now, let me focus on the relative amount of skilled and unskilled human capital in the innovation-only regime. From eq. (18), I get,

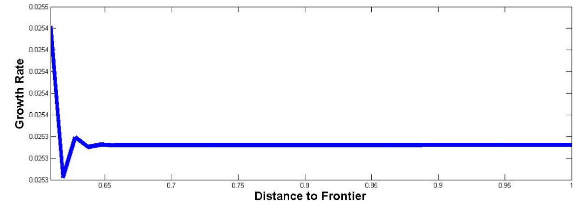
$$\frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}} = \frac{(1-\phi)\left[1 - 2H\left(\widehat{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widehat{S}_{\mathrm{t}}\right)\right]}{\phi + 2H\ (1-\phi)\left(\widehat{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widehat{S}_{\mathrm{t}}\right)} \tag{19}$$

From eq. (17) and (19), I get the relative wage rate of unskilled and skilled human capital. That is,

$$\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}} = \frac{\phi}{(1-\phi)}\frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}} = \frac{\phi\left[1 - 2H\ \left[\widehat{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widehat{S}_{\mathrm{t}}\right]\right]}{\phi + 2H\ (1-\phi)\left[\widehat{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widehat{S}_{\mathrm{t}}\right]}$$

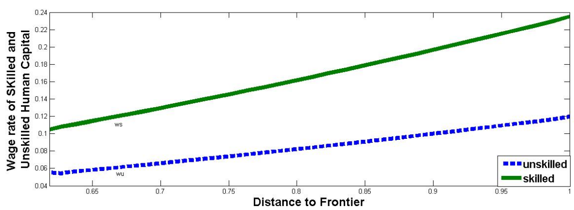


(a) Skilled and Unskilled Human Capital

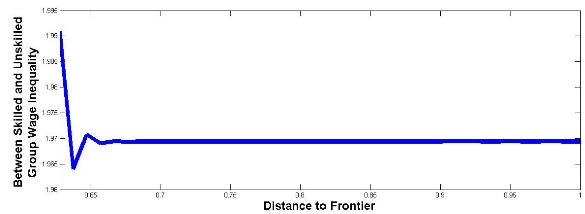


(b) Growth Rate

Figure 3: Innovation Only – Composition of Skilled and Unskilled Human Capital and Growth Rate



(a) Wage Rate



(b) Between Group wage Inequality

Figure 4: Innovation Only – Wage Rate and Wage Inequality between Skilled and Unskilled Human Capital

3.3.3 Growth Rate

Now, let me talk about the growth rate of an economy in the innovation-only regime.

$$\begin{aligned}\widehat{g}_{t+1} &= \lambda\gamma\widehat{U}^{\phi}_{t+1}\widehat{S}^{(1-\phi)}_{t+1} \\ &= \lambda\gamma(1-\phi)^{(1-\phi)}\left[1-2H\left[\widehat{U}_{t}+\frac{x_{u,t}}{x_{s,t}}\widehat{S}_{t}\right]\right]^{(1-\phi)}\left[\phi+2H\ (1-\phi)\left[\widehat{U}_{t}+\frac{x_{u,t}}{x_{s,t}}\widehat{S}_{t}\right]\right]^{\phi} \\ &\qquad\text{[Using eq.(18)]}\end{aligned} \tag{20}$$

It depends on the composition of human capital. So, it is also history dependent. From fig. (3b), after initial adjustment there exists a constant level of growth rate in the innovation-only regime.

3.3.4 Wage Rate

Now, I discuss the wage path of skilled and unskilled human capital as an economy progresses. From eq. (16), I get,

$$\widehat{w}_{\mathrm{u,t+1}} = \lambda\delta_1\phi\widehat{U}_{\mathrm{t+1}}^{\phi-1}\widehat{S}_{\mathrm{t+1}}^{1-\phi}a_{\mathrm{t}}\overline{A}_{\mathrm{t}}; \qquad \widehat{w}_{\mathrm{s,t+1}} = \lambda\delta_1(1-\phi)\widehat{U}_{\mathrm{t+1}}^{\phi}\widehat{S}_{\mathrm{t+1}}^{-\sigma}a_{\mathrm{t}}\overline{A}_{\mathrm{t}}$$

As an economy progresses, technology level increases, which entails an increment in efficiency of the innovation activity. That raises marginal productivity of skilled and unskilled human capital and consequently raises wage rate of skilled and unskilled human capital as illustrate in fig. (4a). Now, the question is what happens to the relative wage gap of skilled and unskilled human capital. After initial adjustment there exists a constant level of wage inequality in the innovation-only regime, as shown in fig. (4b).

3.4 Imitation-Innovation Regime

In this subsection, first, I find out the demand curve for skilled and unskilled human capital and also demand for those in the imitation and in the innovation activity. Subsequently, I figure out the equilibrium allocation of those in these two activities. Also, I get the growth rate and wage path of skilled and unskilled human capital of an economy which is in the diversified regime.

3.4.1 Demand for Skilled and Unskilled Human Capital

First, look at the demand curve of skilled and unskilled human capital in the diversified regime. Maximization exercise of the intermediate input producer in the diversified regime is

$$\max_{u_{mi,t+1},u_{\mathrm{ni,t+1}},s_{mi,t+1},s_{\mathrm{ni,t+1}}} \lambda\delta_1\left[u_{\mathrm{mi,t+1}}^{\sigma}s_{\mathrm{mi,t+1}}^{1-\sigma}\frac{1}{\overline{A}_{\mathrm{t+1}}}(\overline{A}_{\mathrm{t}}-A_{\mathrm{t}})+\gamma u_{\mathrm{ni,t+1}}^{\phi}s_{\mathrm{ni,t+1}}^{1-\phi}A_{\mathrm{t}}\right] - \left[w_{\mathrm{u,t+1}}(u_{\mathrm{mi,t+1}}+u_{\mathrm{ni,t+1}})+w_{\mathrm{s,t+1}}(s_{\mathrm{mi,t+1}}+s_{\mathrm{ni,t+1}})\right]. \tag{21}$$

where $w_{\mathrm{i,t+1}} = [w_{\mathrm{u,t+1}}(u_{\mathrm{mi,t+1}}+u_{\mathrm{ni,t+1}})+w_{\mathrm{s,t+1}}(s_{\mathrm{mi,t+1}}+s_{\mathrm{ni,t+1}})]$ measures the labor cost of R & D activity in the diversified regime.

Now, I derive the first order maximizing condition of the R & D activity in the imitation-innovation regime. From eq. (21), I get,

$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial u_{mi,t+1}} = \lambda\delta_1\sigma u_{mi,t+1}^{\sigma-1}s_{mi,t+1}^{1-\sigma}\frac{1}{\overline{A}_{t+1}}(\overline{A}_t - A_t) - w_{u,t+1} = 0;$$
$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial u_{ni,t+1}} = \lambda\delta_1\gamma\phi u_{ni,t+1}^{\phi-1}s_{ni,t+1}^{1-\phi}A_t - w_{u,t+1} = 0;$$
$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial s_{mi,t+1}} = \lambda\delta_1(1-\sigma) u_{mi,t+1}^{\sigma}s_{mi,t+1}^{-\sigma}\frac{1}{\overline{A}_{t+1}}(\overline{A}_t - A_t) - w_{s,t+1} = 0;$$
$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial s_{ni,t+1}} = \lambda\delta_1\gamma(1-\phi) u_{ni,t+1}^{\phi}s_{ni,t+1}^{-\phi}A_t - w_{s,t+1} = 0. \tag{22}$$

Given that all intermediate producers face the same maximization problem, in equilibrium I have

$$u_{mi,t+1} = u_{m,t+1}, \qquad u_{ni,t+1} = u_{n,t+1}, \qquad s_{mi,t+1} = s_{m,t+1}, \qquad s_{ni,t+1} = s_{n,t+1}. \tag{23}$$

There is mass 1 of intermediate firms, so that labor market equilibrium condition is

$$S_{t+1} = s_{m,t+1} + s_{n,t+1}, \qquad U_{t+1} = u_{m,t+1} + u_{n,t+1}. \tag{24}$$

From the first order condition in eq. (22) and using eq. (23), I get the relative demand curves for skilled and unskilled human capital in the imitation and innovation activities respectively. That is,

$$\frac{w_{s,t+1}}{w_{u,t+1}} = \frac{(1-\sigma)}{\sigma}\frac{u_{m,t+1}}{s_{m,t+1}}; \qquad \frac{w_{s,t+1}}{w_{u,t+1}} = \frac{(1-\phi)}{\phi}\frac{u_{n,t+1}}{s_{n,t+1}}. \tag{25}$$

Equalization of the relative wage rate in eq. (25) implies:

$$\psi\ \frac{s_{m,t+1}}{u_{m,t+1}} = \frac{s_{n,t+1}}{u_{n,t+1}}, \tag{26}$$

where, $\psi = \frac{\sigma(1-\phi)}{\phi(1-\sigma)} > 1$, by **A1**.

Demand for skilled and unskilled human capital in the imitation and innovation activities are[12]

$$s_{n,t+1} = \frac{\psi\ S_{t+1} - h(a_t)\ U_{t+1}}{\psi - 1}; \qquad s_{m,t+1} = \frac{h(a_t)\ U_{t+1} - S_{t+1}}{\psi - 1};$$
$$u_{n,t+1} = \frac{\psi S_{t+1} - h(a_t)\ U_{t+1}}{(\psi-1)\ h(a_t)}; \qquad u_{m,t+1} = \frac{\psi[h(a_t)\ U_{t+1} - S_{t+1}]}{(\psi-1)\ h(a_t)}. \tag{27}$$

where $h(a_t) = \left[\frac{(1-\sigma)\psi^{\sigma}(1-a_t)}{\gamma(1-\phi)(1+\bar{g})\overline{A}_t a_t}\right]^{\frac{1}{(\sigma-\phi)}}$.

Now, I find out the relative demand for skilled and unskilled human capital in the imitation and in the innovation activity. From eq. (27), I get,

$$\frac{s_{m,t+1}}{u_{m,t+1}} = \frac{h(a_t)}{\psi}; \qquad \frac{s_{n,t+1}}{u_{n,t+1}} = h(a_t). \tag{28}$$

[12] Detailed mathematical derivations are provided in eqs. (D1-D3) in the Appendix D.

3.4.2 Equilibrium

Now, equating the demand and supply curve of skilled and unskilled human capital, I get the equilibrium level of skilled and unskilled human capital in the diversified regime. Also, I derive the equilibrium allocation of skilled and unskilled human capital in the imitation-innovation regime.

First I obtain the cutoff level of cognitive ability above which an individual goes for education given that his/ her parent was educated or not. Now, substituting eq. (25) and eq. (28) in eq. (8), I get,

$$\theta^{\mathrm{u}}_{\mathrm{t+1}} = \frac{2H}{1 - \frac{\phi}{(1-\phi)} h(a_{\mathrm{t}})}; \qquad \theta^{\mathrm{s}}_{\mathrm{t+1}} = \frac{2H \ \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}}{1 - \frac{\phi}{(1-\phi)} h(a_{\mathrm{t}})}. \tag{29}$$

Now, from eq. (29), I get the total number of unskilled human capital in the imitation-innovation regime

$$U_{\mathrm{t+1}} = \theta^{\mathrm{s}}_{\mathrm{t+1}} S_{\mathrm{t}} + \theta^{\mathrm{u}}_{\mathrm{t+1}} U_{\mathrm{t}} = \frac{2H(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{[(1-\phi) - \phi h(a_{\mathrm{t}})]} \tag{30}$$

Now, I find out the equilibrium level of skilled human capital in the imitation-innovation regime. From eq. (30), I get,

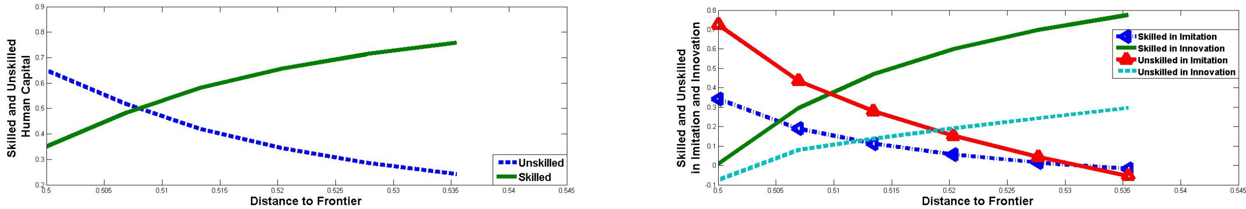
$$S_{\mathrm{t+1}} = 1 - U_{\mathrm{t+1}} = 1 - \frac{2H(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{[(1-\phi) - \phi h(a_{\mathrm{t}})]} \tag{31}$$

For essentiality of both the inputs I need the following conditions:

$$U_{t+1} > 0 \quad \Rightarrow \quad [(1-\phi) - \phi h(a_{\mathrm{t}})] > 0$$

$$S_{\mathrm{t+1}} > 0 \quad \Rightarrow \quad 1 - \frac{2H(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{[(1-\phi) - \phi h(a_{\mathrm{t}})]} > 0 \quad \Rightarrow \quad H < \frac{[(1-\phi) - \phi h(a_{\mathrm{t}})]}{2(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}$$

Now, I do some comparative static analysis. I look at the change in the total stock of skilled and unskilled human capital as an economy progresses. Catch-up component being higher for a technologically backward economy. As an economy progresses relative gap from the world technology frontier decreases. As an outcome of which relative importance of the imitation activity decreases and the innovation activity increases. From **A1**, unskilled human capital is more efficient in imitation than innovation activity. Therefore, in equilibrium total number of unskilled human capital falls and skilled human capital rises as an economy progresses, which is also examined in fig. (5a). That is, $\frac{\partial\ U_{\mathrm{t+1}}}{\partial\ a_{\mathrm{t}}} < 0$ and $\frac{\partial\ S_{\mathrm{t+1}}}{\partial\ a_{\mathrm{t}}} > 0$.



(a) (b)

Figure 5: Diversified Regime – Skilled-Unskilled Human Capital and Allocation of it Imitation and Innovation Activities

Lemma 1

Under **A1**,

In the imitation-only regime equilibrium level of unskilled human capital is higher than the equilibrium level of skilled human capital. Also, after initial adjustment there exists a constant composition of skilled and unskilled human capital.

Similar to Imitation-only regime, in the innovation-only regime also there exists a constant composition of skilled and unskilled human capital, after initial adjustment. But, the equilibrium level of skilled human capital is higher than the equilibrium level of unskilled human capital in the innovation-only regime.

The stock of skilled human capital increases and unskilled human capital decreases as a country moves to the the world technology frontier, for the country which is in the imitation-innovation regime.

From eq. (27), eq. (30) and eq. (31), I get the equilibrium allocation of skilled and unskilled human capital in the imitation and in the innovation activities.

$$s_{\mathrm{m,t+1}} = \frac{2\ H\ (1-\phi)[1+h(a_{\mathrm{t}})]\ \left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{(\psi-1)\left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]} - \frac{1}{(\psi-1)}$$

$$u_{\mathrm{m,t+1}} = \frac{2\ H\ \psi\ (1-\phi)[1+h(a_{\mathrm{t}})]\ \left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} U_{\mathrm{t}}\right]}{(\psi-1)h(a_{\mathrm{t}})\left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]} - \frac{\psi}{(\psi-1)\ h(a_{\mathrm{t}})}$$

$$s_{\mathrm{n,t+1}} = \frac{\psi}{(\psi-1)} - \frac{2\ H\ (1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right][\psi+h(a_{\mathrm{t}})]}{(\psi-1)\ \left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]}$$

$$u_{\mathrm{n,t+1}} = \frac{\psi}{(\psi-1)h(a_{\mathrm{t}})} - \frac{2\ H\ (1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right][\psi+h(a_{\mathrm{t}})]}{(\psi-1)\ h(a_{\mathrm{t}})\ \left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]} \tag{32}$$

Now, let me look at the comparative dynamics. First I discuss the change in the allocation of skilled and unskilled human capital in the imitation and innovation activity as an economy progresses. By **Lemma 1**, total stock of skilled (resp. unskilled) human capital increases (resp. decreases). By **A1**, innovation is more skilled intensive. Therefore, innovation attracts more skilled human capital than imitation as gap from the world technology frontier falls. Due to complementary effect unskilled human capital also shifts from the imitation to the innovation activity. This attracts more skilled human capital and so on. Therefore, in equilibrium total stock of skilled and unskilled human capital increases in the innovation activity and decreases in the imitation activity, as shown in fig. (5b). That is, $\frac{\mathrm{d}\ s_{\mathrm{m,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} < 0$, $\frac{\mathrm{d}\ u_{\mathrm{m,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} < 0$, $\frac{\mathrm{d}\ s_{\mathrm{n,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} > 0$ and $\frac{\mathrm{d}\ u_{\mathrm{n,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} > 0$.

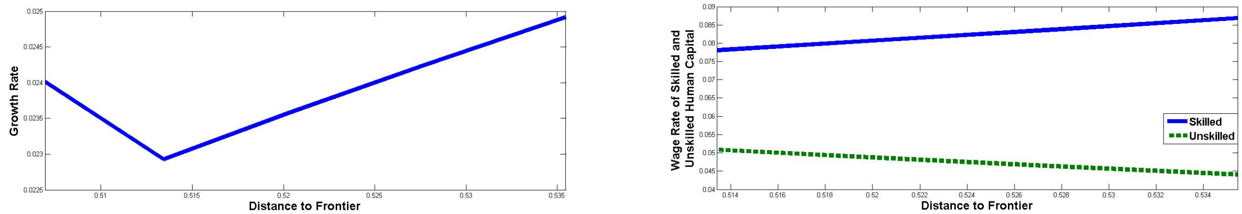
Lemma 2

Under **A1***,*

The stock of both skilled and unskilled human capital shifts from the imitation activity to the innovation activity as an economy bridges gap from the world technology frontier in the imitation-innovation regime.

Now, I discuss the regularity condition for the existence of the positive amount of skilled and unskilled human capital in the imitation and in the innovation activity. That is, $s_{\mathrm{m,t+1}} > 0$, $s_{\mathrm{m,t+1}} < S_{\mathrm{t+1}}$, $s_{\mathrm{n,t+1}} > 0$, $s_{\mathrm{n,t+1}} < S_{\mathrm{t+1}}$, $u_{\mathrm{m,t+1}} > 0$, $u_{\mathrm{m,t+1}} < U_{\mathrm{t+1}}$, $u_{\mathrm{n,t+1}} > 0$ and $u_{\mathrm{n,t+1}} < U_{\mathrm{t+1}}$. The regularity condition is

$$\frac{\left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]}{2\ (1-\phi)\ [1+h(a_{\mathrm{t}})]\ \left[S_{\mathrm{t}} + \frac{x_{\mathrm{st}}}{x_{\mathrm{ut}}}\ U_{\mathrm{t}}\right]} < H < \frac{\psi\ \left[(1-\phi)-\phi\ h(a_{\mathrm{t}})\right]}{2\ (1-\phi)[\psi+h(a_{\mathrm{t}})]\ \left[S_{\mathrm{t}} + \frac{x_{\mathrm{st}}}{x_{\mathrm{ut}}}\ U_{\mathrm{t}}\right]}. \tag{33}$$



(a) Growth Rate (b) Wage Rate

Figure 6: Diversified Regime – Growth Rate and Wage Rate of Skilled and Unskilled Human Capital

3.4.3 Growth Rate

Now, I try to figure out the growth rate of an economy in the diversified regime. From eq. (3), I get,[13]

$$\begin{aligned} g_{\mathrm{t+1}} &= \lambda \left[u_{\mathrm{m,t+1}}^{\sigma}\ s_{\mathrm{m,t+1}}^{1-\sigma} \frac{1}{\overline{A}_{\mathrm{t+1}}} \left(\frac{1-a_{\mathrm{t}}}{a_{\mathrm{t}}} \right) + \gamma\ u_{\mathrm{n,t+1}}^{\phi} s_{\mathrm{n,t+1}}^{1-\phi} \right] \\ &= \lambda \left[\left(\frac{u_{\mathrm{m,t+1}}}{s_{\mathrm{m,t+1}}} \right)^{\sigma} \ \frac{s_{\mathrm{m,t+1}}}{\overline{A}_{\mathrm{t+1}}} \ \frac{(1-a_{\mathrm{t}})}{a_{\mathrm{t}}} + \gamma \ \left(\frac{u_{\mathrm{n,t+1}}}{s_{\mathrm{n,t+1}}} \right)^{\phi} \ s_{\mathrm{n,t+1}} \right] \\ &= \lambda\ \gamma(1-\phi)\ h^{-\phi}(a_{\mathrm{t}}) \left[1 - 2H \left(S_{\mathrm{t}} + \frac{x_{\mathrm{st}}}{x_{\mathrm{ut}}} U_{\mathrm{t}} \right) \right]. \end{aligned} \tag{34}$$

Growth rate of an economy in the diversified regime depends on the relative intensity of skilled and unskilled human capital, allocation of skilled human capital on these two activities and the distance of an economy from the world technology frontier. From eq. (28) and (D4), I get that relative intensity of skilled-unskilled human capital depends positively w.r.t distance to frontier. Allocation of skilled human capital in the imitation (resp. innovation) activity depends negatively (resp. positively) on distance to frontier. Obviously, the relative gap from frontier decreases as an economy progresses. Now, I want to study the growth rate of an economy as an economy bridging the gap from the world technology frontier. From fig. (6a), I get, growth rate initially falls and after that increases as an economy progresses. That is, I get a U-shaped growth curve in the diversified regime.

[13] Detailed mathematical derivations are provided in eq. (D5) in the Appendix C.

Proposition 1

Under **A1**,

In the imitation-only regime, growth rate falls as an economy progresses.

In the Diversified regime, growth rate initially falls and after that rises as an economy progresses. That is, there exists an U-shaped growth path in the diversified regime.

In the long-run there exists constant growth rate in the innovation-only regime.

3.4.4 Wage Rate

Now, let me talk about the dynamic path of the wage rate of skilled and unskilled human capital as an economy bridges gap from the world frontier. Substituting eq. (28) in eq. (22), I get,

$$w_{\mathrm{u,\ t+1}} = \lambda\delta_1\gamma\phi h^{1-\phi}(a_{\mathrm{t}})a_{\mathrm{t}}\overline{A}_{\mathrm{t}}; \qquad w_{\mathrm{s,\ t+1}} = \lambda\delta_1\gamma(1-\phi)h^{-\phi}(a_t)a_{\mathrm{t}}\overline{A}_{\mathrm{t}}.$$

In the diversified regime, as an economy progresses relative importance of innovation (resp. imitation) increases (resp. decreases). From **A1**, marginal productivity of skilled human capital increases and unskilled human capital decreases and so does the wage rate of skilled human capital rises and unskilled human capital falls. Consequently, wage gap between skilled and unskilled human capital increases, as demonstrated in fig. (6b).

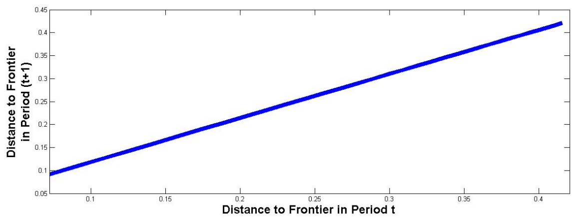
3.5 Steady State

In this subsection I look at the long run equilibrium condition of an economy. That is, as time progresses whether an economy converges its gap from the world technology frontier, depending on its distance to frontier. From the very definition of growth rate, I know that,

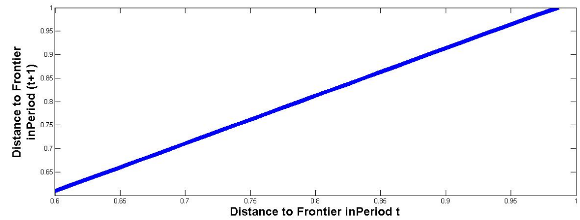
$$g_{\mathrm{t+1}} = \frac{A_{\mathrm{t+1}} - A_{\mathrm{t}}}{A_{\mathrm{t}}} = \frac{A_{\mathrm{t+1}}}{A_{\mathrm{t}}} - 1 = \frac{A_{\mathrm{t+1}}}{\overline{A}_{\mathrm{t+1}}}\frac{\overline{A}_{\mathrm{t}}}{A_{\mathrm{t}}}(1+\bar{g}) - 1 = \frac{a_{\mathrm{t+1}}(1+\bar{g})}{a_{\mathrm{t}}} - 1$$

$$a_{\mathrm{t+1}} = \frac{(1+g_{\mathrm{t+1}})}{(1+\bar{g})}a_{\mathrm{t}} \tag{35}$$

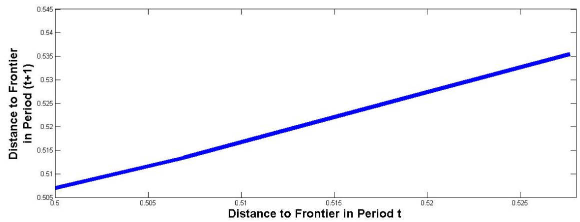
That is, if growth rate of an economy be higher that the growth rate of the world leader, then the economy will be able to converge its gap from the frontier. In the long run, it will catch up with the frontier technology level. From fig. (7a), fig. (7c) and fig. (7b), it is clear that as an economy



(a) Imitation-only Regime



(b) Innovation-only Regime



(c) Diversified Regime

Figure 7: Steady State

progresses it is converge its distance from the world technology frontier. In the long run all the economies will converge to the same technology level.

In the stead state a_{t} will converge to a^*, that is, $a_{\mathrm{t}} \to a^*$ and growth rate of the economy will converge to g^*, that is, $g_{\mathrm{t}} \to g^*$. Therefore, from eq. (35), I get that either $g^* = \bar{g}$ or $a^* = 0$. That is, in the long run all the economies will grow at the same rate. That is, there exists absolute convergence of the economies in the long run.

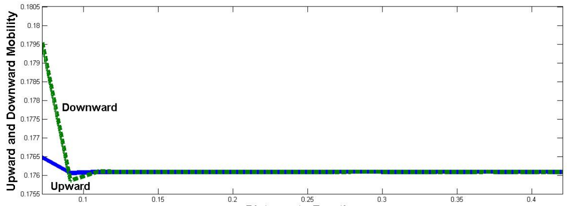
Proposition 2

In the long-run all the economies will converge to the world technology frontier irrespective of its distance to frontier. Moreover, in the steady state all the economies will grow at the same rate.

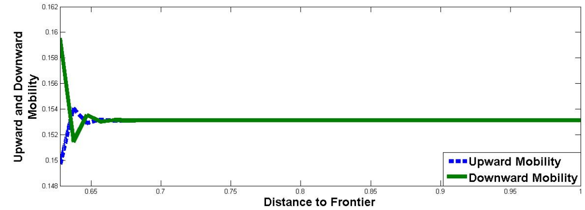
3.6 Intergenerational Mobility

In this subsection, I look at the upward and downward mobility of individuals as an economy progresses. Upward mobility implies that an individual works as a skilled worker given that his/ her parent was unskilled. That is,

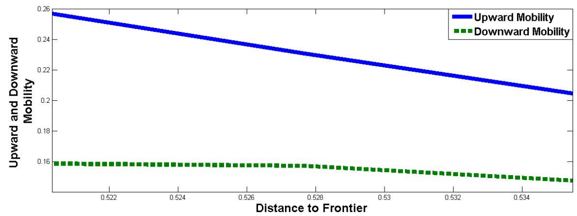
$$\mathrm{UM}_{\mathrm{t+1}} = U_{\mathrm{t}}(1 - \theta^{\mathrm{u}}_{\mathrm{t+1}}), \tag{36}$$



(a) Imitation-only Regime



(b) Innovation-Only Regime



(c) Diversified Regime

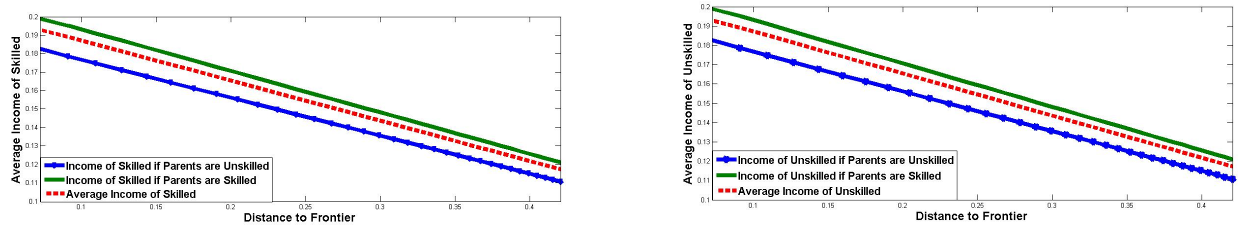
Figure 8: Upward and Downward Mobility

where, $\mathrm{UM}_{\mathrm{t+1}}$ measures upward mobility in period $(t+1)$. It captures the chance of moving from low equilibrium to high equilibrium. Whereas downward mobility implies that parent was skilled but child is working as an unskilled worker. That is,

$$\mathrm{DM}_{\mathrm{t+1}} = S_{\mathrm{t}} \theta^{\mathrm{s}}_{\mathrm{t+1}}, \tag{37}$$

where $\mathrm{DM}_{\mathrm{t+1}}$ measures downward mobility in period $(t+1)$. Intergenerational mobility helps me to capture the correlation between cognitive ability and income of an individual. Low mobility implies that income of an individual is less correlated with the cognitive ability and highly correlated with the parental income. From fig. (8a) and fig. (8b), I get that after initial adjustment there exists a constant level of upward and downward mobility in the imitation-only and in the innovation-only regimes.

Now, I look at the dynamic path of upward and dowanward mobility in the diversified regime. From fig. (6b), I know that in the diversified regime, wage gap between skilled and unskilled human capital rises as an economy progresses. This implies that, individual's whose parents are unskilled (resp. skilled) leave less (resp. more) bequest. As a result of which chance of becoming educated falls (resp. rises) for them. Subsequently, upward (resp. downward) mobility falls as an economy progresses. From fig. (8c), one can verify that, as an economy progresses both upward



(a) Average Income of Skilled

(b) Average Income of Unskilled

Figure 9: Imitation only Regime – Average Income of Skilled and Unskilled Human Capital

and downward mobility falls. That is, as gap from the world frontier falls chance of shifting from low equilibrium to high equilibrium falls. Also, the chance of moving from high equilibrium to low equilibrium falls as an economy progresses. That is, if parent was educated then chance of being educated rises and if parent was unskilled then opportunity of becoming educated falls. That is, education becomes more correlated with the parental income and less related with the cognitive ability.

Proposition 3

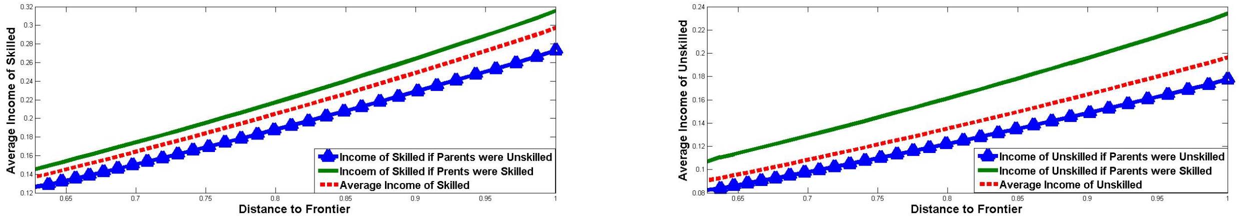
Under **A1***,*

In the imitation-only and innovation-only regimes, there exists a constant level of upward and downward mobility as an economy progresses.

In the Diversified regime, upward mobility and downward mobility falls as an economy progresses.

3.7 Income Inequality

In this subsection, first I find out the average income of skilled and unskilled human capital. Also, I derive the income inequality between skilled and unskilled human capital. Furthermore, I also work out the within skilled and unskilled group income inequality due to parental income differences. Moreover, I figure out the within skilled group welfare gap due to difference in the cognitive ability among individual's. Now, from eq. (6), I define the income of skilled and unskilled human capital depending on his/ her parental education status. Average income an individual depends his/ her



(a) Average Income of Skilled

(b) Average Income of Unskilled

Figure 10: Innovation only – Average Income of Skilled and Unskilled Human Capital

education level and the level of bequest that he/ she gets from his/ her parent.

$$I^{ss}_{\mathrm{t}+1} = \left[\frac{ws_{\mathrm{t}}}{2} - \frac{H\ wu_{\mathrm{t}}}{2} - \frac{H\ wu_{\mathrm{t}}}{2\ \theta^{\mathrm{s}}_{\mathrm{t}+1}}\right] + ws_{\mathrm{t}+1};$$

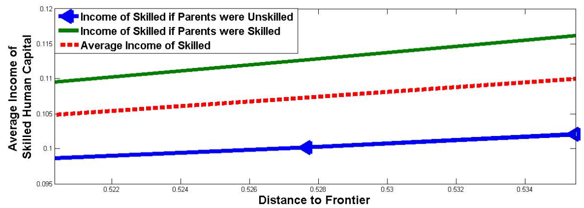
$$I^{su}_{\mathrm{t}+1} = \left[\frac{wu_{\mathrm{t}}}{2} - \frac{H\ wu_{(\mathrm{t})}}{2} - \frac{H\ wu_{\mathrm{t}}}{2\ \theta^{\mathrm{u}}_{\mathrm{t}+1}}\right] + ws_{\mathrm{t}+1};$$

$$I^{me}_{\mathrm{t}+1} = \frac{ws_{\mathrm{t}}}{2} + wu_{\mathrm{t}+1}; \qquad I^{uu}_{\mathrm{t}+1} = \frac{wu_{\mathrm{t}}}{2} + wu_{\mathrm{t}+1};$$

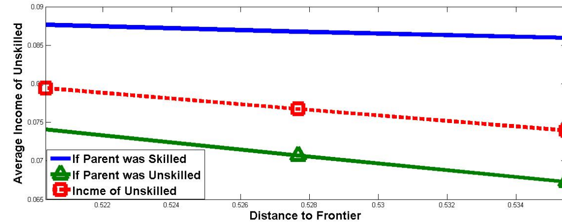
$$\overline{I}^{s}_{\mathrm{t}+1} = \frac{(1-\theta^{\mathrm{s}}_{\mathrm{t}+1})I^{ss}_{\mathrm{t}+1} + (1-\theta^{\mathrm{u}}_{\mathrm{t}+1})I^{su}_{\mathrm{t}+1}}{(1-\theta^{\mathrm{s}}_{\mathrm{t}+1}) + (1-\theta^{\mathrm{u}}_{\mathrm{t}+1})}; \qquad \overline{I}^{u}_{\mathrm{t}+1} = \frac{\theta^{\mathrm{s}}_{\mathrm{t}+1}I^{me}_{\mathrm{t}+1} + \theta^{\mathrm{u}}_{\mathrm{t}+1}I^{uu}_{\mathrm{t}+1}}{\theta^{\mathrm{s}}_{\mathrm{t}+1} + \theta^{\mathrm{u}}_{\mathrm{t}+1}}; \tag{38}$$

where, $I^{ss}_{\mathrm{t}+1}$ (resp. $I^{su}_{\mathrm{t}+1}$) measures the income of skilled human capital if parent was skilled (resp. unskilled) and $I^{me}_{\mathrm{t}+1}$ (resp. $I^{uu}_{\mathrm{t}+1}$) measures the income of unskilled human capital if parent was skilled (resp. unskilled). $\overline{I}^{s}_{\mathrm{t}+1}$ (resp. $\overline{I}^{u}_{\mathrm{t}+1}$) measures the average income of skilled (resp. unskilled) human capital. That is, average income of skilled (resp. unskilled) is the weighted average of the income of skilled (resp. unskilled) whose parent was skilled and whose parent was unskilled. From fig. (9a) and (9b), I show that average income of skilled and unskilled human capital falls as an economy progresses in the imitation-only regime. Whereas from fig. (10a) and fig. (resp. (10b), I get that average income of skilled and unskilled human capital rises as an economy progresses in the innovation-only regime. Moreover, in the diversified regime, average income of skilled human capital rises and unskilled human capital falls as gap from the world technology frontier falls, as shown in fig. (11a) and fig. (11b). That is, between skilled and unskilled group income inequality rises. That is, income path of skilled and unskilled human capital follows the same pattern as like the wage path of skilled and unskilled human capital in all the three regimes.

Now, I define the income inequality within skilled and unskilled human capital. First, I figure

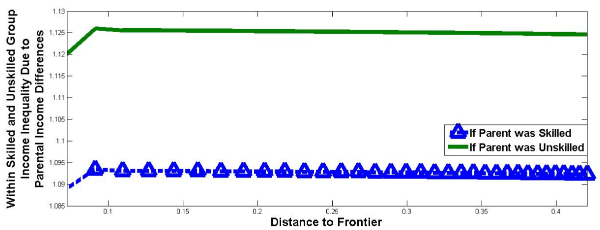


(a) Average Income of Skilled

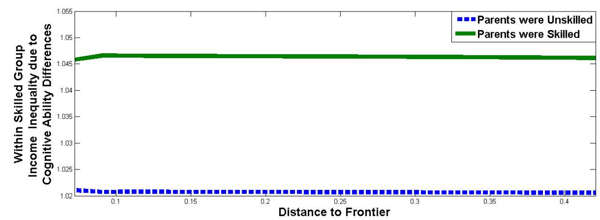


(b) Average Income of Unskilled

Figure 11: Diversified Regime – Average Income of Skilled and Unskilled Human Capital



(a) Due to Parental Income Differences



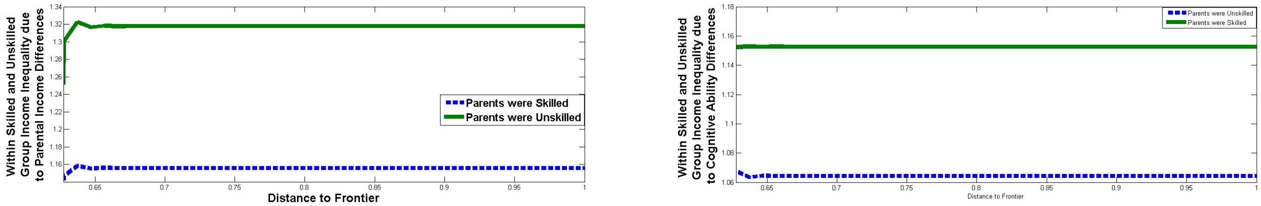
(b) Due to Cognitive Ability Difference

Figure 12: Imitation only Regime – Income Inequality with in Skilled and Unskilled Human Capital

out the income inequality due to the parental income differences.

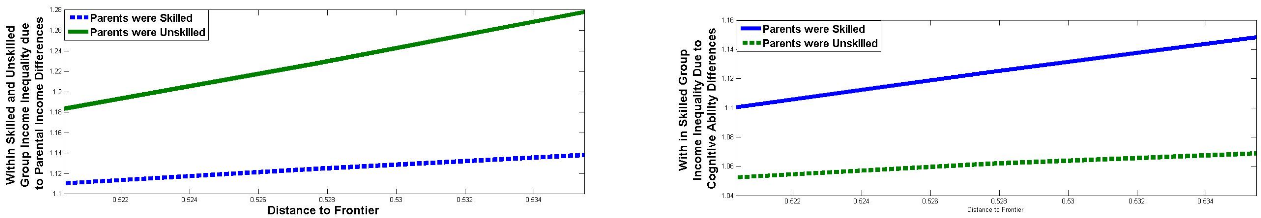
$$\mathrm{Iin}^{s}_{t+1} = \frac{I^{ss}_{t+1}}{I^{su}_{t+1}}; \qquad \mathrm{Iin}^{u}_{t+1} = \frac{I^{me}_{t+1}}{I^{uu}_{t+1}}, \tag{39}$$

where Iin^{s}_{t+1} (resp. Iin^{u}_{t+1}) measures the income inequality within skilled (resp. unskilled) human capital due to difference in the parental education level. In the imitation-only and innovation-only regimes, there exists a constant level of income inequality within skilled and unskilled human capital due to parental income differences. Income inequality within unskilled human capital is higher than skilled human capital, as shown in fig. (12a) and fig. (13a). Whereas, in the diversified regime, due to parental income differences within skilled and unskilled human capital income inequality rises as an economy progresses, as illustrated in fig. (14a). As an economy moves toward the world technology frontier, as mentioned in fig. (6b), wage gap between skilled and unskilled human capital rises. Therefore, the gap between the level of bequest that a skilled (resp. unskilled) individual



(a) Due to Parental Income Differences (b) Due to Cognitive Ability Differences

Figure 13: Innovation only Regime – Income Inequality with in Skilled and Unskilled Human Capital



(a) Due to Parental Income Differences (b) Due to Cognitive Ability Differences

Figure 14: Diversified Regime – Income Inequality with in Skilled and Unskilled Human Capital

gets from his/ her parent due to the difference in the parental income level rises. This leads to a higher income inequality within skilled (resp. unskilled) human capital due to difference in the parental income.

Now, I illustrate the income inequality among skilled human capital due to differences in the cognitive ability.

$$\mathrm{Iin}^{\theta s}_{t+1} = \frac{I^{ss}_{t+1}|_{\theta=1}}{I^{ss}_{t+1}|_{\theta=\theta^s}}; \qquad \mathrm{Iin}^{\theta u}_{t+1} = \frac{I^{su}_{t+1}|_{\theta=1}}{I^{su}_{t+1}|_{\theta=\theta^u}}, \tag{40}$$

where $\mathrm{Iin}^{\theta s}_{t+1}$ (resp. $\mathrm{Iin}^{\theta s}_{t+1}$) measures the income inequality due to the cognitive ability differences among skilled human capital even if their parents are skilled (resp. unskilled). That is, it measures the income gap of skilled human capital with highest and lowest cognitive ability. Also, all of their parents have the same income level, since all of them were educated (resp. uneducated). There also exists constant level of inequality within skilled human capital due to difference in cognitive ability in the imitation-only and innovation-only regime. This is true for both the cases – where parents were either skilled or unskilled, as shown in fig. (12b) and fig. (13b). In the imitation-innovation regime, due to difference in cognitive ability, income inequality within skilled human capital rises irrespective of their parent's education status as an economy progresses, as depicted in fig. (14b). By **Lemma1**, as an economy progresses skilled human rises in the diversified regime. That is, individual's with relative low cognitive ability now become educated. As a result of which of cost gap among the skilled human capital rises irrespective of their parental income level. This leads to a higher income inequality within skilled human capital due to difference in cognitive ability.

Proposition 4

Under **A1**,

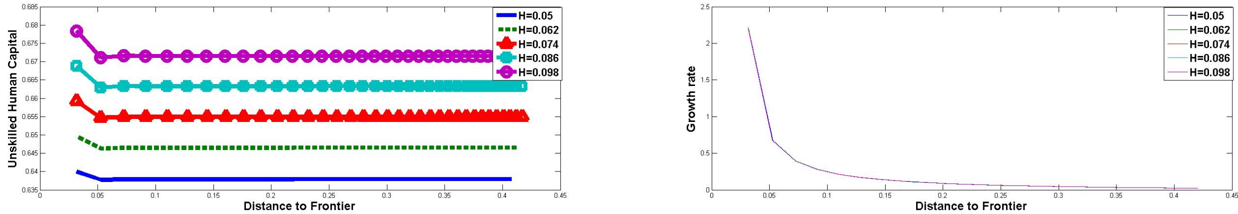
In the imitation-only regime, wage rate and average income of skilled and unskilled human capital falls as an economy progresses. There exists a constant level of wage and income inequality between skilled and unskilled group.

In the Diversified regime, wage rate and average income of skilled human capital rises and unskilled human capital falls as an economy steps forward to the world frontier. Wage inequality and income inequality between skilled and unskilled human capital rises as an economy bridges the gap from the world technology frontier.

In the innovation-only regime, wage rate and average income of skilled and unskilled human capital rises as an economy progresses. There exists a constant level of wage and income inequality between skilled and unskilled human capital.

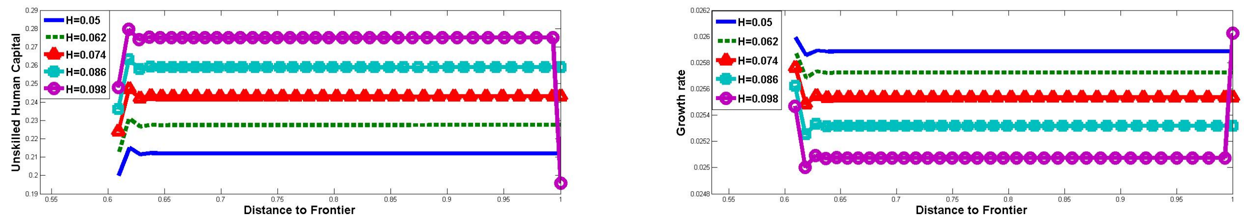
In the imitation-only and in the innovation-only regimes, there exists constant level of income inequality within skilled and unskilled human capital due to parental income differences. Whereas there exists constant level of income inequality within skilled human capital due to cognitive ability differences, in the imitation-only and innovation-only regimes.

In the imitation-innovation regime, income inequality within skilled and unskilled human capital rises due to parental income differences. Moreover, income inequality within skilled human capital rises due to cognitive ability differences.



(a) Unskilled Human Capital (b) Growth Rate

Figure 15: Imitation-Only Regime– Comparative Static wrt Cost of Education



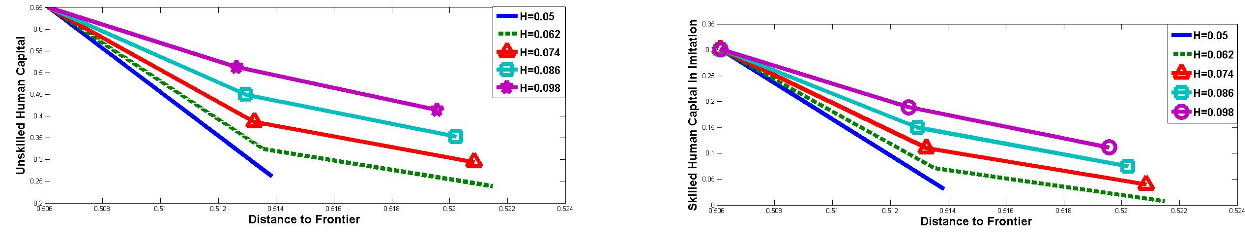
(a) Unskilled Human Capital (b) Growth Rate

Figure 16: Innovation-Only Regime– Comparative Static wrt Cost of Education

3.8 Cost of Education

In this subsection, I do some comparative static analysis. Our main focus is to look at the impact of an increment in the cost of education on the composition of human capital and also on the growth rate of an economy. As cost of education increases, income of an individual who works as a skilled worker decreases whereas income as an unskilled worker remains unchanged. Therefore, increment in cost of education reduces total stock of skilled human capital and increases unskilled human capital, irrespective of its distance to frontier, as shown in fig. (15a), in fig. (16a) and in fig. (17a). As an outcome of which growth rate also falls in the innovation-only regime, as cost of education rises, as illustrated in fig. (16b). This in turn implies that skilled human capital is growth enhancing in the innovation-only regime.

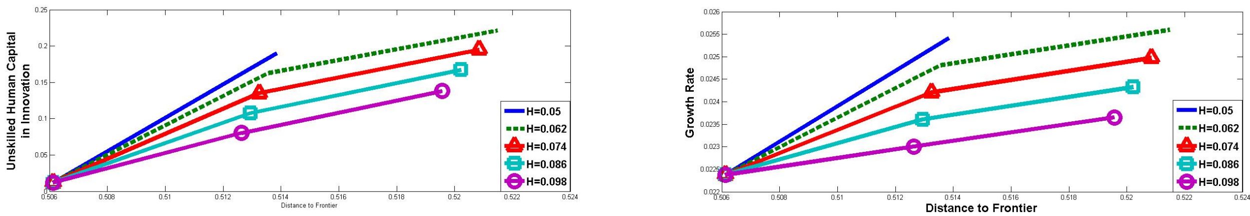
Now, I talk about the impact of higher cost of education in the allocation of skilled and unskilled human capital in the imitation and in the innovation activities. By **A1**, both skilled and unskilled human capital shifts from innovation to imitation activity and consequently growth rate falls as cost of education rises, as shown in fig. (17b), in fig. (18a) and in fig. (18b). That is, skilled human



(a) Unskilled Human Capital

(b) Skilled in Imitation

Figure 17: Diversified Regime– Comparative Static wrt Cost of Education



(a) Unskilled in Innovation

(b) Growth Rate

Figure 18: Diversified Regime– Comparative Static wrt Cost of Education

capital is growth enhancing in the imitation-innovation regime.

4 Conclusion

Technological progress is a dual phenomenon. A country can improve its technology level by imitating from the frontier or by innovating new knowledge. An economy which is lagging far behind the world technology frontier can improve its technology level by allocating its labor force mainly into imitation. Similarly, an advanced economy can progress technologically by innovating new knowledge. Under the assumption that different types of human capital are efficient in different activities, Vandenbussche, Aghion and Meghir (2006), Aghion, Boustan, Hoxby and Vandenbussche (2009) and Basu and Mehra (2014) show that unskilled human capital is the main source of growth for the technologically backward economy and skilled human capital is the main source of growth for the technologically advanced economy. Now, by utilizing an endogenous growth model, with complete absence of credit market I show that growth maximizing level of skilled and unskilled human capital is different for economies depending on its distance from the world technology frontier. Moreover, in the diversified regime opportunity of becoming rich (resp. poor) given that parent was rich (resp. poor) rises as an economy progresses. That is, as an economy progresses, correlation between income and cognitive ability falls. Also, between and within skilled and unskilled group income inequality rises in the diversified regime as an economy bridges its gap from the frontier.

Our work can be extended in several directions. First, one can allow the possibility of outsourcing of the R and D activity by a developed economy to a less developed economy. Wage rate of skilled human capital is relatively lower and the average cognitive ability of skilled human capital is higher in the less developed economy. This increases the profit of the R and D producer in the developed economy and also raises the income of skilled and unskilled human capital in the underdeveloped economy. Second, in this entire work, I assume that new knowledge is freely available to all the economies. Instead, one can characterize the growth path and the convergence condition of the economy by ruling out the assumption that world technology level is freely accessible. Third, till now all the work in this area has abstracted from international trade in commodities. One can develop a dynamic Ricardian model of international trade around the core idea of our work and can study cross-sectoral allocation of skilled and unskilled human capital in the context of international specialization in goods production and trade. Forth, one can analyze the consequences of heterogeneous cost of education depending on his/ her parental education level and can study the impact of that on growth rate, inequality and intergenerational mobility of an economy depending on its distance to frontier. This would certainly yield further insights on the relationship between distance to frontier and composition of human capital and economic growth.

Appendix A

Maximization exercise of an individual

$$\begin{aligned}
\mathbb{L}_k &= \sqrt{c_{\mathrm{k,t,t+1}}\ x_{\mathrm{k,t,t+1}}} + \mu[w_{\mathrm{k,t+1}} - c_{\mathrm{k,t,t+1}} - x_{\mathrm{k,t,t+1}}]; \\
\frac{\partial \mathbb{L}_k}{\partial c_{\mathrm{k,t,t+1}}} &= \frac{1}{2}\sqrt{\frac{x_{\mathrm{k,t,t+1}}}{c_{\mathrm{k,t,t+1}}}} - \mu = 0; \\
\frac{\partial \mathbb{L}_k}{\partial x_{\mathrm{k,t,t+1}}} &= \frac{1}{2}\sqrt{\frac{c_{\mathrm{k,t,t+1}}}{x_{\mathrm{k,t,t+1}}}} - \mu = 0; \\
\frac{\partial \mathbb{L}_k}{\partial x_{\mathrm{k,t,t+1}}} &= w_{\mathrm{k,t+1}} - c_{\mathrm{k,t,t+1}} - x_{\mathrm{k,t,t+1}} = 0 \\
\Rightarrow \quad c_{\mathrm{k,t,t+1}} &= x_{\mathrm{k,t,t+1}} = \frac{w_{\mathrm{k,t+1}}}{2} \qquad \text{(A1)}
\end{aligned}$$

Education decision of an individual

$$\begin{aligned}
&\mathbb{W}_s \geq \mathbb{W}_u \\
\Rightarrow \quad &\left[x_{\mathrm{t,t}} - \frac{Hw_{\mathrm{ut}}}{\theta}\right]\frac{w_{\mathrm{s,t+1}}}{2} \geq \ x_{\mathrm{t,t}}\frac{w_{\mathrm{u,t+1}}}{2} \qquad \text{[Using eq.(6)]} \\
\Rightarrow \quad &\theta_{\mathrm{t+1}} \geq \frac{Hw_{\mathrm{ut}}}{x_{\mathrm{t,t}}\left[1 - \left(\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}\right)\right]} \qquad \text{(A2)}
\end{aligned}$$

Appendix C

Change in the relative stock of skilled-unskilled human capital w.r.t skilled

$$\text{let,} \quad z = \frac{\widetilde{S}_{\mathrm{t+1}}}{\widetilde{U}_{\mathrm{t+1}}}$$

$$\Rightarrow \quad \ln z = \ln\widetilde{S}_{\mathrm{t+1}} - \ln\widetilde{U}_{\mathrm{t+1}}$$

$$\Rightarrow \quad \frac{1}{z}\frac{\mathrm{d}\ z}{\mathrm{d}\ \widetilde{S}_{\mathrm{t+1}}} = \frac{1}{\widetilde{S}_{\mathrm{t+1}}} + \frac{1}{\widetilde{U}_{\mathrm{t+1}}}$$

$$= \frac{1}{\widetilde{S}_{\mathrm{t+1}}\widetilde{U}_{\mathrm{t+1}}}$$

$$\Rightarrow \quad \frac{\mathrm{d}\ z}{\mathrm{d}\ \widetilde{S}_{\mathrm{t+1}}} = \frac{1}{\widetilde{U}^2_{\mathrm{t+1}}}$$

$$\Rightarrow \quad \frac{\mathrm{d}\ \widetilde{S}_{\mathrm{t+1}}}{\mathrm{d}\ \frac{\widetilde{S}_{\mathrm{t+1}}}{\widetilde{U}_{\mathrm{t+1}}}} = \widetilde{U}^2_{\mathrm{t+1}} \tag{C1}$$

Innovation-only Regime – Growth Rate

From eq. (20), I get,

$$\frac{\mathrm{d}\ \widehat{g}_{\mathrm{t+1}}}{\mathrm{d}\ \left(\frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}}\right)} = \lambda\gamma \left[-\phi \left(\frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}}\right)^{-1-\phi} \widehat{S}_{\mathrm{t+1}} + \left(\frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}}\right)^{-\phi} \frac{\mathrm{d}\ \widehat{S}_{\mathrm{t+1}}}{\mathrm{d}\ \frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}}}\right]$$

$$= \lambda\gamma \left[-\phi \left(\frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}}\right)^{-1-\phi} S_{\mathrm{t+1}} + \left(\frac{\widehat{S}_{\mathrm{t+1}}}{\widehat{U}_{\mathrm{t+1}}}\right)^{-\phi} \widehat{U}^2_{\mathrm{t+1}}\right] \qquad \text{[Using eq. (C1)]}$$

$$= \lambda\gamma\widehat{S}^{-\phi}_{\mathrm{t+1}}\widehat{U}^{(1+\phi)}_{\mathrm{t+1}} \left(\widehat{U}_{\mathrm{t+1}} - \phi\right) > 0 \quad \text{[Using eq. (18)]}$$

Appendix D

Imitation-Innovation Regime – Allocation of skilled and unskilled human capital in imitation and innovation activities

Next by using eq. (24) and eq. (26), I get the following conditions

$$\psi \left(\frac{s_{\mathrm{m,t+1}}}{u_{\mathrm{m,t+1}}}\right) = \left(\frac{s_{\mathrm{n,t+1}}}{u_{\mathrm{n,t+1}}}\right)$$

$$\Rightarrow \quad \psi \left(\frac{S_{\mathrm{t+1}} - s_{\mathrm{n,t+1}}}{U_{\mathrm{t+1}} - u_{\mathrm{n,t+1}}}\right) = \left(\frac{s_{\mathrm{n,t+1}}}{u_{\mathrm{n,t+1}}}\right)$$

$$\Rightarrow \qquad u_{\mathrm{n,t+1}} = \left[\frac{s_{\mathrm{n,t+1}}\ U_{\mathrm{t+1}}}{\psi\ S_{\mathrm{t+1}} - (\psi - 1)\ s_{\mathrm{n,t+1}}}\right];$$

$$\text{Again,} \quad u_{\mathrm{m,t+1}} = (U_{\mathrm{t+1}} - u_{\mathrm{n,t+1}})$$

$$= \left[\frac{\psi\ s_{\mathrm{m,t+1}}\ U_{\mathrm{t+1}}}{S_{\mathrm{t+1}} - (\psi - 1)\ s_{\mathrm{n,t+1}}}\right]; \qquad \text{(D1)}$$

From eq. (D1) in eq. (22) and by using (24), I get the demand for skilled human capital in the innovation activity

$$(1-\sigma) \left(\frac{s_{\mathrm{m,t+1}}}{u_{\mathrm{m,t+1}}}\right)^{-\sigma} \frac{1}{(1+\bar{g})\overline{A}_{\mathrm{t}}} \frac{(1-a_{\mathrm{t}})}{a_{\mathrm{t}}} = \gamma\ (1-\phi) \left(\frac{s_{\mathrm{n,t+1}}}{u_{\mathrm{n,t+1}}}\right)^{-\phi}$$

$$\Rightarrow \quad (1-\sigma) \left[\frac{\psi U_{\mathrm{t+1}}(S_{\mathrm{t+1}} - s_{\mathrm{n,t+1}})}{\psi S_{\mathrm{t+1}} - (\psi-1)s_{\mathrm{n,t+1}}}\right]^{\sigma} (S_{\mathrm{t+1}} - s_{\mathrm{n,t+1}})^{-\sigma} \frac{1}{(1+\bar{g})\overline{A}_{\mathrm{t}}} \frac{(1-a_{\mathrm{t}})}{a_{\mathrm{t}}}$$

$$= \gamma(1-\phi) \left[\frac{U_{\mathrm{t+1}} s_{\mathrm{n,t+1}}}{[\psi S_{\mathrm{t+1}} - (\psi-1)s_{\mathrm{n,t+1}}]}\right]^{\phi} s_{\mathrm{n,t+1}}^{-\phi}$$

$$\Rightarrow \quad \left[\frac{(1-\sigma)\psi^{\sigma}(1-a_{\mathrm{t}})}{\gamma(1-\phi)(1+\bar{g})\overline{A}_{\mathrm{t}} a_{\mathrm{t}}}\right] U_{\mathrm{t+1}}^{\sigma-\phi} = [\psi S_{\mathrm{t+1}} - (\psi-1)s_{\mathrm{n,t+1}}]^{\sigma-\phi}$$

$$\Rightarrow s_{\mathrm{n,t+1}} = \left[\frac{\psi\ S_{\mathrm{t+1}} - h(a_{\mathrm{t}})\ U_{\mathrm{t+1}}}{\psi - 1}\right], \qquad \text{(D2)}$$

where $h(a_{\mathrm{t}}) = \left[\frac{(1-\sigma)\psi^{\sigma}(1-a_{\mathrm{t}})}{\gamma(1-\phi)(1+\bar{g})\overline{A}_{\mathrm{t}} a_{\mathrm{t}}}\right]^{\frac{1}{(\sigma-\phi)}}$.

From eq. (D1) and (D2), I get the demand for skilled human capital in the imitation activity

$$s_{\mathrm{m,t+1}} = (S_{\mathrm{t+1}} - s_{\mathrm{n,t+1}}) = \left[\frac{h(a_{\mathrm{t}})\ U_{\mathrm{t+1}} - S_{\mathrm{t+1}}}{\psi - 1}\right];$$

$$\text{Now,}\quad [\psi S_{\mathrm{t+1}} - (\psi - 1)s_{\mathrm{n,t+1}}] = h(a_{\mathrm{t}})U_{\mathrm{t+1}};$$

$$\text{Therefore,}\quad u_{\mathrm{n,t+1}} = \left[\frac{\psi S_{\mathrm{t+1}} - h(a_{\mathrm{t}})\ U_{\mathrm{t+1}}}{(\psi - 1)\ h(a_{\mathrm{t}})}\right];$$

$$u_{\mathrm{m,t+1}} = \left[\frac{\psi[h(a_{\mathrm{t}})\ U_{\mathrm{t+1}} - S_{\mathrm{t+1}}]}{(\psi - 1)\ h(a_{\mathrm{t}})}\right]. \tag{D3}$$

Now, differentiating $h(a_t)$ w.r.t a_t, I get,

$$\begin{aligned} h'(a_{\mathrm{t}}) &= -\frac{1}{(\sigma - \phi)}\left[\frac{(1-\sigma)\psi^{\sigma}(1-a_{\mathrm{t}})}{\gamma(1-\phi)(1+\bar{g})\overline{A}_{\mathrm{t}}a_{\mathrm{t}}}\right]^{\frac{1}{(\sigma-\phi)}-1}\left[\frac{(1-\sigma)\psi^{\sigma}}{\gamma(1-\phi)(1+\bar{g})\overline{A}_{\mathrm{t}}}\right]\frac{1}{a^2_{\text{t-1}}} \\ &= -\frac{h(a_{\mathrm{t}})}{(\sigma-\phi)a_t(1-a_{\mathrm{t}})} < 0. \end{aligned} \tag{D4}$$

Imitation-Innovation Regime – Growth Rate

From eq. (3), I get,

$$\begin{aligned} g_{\mathrm{t+1}} &= \lambda\left[\frac{u^{\sigma}_{\mathrm{m,t+1}}\ s^{1-\sigma}_{\mathrm{m,t+1}}}{(1+\bar{g})\overline{A}_{\mathrm{t}}}\left(\frac{1-a_{\mathrm{t}}}{a_{\mathrm{t}}}\right) + \gamma\ u^{\phi}_{\mathrm{n,t+1}}s^{1-\phi}_{\mathrm{n,t+1}}\right] \\ &= \lambda\left[\frac{\psi^{\sigma}}{(1+\bar{g})\overline{A}_{\mathrm{t}}}h^{-\sigma}(a_{\mathrm{t}})s_{\mathrm{m,t+1}}\left(\frac{1-a_{\mathrm{t}}}{a_{\mathrm{t}}}\right) + \gamma h^{-\phi}(a_{\mathrm{t}})\ s_{\mathrm{n,t+1}}\right] \quad \text{[Using eq. (28)]} \end{aligned}$$

$$\text{Now,}\quad h(a_{\mathrm{t}}) = \left[\frac{(1-\sigma)\psi^{\sigma}(1-a_{\mathrm{t}})}{\gamma(1-\phi)(1+\bar{g})\overline{A}_{\mathrm{t}}a_{\mathrm{t}}}\right]^{\frac{1}{(\sigma-\phi)}}$$

$$\Rightarrow\quad \frac{(1-a_{\mathrm{t}})}{a_{\mathrm{t}}} = \frac{\gamma(1-\phi)(1+\bar{g})\overline{A}_{\mathrm{t}}}{(1-\sigma)\psi^{\sigma}}h^{(\sigma-\phi)}(a_{\mathrm{t}})$$

$$\begin{aligned} \text{Therefore,}\quad g_{\mathrm{t+1}} &= \lambda\left[\psi^{\sigma}\ h^{-\sigma}(a_{\mathrm{t}})\frac{\gamma\ (1-\phi)}{(1-\sigma)\ \psi^{\sigma}}\ h^{\sigma-\phi}(a_{\mathrm{t}})\ s_{\mathrm{m,t+1}}\ +\ \gamma\ h^{-\phi}(a_{\mathrm{t}})\ s_{\mathrm{n,t+1}}\right] \\ &= \lambda\ \gamma\ h^{-\phi}(a_{\mathrm{t}})\left[\frac{(1-\phi)}{(1-\sigma)}\ s_{\mathrm{m,t+1}}\ +\ s_{\mathrm{n,t+1}}\right] \\ &= \lambda\ \gamma(1-\phi)\ h^{-\phi}(a_{\mathrm{t}})\left[1 - 2H\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}}S_{\mathrm{t}}\right]\right] \end{aligned} \tag{D5}$$

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