# Composition of Public Education Expenditures and Human Capital Accumulation

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# Composition of Public Education Expenditures and Human Capital Accumulation

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**Abstract**

This paper provides a simple theory to study how the allocation of public funds between primary and higher education affects human capital accumulation. The allocation is endogenously determined through majority voting. Public funding for higher education is not supported when a majority is poor. In some cases, higher education starts to be realized as a majority of individuals accumulate enough human capital through primary education. Although the emergence of higher education can accelerate aggregate human capital accumulation, it widens income inequality because the very poor are excluded from higher education and the declined budget share for primary education decreases its quality.

**JEL Classifications:** D72, O11, O15, O40
**Keywords:** Public Education, Economic Development, Income Inequality, Majority Voting

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# 1 Introduction

Human capital accumulation can be an engine of economic growth, and the government has significant roles in providing formal education. In many existing studies, human capital is produced in a single education sector, where an amount of government expenditure is a direct input. In reality, however, an education system is divided into multiple stages, such as primary, secondary and tertiary education, and the government supports each of them. Primary and secondary (K-12) education has a particularly big difference from tertiary education in the sense that it is mandatory but serves everyone for free although college education is optional and requires private spending.

It is relatively recently, however, that a number of studies have begun to analyze how the allocation of government expenditures for different education sectors affects economic growth and income distribution (Restuccia and Urrutia, 2004; Su, 2004; Blankenau, 2005; Blankenau et al., 2007; Arcalean and Schiopu, 2010). In particular, with heterogeneity in income or innate ability, a policy that changes budget allocations across different education sectors may increase the welfare of some individuals at the expense of others as shown by Su (2004) and Blankenau et al. (2007), which provides fertile ground for politico-economic analysis. For example, enhancing college education may most benefit the rich but may not benefit the poor who cannot afford to attend there or those whose private return from college is low due to their low innate ability.

This paper proposes an overlapping generations model in which the allocation of a given level of public education expenditures for multiple education sectors is determined via majority voting and analyzes its effects on human capital accumulation. Individuals live for two periods, childhood and adulthood, and they have children when they are adult. Individuals in adulthood make all the economic and political decisions but they care about human capital of their children. The government operates two education sectors, which we call primary and higher education sectors. Primary education is compulsory but serves all childhood individuals free of charge. On the other hand, higher education is optional and requires private expenditures. In each education, private return on education depends on resource allocations from the government and parental human capital. Parents with low human capital are unable to or unwilling to have their children receive higher education because its private return is low or they are too poor to cover its private costs. Parents whose human capital is above some threshold are willing and able to have their children obtain higher education. We show that the individual with median human capital is the decisive voter although preferences for the allocation policy are not necessarily single-peaked.

Our model generates various dynamics of human capital accumulation and steady state income inequalities. As long as human capital of the median voter is below some threshold, all the resources are provided for primary education. In some cases, the median voter accumulates sufficient human capital through primary education and a positive share of government expenditure is allocated to higher education. Although the emergence of higher education can facilitate aggregate human capital accumulation, it widens steady state income inequality because the very poor are excluded from higher education and declined share for primary education diminishes its quality. In other cases, public support for higher education is never realized if the initial human capital level of the median voter is low.

The relationship between the political economy of education policies and economic growth has been studied at least since Glomm and Ravikumar (1992) and Saint-Paul and Verdier (1993). A more recent example includes Galor et al. (2009), who examine an incentive of agents who are better endowed with production factors to prevent the implementation of public education. They argue that landowners wish to block policy changes that promote human capital formation and unequal distribution of landownership has an adverse effect on growth.

The rest of this paper is organized as follows. Section 2 sets up a model, and Section 3 analyzes individuals' decisions of investment in higher education. Section 4 studies how the share of government budget for primary and higher education is determined under majority voting. Section 5 examines interactions between the education policy and human capital accumulation. Section 6 concludes.

# 2 The Model

We study an overlapping generations model. Each generation consists of a unit mass of individuals who live for two periods, childhood and adulthood. Every individual, who belongs to a lineage indexed by $i$, has one child in the second period. The only source of heterogeneity across individuals is the human capital of their parents. Individual $i$ born in period $t$ derives utility from consumption in her adulthood and human capital of her child:

$$U(c_{it+1}, h_{it+1}) = c_{it+1} + h_{it+1}, \tag{1}$$

where $c_{it+1}$ is the consumption in her adulthood and $h_{it+1}$ is the human capital of her child.

In the childhood, individuals make no decisions but receive compulsory primary education supplied by the government. After the completion of the primary education, their parents may decide to have them receive further education, or higher education, which is also supplied by the government. Higher education requires a private cost, whose size is fixed and normalized to one. Parents cannot borrow against future human capital of their children and therefore must self-finance the cost. In the adulthood, individuals make all the economic and political decisions. They supply their human capital to a firm that transforms one unit of human capital to one unit of a final good. The final good market is perfectly competitive and therefore the wage per unit of human capital is one. Individuals also vote for an allocation of government expenditures on primary and higher education, which determines the quality of each education. After that, they decide, as parents, whether they have their children receive higher education and then consume all the remaining wealth.

If individual $i$ born in period $t+1$ receives only primary education, she accumulates human capital according to

$$h_{it+1} = B^P \left( \frac{G^P_{t+1}}{\overline{h}_t} \right)^{\beta} h^{\gamma}_{it}, \quad \beta \in (0,1), \gamma \in (0,1), \tag{2}$$

where $h_{it+1}$ is the human capital output, $B^P$ is the productivity parameter of primary education, $G^P_{t+1}$ is the government expenditure on primary education, $h_{it}$ is her parental human capital, and $\overline{h}_t$ is the average (and aggregate) human capital of the parental generation. The term $G^P_{t+1}/\overline{h}_t$ should be interpreted as the number of teachers hired at primary schools. Since the wage per unit of human capital paid by the final good sector is one, the government must compensate the same wage rate to hire an old individual as a teacher. The cost of hiring teachers are thus $\overline{h}_t$ and $G^P_{t+1}/\overline{h}_t$ represents the number of teachers.

If individual $i$ receives higher education, she accumulates human capital according to

$$h_{it+1} = B^P \left( \frac{G^P_{t+1}}{\overline{h}_t} \right)^{\beta} h^{\gamma}_{it} + B^A \left( \frac{G^A_{t+1}}{\overline{h}_t} \right)^{\beta} h^{\gamma}_{it}, \tag{3}$$

where $G^A_{t+1}$ is the government expenditure on higher education and $B^A$ is its productivity parameter. The term $G^A_{t+1}/\overline{h}_t$ represents the number of teachers employed at higher education institutions. We assume that

$$B^A \geq B^P. \tag{A.1}$$

Income is taxed at the rate of $\tau \in (0, 1)$, and the tax revenue is $\tau \overline{h}_t$. The government allocates the revenue to primary and higher education, which leads to

$$\begin{aligned} G^P_{t+1} &= x_{t+1} \tau \overline{h}_t, & (4) \\ G^A_{t+1} &= (1 - x_{t+1}) \tau \overline{h}_t, & (5) \end{aligned}$$

where $x_{t+1} \in [0, 1]$ is the share of government expenditure on primary education. In period $t + 1$, individuals born in period $t$ vote for the value of $x_{t+1}$, which affects the human capital of their children. Substituting out $G^P_{t+1}$ and $G^A_{t+1}$ from (2) and (3) gives the human capital level of individual $i$ born in period $t + 1$:

$$h_{it+1} = B^P x^{\beta}_{t+1} \tau^{\beta} h^{\gamma}_{it} \tag{6}$$

if she receives only primary education, and

$$h_{it+1} = B^P x^{\beta}_{t+1} \tau^{\beta} h^{\gamma}_{it} + B^A (1 - x_{t+1})^{\beta} \tau^{\beta} h^{\gamma}_{it} \tag{7}$$

if she receives higher education.

# 3 Investment in Education

Given the share of government expenditure on primary and higher education, each individual decides whether she invests in higher education for her child. When individual $i$ with $h_{it}$ has her child obtain higher education, (1) and (7) give her utility, $V^E(x_{t+1}, h_{it})$, as

$$V^E(x_{t+1}, h_{it}) = [(1 - \tau) h_{it} - 1] + [B^P x^{\beta}_{t+1} + B^A (1 - x_{t+1})^{\beta}] \tau^{\beta} h^{\gamma}_{it}. \tag{8}$$

The first term is the consumption in her adulthood and the second term is human capital of her child. When individual $i$ decides not to have her child receive higher education, (1) and (6) give

$$V^N(x_{t+1}, h_{it}) = (1-\tau)h_{it} + B^P x_{t+1}^{\beta}\tau^{\beta}h_{it}^{\gamma}, \tag{9}$$

where $V^N(x_{t+1}, h_{it})$ is her utility.

Individual $i$ with $h_{it}$ is able to invest in higher education for her child if $(1-\tau)h_{it} \geq 1$, which is equivalent to

$$h_{it} \geq \frac{1}{1-\tau} \equiv \bar{H}. \tag{10}$$

Individual $i$ is willing to have her child receive higher education if and only if $V^E(x_{t+1}, h_{it}) \geq V^N(x_{t+1}, h_{it})$. Simple calculations show

$$h_{it} \geq \left[\frac{1}{B^A(1-x_{t+1})^{\beta}\tau^{\beta}}\right]^{\frac{1}{\gamma}} \equiv H(x_{t+1}). \tag{11}$$

$H'(x) > 0$ since smaller share of government expenditure on higher education lowers its productivity, which makes more individuals unwilling to invest in education for their children. $H(x)$ satisfies

$$H(0) = \left(\frac{1}{B^A\tau^{\beta}}\right)^{\frac{1}{\gamma}} \quad \text{and} \quad \lim_{x\to 1} H(x) = \infty. \tag{12}$$

We assume that

$$\bar{H} > H(0), \tag{A.2}$$

which means that higher education is not affordable for some individuals. In the following analysis, it is useful to define $x(h_{it})$ by

$$H\left[x(h_{it})\right] = h_{it} \Leftrightarrow x(h_{it}) = 1 - \frac{1}{\tau}\left(\frac{1}{B^A h_{it}^{\gamma}}\right)^{\frac{1}{\beta}}. \tag{13}$$

Figure 1 sketches $H(x)$ as well as $\bar{H}$.

Let us consider decisions of parents born in period $t$ and their resulting utility under given $x_{t+1}$. If $0 \leq h_{it} < \bar{H}$, individual $i$ is unable or unwilling to have her child receive education and her utility is therefore $V^N(x_{t+1}, h_{it})$. If $h_{it} \geq \bar{H}$, then there are two cases that must be considered; (i) $0 \leq x_{t+1} \leq$

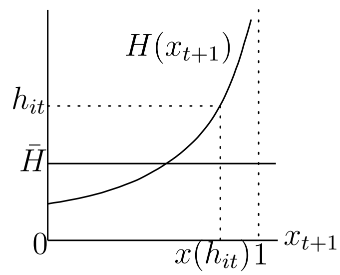

Figure 1: Feature of $H(x)$

$x(h_{it})$ and (ii) $x(h_{it}) < x_{t+1} \leq 1$. In the case where $0 \leq x_{t+1} \leq x(h_{it})$, individual $i$ is willing and able to have her child get higher education, and her utility is $V^E(x_{t+1}, h_{it})$. On the other hand, if $x(h_{it}) < x_{t+1} \leq 1$, she is able but unwilling to have her child receive education because small share on higher education, $1-x$, makes its productivity low. Her utility is hence $V^N(x_{t+1}, h_{it})$.

# 4 Political Equilibrium

Based on the analysis so far, this section identifies the most preferred share for each individual and characterizes political equilibrium. First of all, higher education is not affordable for any $x_{t+1} \in [0,1]$ if $0 \leq h_{it} < \bar{H}$. Since $V^N(x_{t+1}, h_{it})$ is increasing in $x_{t+1}$, an individual with $h_{it} \in [0, \bar{H})$ prefers $x_{t+1} = 1$. In order to identify the most preferred share of an individual with $h_{it} \geq \bar{H}$, let us define $x^*$ by

$$x^* = \underset{x_{t+1}}{\arg\max}\ V^E(x_{t+1}, h_{it}). \tag{14}$$

By the first order condition,

$$x^* = \frac{(B^P/B^A)^{\frac{1}{1-\beta}}}{1 + (B^P/B^A)^{\frac{1}{1-\beta}}} \in (0,1). \tag{15}$$

The remaining task in order to find out the most preferred share is to compare $V^E(x^*, h_{it})$ with $V^N(1, h_{it})$. Although individuals prefer $x_{t+1} = x^*$ under the presumption that they invest in higher education for their children, the individuals may prefer to set $x_{t+1} = 1$ and to choose not to invest in

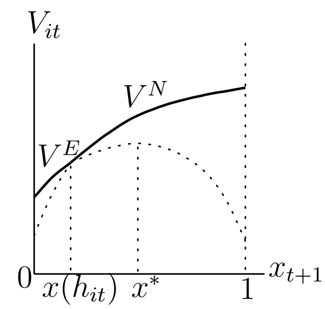


Figure 2: Welfare of individuals with $h_{it} \in [\bar{H}, H(x^*))$

higher education. If $x^* > x(h_{it})$, or equivalently $h_{it} < H(x^*)$, then individual $i$ with $h_{it}$ is unwilling to have her child receive higher education. Hence, she obviously prefers $x_{t+1} = 1$ to $x_{t+1} = x^*$. Figure 2 draws the welfare of such individuals as a function of $x_{t+1}$. If $x^* \leq x(h_{it})$, or $h_{it} \geq H(x^*)$, then an individual with $h_{it}$ is willing to have her child obtain higher education at $x_{t+1} = x^*$. However, $V^N(1, h_{it})$ may exceed $V^E(x^*, h_{it})$. Simple calculations show that $V^E(x^*, h_{it}) \geq V^N(1, h_{it})$ is equivalent to

$$\left\{B^A(1-x^*)^\beta - B^P\left[1-(x^*)^\beta\right]\right\}\tau^\beta h_{it}^\gamma \geq 1. \tag{16}$$

Since we assume $B^A \geq B^P$, the inequality can be rewritten as

$$h_{it} \geq \left[\frac{1}{\left\{B^A(1-x^*)^\beta - B^P\left[1-(x^*)^\beta\right]\right\}\tau^\beta}\right]^{\frac{1}{\gamma}} \equiv \tilde{H} > H(x^*). \tag{17}$$

Figure 3 (a) and (b) depict the welfare of an individual with $h_{it} < \tilde{H}$ and $h_{it} \geq \tilde{H}$, respectively. The following lemma summarizes the obtained results.

**Lemma 1** *Individual $i$ with $h_{it}$ prefers $x_{t+1} = x^*$ if $h_{it} \geq \tilde{H}$, while prefers $x_{t+1} = 1$ if $h_{it} < \tilde{H}$.*

It is easy to show that the individual with median income is the decisive voter although $V(x_{t+1}, h_{it}) \equiv \max\left\{V^N(x_{t+1}, h_{it}), V^E(x_{t+1}, h_{it})\right\}$ is not necessarily single peaked. This result immediately follows from lemma 1. Let $h_{mt}$ denote the human capital of the individual with median income. If $h_{mt} < \tilde{H}$, then she prefers $x_{t+1} = 1$ the most. Since individuals whose human capital is less than $\tilde{H}$, who comprises of 50 percent of the total population, also prefer $x_{t+1} = 1$ the most, it is chosen in majority voting. Similarly, if $h_{mt} \geq \tilde{H}$,

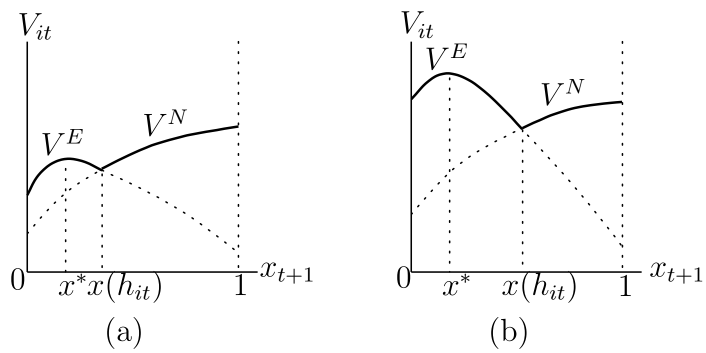


Figure 3: Welfare of individuals with $h_{it} \geq H(x^*)$

then the median voter prefers $x_{t+1} = x^*$. Since individuals whose human capital is greater than $\tilde{H}$, who consist of 50 percent of the total population, also prefer $x_{t+1} = x^*$, it is chosen in majority voting.

**Proposition 1** *Under majority voting, $x_{t+1} = 1$ if $h_{mt} < \tilde{H}$, and $x_{t+1} = x^*$ if $h_{mt} \geq \tilde{H}$.*

# 5 Dynamic Analysis

This section analyzes equilibrium dynamics. It should be remembered that the realization of higher education only relies on whether or not $h_{mt}$ exceeds $\tilde{H}$. Let $h_1^*$, $h_2^*$, and $h_3^*$ denote

$$F^P(h_1^*, 1) = h_1^* \quad \Leftrightarrow \quad h_1^* = (B^P \tau^\beta)^{\frac{1}{1-\gamma}}, \tag{18}$$

$$F^P(h_2^*, x^*) = h_2^* \quad \Leftrightarrow \quad h_2^* = \left[B^P \tau^\beta (x^*)^\beta\right]^{\frac{1}{1-\gamma}}, \tag{19}$$

$$F^A(h_3^*, x^*) = h_3^* \quad \Leftrightarrow \quad h_3^* = \left[B^P \tau^\beta (x^*)^\beta + B^A \tau^\beta (1 - x^*)^\beta\right]^{\frac{1}{1-\gamma}}, \tag{20}$$

respectively. There are three cases that need to be considered; (i) $\tilde{H} \leq h_1^*$, (ii) $h_1^* < \tilde{H} \leq h_3^*$, and (iii) $h_3^* < \tilde{H}$.

First, let us consider the case where $\tilde{H} < h_1^*$. Figure 4 (a) sketches the dynamics of $h_{mt}$. If $h_{m0} < \tilde{H}$, higher education is not realized at the first place and all individuals consequently accumulate their human capital according to $F^P(h_{it}, 1)$. However, human capital of lineage $m$ eventually exceeds $\tilde{H}$ at some time period, say $\hat{t}$, following which higher education is always realized and $h_{mt}$ converges to $h_3^*$. Human capital dynamics of individuals in the other lineages can also be easily observed. As shown in Figure 4 (b), individuals in lineage $i$ with $h_{i\hat{t}} \geq H(x^*)$ always have their children receive higher education since period $\hat{t}$, and human capital in such lineages converges to $h_3^*$.

For an individual in lineage $i$ such that $h_{i\hat{t}} < H(x^*)$, there are two possibilities. If $h_2^* \geq H(x^*)$ as in Figure 4 (c), then human capital in the lineage converges to $h_3^*$ although individuals in the lineage obtain only primary education for some periods after period $\hat{t}$. In this case, human capital in any lineage converges to $h_3^*$ and there is hence no steady state income inequality. If $h_2^* < H(x^*)$, human capital in the lineage converges to $h_2^*$ because it evolves according to $F^P(h_{it}, x^*)$ at period $\hat{t}$ and the subsequent periods. This dynamics is shown in Figure 4 (d). As contrasted with the case of $h_2^* \geq H(x^*)$, there is considerable income inequality in the long-run. In the steady state, some individuals obtain both primary and higher education while the others obtain only primary education, and moreover, the quality of primary education is diminished since a positive share of the government budget is used to finance higher education. This creates high income inequality. Notice that if $h_{m0} \geq \tilde{H}$, then $\hat{t} = 0$ and the above analysis remains intact.

Second, we need to discuss the case where $h_1^* < \tilde{H} \leq h_3^*$. In this case, the realization of higher education is dependent on the value of $h_{m0}$. Figure 5 (a) depicts the dynamics of $h_{mt}$. If $h_{m0} < \tilde{H}$, $h_{mt}$ converges to $h_1^* < \tilde{H}$. Higher education is hence never realized, and human capital of all lineages converges to $h_1^*$. There is no income inequality in the long-run. If $h_{m0} \geq \tilde{H}$, on the other hand, higher education is realized at the beginning, and $h_{mt}$ converges to $h_3^*$. Dynamic analysis on the other lineages is essentially the same as that in the case where $\tilde{H} \leq h_1^*$ and $h_{m0} \geq \tilde{H}$ ($\hat{t} = 0$). For lineage $i$ such that $h_{i0} \geq H(x^*)$, all individuals obtain higher education all the time and $h_{it}$ converges to $h_3$. For lineage $i$ such that $h_{i0} \leq H(x^*)$, the dynamic behavior of $h_{it}$ depends on the value of $h_2^*$ and $H(x^*)$. If $h_2^* \geq H(x^*)$, $h_{it}$ follows the path illustrated in Figure 4 (c). Every individual in the economy obtain $h_3^*$ in the steady state and there is income inequality in the long-run. If $h_2^* < H(x^*)$, all individuals in a lineage with $h_{i0} < H(x^*)$ receive only primary education and $h_{it}$ converges to $h_2^*$ as shown in Figure 4 (d). This generates substantial long-run income inequality.

Finally, in the case where $h_3^* < \tilde{H}$, higher education is not realized in the long-run. If $h_{m0} \geq \tilde{H}$, then higher education obtains majority support in the beginning, but $h_{mt}$ eventually falls below $\tilde{H}$, following which higher

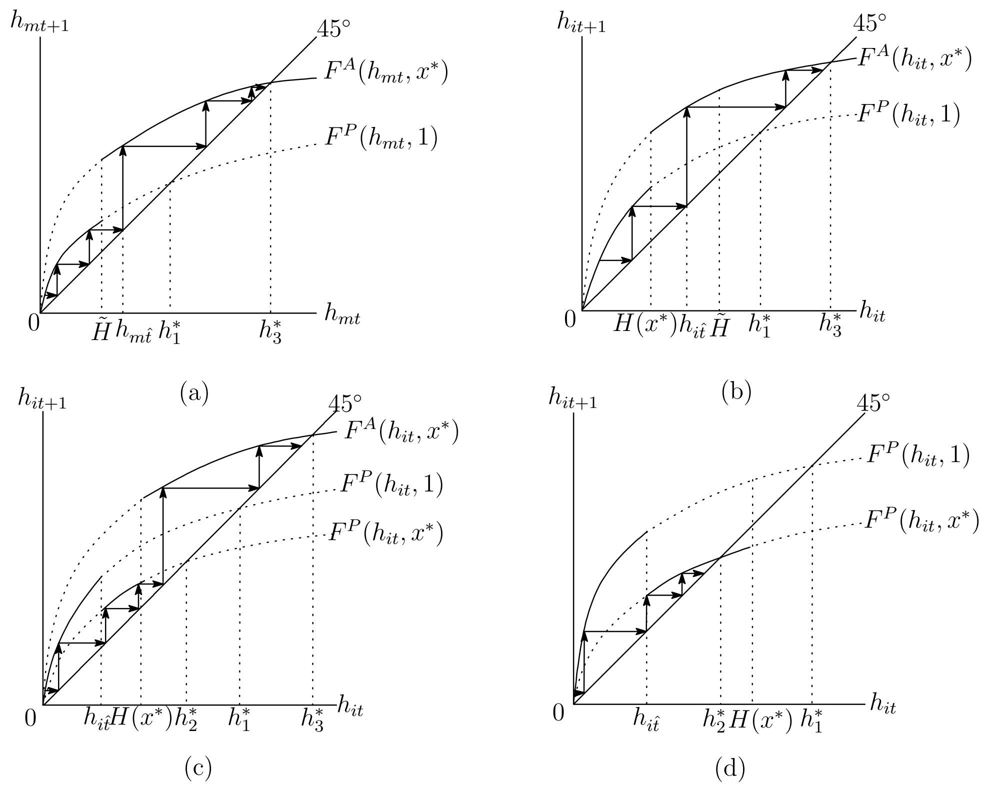


Figure 4: Dynamics of $h_{mt}$ and $h_{it}$ in the case of $\tilde{H} \leq h_1^*$

education is not realized as illustrated in Figure 5 (b). The level of human capital in any lineage consequently converges to $h_1^*$ and there exists no income inequality in the long-run.

# 6 Conclusion

The government plays an important role of funding both primary and higher education. This paper provides a theoretical examination on how the allocation of public expenditures across the each education influences the dynamics of human capital accumulation if the allocation is determined through majority voting. Primary education is compulsory while higher education is optional and requires private expenditures. When a majority of individuals

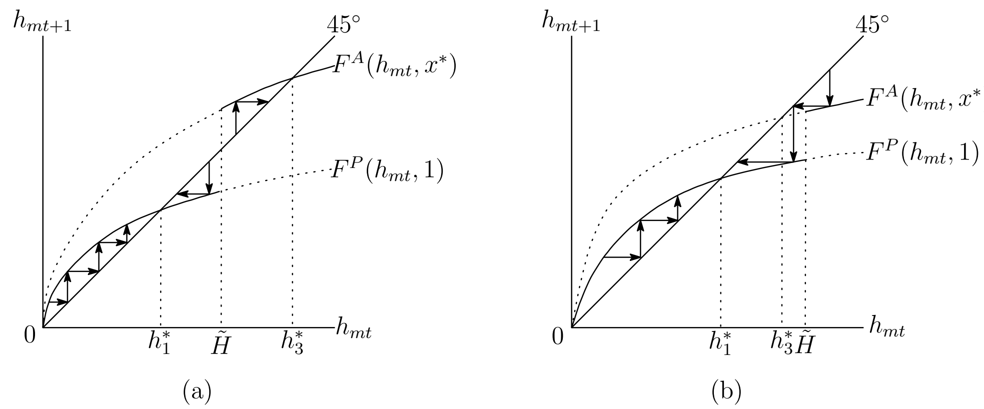


Figure 5: Dynamics of $h_{mt}$ in the case of $h_1^* < \tilde{H} \leq h_3^*$ and $h_3^* < \tilde{H}$

have not accumulated enough human capital, public funding for higher education is not realized. In some cases, however, the individuals accumulate human capital sufficiently, and higher education begins to obtain majority support. This enables a majority of individuals to accelerate their human capital accumulation although it decreases the budget share for primary education and declines its quality. As an economy grows, income inequality can expand between individuals who obtain higher education and those who do not.

We have assumed that individuals must cover private costs to obtain higher education supplied by the government. The logic of our model would be applied to other public policies by considering a situation in which access to publicly provided services requires private spending. For example, the government may consider two policies: (i) public support to enhance productivity in high-tech industries that employ skilled workers, and (ii) lump-sum transfer of a tax revenue to all individuals. Rich individuals who can cover education costs to be a skilled worker would prefer public support for high-tech industries while poor individuals would prefer lump-sum transfer. The implication of such a situation for economic growth would be a topic of future research.

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