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Chu, Angus C. and Furukawa, Yuichi and Ji, Lei

University of Liverpool, Chukyo University, OFCE Sciences-Po and SKEMA Business School

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Patents, R&D Subsidies and Endogenous Market Structure in a Schumpeterian Economy

Angus C. Chu, University of Liverpool
Yuichi Furukawa, Chukyo University
Lei Ji, OFCE Sciences-Po and SKEMA Business School

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Abstract

This study explores the different implications of patent breadth and R&D subsidies on economic growth and endogenous market structure in a Schumpeterian growth model. We find that when the number of firms is fixed in the short run, patent breadth and R&D subsidies serve to increase economic growth as in previous studies. However, when market structure adjusts endogenously in the long run, R&D subsidies increase economic growth but decrease the number of firms, whereas patent breadth expands the number of firms but reduces economic growth. Therefore, in accordance with empirical evidence, R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating long-run economic growth.

JEL classification: O30, O40

Keywords: economic growth, endogenous market structure, patents, R&D subsidies

Chu: angusccc@gmail.com. University of Liverpool Management School, University of Liverpool, UK.

Furukawa: you.furukawa@gmail.com. School of Economics, Chukyo University, Nagoya, Japan.

Ji: lei.ji@skema.edu. OFCE Sciences-Po and SKEMA Business School, Sophia Antipolis, France.

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1 Introduction

What are the different implications of patent breadth and R&D subsidies on economic growth and market structure? To explore this question, we consider a second-generation R&D-based growth model, pioneered by Peretto (1998), Young (1998), Howitt (1999) and Segerstrom (2000). To our knowledge, this is the first study that analyzes patent breadth in a second-generation R&D-based growth model.¹ The model features two dimensions of technological progress. In the vertical dimension, firms improve the quality of existing products. In the horizontal dimension, firms invent new products. In this Schumpeterian growth model with endogenous market structure (EMS) measured by the equilibrium number of firms, we find some interesting differences between patent breadth and R&D subsidies. At the first glance, these two policy instruments should have similar effects on innovation and economic growth. Patent breadth improves incentives for innovation by increasing the private return to R&D investment, whereas R&D subsidies improve incentives for innovation by reducing the private cost of R&D investment. Previous studies, such as Grossman and Helpman (1991), Segerstrom (1998), Li (2001), Zeng and Zhang (2007) and Iwaisako and Futagami (2013), often find that these two policy instruments have positive effects on innovation in R&D-based growth models. However, in a Schumpeterian growth model with EMS, we find that patent breadth and R&D subsidies have drastically different implications on economic growth and market structure. Specifically, when the number of firms is fixed in the short run, patent breadth and R&D subsidies both have positive effects on economic growth as in previous studies. Interestingly, when market structure adjusts endogenously in the long run, patent breadth expands the number of firms but decreases economic growth, whereas R&D subsidies increase economic growth but reduce the number of firms.

Intuitively, R&D subsidies decrease the cost of R&D investment and improve incentives for R&D; therefore, a higher rate of R&D subsidies increases economic growth in the short run and in the long run. As for an increase in patent breadth, it raises the profit margin of monopolistic firms and provides more incentives for R&D in the short run. In the long run, it encourages the entry of new firms, which in turn reduces the market size of each firm. Given that the market size of a firm determines its incentives for innovation in the second-generation R&D-based growth model,² a larger patent breadth decreases long-run economic growth. These contrasting long-run implications of patent breadth and R&D subsidies suggest that R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating long-run economic growth. The negative effect of patent protection on innovation is consistent with the evidence discussed in Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008). As for the positive effect of R&D subsidies on innovation, it is also consistent with empirical evidence; see for example, Hall and Van Reenen (2000) for a survey of empirical studies.

This study relates to the literature on R&D-driven economic growth; see Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) for

¹See also Cozzi and Spinesi (2006) for an analysis of an alternative patent policy instrument, namely intellectual appropriability, in the second-generation model in Howitt (1999).

²Laincz and Peretto (2006) provide empirical evidence for a positive relationship between average firm size and economic growth. See also Ha and Howitt (2007), Madsen (2008), Madsen *et al.* (2010) and Ang and Madsen (2011) for other empirical studies that support the second-generation R&D-based growth model.

seminal studies. Subsequent studies in this literature often apply variants of the R&D-based growth model to analyze the effects of policy instruments, such as R&D subsidies and patent breadth, on economic growth and innovation; see for example, Segerstrom (1998, 2000), Li (2001), Goh and Olivier (2002), Lin (2002), Impullitti (2007, 2010), Zeng and Zhang (2007), Chu (2011), Chu and Furukawa (2011) and Iwaisako and Futagami (2013).³ However, these studies do not analyze the effects of patent policy on market structure.⁴ Therefore, the present study contributes to the literature with a novel analysis of patent breadth in a Schumpeterian growth model in which market structure is endogenous. Furthermore, we compare the effects of patent breadth and R&D subsidies and find that in a Schumpeterian growth model with EMS, the long-run effects of patent breadth and R&D subsidies are drastically different suggesting the importance of taking into consideration the endogeneity of market structure when performing policy analysis in R&D-based growth models. O’Donoghue and Zweimuller (2004), Horii and Iwaisako (2007), Furukawa (2007, 2010), Chu (2009), Chu *et al.* (2012), Chu and Pan (2013) and Yang (2014) also find that increasing the strength of other patent policy levers, such as blocking patents and patentability requirement, could have negative effects on economic growth. Acemoglu and Akcigit (2012) consider the interesting case of state-dependent patent length and show that full patent protection does not maximize economic growth. The present study differs from these previous studies that mostly focus on the long-run effects of patent policy and contributes to the literature by showing that EMS leads to different short-run and long-run implications of patent protection on economic growth. Cozzi and Galli (2014) simulate the transitional effects of increasing the strength of basic research patents and find that it has different effects on economic growth at different time horizons. Our study complements their interesting analysis by exploring the effects of patent policy via an alternative mechanism, namely EMS, and by analytically showing the opposite short-run and long-run effects of patent breadth.

The rest of this study is organized as follows. Section 2 presents the Schumpeterian growth model with EMS. Section 3 analyzes the effects of patent breadth and R&D subsidies. Section 4 concludes.

2 A Schumpeterian growth model with EMS

In summary, the growth-theoretic framework is based on the Schumpeterian model with in-house R&D and EMS in Peretto (2007, 2011). In this model, labor is used as a factor input for the production of final goods. Final goods are either consumed by the household or used as a factor input for R&D, entry and the production of intermediate goods. We incorporate patent breadth into the model and analyze its different implications from R&D subsidies on economic growth and market structure. In our analysis, we provide a complete closed-form solution for the balanced growth path and transition dynamics.

³For studies that explore the effects of patent length on economic growth, see for example Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), Lin (2014) and Zeng *et al.* (2014).

⁴See Peretto (1996, 1999) for seminal studies in the R&D-based growth model with EMS and Etro (2012) for an excellent textbook treatment of this topic.

2.1 Household

In the economy, the population size is normalized to unity, and there is a representative household who has the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \ln C_t dt, \quad (1)$$

where C_t denotes consumption of final goods (numeraire) at time t . The parameter $\rho > 0$ determines the rate of subjective discounting. The household maximizes (1) subject to the following asset-accumulation equation:

$$\dot{A}_t = r_t A_t + (1 - \tau)w_t L - C_t. \quad (2)$$

A_t is the real value of assets owned by the household, and r_t is the real interest rate. The household has a labor endowment of L units and supplies them inelastically to earn a real wage rate w_t . The household also pays a wage-income tax $\tau w_t L$ to the government. From standard dynamic optimization, the familiar Euler equation is

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \quad (3)$$

2.2 Final goods

We follow Aghion and Howitt (2005, 2008) and Peretto (2007, 2011) to assume that final goods Y_t are produced by competitive firms using the following production function:⁵

$$Y_t = \int_0^{N_t} X_t^\theta(i) [Z_t^\alpha(i) Z_t^{1-\alpha} L/N_t]^{1-\theta} di, \quad (4)$$

where $\{\theta, \alpha\} \in (0, 1)$ and $X_t(i)$ denotes intermediate goods $i \in [0, N_t]$. The productivity of intermediate good $X_t(i)$ depends on its quality $Z_t(i)$ and also on the average quality $Z_t \equiv \frac{1}{N_t} \int_0^{N_t} Z_t(i) di$ of all intermediate goods capturing R&D spillovers. The degree of technology spillovers is determined by $1 - \alpha$. From profit maximization, the equilibrium wage rate is determined by

$$w_t = (1 - \theta)Y_t/L, \quad (5)$$

and the conditional demand function for $X_t(i)$ is

$$X_t(i) = \left(\frac{\theta}{p_t(i)} \right)^{1/(1-\theta)} Z_t^\alpha(i) Z_t^{1-\alpha} L/N_t, \quad (6)$$

where $p_t(i)$ is the price of $X_t(i)$ and the price of Y_t is normalized to unity. Perfect competition implies that final goods producers pay $\theta Y_t = \int_0^{N_t} p_t(i) X_t(i) di$ to intermediate goods firms.

⁵Peretto (2007, 2011) consider a slightly different production function that replaces L/N_t by $l_t(i)$, which denotes labor that uses intermediate goods $X_t(i)$. Given that $l_t(i) = L/N_t$ in equilibrium, we follow Aghion and Howitt (2005, 2008) to consider the specification with L/N_t , which has the advantage of being generalizable. Peretto (2013) considers a more general specification with L/N_t^σ , where $\sigma \in (0, 1)$ inversely measures the social return to varieties. In Section 3.1, we discuss the robustness of our main results under this general specification.

2.3 Intermediate goods and in-house R&D

Existing intermediate goods firms produce differentiated goods with a technology that requires one unit of final goods to produce one unit of intermediate goods $X_t(i)$. Following Peretto (2011), we assume that the firm in industry i incurs ϕZ_t units of final goods as a fixed operating cost, where Z_t is aggregate technology as defined above. This specification implies that managing facilities are more expensive to operate in a technologically more advanced economy. To improve the quality of its products, the firm invests $R_t(i)$ units of final goods in R&D. The innovation process is

$$\dot{Z}_t(i) = R_t(i). \quad (7)$$

The value of the monopolistic firm in industry i is

$$V_t(i) = \int_t^\infty \exp\left(-\int_t^u r_v dv\right) \pi_u(i) du. \quad (8)$$

The dividend flow $\pi_t(i)$ at time t is

$$\pi_t(i) = [p_t(i) - 1]X_t(i) - \phi Z_t - (1 - s)R_t(i), \quad (9)$$

where the parameter $s \in (0, 1)$ is the rate of R&D subsidies. The monopolistic firm maximizes (8) subject to (6) and (7). The current-value Hamiltonian for this optimization problem is

$$H_t(i) = \pi_t(i) + \lambda_t(i)\dot{Z}_t(i). \quad (10)$$

We solve this optimization problem in the Appendix and find that the unconstrained profit-maximizing markup ratio is $1/\theta$. To analyze the effects of patent breadth, we impose an upper bound $\mu > 1$ on the markup ratio.⁶ Therefore, the equilibrium price becomes

$$p_t(i) = \min\{\mu, 1/\theta\}. \quad (11)$$

For the rest of this study, we assume that $\mu < 1/\theta$. In this case, a larger patent breadth μ leads to a higher markup, and this implication is consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price". Finally, Lemma 1 shows that the return to in-house R&D is increasing in the market size of each firm measured by employment per variety L/N_t .

Lemma 1 *The return to in-house R&D is given by*

$$r_t = \frac{\alpha}{1 - s} \left[(\mu - 1) \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L}{N_t} \right]. \quad (12)$$

Proof. See the Appendix. ■

⁶Intuitively, the presence of monopolistic profits attracts potential imitators. However, stronger patent protection increases the production cost of imitative products and allows monopolistic firms to charge a higher markup without losing market share to these potential imitators; see also Li (2001), Goh and Olivier (2002), Chu (2011), Chu and Furukawa (2011) and Iwaisako and Futagami (2013) for a similar formulation.

2.4 Entrants

A firm that is active at time t must have been born at some earlier date. Following the standard treatment in the literature, we consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0, N_t]$, by assuming that any new entry at time t has access to the level of aggregate technology Z_t .⁷ A new firm pays a setup cost $X_t(i)F$, where $F > 0$ is a cost parameter, to set up its operation and introduce a new variety of products to the market.⁸ We refer to this process as entry. Suppose entry is positive (i.e., $\dot{N}_t > 0$). The no-arbitrage condition is⁹

$$V_t(i) = X_t(i)F. \quad (13)$$

The familiar Bellman equation implies that the return to entry is

$$r_t = \frac{\pi_t}{V_t} + \frac{\dot{V}_t}{V_t}. \quad (14)$$

2.5 Government

The government chooses an exogenous rate of R&D subsidies $s \in (0, 1)$. The government collects tax revenue T_t from the household, and the amount of tax revenue is

$$T_t = \tau w_t L = \tau(1 - \theta)Y_t, \quad (15)$$

where $\tau \in (0, 1)$ is an exogenous tax rate on wage income. The balanced-budget condition is

$$T_t = G_t + s \int_0^{N_t} R_t(i) di, \quad (16)$$

where G_t is unproductive government consumption that changes endogenously to balance the fiscal budget as in Peretto (2007).

2.6 General equilibrium

The equilibrium is a time path of allocations $\{A_t, C_t, Y_t, X_t(i), R_t(i)\}$ and prices $\{r_t, w_t, p_t(i), V_t(i)\}$ such that the following conditions are satisfied:

- the household maximizes utility taking $\{r_t, w_t\}$ as given;

⁷See Peretto (1998, 1999, 2007) for a discussion of the symmetric equilibrium being a reasonable equilibrium concept in this class of models.

⁸The setup cost is proportional to the new firm's initial volume of output. This assumption captures the idea that the setup cost depends on the amount of productive assets required to start production; see Peretto (2007) for a discussion.

⁹We follow the standard approach in this class of models to treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also $X_t(i)F$); therefore, $V_t(i) = X_t(i)F$ always holds. If $V_t(i) > X_t(i)F$ ($V_t(i) < X_t(i)F$), then there would be an infinite number of entries (exits).

- competitive final goods firms maximize profits taking $\{p_t(i), w_t\}$ as given;
- incumbents in the intermediate goods sector choose $\{p_t(i), R_t(i)\}$ to maximize $\{V_t(i)\}$ taking $\{r_t\}$ as given;
- entrants make entry decisions taking $\{V_t(i)\}$ as given;
- the value of all existing monopolistic firms adds up to the value of the household's assets such that $A_t = N_t V_t$; and
- the market-clearing condition of final goods holds.

The market-clearing condition of final goods is

$$Y_t = C_t + N_t(X_t + \phi Z_t + R_t) + \dot{N}_t X_t F + G_t. \quad (17)$$

Substituting (6) into (4) and imposing symmetry yield the aggregate production function:¹⁰

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} Z_t L, \quad (18)$$

which also uses markup pricing $p_t(i) = \mu$.

We now analyze the dynamics of the economy. In the Appendix, we show that the consumption-output ratio C_t/Y_t jumps to a unique and stable steady-state value. This equilibrium property simplifies the analysis of transition dynamics.

Lemma 2 *The consumption-output ratio jumps to a unique and stable steady-state value:*

$$(C/Y)^* = (1 - \tau)(1 - \theta) + \frac{\rho\theta F}{\mu}. \quad (19)$$

Proof. See the Appendix. ■

Equations (18) and (19) imply that for any given μ and τ ,

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = r_t - \rho, \quad (20)$$

where the last equality uses the Euler equation in (3). Combining (12) and (20), we derive the equilibrium growth rate given by

$$g_t \equiv \frac{\dot{Z}_t}{Z_t} = \max \left\{ \frac{\alpha}{1-s} \left[(\mu - 1) \left(\frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{L}{N_t} \right] - \rho, 0 \right\}, \quad (21)$$

¹⁰As discussed in footnote 4, introducing a social return to varieties would not change our results.

which is increasing in the market size of each firm measured by employment per variety L/N_t .¹¹ From (21), the growth rate g_t is strictly positive if and only if

$$N_t < \bar{N} \equiv \frac{\alpha(\mu - 1)(\theta/\mu)^{1/(1-\theta)} L}{(1-s)\rho}.$$

Intuitively, innovation requires each firm's market size to be large enough so that it is profitable for firms to do in-house R&D. A sufficient market size requires the number of firms to be below a critical level \bar{N} . If $N_t > \bar{N}$, then there are too many firms diluting the return to R&D. As a result, firms do not invest in R&D, and the growth rate of vertical innovation is zero. In the Appendix, we provide the derivations of the dynamics of N_t , which is a state variable.

Lemma 3 *The dynamics of N_t is determined by a one-dimensional differential equation:*¹²

$$\frac{\dot{N}_t}{N_t} = \begin{cases} \frac{\mu-1}{F} - \left[\phi + (1-s)\frac{\dot{Z}_t}{Z_t} \right] \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} - \rho & \text{if } N_t < \bar{N} \\ \frac{\mu-1}{F} - \phi \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} - \rho & \text{if } N_t > \bar{N} \end{cases}. \quad (22)$$

Proof. See the Appendix. ■

The differential equation in (22) shows that given any initial value, N_t gradually converges to its steady-state value denoted as N^* .¹³ On the transition path, the number of firms determines each firm's market size L/N_t and the equilibrium growth rate g_t according to (21). When N_t evolves toward the steady state, g_t also gradually converges to its steady-state value g^* . The following proposition derives the steady-state values $\{N^*, g^*\}$.

Proposition 1 *Under the parameter restrictions $\rho < \min\{\phi/(1-s), (1-\alpha)(\mu-1)/F\}$,¹⁴ the dynamics of N_t is globally stable and N_t gradually converges to a unique positive steady-state value. The steady-state values $\{N^*, g^*\}$ are given by*

$$N^*_{+,-}(\mu, s) = \left[(1-\alpha) \frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\rho F}{\mu^{1/(1-\theta)}} \right] \frac{\theta^{1/(1-\theta)}}{\phi - (1-s)\rho} L > 0, \quad (23)$$

$$g^*_{-,+}(\mu, s) = \frac{\alpha(\mu-1)}{(1-\alpha)(\mu-1) - \rho F} \left(\frac{\phi}{1-s} - \rho \right) - \rho > 0. \quad (24)$$

Proof. See the Appendix. ■

¹¹Considering data on employment, R&D personnel, and the number of establishments in the US for the period from 1964 to 2001, Laincz and Peretto (2006) provide empirical evidence that is consistent with the theoretical prediction from this class of models that economic growth is increasing in average firm size.

¹²It is useful to note that \dot{Z}_t/Z_t is a function of N_t given by (21).

¹³In this model, we have assumed zero population growth, so that N_t converges to a steady state. If we assume positive population growth, it would be the number of firms per capita that converges to a steady state instead, and our main results would be unchanged.

¹⁴These parameter restrictions would depend on a larger set of parameters if we parameterize R&D productivity in (7) and the productivity in producing intermediate goods from final goods. For simplicity, we have implicitly normalized these productivity parameters to unity.

3 Patent breadth versus R&D subsidies

In this section, we analyze the effects of patent breadth and R&D subsidies. In Section 3.1, we analyze the effects of patent breadth on the number of firms, the market size of each firm and economic growth. In Section 3.2, we analyze the effects of R&D subsidies.

3.1 Effects of patent breadth

In this subsection, we analyze the effects of patent breadth. Equation (21) shows that the initial impact of a larger patent breadth μ on the equilibrium growth rate g_t is positive because N_t is fixed in the short run. This is the standard positive *profit-margin* effect, captured by $(\mu - 1)/\mu^{1/(1-\theta)}$ in (21), of patent breadth on monopolistic profits and innovation as in previous studies, such as Li (2001), Chu (2011), Chu and Furukawa (2011) and Iwaisako and Futagami (2013). However, in the long run, market structure is endogenous and the number of firms adjusts. In particular, the higher profit margin attracts the entry of firms, which in turn reduces each firm's market size L/N_t and decreases incentives for innovation. This negative *entry* effect dominates the positive profit-margin effect such that the steady-state equilibrium growth rate g^* becomes lower than the original steady-state level. Therefore, allowing for the endogeneity of market structure, the present study extends previous studies in the literature by demonstrating the opposite short-run and long-run effects of patent breadth on economic growth. Proposition 2 summarizes the results. Figures 1 and 2 plot the transition paths of $\{g_t, N_t\}$ when μ increases at time t .

Proposition 2 *The initial effect of a larger patent breadth on economic growth is positive as a result of increased monopolistic profits. In the long run, higher profit margin attracts the entry of firms and reduces the market size of each firm. The smaller market size decreases incentives for innovation and the steady-state growth rate.*

Proof. Equation (21) shows that for a given N_t , $\partial g_t/\partial\mu > 0$. Equations (23) and (24) show that $\partial N^*/\partial\mu > 0$ and $\partial g^*/\partial\mu < 0$. ■

[Insert Figures 1 and 2 here]

Proposition 2 shows that the long-run effect of patent breadth is monotonically negative. This result is based on the parameter restriction in Proposition 1 that ensures the global stability of N_t , and this parameter restriction can be reexpressed as

$$\mu > 1 + \frac{\rho F}{1 - \alpha} \equiv \bar{\mu}.$$

Within this parameter space $\mu > \bar{\mu}$, we find that a larger patent breadth μ increases the number of firms in the long run, which in turn decreases the market size of each firm (i.e.,

L/N^*) and long-run growth. In our model, increasing patent breadth triggers two competition effects. First, increasing patent breadth allows a firm to charge a higher markup by preventing other firms from imitating its product. Second, increasing patent breadth encourages more firms to enter the market with new products. If we refer to the first effect as vertical competition and the second effect as horizontal competition, then an alternative way to describe our result would be that increasing patent breadth weakens vertical competition but strengthens horizontal competition. It is this strengthening of horizontal competition that gives rise to the negative entry effect of patent breadth in our analysis.¹⁵

In our model, the negative entry effect of patent breadth dominates the positive profit-margin effect. In an alternative model, it may be the case that the relative magnitude of these two effects is different. Indeed, we find that the relative magnitude of the *entry* and *profit-margin* effects depends on the specification of the entry cost. In this study, we have followed Peretto (2007) to assume an entry cost given by $X_t F$, which is proportional to Z_t/N_t in equilibrium. It is useful to recall that N^* is increasing in μ . Therefore, by attracting entry, a larger patent breadth decreases each firm's output X_t and the cost of entry, which in turn amplifies the magnitude of the negative entry effect on long-run growth. Suppose we consider an alternative entry cost function $Z_t F$. In this case, we find that the negative entry effect and the positive profit-margin effect exactly cancel each other leaving an overall neutral effect of patent breadth on long-run growth.¹⁶ However, we find that an entry cost that depends on the firm's initial output volume X_t to be a more reasonable specification.

Finally, as mentioned in footnote 5, our result is robust to a more general production function given by

$$Y_t = \int_0^{N_t} X_t^\theta(i) [Z_t^\alpha(i) Z_t^{1-\alpha} L/N_t^\sigma]^{1-\theta} di, \quad (4a)$$

where $\sigma \in (0, 1)$ inversely measures the social benefit of variety. Then, the aggregate production function in (18) becomes

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} Z_t N_t^{1-\sigma} L, \quad (18a)$$

where N_t contributes to the production of Y_t . In this case, patent breadth continues to have a negative effect on long-run growth,¹⁷ because even under a positive social return to varieties, increasing the number of varieties raises the level of output but not the long-run growth rate.¹⁸

3.2 Effects of R&D subsidies

In this subsection, we analyze the effects of R&D subsidies. Equation (21) shows that the initial impact of a higher rate of R&D subsidies s on the equilibrium growth rate g_t is positive

¹⁵See also Aghion *et al.* (2005, 2009) on the theoretical and empirical relationship between firm entry, competition and economic growth.

¹⁶Derivations available upon request.

¹⁷Derivations available upon request.

¹⁸This is true even if N_t increases over time in the presence of population growth. In the second-generation model, the long-run growth rate of N_t is determined by the exogenous population growth rate.

given N_t . On the transition path, the higher rate of R&D subsidies makes in-house R&D more attractive relative to entry. As a result, resources reallocate from entry to in-house R&D, and the number of firms decreases. The smaller number of firms increases each firm's market size, which further improves incentives for in-house R&D. This positive market-size effect strengthens the initial positive effect of R&D subsidies such that the steady-state equilibrium growth rate g^* increases further above the initial level. Therefore, the endogeneity of market structure amplifies the positive effects of R&D subsidies on economic growth. Peretto (1998) and Segerstrom (2000) also analyze the effects of R&D subsidies in a second-generation Schumpeterian growth model. Segerstrom (2000) finds that R&D subsidies can have either a positive or negative effect on economic growth, and this interesting result is driven by the tradeoff between quality improvement and variety expansion on economic growth. In contrast, economic growth is solely based on quality improvement in the present study and in Peretto (1998), who also finds a positive effect of R&D subsidies on economic growth. Peretto and Connolly (2007) provide a theoretical justification that quality improvement is the only plausible engine of economic growth in the long run. Proposition 3 summarizes the results. Figures 3 and 4 plot the transition paths of $\{g_t, N_t\}$ when s increases at time t .

Proposition 3 *The initial effect of a higher rate of R&D subsidies on economic growth is positive. In the long run, firms exit the market, and the market size of each firm increases. The larger market size further strengthens incentives for innovation and increases the steady-state growth rate.*

Proof. Equation (21) shows that for a given N_t , $\partial g_t / \partial s > 0$. Equations (23) and (24) show that $\partial N^* / \partial s < 0$ and $\partial g^* / \partial s > 0$. ■

[Insert Figures 3 and 4 here]

We now consider an extension of the baseline model by allowing for a subsidy to entry denoted by $e \in (0, 1)$. In this case, the entry condition in (13) becomes

$$V_t(i) = (1 - e)X_t(i)F. \quad (25)$$

Furthermore, the government's balanced-budget condition is modified to

$$T_t = G_t + s \int_0^{N_t} R_t(i)di + e\dot{N}_t X_t F. \quad (26)$$

The rest of the model is the same as before. Following the same procedures as before,¹⁹ we derive the same equilibrium growth rate in (21) and the steady-state equilibrium number of varieties given by

$$N^*_+(e) = \left[(1 - \alpha) \frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{(1 - e)\rho F}{\mu^{1/(1-\theta)}} \right] \frac{\theta^{1/(1-\theta)}}{\phi - (1 - s)\rho} L > 0, \quad (27)$$

¹⁹Derivations are available upon request.

which is naturally increasing in the entry subsidy rate e . Given that the equilibrium growth rate is given by (21) as before and does not directly depend on e , an increase in entry subsidies does not affect economic growth initially. However, given that entry subsidies attract the entry of firms and reduce the market size of each firm, the equilibrium growth rate gradually decreases during the transition path and converges to a lower steady-state value.

[Insert Figure 5 here]

If we think of entry as horizontal R&D, then the above analysis implies that horizontal R&D subsidies can be harmful to economic growth, and this finding is consistent with Peretto (2007). In other words, in order for R&D subsidies to have a positive effect on economic growth, policymakers need to design a subsidy (or tax-deduction) system that distinguishes between different types of R&D activities, which may be difficult to implement in practice.

In the rest of this subsection, we consider symmetric R&D and entry subsidies by setting $e = s = \bar{s}$. Given that entry subsidies e have no effect on the initial growth rate, an increase in \bar{s} must have the same initial positive effect on the growth rate g_t as R&D subsidies. As for the long-run effect on the number of firms, (27) becomes

$$N^*(\bar{s}) = \left[\frac{(1 - \alpha)(\mu - 1) - (1 - \bar{s})\rho F}{\phi - (1 - \bar{s})\rho} \right] \left(\frac{\theta}{\mu} \right)^{1/(1-\theta)} L > 0, \quad (28)$$

which is decreasing (increasing) in \bar{s} if the following inequality holds:

$$(1 - \alpha)(\mu - 1) > (<) \phi F.$$

If N^* is decreasing in \bar{s} , then the long-run effect of \bar{s} on g^* must be positive, which we refer to as case 1 in Figure 6. If N^* is increasing in \bar{s} , then a higher rate of subsidies \bar{s} would have a negative indirect effect on long-run growth through entry partly offsetting the positive direct effect of \bar{s} on growth. We refer to the case in which the positive direct effect dominates (is dominated by) the negative indirect effect as case 2 (case 3) in Figure 6.

[Insert Figure 6 here]

Substituting (28) into (21) yields

$$g = \underbrace{\frac{\alpha(\mu - 1)}{1 - \bar{s}}}_{\text{direct effect of } \bar{s}} \underbrace{\frac{\phi - (1 - \bar{s})\rho}{(1 - \alpha)(\mu - 1) - (1 - \bar{s})\rho F}}_{\text{indirect effect of } \bar{s}} - \rho, \quad (29)$$

which is increasing in \bar{s} if and only if the following inequality holds:²⁰

$$\underbrace{[(1 - \alpha)(\mu - 1) - (1 - \bar{s})\rho F]}_{+} - \underbrace{[\phi - (1 - \bar{s})\rho]}_{+} (1 - \bar{s})\rho F / \phi > 0.$$

This inequality holds if ρ is sufficiently small. In other words, the overall long-run growth effect of symmetric R&D and entry subsidies \bar{s} is generally ambiguous; however, if the

²⁰It can be shown that $(1 - \alpha)(\mu - 1) > \phi F$ is sufficient (but not necessary) for this inequality to hold.

discount rate ρ is sufficiently small, then an increase in \bar{s} would have a positive effect on long-run growth.

To assess whether an increase in \bar{s} is likely to have a positive long-run effect on economic growth, we provide a simple calibration exercise here. Our analysis involves the following parameters $\{\rho, \mu, \alpha, \bar{s}, \phi, F\}$. We follow Acemoglu and Akgigit (2012) to set the discount rate ρ to a standard value of 0.05. We set the markup ratio μ to 1.30 that implies a 30% markup, which is within the range of empirical estimates summarized in Jones and Williams (2000). For the degree of spillovers $1 - \alpha$, we consider a value of 0.10. As for the R&D subsidy rate \bar{s} , we consider a common value of 15% in developed countries. We consider a range of values for the entry cost parameter $F \in (0, \bar{F})$, where the upper bound $\bar{F} \equiv (1 - \alpha)(\mu - 1)/[\rho(1 - \bar{s})]$ is imposed to ensure the global stability of the dynamics of N_t . For each value of F , we calibrate the value of ϕ by equating the long-run equilibrium growth rate g^* in (29) to a standard value of 2%. Table 1 reports the calibrated parameter values and shows the following implication: for the entire range of $F \in (0, \bar{F})$, the inequality $(1 - \alpha)(\mu - 1) > \phi F$ holds, which is sufficient to imply that the equilibrium growth rate g^* is increasing in R&D subsidies \bar{s} as empirical studies tend to find.

Table 1: Calibrated parameter values

ρ	μ	α	\bar{s}	F	ϕ	$(1 - \alpha)(\mu - 1)$	ϕF
0.05	1.30	0.90	0.15	0.01	0.0490	0.0300	0.0005
0.05	1.30	0.90	0.15	0.10	0.0482	0.0300	0.0048
0.05	1.30	0.90	0.15	0.20	0.0472	0.0300	0.0094
0.05	1.30	0.90	0.15	0.30	0.0463	0.0300	0.0139
0.05	1.30	0.90	0.15	0.40	0.0454	0.0300	0.0181
0.05	1.30	0.90	0.15	0.50	0.0444	0.0300	0.0222
0.05	1.30	0.90	0.15	0.60	0.0435	0.0300	0.0261
0.05	1.30	0.90	0.15	0.70	0.0426	0.0300	0.0298

4 Conclusion

In this study, we have explored the different implications of two important policy instruments, patent breadth and R&D subsidies, on economic growth and market structure in a scale-invariant Schumpeterian growth model with EMS. We find that when the number of firms is fixed in the short run, patent breadth and R&D subsidies serve to increase economic growth as in previous studies. However, when market structure adjusts endogenously in the long run, these two commonly discussed policy instruments have surprisingly opposing effects on economic growth and market structure. Specifically, patent breadth decreases economic growth but expands the number of firms, whereas R&D subsidies reduce the number of firms but increase economic growth. These contrasting effects of patent breadth and R&D subsidies suggest that R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating economic growth. This finding is consistent with evidence

from empirical studies discussed in the introduction. Given our result that the endogeneity of market structure leads to different short-run and long-run effects of patent breadth, it is important for policymakers to take into consideration the different implications of patent policy reform across time horizons.

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Appendix

Proof of Lemma 1. Substituting (6), (9) and the constraint $p_t(i) \leq \mu$ into (10) yields

$$H_t(i) = [p_t(i) - 1] \left(\frac{\theta}{p_t(i)} \right)^{1/(1-\theta)} Z_t^\alpha(i) Z_t^{1-\alpha} L/N_t - \phi Z_t - (1-s)R_t(i) + \lambda_t(j)R_t(i) + \eta_t(j)[p_t(i) - \mu], \quad (\text{A1})$$

where $\eta_t(j)$ is the multiplier on $p_t(i) \leq \mu$ and $\eta_t(j) = 0$ if $p_t(i) < \mu$. The first-order conditions include

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Leftrightarrow p_t(i) = \min \{ \mu, 1/\theta \}, \quad (\text{A2})$$

$$\frac{\partial H_t(i)}{\partial R_t(i)} = 0 \Leftrightarrow \lambda_t(i) = 1 - s, \quad (\text{A3})$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha [p_t(i) - 1] \left(\frac{\theta}{p_t(i)} \right)^{1/(1-\theta)} Z_t^{\alpha-1}(i) Z_t^{1-\alpha} L/N_t = r_t \lambda_t(i) - \dot{\lambda}_t(i). \quad (\text{A4})$$

Substituting (A3) and the constrained monopolistic price $p_t(i) = \mu < 1/\theta$ from (A2) into (A4) yields

$$r_t = \frac{\alpha}{1-s} \left[(\mu - 1) \left(\frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{L}{N_t} \right], \quad (\text{A5})$$

where we have also applied the symmetry condition $Z_t(j) = Z_t$. ■

Proof of Lemma 2. Substituting $V_t = X_t F$ from (13) into $A_t = N_t V_t$ yields

$$A_t = N_t X_t F = \frac{p_t N_t X_t}{p_t} F = \frac{\theta Y_t}{\mu} F, \quad (\text{A6})$$

where the last equality uses $p_t = \mu$ and $p_t X_t N_t = \theta Y_t$. Using (A6) and (2), we obtain

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} = r_t + \mu \frac{(1-\tau)w_t L - C_t}{\theta Y_t F}. \quad (\text{A7})$$

Substituting the Euler equation and $w_t L = (1-\theta)Y_t$ into (A7) yields

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = \mu \frac{C_t/Y_t}{\theta F} - \left[\mu \frac{(1-\tau)(1-\theta)}{\theta F} + \rho \right]. \quad (\text{A8})$$

Therefore, the dynamics of C_t/Y_t is characterized by saddle-point stability such that C_t/Y_t must jump to its steady-state value in (19). ■

Proof of Lemma 3. Substituting (9), (13) and $p_t(i) = \mu$ into (14) yields

$$r_t = \frac{\mu - 1}{F} - \frac{\phi Z_t + (1-s)R_t}{X_t F} + \frac{\dot{X}_t}{X_t}, \quad (\text{A9})$$

where we have applied $\dot{V}_t/V_t = \dot{X}_t/X_t$. Substituting $p_t(i) = \mu$ into (6) yields

$$X_t = \frac{Z_t}{N_t} \left(\frac{\theta}{\mu} \right)^{1/(1-\theta)} L, \quad (\text{A10})$$

where we have applied $Z_t(i) = Z_t$. Substituting (7) and (A10) into (A9) yields

$$r_t = \frac{\mu - 1}{F} - \left[\phi + (1 - s) \frac{\dot{Z}_t}{Z_t} \right] \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} + \frac{\dot{Z}_t}{Z_t} - \frac{\dot{N}_t}{N_t}, \quad (\text{A11})$$

where we have used $\dot{X}_t/X_t = \dot{Z}_t/Z_t - \dot{N}_t/N_t$. Substituting (20) into (A11) yields the dynamics of N_t given by

$$\frac{\dot{N}_t}{N_t} = \frac{\mu - 1}{F} - \left[\phi + (1 - s) \frac{\dot{Z}_t}{Z_t} \right] \frac{N_t/L}{(\theta/\mu)^{1/(1-\theta)} F} - \rho. \quad (\text{A12})$$

Equation (A12) describes the dynamics of N_t when $N_t < \bar{N}$. When $N_t > \bar{N}$, $\dot{Z}_t/Z_t = 0$ as shown in (21). ■

Proof of Proposition 1. This proof proceeds as follows. First, we prove that under $\rho < \min \{ \phi/(1-s), (1-\alpha)(\mu-1)/F \}$, there exists a stable, unique and positive steady-state value of N_t . Substituting (21) into the first equation of (22) yields

$$\frac{\dot{N}_t}{N_t} = \frac{(1-s)\rho - \phi}{(\theta/\mu)^{1/(1-\theta)} F} \frac{N_t}{L} + \frac{(1-\alpha)(\mu-1)}{F} - \rho. \quad (\text{A13})$$

Because N_t is a state variable, the dynamics of N_t is stable if and only if $(1-s)\rho < \phi$. Solving $\dot{N}_t = 0$, we obtain the steady-state value of N_t in an economy with positive in-house R&D given by

$$N^* = \left[\frac{(1-\alpha)(\mu-1)}{F} - \rho \right] \frac{(\theta/\mu)^{1/(1-\theta)} F}{\phi - (1-s)\rho} L. \quad (\text{A14})$$

Given $(1-s)\rho < \phi$, (A14) shows that $N^* > 0$ if and only if $\rho < (1-\alpha)(\mu-1)/F$. Combining this inequality with $(1-s)\rho < \phi$, we have

$$\rho < \min \left\{ \frac{\phi}{1-s}, \frac{(1-\alpha)(\mu-1)}{F} \right\}.$$

Finally, substituting (A14) into (21) yields

$$g_t = \frac{\alpha(\mu-1)}{(1-\alpha)(\mu-1) - \rho F} \left(\frac{\phi}{1-s} - \rho \right) - \rho, \quad (\text{A15})$$

which is positive if and only if the following inequality holds:

$$(1-s)F\rho^2 - (1-s)(\mu-1)\rho + \phi\alpha(\mu-1) > 0,$$

and this inequality holds if ρ is sufficiently small (or sufficiently large). ■

Figures

Figure 1: Transitional effects of patent breadth on economic growth

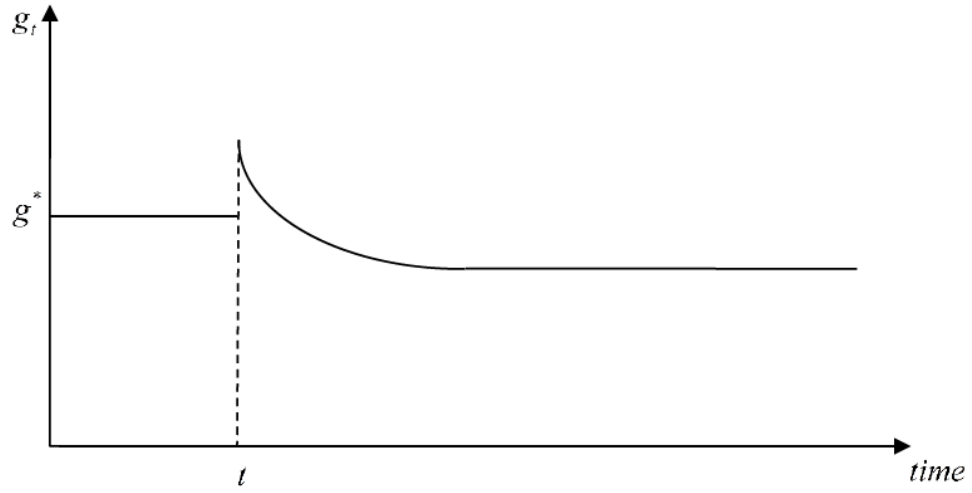


Figure 2: Transitional effects of patent breadth on the number of firms

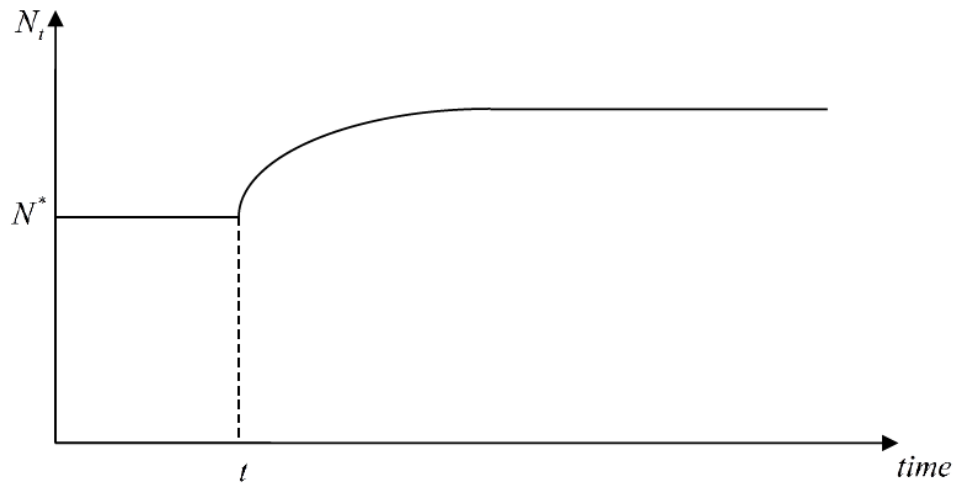


Figure 3: Transitional effects of R&D subsidies on economic growth

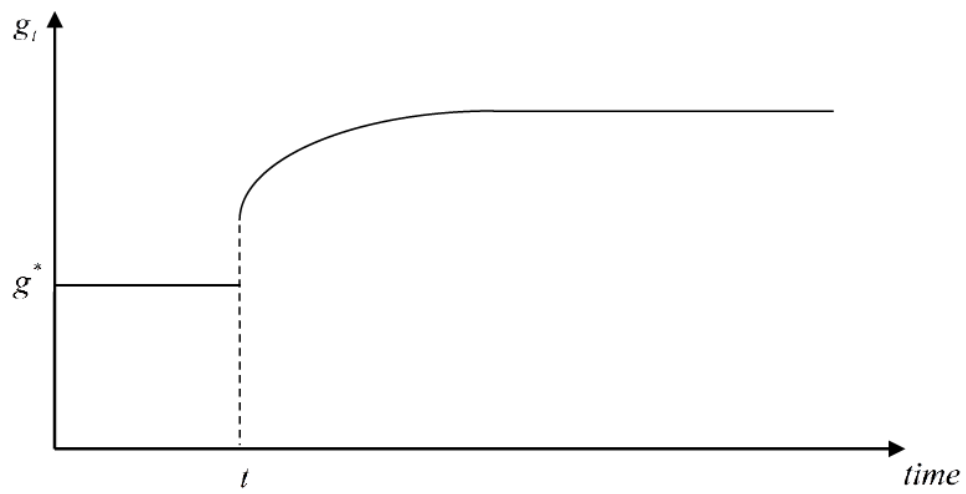


Figure 4: Transitional effects of R&D subsidies on the number of firms

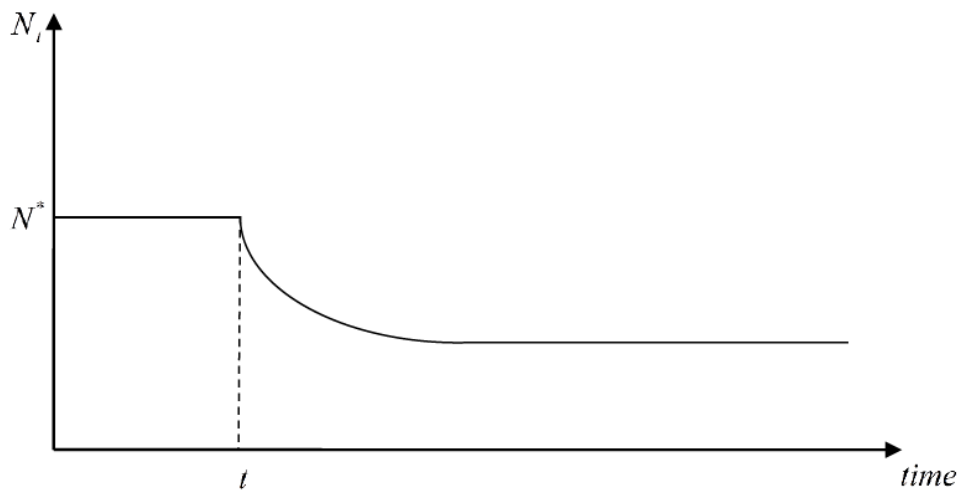


Figure 5: Transitional effects of entry subsidies on economic growth

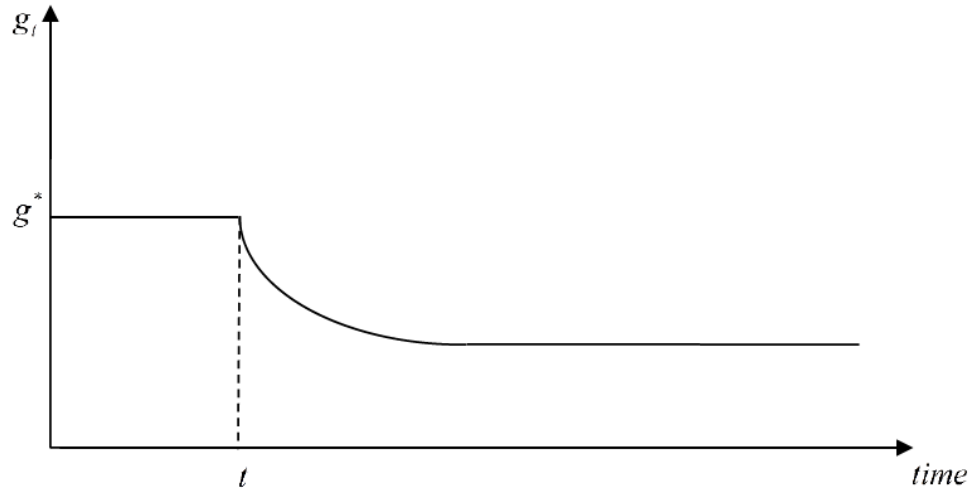


Figure 6: Transitional effects of symmetric subsidies on economic growth

