Top-Down Approach for the Social Capital Development

Shah, Syed Akhter Hussain and Shah, Tari shah and Ahmed, Eatzaz

Pakistan Institute of Development Economics Islamabad, Qaid-e-Azam University Islamabad, Qaid-e-Azam University Islamabad

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TOP-DOWN APPROACH FOR THE SOCIAL CAPITAL DEVELOPMENT: A POLICY FRAMEWORK FOR THE GOVERNMENT

Tariq Shah, Syed Akhter Hussain Shah, Eatzaz Ahmed & Bakhtiar Ali

ABSTRACT: This study provides the compatibility of top-down approach of the collectivist society with the algebraic representation of social capital matrix [22], which establishes algebraic based model having capability of interpreting the state intervention in building social capital. The study also suggests a policy framework for the developing economic strategies and effective planing at government level regarding social capital development in perspective of top-down approach.

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1. Introduction

Social capital is considered as the major source of quality of life [19]. Trust among society members is the most fundamental element of social capital. Despite in-depth research in the field from different perspective, consensus is yet to be develop on the sources of trust creation. Stolle [24] and Hooghe [13] argue that "we do not know whether trust can be created or should be seen as a result of historical traditions". Consequently, two major approaches and processes are identified for the development of social capital, i.e. bottom-up (capacity building) and top-down (resource allocation) (Woolcook [29]). Bottom-up approach emphasizes on the role of individual or small groups (for instance see Bourdieu [2] Coleman [4], Putnam [19]), while the top-down approach considers the state's role as a strong factor for generating social capital (Tarrow [25]). Stolle [24] further adds that for fostering the process of generating social capital, the civic capacity for cooperative ties between individuals is highly influenced by the state policy. Levi [16] contemplates that government is the source of social capital.

Furthermore, various latent factors have been identified for development and strengthening of social capital. This study selects state's role as one of the determinants, which correlated with economic development and hence welfare (Evans and Syrett [9]). While de.ning the term social capital, Portes [18] considers the source of social control as one of its major attributes, which itself indicates the state's role. Most of the studies

are conducted in the western world, which are mostly individualist in nature, thus bottom-up model is best suited to them. But for the collectivist society like Pakistan (Hofstede [11]), the top-down approach may be recommended, where the state required to develop and increase the civic capacities of its members (see also, Hofstede and Hofstede [12]).

Apart from positive impact of individualism, researchers have identified different negative aspects, like alienation, excessive selfi … shness, destruction of family institutions, crime, suicide, divorce, child abuse, emotional stress, physical and mental illness (Allik and Realop [1]). On the contrary such aspects are less visible despite poverty in collectivistic societies. However, to increase the trust level and civic capacities and strengthen the close ties with associates, state needs to pro-vide infrastructure and to develop the sense of welfare state. For such arrangement the "… institutional engineering might indeed be used to foster social capital" (Stolle [24, Page 21]). The individualist societies have bottom-up approach and can be interpreted as; individual-Community-Organization and then State (abbreviated as LCOS), whereas the collectivist societies have top-down approach, i.e. State-Organization-Community and Individual (abbreviated as SOCL) for their social capital development. The algebraic model of social capital matrix [22] proposed in this study establishes that economic and social capital development of a collectivist society can be achieve through a useful and effective role of a state in each layer of interaction of individual, community, organization, that is, intra- and across interactions of categories of organizations, communities and individuals.

Therefore, this study is based on the assumptions that state level policy makers might need a scientific tool for intervening in and acceleration of the process of the social capital development, This proposed model, by using algebraic approach can increase the capability of state to better plan, foster and implement the requisite infrastructure for the social capital, as this model has the capability to interpret intra and across interactions of the players by its three types of operations. This study can also be applied in multi-period analysis.

The next section presents the theoretical aspect of social capital in perspective of within and across interactions of state, organization, community and individual. Section 3 develops a link between algebraic representation of social capital matrix [22] and top-down approach. Section 4 digged the concept that the model interpret government intervention in building social capital in the society. Finally on the basis of this study we suggest a policy framework for the government for the enhancement of social capital and planning for strategies regarding economic development.

2. Social Capital and Social Capital Matrix

In this section first we briefly introduce social capital regarding its major two classes and then indicate the different viewpoints of interactive social capital for state,

organization, community and individuals for understanding the formation of social capital matrix of [21].

2.1 Social Capital

Trust, Reciprocity, Norms and Networks of Civic Engagement that facilitate interactive efforts in a society to achieve desired goals, produces social capital, which is interactive in nature and embedded in the structure of the society. Robinson and Flora [20] state that social capital is an individual's sacrifice (time, effort, consumption) made in an effort to promote cooperation with others and social cohesion is a characteristic of society which depends on the accumulated social capital. Economists and sociologist have categorized social capital with respect to its nature functions and dimensions. Uphoff [27]categorizes social capital into following two types:

1. Structural Social Capital, refers to relatively objective and externally observable social structures, such as networks, associations and institutions and the roles, rules and procedures they embody.
2. Cognitive Social Capital comprises of more subjective and intangible elements such as generally accepted attitude and norms of behavior, shared values, reciprocity and trust. The cognitive social capital may create coherence and homogeneity in the group of people having commonality in their norms, values, behavior, beliefs, reciprocity and attitudes.

2.2 Interactive Social Capital

We classify the interactive social capital as intraactive and across interaction.

2.2.1 *Intra-Action*

According to Coleman [4] social capital constitutes a capital asset for the individual and it consists of some aspect of social structure and facilitates certain action of individuals who are within the structure. However, Sobel [23] adds that, these interactions involve a small number of agents who know each other and interact repeatedly.

Inter-organizational or institutional interaction occurs through their members, in which individuals make investments through interaction and reciprocity with other organizations. Social capital generates reciprocity between organizations in order to develop their mutual trust.

Cognitive and structural social capitals facilitate and regulate patterns of mutual interaction of institutions. Turner [26] states that the term institution "denotes the way that members of a population are organized in order to face fundamental problems of

coordinating their activities to survive within a given environment". Ostrom's [17] contribution regarding common interests has been strengthened by Sobel [23], that common-property resources highlight the importance of institutions.

2.2.2 Across Interaction

Across interaction of individuals and communities also results in accumulation of social capital. Robinson and Flora [20] confirm that individuals and groups can consciously work to strengthen social capital. Castle [3] and Sobel [23] note that even though self-interest is an important motivator, it does not preclude, indeed it may require, participation in groups.

Individuals interact with organization through their members by allocation of resources that accumulate social capital between them. Individuals make investment through interaction and reciprocity with organization that generates social capital. This develops a level of trust among individual and institutions or organizations. Sobel [23] has the opinion that the level of trust on institutions indicate the granted permission to them for taking and influencing over the indivdiuals' welfare.

Social capital is accumulated between an individuals and a state. Individuals make investment through reciprocity with state directly or indirectly through its institutions or organization, which develops their mutual trust. Evidences in literature show that cooperative behavior of citizenship is the sign of the individuallistic on state, which leads to sustained reciprocity between individual and state. Cognitive and structural social capital facilitates patterns of their interaction with each others in the form congnito-structural relations.

Interaction between communities by allocation of time and money accumulates social capital among them, as the individuals and groups can consciously work to strengthen the social capital ([3], [20], [23]). Similarly, Woolcock [29] is of the view that physical capital and human capital are essentially the property of individuals, while social capital and extension inheres in groups. The social capital is embodied within communities, which refers to the relations within a group, including social norms and sanctions, mutual obligations, trust, and information transmission ([4], [5]]).

A group or community interacts with institutions or organizations by allocation of resources that accumulates social capital among them. Castle [3] and Sobel [23] have pointed out existence of social capital among groups due to common interests. Similarly, Woolcock [29] interprets that social capital is property of group. Individuals directly or indirectly on behalf of group make investment through interaction and reciprocity with state or its institutions or organization that accumulates social capital, which develops their reciprocity and trust in each others. Evans [8] is of the view that "Active government and mobilized communities can enhance each other's development efforts".

Similarly, Harris [10] supports Putnam's [19] finding that 'networks of civic engagement is a major determinant of government performance'.

Individuals on behalf of institutions or organizations make investment and develop reciprocity with a state directly or indirectly through its institutions or organizations that promote trust in each others. Cognitive and structural social capital facilitate patterns of interaction of institutions or organizations with state. Evans [8] is of the view that for development purposes, in addition to scaling upmicro-level capital, state-society synergy can give better results. Social trust, norms of reciprocity networks of civic engagement and successful cooperation are mutually rein-forcing. Putnam [19] points out that for effective collaboration, institutions require interpersonal skills and trust, which are also inculcated and reinforced by organized collaborations. Institutions, organizations and state may allocate resources for accumulation of social capital to enhance effectiveness.

2.2.3 Social Capital Matrix

The interactive systems, as discussed above, supports the interactive social capital model proposed by Shah [21], which consists of 16 different situation in a matrix form (Interactive social capital matrix):

Stake holders	*Individual (L)*	*Community (C)*	*Organization (O)*	*State (S)*
Individual (L)	L vs L	L vs C	L vs O	L vs S
Community (C)	C vs L	C vs C	C vs O	C vs S
Organization (O)	O vs L	O vs C	O vs O	O vs S
State (S)	S vs L	S vs C	S vs O	S vs S

3. ALGEBRAIC REPRESENTATION OF SOCIAL CAPITAL MATRIX AND TOP-DOWN APPROACH

The reverse format of social capital matrix of [21] establishes an algebraic representation of social capital matrix [22], and here we have given its short introduction. We determined that this algebraic representation of social capital matrix has the interpretation of top-down approach for the collectivist societies. Social capital has wide range and a number of dimensions, therefore by using special algebraic structures in [22], it is codi.ed and elaborated for contextual understanding as follows.

3.1 Algebra Under Consideration

A non-empty set G with an associative binary operation $*$ is a group if $a * b \in G$ for $a, b \in G$, if $e * r = r * e = r$, we call e, the identity element in G, and for each $g \in G$, there exist $h \in G$ such that $g * h = h * g = e$, whereas we call g and h, the inverses of

each other. A non-empty set R with two binary operations, say "+" and "." is said to be a ring if $(R, +)$ is abelian group (i.e., $a + b = b + a$), $(R, .)$ is semigroup (i.e., binary operation "." is associative in R) and "." is distributive over "+". A ring R is commutative if $a.b = b.a$, for all $a, b \in R$ and it is a ring with identity if $1 \in R$. A commutative ring R with identity is said to be an integral domain if $ab = 0$, where $a, b \in R$, then either $a = 0$ or $b = 0$. In other words, an integral domain has no zero divisors. Let R be a commutative ring with identity. An element $a \in R$ is said to be invertible or unit in R if there exists an element $b \in R$ such that $ab = ba = 1$. We represent $U(R)$, the set of all unit elements in R. A commutative ring F with identity is said to be field if $U(F) = F \setminus \{0\}$. A finite integral domain is a field.

Given nonnegative integers $0 < a$ and b, there exist $q \geq 0$ and r with $0 \leq r \leq a$ such that $b = aq + r$, where q is quotient and r is remainder which are unique. If $r = 0$, we say a divides b (that is $a \mid b$). For a fixed $m \in \mathbb{Z}^+$, we say $a, b \in \mathbb{Z}$ are congruent modulo m, written $a \equiv b \pmod{m}$ if $m \mid a - b$ or equivalently, if $a = b + mt$, where $t \in \mathbb{Z}$. Here m is called the modulus (plural; moduli). $a \equiv 0 \pmod{m}$ means $m \mid a$, $a \equiv b \pmod{1}$ for all $a, b \in \mathbb{Z}$, therefore we consider the positive integer $m > 1$ and $\{b + mt : t \in \mathbb{Z}\}$ is the set of integers to which $b \in \mathbb{Z}$ is congruent modulo m. Every integer is congruent modulo m to exactly one of the numbers in the set $\{0, 1, 2, ..., m - 1\}$.

Let $2 \leq m \in \mathbb{Z}^+$ be the modulus, which is fixed. Define the congruence class of $b \pmod{m}$, written $[b]_m$ as $[b]_m = \{a \in \mathbb{Z} : a \equiv b \pmod{m}\} = \{a \in \mathbb{Z} : m \mid a - b\} = \{a \in \mathbb{Z} : a = b + mt, \text{ where } t \in \mathbb{Z}\}$. $[a]_m = [b]_m$ if and only if $a \equiv b \pmod{m}$.

Every congruence class mod m is equal to one of $[0]_m$, $[1]_m$, $[2]_m$, ..., $[m - 1]_m$. Obviously all these classes are different. Thus there are only m congruence classes modulo m. We represent the set of all congruence classes modulo m by $\mathbb{Z}_m = \{[0]_m, [1]_m, [2]_m, ..., [m - 1]_m\}$.

In $\mathbb{Z}_m$ we define the binary operations $\oplus_m$ and $\odot_m$: For $m = 2$, $\mathbb{Z}_2 = \{[0]_2, [1]_2\} = \{0, 1\}$ and we define

$\oplus_2$	[0]	[1]		$\odot_2$	[0]	[1]
[0]	[0]	[1]	and	[0]	[0]	[0]
[1]	[1]	[0]		[1]	[0]	[1].

$[1]_m$ is the identity in $\mathbb{Z}_m$ with respect to binary operation $\odot_m$ and it is unique. $(\mathbb{Z}_m, \oplus_m, \odot_m)$ is a commutative ring with identity for any $m \geqslant 2$: Moreover $\mathbb{Z}_m$ is an integral domain (and hence a field) if and only if m is prime integer.

An additive abelian group V is said to be a vector space over the field F if the scalar multiplication map $F \times V \to V$, defined as $(\alpha, v) \rightarrowtail \alpha v$ satisfies $\alpha(v + w) = \alpha v + \alpha w$; $(\alpha + \beta)v = \alpha v + \beta v$; $(\alpha\beta)v = \alpha(\beta v)$; $1.v = v$, for all $\alpha, \beta \in F$, $v, w \in V$.

A vector space V is said to be an algebra over the field F if V is ring and $\alpha(vw) = (\alpha v)w = v(\alpha w)$ for all $\alpha \in F$, $v, w \in V$. A field one dimensional algebra over itself. Furthermore for a positive integer n, $F^n = \{(\alpha_1, \alpha_2, ..., \alpha_n) : \alpha_1, \alpha_2, ..., \alpha_n \in F\}$ is an algebra over F with dimension n. If p is prime integer and n be any positive integer, then $\mathbb{Z}_p$ is a one dimensional algebra over the field $\mathbb{Z}_p$ and $\mathbb{Z}_p^n$ is n dimensional algebra over the field $\mathbb{Z}_p$, particularly we may take $p = 2$. Particularly $\mathbb{Z}_2$ is a Boolean algebra, as $a^2 = a$ and $a + a = 0$, for all $a \in \mathbb{Z}_2$. Let V and W be finite dimensional vector spaces over the same field F. A ring homomorphism vector space homomorphism (respectively linear transformation) is a map $\varphi : R \to S$ (respectively $\varphi : V \to W$) which satisfies $\varphi(x + y) = \varphi(x) + \varphi(y)$ and $\varphi(xy) = \varphi(x)\varphi(y)$, for all $x, y \in R$ (respectively $\varphi(x + y) = \varphi(x) + \varphi(y)$ and $\varphi(y) = \varphi(y)$, for all $x, y \in V$, $\alpha \in F$). A homomorphism φ is said to be monomorphism, epimorphism, isomorphism if it is one one, onto and bijective respectively. If φ is an isomorphism, it can be represented as $R \cong S$ (respectively $V \cong W$). For more details one may consult [6, 7, 14, 15, 28].

3.2 Matching in Algebraic Representation of Interactive Systems and Top-Down Approach

In [22], the algebraic representation is devised in view of the systems namely State, organization, community and individuals and their interactions as described in [21] with the understanding that, behind these interactions the economics of spending and welfare work. The framework in [21] is limited to one period information regarding economic activism and creation of social capital but the algebraic representation [22] has capability to provide multperiod analysis.

By different interaction we obtain the 4×4 matrix, which contains 16 different systems, known as social capital matrix (Shah [21]).

3.2.1 *Correspondence*

According to [22], the state S is represented by finite Boolean algebra $\mathbb{Z}_2 = \{0, 1\}$, which have two active categories (vectors) 0, 1 denoted as *S – vectors* or *S – categories*. Furthermore the category 0 represents the investments/spending and 1 represents the return/welfare indicator of the state. Further, we assume that $\mathbb{Z}_2^2$ represents organization O with four *O – vectors* or *O – categories* (categories of organizations): Likewise the linear spaces $\mathbb{Z}_3^2$ and $\mathbb{Z}_4^2$ represent community C with eight *C – vectors* or *C – categories* (categories communities) and individual L with sixteen *L – vectors* or *L – categories* (categories of individuals) respectively.

The collectivist societies have top-down approach, that is State-Organization-Community and individual (abbreviated as SOCL) for their social capital development. The model used in [22] provides a coding of top-down approach. In line with he

previously discussed state's role, [22] also assume that the business of a state depends on two indicators which are running all other systems or organs of a society, and therefore it requires the reverse format of social capital matrix [21]. It is strengthened by [Wollcook, 1998] that top-down is resource allocation approach. Follows by [22].

$$\begin{array}{ccccc} \times & S & O & C & L \\ S & SS & SO & SC & SL \\ O & OS & OO & OC & OL \\ C & CS & CO & CC & \mathrm{CL} \\ L & LS & LO & LC & LL \end{array} \equiv \begin{array}{ccccc} \times & \mathbb{Z}_2 & \mathbb{Z}_2^2 & \mathbb{Z}_2^3 & \mathbb{Z}_2^4 \\ \mathbb{Z}_2 & \mathbb{Z}_2 \times \mathbb{Z}_2 & \mathbb{Z}_2 \times \mathbb{Z}_2^2 & \mathbb{Z}_2 \times \mathbb{Z}_2^3 & \mathbb{Z}_2 \times \mathbb{Z}_2^4 \\ \mathbb{Z}_2^2 & \mathbb{Z}_2^2 \times \mathbb{Z}_2 & \mathbb{Z}_2^2 \times \mathbb{Z}_2^2 & \mathbb{Z}_2^2 \times \mathbb{Z}_2^3 & \mathbb{Z}_2^2 \times \mathbb{Z}_2^4 \\ \mathbb{Z}_2^3 & \mathbb{Z}_2^3 \times \mathbb{Z}_2 & \mathbb{Z}_2^3 \times \mathbb{Z}_2^2 & \mathbb{Z}_2^3 \times \mathbb{Z}_2^3 & \mathbb{Z}_2^3 \times \mathbb{Z}_2^4 \\ \mathbb{Z}_2^4 & \mathbb{Z}_2^4 \times \mathbb{Z}_2 & \mathbb{Z}_2^4 \times \mathbb{Z}_2^2 & \mathbb{Z}_2^4 \times \mathbb{Z}_2^3 & \mathbb{Z}_2^4 \times \mathbb{Z}_2^4 \end{array}$$

4. THE MODEL INTERPRET GOVERNMENT INTERVENTION IN BUILDING SOCIAL CAPITAL IN THE SOCIETY

In this work by economic functionality/activism (social capital building/creation) we mean the economic functionality/activism (social capital building/creation) in the respective categories of organization, community and individual.

4.1 The Algebra Involved in Categories of the Systems

The following is a look of the components of categories of all 4 systems.

System	
L – categories	$\{a_i b_j c_k d_l : a_i, b_j, c_k, d_l \in \mathbb{Z}_2, i, j, k, l \in \{1, 2\}$
C – categories	$\{a_i b_j c_k : a_i, b_j, c_k \in \mathbb{Z}_2, i, j, k, \in \{1, 2\}$
O – categories	$\{a_i b_j : a_i, b_j \in \mathbb{Z}_2, i, j \in \{1, 2\}$
S – categories	$\{a_i : a_i \in \mathbb{Z}_2, i \in \{1, 2\}.$

Convention:

The number of 1's in a category of any system determines its stability regarding economic development and hence in social capital.

Before going ahead, we first establishing the compatibility of social capital and economic development due to investment/spending and in return, the welfare characteristized by the Boolean algebra $\mathbb{Z}_2$.

Components behavior:

1. Investment/spending by both ends causes again investment/spending, i.e. $0 + 0 = 0$.
2. Investment/spending by one end and welfare by other end causes welfare, i.e. $0 + 1 = 1 + 0 = 1$.

3. Welfare by both ends causes investment/spending power, i.e. 1 + 1 = 0.
4. (1), (2), (3) can be viewed as in terms of investment of time and money (i.e. 0) to obtain the social capital (i.e. 1).

On the basis of size of the systems we may say *L* is larger than *C*, *C* is larger than *O* and *O* is larger than *S*.

4.2 Interaction of Categories of Systems

4.2.1 *Components in the Categories of the Systems*

- The state $\mathbb{Z}_2$ has two categories representing the investment/spending and return/welfare indicators and their permutations constitute four different categories of the Organization $\mathbb{Z}_2^2$ and each one consists of two components in the form of investments/spending and return/welfare indicators.
- The permutations of two categories of state $\mathbb{Z}_2$ constitute eight different categories of the Community $\mathbb{Z}_2^3$ and each one consists of three components in the form of investments/spending and return/welfare indicator.
- The permutations of two categories of state $\mathbb{Z}_2$ constitute sixteen different categories of the Individual $\mathbb{Z}_2^4$ and each one consists of four components in the form of investments/spending and return/welfare indicators.

These systems in algebraic representation of social capital matrix [22] have the following positions.

$$\begin{array}{cccccc} \times & S & O & C & L \\ S & & & & & \\ O & & & & & \\ C & & & & & \\ L & & & & & \end{array} \quad \equiv \quad \begin{array}{ccccc} \times & \mathbb{Z}_2 & \mathbb{Z}_2^2 & \mathbb{Z}_2^3 & \mathbb{Z}_2^4 \\ \mathbb{Z}_2 & & & & \\ \mathbb{Z}_2^2 & & & & \\ \mathbb{Z}_2^3 & & & & \\ \mathbb{Z}_2^4 & & & & \end{array}$$

4.2.2 *Intra-Action of Categories of a System*

According to [22] the interaction of a system with itself means State vs. State, Organization vs. Organization, Community vs. Community and Individual vs. Individual.

Following [22] an intra-active function $\delta_{m \hookrightarrow m}$ or δ is defined as:

$$\delta : \mathbb{Z}_2^m \times \mathbb{Z}_2^m \to \mathbb{Z}_2^m, \quad \text{where} \quad 1 \leq m \leq 4 \text{ by}$$
$$\delta(a_1 \ldots a_m, b_1 \ldots b_m) = c_1 \ldots c_m \in \mathbb{Z}_2^m, \quad \text{for any} \quad a_1 \ldots a_m, b_1 \ldots b_m \in \mathbb{Z}_2^m,$$

where $c_i = a_i + b_i$, $1 \leq i \leq 4$: The intra-active function δ is interpreted as the economic trade-off among the categories of a system. However, in resulting one can obtain again

a category of the same system, which may have $m!$ number of possibilities regarding its status in respect of economic development or accumulation of social capital.

The following are the possible outcomes of this type of interaction.

(1) (i) Put no effect, e.g. $(0000, 0000) = 0000 + 0000 = 0000 \in \mathbb{Z}_2^4$, the infimum level of welfare and hence social capital.

(ii) In loss, e.g. $\delta(1111, 1111) = 1111 + 1111 = 0000 \in \mathbb{Z}_2^4$, the infimum level of welfare and hence the social capital.

(2) In gain, e.g. $\delta(0110, 1001) = 0110 + 1001 = 1111 \in \mathbb{Z}_2^4$, the optimum level of welfare and hence social capital.

(3) Mixed effect, e.g. $\delta(0110, 1010) = 0110 + 1010 = 1100 \in \mathbb{Z}_2^4$, the average level of welfare and hence social capital.

Particularly, if we consider $\delta : \mathbb{Z}_2^3 \times \mathbb{Z}_2^3 \to \mathbb{Z}_2^3$, then

$$(a_1 a_2 a_3, b_1 b_2 b_3) = c_1 c_2 c_3 \in \mathbb{Z}_2^3, \quad \text{where} \quad c_i = a_i + b_i, \quad 1 \leq i \leq 3.$$

Remark 1: During intra-action of any two categories, the following possible outcomes can be observed.

(i) As a consequence of interaction of a category of each of the system with itself we obtain the category 0, 00, 000 and 0000 respectively, *which is an extremely minimum level of a category in S, O, C and L regarding economic development* and social capital.

(ii) Interaction of a category of each of the system with a category having contrast components provided the category 1, 11, 111 and 1111 respectively, *an optimum level of a category in S, O, C and L regarding economic development* and social capital.

(iii) The interactions of different categories of each system not like (i) and (ii).

In [22] it is established that due to intra-active function all components of two categories of the community are doing business with all of their corresponding components. This also reflects that the total assets of interactive categories of the community are fully operationalized and no part left for its own survival which indicates the consumption and spending of all assets/resources in one period. This also indicates that intra-action of any system provides a high level of trust among the categories of the same system, which causes in economic functionality and improvements in social capital stock of the categories and hence the improvements in the stock of social capital of the system under consideration.

4.2.3 *Across Interaction of the Categories of different Systems in SOCL*

These can be observed on the lower and upper diagonal of algebraic representation of SOCL that is these are not on the main diagonal of SOCL. These are representing

12 across interactions of the systems, i.e. State vs. Organization and vice versa, State vs. Community and vice versa, Community vs. Individual and vice versa. In [22] it is represented as

$$\begin{array}{cccc} & SO & SC & SL \\ OS & & OC & OL \\ CS & CO & & CL \\ LS & LO & LC & \end{array} \equiv \begin{array}{ccc} & \mathbb{Z}_2 \times \mathbb{Z}_2^2 & \mathbb{Z}_2 \times \mathbb{Z}_2^3 \quad \mathbb{Z}_2 \times \mathbb{Z}_2^4 \\ \mathbb{Z}_2^2 \times \mathbb{Z}_2 & & \mathbb{Z}_2^2 \times \mathbb{Z}_2^3 \quad \mathbb{Z}_2^2 \times \mathbb{Z}_2^4 \\ \mathbb{Z}_2^3 \times \mathbb{Z}_2 & \mathbb{Z}_2^3 \times \mathbb{Z}_2^2 & \quad \mathbb{Z}_2^3 \times \mathbb{Z}_2^4 \\ \mathbb{Z}_2^4 \times \mathbb{Z}_2 & \mathbb{Z}_2^4 \times \mathbb{Z}_2^2 & \mathbb{Z}_2^4 \times \mathbb{Z}_2^3 \end{array}$$

Remark 2: Lower Diagonal interactions and Upper Diagonal interactions having symmetries due to this model, for example $\mathbb{Z}_2^l \times \mathbb{Z}_2^m$ and $\mathbb{Z}_2^m \times \mathbb{Z}_2^l$, $1 \leq l, m \leq 4$, are same in nature in algebraic perspective.

In [22] it is established that if 0 is zero vector space, consisting on 0 only. So, for $l \leq m$, $\mathbb{Z}_2^l \hookrightarrow \mathbb{Z}_2^m$ is imbedding of $\mathbb{Z}_2^l$ in $\mathbb{Z}_2^m$, i.e. $\mathbb{Z}_2^l \cong \mathbb{Z}_2^l \times 0 \times ... \, 0 \subset \mathbb{Z}_2^m$, this means $a_1 ... \, a_1 = a_1 ... \, a_l 0_{l+1} ... \, 0_m \in \mathbb{Z}_2^m$. Similarly $m \leq l$, $\mathbb{Z}_2^m \hookrightarrow \mathbb{Z}_2^l$ is imbedding of $\mathbb{Z}_2^m$ in $\mathbb{Z}_2^l$, i.e. $\mathbb{Z}_2^m \cong 0_1 \times ... \times 0_{l-m} \times \mathbb{Z}_2^m \subset \mathbb{Z}_2^l$, this means $a_1 ... \, a_1 = 0_1 ... \, 0_{l-m} a_1 ... \, a_l \in \mathbb{Z}_2^l$.

Following [22] the across interactive functions $\delta_{l \hookrightarrow m}$ and $\delta_{l \hookleftarrow m}$ are defined as follow:

$$\delta_{l \hookrightarrow m} : \mathbb{Z}_2^{\,l} \ \mathbb{Z}_2^{\,m} \to \mathbb{Z}_2^{\,m}, \text{ where } 1 \leq m \leq 4, \text{ and } 1 \leq m$$

by $\delta_{l \hookrightarrow m}(a_1 ... \, a_l a_{l+1} ... \, a_m, b_1 ... \, b_m) = c_1 ... \, c_l c_{l+1} ... \, c_m \in \mathbb{Z}_2^m$, $c_i = a_i + b_i$, $1 \leq i \leq l$,

for any a1 $... \, a_l \in \mathbb{Z}_2^l$, $b_1 ... \, b_m \in \mathbb{Z}_2^m$ and $a_{l+1} = ... = a_m = 0$

and

$$\delta_{l \hookleftarrow m} : \mathbb{Z}_2^{\,l} \ \mathbb{Z}_2^{\,m} \to \mathbb{Z}_2^{\,m}, \text{ where } 1 \leq m \leq 4, \text{ and } m \leq l$$

by $\delta_{l \hookleftarrow m}(a_1 ... \, a_l, b_1 ... \, b_m b_{m+1} ... \, b_l) = c_1 ... \, c_m c_{m+1} ... \, c_l \in \mathbb{Z}_2^m$, $c_k = a_k + b_k$, $1 \leq k \leq m$,

for any $a_1 ... \, a_l \in \mathbb{Z}_2^l$, $b_1 ... \, b_m \in \mathbb{Z}_2^m$ and $b_{m+1} = ... = b_l = 0$.

Whereas $c_i = a_i + b_i$, $1 \leq i \leq 4$. $\delta_{l \hookrightarrow m}$ and $\delta_{l \hookleftarrow m}$ are interpreted as the economic trade-off among the categories of different systems. However in result of this trade-off, again a category is obtained, which is infact belongs to larger systems of across inter-active systems (see [22]).

The algebraic model of social capital matrix [22] provides that the economic development and hence social capital of a category of individual reflects the presence of 3 indicators of a category of the community. As well in the category of community there is a reflection of 2 components of a category of organization. Similarly in the category of organization there is a reflection of 1 component of a category of state.

This confirms the role of state in making social capital and reflects the fact that it seems resource allocation and hence top-down approach.

Also following [22], for $l \leq m$, $\mathbb{Z}_2^l \hookrightarrow \mathbb{Z}_2^m$ is imbedding of $\mathbb{Z}_2^l$ in $\mathbb{Z}_2^m$, i.e. $\mathbb{Z}_2^l \cong \mathbb{Z}_2^l \times 0 \times ... \times 0 \subset \mathbb{Z}_2^l \times \mathbb{Z}_2^{m-1} \cong \mathbb{Z}_2^m$, the $m - l$ components are inactive during interaction of $\mathbb{Z}_2^l$ and $\mathbb{Z}_2^m$. This indicates that personal character or actual having of a category of a larger system is totally protected by the owner category during interaction with a category of a smaller system. More precisely if $m - l = 0$, then there will be the activism of each component of each category whenever it is interactive with the category of the same system. If $m - l$ is 1, 2 or 3, then there will be activism of each component of each category whenever it is interactive with the category of the same system.

The following table provides a complete information under the assumption that:

The # of components in a category of smaller system = l

The # of components in a category of larger system = m

The # of components inactive in a category of larger system = $m - l$

Interaction	l	m	$m - l$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	1	1	0
$\mathbb{Z}_2 \times \mathbb{Z}_2^2$	1	2	1
$\mathbb{Z}_2 \times \mathbb{Z}_2^3$	1	3	2
$\mathbb{Z}_2 \times \mathbb{Z}_2^4$	1	4	3
$\mathbb{Z}_2^2 \times \mathbb{Z}_2^2$	2	2	0
$\mathbb{Z}_2^2 \times \mathbb{Z}_2^3$	2	3	1
$\mathbb{Z}_2^2 \times \mathbb{Z}_2^4$	2	4	2
$\mathbb{Z}_2^3 \times \mathbb{Z}_2^3$	3	3	0
$\mathbb{Z}_2^3 \times \mathbb{Z}_2^4$	3	4	1
$\mathbb{Z}_2^4 \times \mathbb{Z}_2^4$	4	4	0

We may interpret this discussion for $0 \leq m - l \leq 3$.

1. $0 = m - l$ provides $m = l$ and hence a system is interacting with itself (i.e. intra-action of a system). In this type of interaction all components of two interactive categories are doing business with all of their corresponding components, which reflects that the total assets of interactive categories of the same system are fully operationalized and no part left for substance for its own survival. This also indicates that intra-action of any system provides a high level of trust among the categories of system under consideration (see [22]).

2. $1 \leq m - l \leq 3$ provides $l \leq m$ and a smaller system is interacting with a larger system (i.e. across interaction of different systems). This shows that not all

the components of a category of the larger system are doing business with the components of the smaller system. This means the total assets/resources are not operationalized by the larger system rather a part left for substance for its own survival, which reflects extreme case, that is in contradiction to this finding that the categories of individual, community or organization that consumes/ spend all of its assets/resources in one period do not survive for next period (see [22]).

5. A POLICY FRAMEWORK FOR THE GOVERNMENT

These findings strengthened our format of Social Capital Matrix, that is *SOCL*, which compels for the leading role of state in all types of activities of categories of organization, community and finally, individual.

The following represent the *SOCL*.

L	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
C					...	...	...	...	...	...	...	...				
O							...	...	...	...						
S								...	...							

5.1 How Achievable is the Social Cohesion of Society in Multiperiod?

5.1.1 *Status of a Category Regarding Welfare*

In a category $a_1 \ldots a_m$, $1 \leq m \leq 4$, where $a_1, \ldots, a_m \in \{0, 1\}$, of any system of *SOCL*, the # of 0's and # of 1's define the welfare level and hence the level of social capital of the category under consideration.

That is maximum # of 1's indicates the higher welfare level and hence the social capital.

The # of 1's in categories of the systems

$$\#\ 1\text{'s in } a_i = n_{S\alpha},\ 1 \leq \alpha \leq 2^1,\ i \in \{1, 2\},$$
$$\#\ 1\text{'s in } a_i a_j = nO,\ 1 \leq \alpha \leq 2^2,\ i,\ j \in \{1, 2\},$$
$$\#\ 1\text{'s in } a_i a_j a_k = n_{C\alpha},\ 1 \leq \alpha \leq 2^3,\ i, j, k \in \{1, 2\},$$
$$\#\ 1\text{,s in } a_i a_j a_k a_l = n_{L\alpha},\ 1 \leq \alpha \leq 2^4,\ i, j, k, l \in \{1, 2\}.$$

The rotation of cone indicates the intra-action of categories of each system in a period r of time. Since social cohesion is a characteristic of a society which depends on the accumulated social capital, therefore for a period $r (\in \{1, 2, \ldots\})$ of time we obtained the following.

$$\text{Social cohesion in period } r\,((SC)_r) = sc\,(S)\,r + sc\,(O)\,r + sc\,(C)\,r + sc\,(L)\,r,$$

where $sc(S)r$, $sc(O)r$, $sc(C)r$ and $sc(L)r$ represent social capital of state, organization, community and individual at period r.

Particularly if we consider the period $r = 1$, then initially

$$\text{Social cohesion} = (SC)_1 = sc(S)_1 + sc(O)_1 + sc(C)_1 + sc(L)_1.$$

$$sc(S)_1 = 0 + 1 = 1, \text{ at period } r = 1,$$

$$sc(O)_1 = \sum_{\alpha=1}^{4} n_{O\alpha}, \text{ at period } r = 1,$$

$$sc(C)_1 = \sum_{\alpha=1}^{8} n_{C\alpha}, \text{ at period } r = 1,$$

$$sc(L)_1 = \sum_{\alpha=1}^{16} n_{L\alpha}, \text{ at period } r = 1.$$

Hence

$$\text{Social cohesion at starting} = (SC)_1 = 1 + \sum_{\alpha=1}^{4} n_{O\alpha} + \sum_{\alpha=1}^{6} n_{C\alpha} + \sum_{\alpha=1}^{16} n_{L\alpha}.$$

As a consequent we obtain the following:

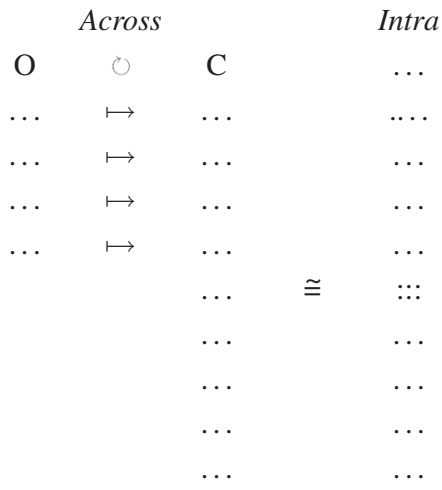
Theorem 1: Let a period r be a period of time and Social cohesion S_{Cr} represents the Social cohesion of the society. Then

1. $SC_r = sc(S)r + sc(O)r + sc(C)r + sc(L)r$, where $sc(S)r$, $sc(O)r$, $sc(C)r$ and sc(L)r represent social capital of state, organization, community and individual at period r.
2. $1 \leq SC_r \leq 97$.

Remark 3: As a consequence of intra-action of each system in *SOCL* the government can devise a policy framework to evaluate the intertemporal social cohesion of the society, which may help in planning.

Limitations of across interaction of systems under given model

Ultimately the across interaction of the systems converges to inrta-action for the larger system but in this process we observe change only in the components of the categories of larger system which are coincides with the components of categories of smaller systems.



Remark 4: (i) At the same time each system is involved in across and intra-actions.

(ii) The social cohesion of society reflects the both, that is the across interaction and intra-action of the systems for multiperiods.

6. CONCLUSION

Based on algebraic representation of social capital matrix which emerges through interaction of state (S), organization (O), community (C) and the individual (L), that proposed top-down approach is more compatible for the collectivist society to strengthen the social capital.

The intra-action and inter-action of each system, the study concludes that the social cohesion of the society reflects both across interaction and intra-action of the systems for multiperiods. Furthermore the study suggests a policy framework for the government regarding social capital development in perspective of top-down approach.

This study will be helpful in planning economic developments strategies for the government in particular perspective of a collectivist society. Furthermore these findings indicates that a compact check and balance is in.ltrated in each set up, which provides a strength in whole economic and social capital development phenomena.

This study may be generalized if we increase the state indicators, that is $n \geq 2$, then state S should be $\mathbb{Z}_n$. Since $\mathbb{Z}_n$ is a field if n is prime, so almost same algebraic construction applies as considered in this paper and the behavior of *SOCL* regarding top-down approach can be characterized with complexities. On the other hand, $\mathbb{Z}_n$ behaves as a commutative ring with identity whenever n is not prime, which is not an integral domain. This would of course be more suitable option in analyzing the top-down approach in a rational way and this extended approach may provide a rationale regarding non-availibility of smooth environment for interaction of categories of the systems.

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Tariq Shah
Department of Mathematics,
Quaid-i-Azam University,
Islamabad, Pakistan.
E-mail: stariqshah@gmail.com

Syed Akhter Hussain Shah
Small Industries Development Board,
Government of N.-W. F. P., Pakistan.
E-mail: akhtar_hshah@yahoo.com

Eatzaz Ahmed
Department of Economics,
Quaid-i-Azam University,
Islamabad-Pakistan.
E-mail: eatzaz@qau.edu.pk

Bakhtiar Ali
Department of Management Science,
Shaheed Zul.kar Ali Bhutto Institute of
Science and Technology (SZABIST),
Islamabad Campus, Pakistan.
E-mail: bakhtiar@szabist-isb.edu.pk