Exploiting of interest rates fundamental inefficiency

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Abstract

This article is a supplement to previously published paper [1]. It describes a paradox, which shows that arbitrage opportunities almost always exist. Markets that do not allow such opportunities differ from current significantly.

Introduction

Presented here paradox exploit the same market inefficiency as Siegel's Paradox. It shows that this inefficiency could be used for arbitrage.

There are prepaid forward contracts F_1, F_2, F_3 with delivery dates $T_1 < T_2 < T_3$. No default risk assumed.

S_t is exchange rate between F_1 and $F_2 : 1 \cdot F_2 = S_t \cdot F_1$. There are three possible scenarios: S_T is equal to S_T^1, S_T^2 or S_T^3 at some moment T. Let current price $S_0 = S_T^2$.

S'_t is exchange rate between F_2 and $F_3 : 1 \cdot F_3 = S'_t \cdot F_2$.

There are options with next properties:

1. Initial moment t_0.
2. Expiration date $T, t_0 < T < T_1$.
3. Numeraire is F_2.
4. Options have analogous to Arrow-Debreu securities payoff $H_T^i = 1$ if $S_T = S_T^i$ and 0 otherwise, $i = 1, 2, 3$. There are three options with premiums H_0^i.

If someone borrows prepaid forward contract then he has to return one forward contract. One forward contract tomorrow cost one forward contract today.

The paradox

If we buy one option of each type, i.e. use strategy (1;1;1), then payoff is equal to 1 F_2 independently from price S_T. Premium has to be equal to 1:

$$H_0^1 + H_0^2 + H_0^3 = 1 \tag{1}$$

Arbitrage is possible otherwise.

If we want payoff to be equal to 1 F_1 then we should use strategy ($\frac{1}{S_T^1}; \frac{1}{S_T^2}; \frac{1}{S_T^3}$). After exercise we exchange payoff to F_1 ($\frac{1}{S_T^i} \cdot S_T^i = 1$). Premium is

$$\frac{1}{S_T^1}\cdot H_0^1+\frac{1}{S_T^2}\cdot H_0^2+\frac{1}{S_T^3}\cdot H_0^3=\frac{1}{S_0}=\frac{1}{S_T^2} \tag{2}$$

Consider the case when delivery date T_3 is variable and unknown at t_0. It is being fixed at moment T. Let

$$\lim_{T_3\to T_2} S'_T=1$$
$$\lim_{T_3\to\infty} S'_T=0 \tag{3}$$

Such situation is normal for non-zero interest rate.

By choosing T_3 we can change S'_T. At moment T we choose such T_3 that $S'_T=S_T$. If we want payoff to be equal to 1 F_3 then we should use strategy ($S_T^1;S_T^2;S_T^3$).

Buying F_3 using numeraire F_2 is equivalent to buying F_2 using numeraire F_1. In both cases prices at moment T are equal, prices of Arrow-Debreu securities are also equal. Consequently, initial prices at t_0 are also equal and

$$S_T^1\cdot H_0^1+S_T^2\cdot H_0^2+S_T^3\cdot H_0^3=S'_0=S_0=S_T^2 \tag{4}$$

System of equations (1), (2) and (4) has only one solution for premiums H_0^1, H_0^2 and H_0^3. It is (0;1;0). In fact, premium of Arrow-Debreu security is market estimation of probability of scenario. This means that S_t is expected to be constant. Arbitrage is possible otherwise.

However, this situation is impossible in real world. Interest rates are changing. Moreover, it is possible to make exchange rate S_t not constant artificially. Dividends, for example, affect such exchange rates. Consider some portfolio. It has some current price. If we are going to sell part of it then we expect that its price will be lower than current one. This decision affects exchange rates on forward contracts with the portfolio as underlying asset.

Conclusion

The paradox could be easily extended to continuous general case. If arbitrageurs are able to use presented paradox then markets will change very seriously.

Described paradox is theoretical. It is too complex for practical use, but it demonstrates that important inefficiencies exist. They could be used in a very profitable way.

References

1. Ivanov, Sergei A. (2014) Implied-in-prices expectations: Their role in arbitrage. AAPP | Physical, Mathematical, and Natural Sciences, vol. 92, S1, B1. doi:10.1478/AAPP.92S1B1.