Exploiting of interest rates fundamental inefficiency

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Abstract
This article is a supplement to previously published paper [1]. It describes a paradox, which shows that arbitrage opportunities almost always exist. Markets that do not allow such opportunities differ from current significantly.

Introduction
Presented here paradox exploit the same market inefficiency as Siegel’s Paradox. It shows that this inefficiency could be used for arbitrage.

There are prepaid forward contracts $F_1, F_2, F_3$ with delivery dates $T_1 < T_2 < T_3$. No default risk assumed.

$S_i$ is exchange rate between $F_1$ and $F_2: 1 \cdot F_2 = S_i \cdot F_1$. There are three possible scenarios: $S_T$ is equal to $S_T^1, S_T^2$ or $S_T^3$ at some moment $T$. Let current price $S_0 = S_T^2$.

$S_T'$ is exchange rate between $F_2$ and $F_3: 1 \cdot F_3 = S_T' \cdot F_2$.

There are options with next properties:

1. Initial moment $t_0$.
2. Expiration date $T, t_0 < T < T_i$.
3. Numeraire is $F_2$.
4. Options have analogous to Arrow-Debreu securities payoff $H_i^T = 1$ if $S_T = S_T^i$ and 0 otherwise, $i = 1, 2, 3$. There are three options with premiums $H_i^0$.

If someone borrows prepaid forward contract then he has to return one forward contract. One forward contract tomorrow cost one forward contract today.

The paradox

If we buy one option of each type, i.e. use strategy $(1;1;1)$, then payoff is equal to $1 F_2$ independently from price $S_T$ . Premium has to be equal to $1$:

$$H_0^1 + H_0^2 + H_0^3 = 1$$

Arbitrage is possible otherwise.

If we want payoff to be equal to $1 F_1$ then we should use strategy $\left(\frac{1}{S_T^1}, \frac{1}{S_T^2}, \frac{1}{S_T^3}\right)$. After exercise we exchange payoff to $F_1 \left(\frac{1}{S_T^i} \cdot S_T^i = 1\right)$. Premium is
Consider the case when delivery date $T_3$ is variable and unknown at $t_0$. It is being fixed at moment $T$. Let

$$\lim_{T_3 \to T} S_{T_3} = 1$$

$$\lim_{T_3 \to \infty} S_{T_3} = 0$$

Such situation is normal for non-zero interest rate.

By choosing $T_3$, we can change $S_{T_3}$. At moment $T$ we choose such $T_3$ that $S_{T_3} = S_T$. If we want payoff to be equal to 1 $F_3$ then we should use strategy $(S^1_T, S^2_T, S^3_T)$.

Buying $F_3$ using numeraire $F_2$ is equivalent to buying $F_2$ using numeraire $F_1$. In both cases prices at moment $T$ are equal, prices of Arrow-Debreu securities are also equal. Consequently, initial prices at $t_0$ are also equal and

$$S^1_T \cdot H^1_0 + S^2_T \cdot H^2_0 + S^3_T \cdot H^3_0 = S^1_{t_0} = S^2_{t_0} = S^3_{t_0}$$

System of equations (1), (2) and (4) has only one solution for premiums $H^1_0$, $H^2_0$ and $H^3_0$. It is $(0;1;0)$. In fact, premium of Arrow-Debreu security is market estimation of probability of scenario. This means that $S_T$ is expected to be constant. Arbitrage is possible otherwise.

However, this situation is impossible in real world. Interest rates are changing. Moreover, it is possible to make exchange rate $S_T$ not constant artificially. Dividends, for example, affect such exchange rates. Consider some portfolio. It has some current price. If we are going to sell part of it then we expect that its price will be lower than current one. This decision affects exchange rates on forward contracts with the portfolio as underlying asset.

**Conclusion**

The paradox could be easily extended to continuous general case. If arbitrageurs are able to use presented paradox then markets will change very seriously.

Described paradox is theoretical. It is too complex for practical use, but it demonstrates that important inefficiencies exist. They could be used in a very profitable way.

**References**