# Electoral involvement and appreciation for democracy under a compulsory voting rule

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**Abstract**

This paper proposes a theoretical model for the decision of voter registration, which recovers the classical notion that democracy is a public good. The solution of the citizen's problem implies three types of Nash equilibrium (null, partial, and full enrollment), where the real cost for voter enrollment and appreciation for democracy are the key variables. In the partial-enrollment equilibrium, the citizens' democratic valuation has a threshold that encourages a free-rider behavior even when the homogeneous-citizens assumption is not met. In turn, a policy maker could avoid this threat of representativeness crisis by setting an optimal enrollment cost that depends on electorate size and citizens' heterogeneity. Finally, an empirical model is outlined from the policy maker's problem, which is coherent with classical literature on voting behavior.

**Keywords**: electoral engagement, compulsory voting, voting behavior

**JEL classification**: D72

## 1. Introduction

The voting behavior theory is based on the pioneer work of Anthony Downs (1957) and Gordon Tullock in 1967 (cited by Barzel and Silberberg, 1973), but it was mainly developed in the 1970s and the 1980s. In these decades, the efforts were focused on giving a theoretical background to the electoral involvement, where the inclusion of economic rationality into the analysis was crucial (Frey, 1971; Stigler, 1972; Barzel and Silberberg, 1973; Tollison and Willett, 1973; Ferejohn and Fiorina, 1974; May and Martin, 1975; Palfrey and Rosenthal, 1983, 1985). Afterward, the first empirical tests aimed to identify the relevance of social and economic variables over voter turnout (Silberman and Durden, 1975; Tollison et al., 1975; Settle and Abrams, 1976).

Thenceforth, the vast evidence about electoral participation and election outcomes has been inconclusive. The ecological fallacy critique stated by Matsusaka and Palda (1993) called into question most of the previous evidence obtained from aggregated data. Lichtman (1974) and Kramer (1983) previously noted this drawback and pointed out that macro-level regressions exhibit more suitable properties than individual survey estimates. In fact, Kramer remarked that these properties were especially observed when the aggregated model was well-specified.

Nonetheless, recent empirical articles (e.g., Brender, 2003; Cerda and Vergara, 2008; Aleksynska, 2011) are still hypothesizing about the relationship between a set of economic, social, or political variables and turnout, abstention, electoral performance, or civic participation. On the other hand, Ansolabehere and Konisky (2006) stressed the long-term impact of registration laws on turnout in the United States. Although the above evidence on turnout is interesting, the underlying assumptions used in these empirical models are not always revealed.

This paper addresses the lack of theoretical discussion on voter registration and proposes a model for the voter enrollment decision. The background of the model considers a democratic society with a mandatory voting rule[1]. In addition, the optimal solution of the model enables to outline an empirical model where a set of structural regressors are identified.

The structure of the paper is as follows. Section 2 outlines the model for the voter registration decision. Section 3 presents an explicit solution for the citizen's problem. Section 4 develops the policy maker's problem, identifies some policy implications, and proposes an empirical test for the model. Section 5 concludes.

## 2. Modeling the decision of voter registration

Suppose that a democratic society allows voluntary voter registration but mandatory voting. This society is populated by $n$ citizens where each citizen $i$ has preferences over a consumption good, leisure, and democracy.

The demand for the consumption good is denoted by $x$, which price is $p_x$.

The available time is allocated as follows. Each citizen $i$ spends a fraction $h_i$ in the labor market and earns a nominal wage, $w$. On the other hand, the individual enjoys a fraction $l_i$ of leisure

[1] Such as the one adopted by Australia, Singapore, Brazil, or Chile until 2012.

in the form of costless activities that are linked to citizen participation. Thus, the citizen's time constraint is: $l_i + h_i = 1$.

Since the citizens value the democratic institutions that prevail in the society, then they contribute to strengthening democracy, $d$, by being part of the electoral roll. This insight is coherent with the classical notion exposed by Downs (1957) that democracy is a public good. Thus, a higher political engagement will guarantee the stability of the political system.

Therefore, the utility function for citizen $i$ is given by:

$$u_i(x_i, l_i, d) \tag{1}$$

Where $u(\cdot)$ is a continuous, increasing, and at least twice differentiable function in $x$, $l$, and $d$.

Under this democratic context, each citizen must decide whether being or not part of civic life through voter enrollment, which decision is represented by variable $r_i$. If citizen $i$ decides to be enrolled in the voter register, then $r_i$ will be equal to one and zero otherwise. Moreover, the enrollment decision implies a nominal cost $c_r$, which summarizes all the costs related to mandatory voting such as transportation costs or a fine for not attending to vote.[2] Thus, each citizen faces the following budget constraint:

$$p_x x_i + c_r r_i \leq h_i w \tag{2}$$

This budget constraint can be expressed in real terms as follows:

$$x_i + \delta r_i \leq h_i \omega, \tag{3}$$

Where $\delta = c_r/p_x$ is the real cost for being enrolled as a voter and $\omega = w/p_x$ the real wage.

The electoral roll, $R$, is composed by the sum of all those individuals that decided to be enrolled on. Since democracy is treated as a public good, assume that $d$ is a function of the electoral register. That is, $d = f(R)$, where $f(\cdot)$ is a continuous, increasing, and at least twice differentiable function.

In sum, each citizen $i$ must solve the following problem:

$$\begin{aligned}
&\max_{\{x,l,r\}} u_i(x_i, l_i, d) \\
&\text{s.t.:} \quad \begin{aligned} x_i + \delta r_i &\leq h_i \omega \\ l_i + h_i &= 1 \\ R &= \sum_{j=1}^{n} r_j = r_i + \sum_{j \neq i}^{n} r_j \\ d &= f(R) \end{aligned}
\end{aligned} \tag{4}$$

Where $x_i \geq 0$, $l_i, h_i \in [0, 1]$, $r_i = 0, 1$, $R \geq 0$, and $d \geq 0$.

[2]Krasa and Polborn (2009) evaluated the effects of asymmetric voting costs (and information about candidates) and subsidies over voter participation rate in a compulsory voting system.

The optimal values for $x$, $l$, $h$, and $r$ are obtained from the following equation:

$$(5) \quad \frac{\partial u_i}{\partial x_i} = \frac{1}{\omega}\frac{\partial u_i}{\partial l_i} = \frac{1}{\delta}\frac{\partial u_i}{\partial d} f'(R)$$

Where $\partial u_i/\partial x_i$, $\partial u_i/\partial l_i$, and $\partial u_i/\partial d$ are the marginal utility of consumption, leisure, and democracy, and $f'(R)$ is the partial derivative of $f$ with respect to $R$.
Next section introduces an explicit solution that explores the Nash equilibrium under a society populated by homogeneous citizens.

## 3. An explicit private solution

In order to obtain an explicit solution for the model outlined in the previous section, assume that citizens' preferences can be represented by a Cobb-Douglas utility function:

$$(6) \quad u_i(x_i, l_i, d) = x_i^{\phi_1^i} l_i^{\phi_2^i} d^{\phi_3^i}$$

Where the superscript $i$ in the $\phi$'s allows to evaluate if the democratic society is populated by citizens with homogeneous or heterogeneous preferences over consumption, leisure, and/or democracy.
If the society is populated by homogeneous citizens, then they have the same preferences over consumption, leisure, and democracy. Thus, the citizens share the following utility function:

$$(7) \quad u_i(x_i, l_i, d) = x_i^{\phi_1} l_i^{\phi_2} d^{\phi_3}$$

In addition, suppose that the democracy function is given by:

$$(8) \quad d = f(R) = \frac{R^2}{n}$$

Under this particular function, the supply of democracy as a public good is bounded. That is, a full electoral enrollment would bind the supply of democracy to $n$. Alternatively, the supply of this public good would tend to zero if a representativeness crisis takes place.
After solving the equation system derived from (5), citizen $i$ will choose the following optimal values for the voter enrollment ($r^\star$), consumption ($x^\star$), leisure ($l^\star$), and labor supply ($h^\star$):

$$(9) \quad r_i^\star = \frac{\omega}{\delta}\left(\frac{2\phi_3}{\phi_1+\phi_2+2\phi_3}\right) - \left(\frac{\phi_1+\phi_2}{\phi_1+\phi_2+2\phi_3}\right)\sum_{j\neq i}^{n} r_j$$

$$(10) \quad x_i^\star = \frac{\delta\phi_1}{2\phi_3}\left(r_i^\star + \sum_{j\neq i}^{n} r_j\right)$$

$$(11) \quad l_i^\star = \frac{\delta\phi_2}{2\omega\phi_3}\left(r_i^\star + \sum_{j\neq i}^{n} r_j\right)$$

$$(12)\quad h_i^\star = 1 - \frac{\delta\phi_2}{2\omega\phi_3}\left(r_i^\star + \sum_{j\neq i}^{n} r_j\right)$$

From equation (9) it is evident that the optimal enrollment decision of citizen $i$ is a linear reaction function. Since this reaction function maps the closed unit interval of $r$ into itself, then a Nash equilibrium can be obtained. Moreover, remember that $r$ is a binary choice variable, then it can be redefined using the floor function in the following way :

$$(13)\quad r_i^\star = \left\lfloor \max\left\{ \min\left\{ \frac{\omega}{\delta}\left(\frac{2\phi_3}{\phi_1+\phi_2+2\phi_3}\right) - \left(\frac{\phi_1+\phi_2}{\phi_1+\phi_2+2\phi_3}\right)\sum_{j\neq i}^{n} r_j ; 1 \right\} ; 0 \right\} \right\rfloor$$

For illustrative purposes, first suppose that the society is populated by two citizens (i.e., $n = 2$) and $u(\cdot)$ is homogeneous of degree one in consumption and leisure (i.e., $\phi_1 + \phi_2 = 1$). Accordingly, (13) is now given by:

$$(14)\quad r_i^\star = \left\lfloor \max\left\{ \min\left\{ \frac{\omega}{\delta}\left(\frac{2\phi_3}{1+2\phi_3}\right) - \left(\frac{1}{1+2\phi_3}\right)\sum_{j\neq i}^{n} r_j ; 1 \right\} ; 0 \right\} \right\rfloor$$

Therefore, the reaction function equilibrium is $r_1^\star = r_2^\star = \omega\phi_3/\delta(1+\phi_3)$. The equilibrium implies two extreme cases for a unique Nash equilibrium, which are identified and shown in Figure 1. In this figure, the best response of citizen one ($r_1$) is represented by a square and a circle represents the best response of citizen two ($r_2$).

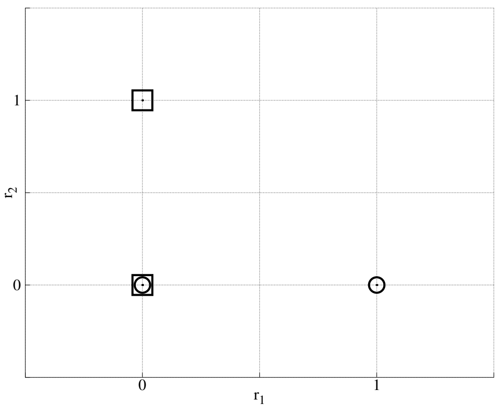
In the first extreme equilibrium, the voter enrollment rate is zero if the dominant strategy is not to be enrolled. This is true if both citizens do not value the democratic principles, $\phi_3 = 0$, or their appreciation for democracy is "too low" and less than $\delta/2(\omega-\delta)$ (see Figure 1(a))

The second case implies a full-enrollment equilibrium since dominant strategy for both citizens is to be enrolled. This outcome is feasible if both citizens have a relative "high" appreciation for democracy, which is equal or greater than $\delta/(\omega-\delta)$ (see Figure 1(b)).

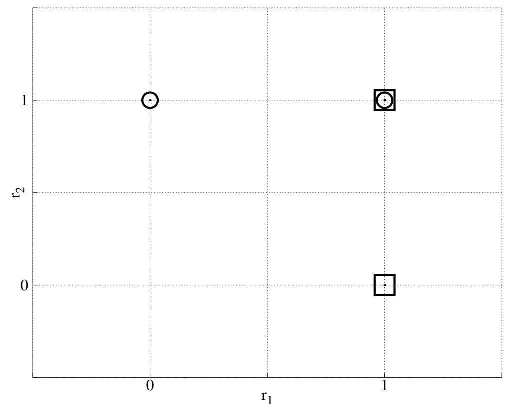
On the other hand, the model allows a multiple Nash equilibrium if the valuation of democracy belongs to the interval $[\delta/2(\omega-\delta); \delta/(\omega-\delta)[$, which is equivalent to a partial enrollment rate (see Figure 1(c)). Since citizens do not have contact to each other when they make a decision about their voter enrollment status, then this equilibrium could be interpreted as a free-rider behavior.

Now, suppose that $n = 3$, then the reaction function equilibrium is $r^\star = 2\omega\phi_3/\delta(3+2\phi_3)$ for all $i = 1, 2, 3$. In this case, the zero-enrollment equilibrium is achieved if all citizens do not value democracy or $\phi_3 < \delta/2(\omega-\delta)$. Alternatively, the full-enrollment equilibrium is possible if $\phi_3 \geq 3\delta/2(\omega-\delta)$.

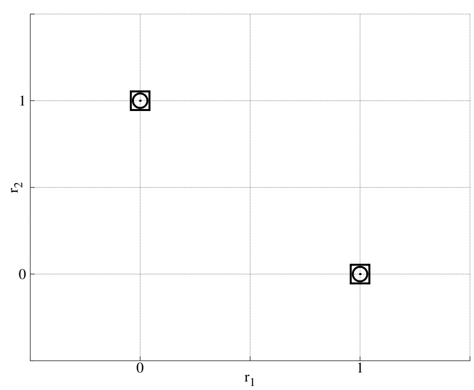
The partial-enrollment equilibrium takes place if $\delta/2(\omega-\delta) \leq \phi_3 < 3\delta/2(\omega-\delta)$ and it involves two particular situations. First, if $\delta/2(\omega-\delta) \leq \phi_3 < \delta/(\omega-\delta)$, then the best response of citizen $i$ will be "being enrolled" if and only if the remaining citizens are not enrolled (i.e., $\sum_{j\neq i}^{3} r_j = 0$). And second, if $\delta/(\omega-\delta) \leq \phi_3 < 3\delta/2(\omega-\delta)$, then the best response of citizen $i$ will be "being enrolled" if and only if the other citizens are not fully enrolled (i.e., $\sum_{j\neq i}^{3} r_j < 2$). Therefore, the free-rider behavior will be most likely if the voter registration rate is high and will be discarded if it is near to zero.




(a) Zero-enrollment equilibrium: $\phi_3 < \frac{\delta}{2(\omega-\delta)}$




(b) Full-enrollment equilibrium: $\phi_3 \geq \frac{\delta}{(\omega-\delta)}$




(c) Partial enrollment equilibrium: $\frac{\delta}{2(\omega-\delta)} \leq \phi_3 < \frac{\delta}{(\omega-\delta)}$

Figure 1: Representation of Nash equilibrium when $n = 2$.

After some algebra, it is straightforward to show that zero-enrollment and full-enrollment equilibria are achieved if $\phi_3 < \delta/2(\omega-\delta)$ and $\phi_3 \geq n\delta/2(\omega-\delta)$, respectively. Regarding to the partial-enrollment equilibrium, the best response of citizen $i$ will be "being enrolled" when the remaining citizens are not enrolled if $\phi_3 < (n-1)\delta/2(\omega-\delta)$, and when at least one of the remaining citizens is enrolled if $(n-1)\delta/2(\omega-\delta) \leq \phi_3 < n\delta/2(\omega-\delta)$.

## 4. Social solution, policy implications, and empirical tests

From a social perspective, suppose that a policy maker seeks to maximize the common good in this democratic society. Therefore, the problem to be solved can be outlined as follows:

$$
\begin{aligned}
&\max_{\{x,l,r,d\}} \sum_{i=1}^{n} \alpha_i \cdot u_i(x_i, l_i, d) \\
&\text{s.t.:} \quad \begin{array}{rcl}
\sum_{i=1}^{n} x_i + \delta \sum_{i=1}^{n} r_i & \leq & \sum_{i=1}^{n} h_i \omega \\
l_i + h_i & = & 1 \\
\sum_{i=1}^{n} r_i & = & R \\
d & = & f(R)
\end{array}
\end{aligned} \tag{15}
$$

Where $x_i \geq 0$, $l_i, h_i \in [0,1]$, $r_i = 0, 1$, $R \geq 0$, $d \geq 0$, and $\alpha_i$ represents the relative weight of citizen $i$ in the society. Therefore, $\sum_{i=1}^{n} \alpha_i = 1$.
Combining the first-order necessary conditions, we have that:

$$
f'(R) = \left(\frac{\delta}{\omega}\right) \sum_{i=1}^{n} \frac{\partial u_i / \partial l_i}{\partial u_i / \partial d} = \delta \sum_{i=1}^{n} \frac{\partial u_i / \partial x_i}{\partial u_i / \partial d} \tag{16}
$$

The above equilibrium condition yields the optimal values for $x_i^\star$, $l_i^\star$, $h_i^\star$ $r_i^\star$, $d^\star$, and $R^\star$. From these values, the following concepts can be defined. The voter registration rate, $\nu$, is given by:

$$
\nu = \frac{1}{n} \sum_{i=1}^{n} r_i^\star \tag{17}
$$

The citizen participation rate, $\kappa$, is defined as follows:

$$
\kappa = \frac{1}{n} \sum_{i=1}^{n} l_i^\star \tag{18}
$$

### 4.1. Policy implications

From equation (13), it is straightforward to show that the reaction function equilibrium depends on real wage ($\omega$), democracy appreciation ($\phi_3$), population ($n$), and the real enrollment cost ($\delta = c_r/p_x$). Thus, the policy maker can choose $c_r$ and reorient the private decision in order to maximize the electoral roll or avoid a representativeness crisis.
In order to evaluate the policy alternatives, suppose for simplicity that citizens are homogeneous, $\omega = 6$, and $\delta = 2$. If we assume that the appreciation for democracy is too weak, let us say $\phi_3 = 0.1$, then the Nash equilibrium will be the zero-enrollment no matter what population size has been considered. Hence, a real enrollment cost equals to two is not optimal, then the policy maker could

encourages the enrollment process after reducing $\delta$ below 0.5, which resembles what is depicted in Figure 2.

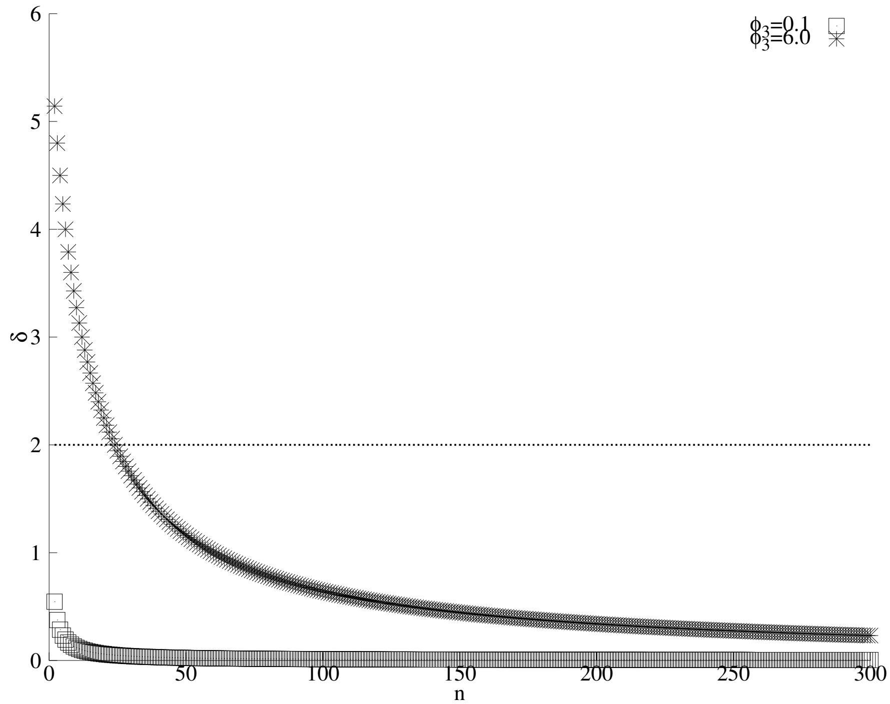



Figure 2: Optimal real enrollment cost for several population sizes and democracy appreciation.

Furthermore, if $\phi_3 = 6$, then the Nash equilibrium is the full-enrollment scenario. In this case, the starting value of two for $\delta$ is optimal only if the population is less than 24 citizens. Nevertheless, this apparent high-valuation of democracy will provoke a free-rider behavior between citizens once population size departs from 24. Therefore, if the policy maker wants to eradicate this kind of behavior, then $\delta$ has to be reduced at a faster rate than under a weaker appreciation for democratic principles.

Finally, if the society is populated by heterogeneous citizens, where a fraction $\theta$ has a low democracy valuation and $(1 - \theta)$ has a strong valuation, then the policy advice is to charge a differential enrollment cost only if $\phi_3$ is known and the potential electorate is small. Otherwise, if the potential electorate is fairly large, then the optimal value for $\delta$ must be close to zero in order to avoid a representativeness crisis.

### 4.2. Empirical tests

In order to obtain an empirical model from the above microfoundations, suppose for the sake of simplicity that citizens are homogeneous, utility function is Cobb-Douglas, and the democracy function is given by (8). Therefore, by substituting the partial derivatives of (7) and (8) into (16), it is trivial to verify that:

$$(19)\quad \frac{2R^\star}{n} = \left(\frac{\delta}{\omega}\right)\sum_{i=1}^{n}\frac{\phi_2}{l_i^\star}\frac{d^\star}{\phi_3}$$

Where $l_i^\star$, $R^\star$, and $d^\star$ denote the optimal values for leisure, electoral roll, and democracy. After rearranging equation (19), we have that:

$$(20)\quad R^\star = \sum_{i=1}^{n} r_i^\star = \left(\frac{2\omega\phi_3}{\delta\phi_2}\right)\sum_{i=1}^{n} l_i^\star$$

Dividing both sides of (20) by population, $n$, then the optimal enrollment rate, $\nu^\star$, can be expressed as:

$$(21)\quad \nu^\star = \left(\frac{2\omega\phi_3}{\delta\phi_2}\right)\kappa^\star$$

Where $\nu^\star = R^\star/n$ and $\kappa^\star = (1/n)\sum_{i=1}^{n} l_i^\star$. The log-linearization of equation (21) implies that:

$$(22)\quad \ln \nu^\star = \ln\left(\frac{2\phi_3}{\phi_2}\right) + \ln(\omega) + \ln(\kappa^\star) - \ln(\delta)$$

Therefore, an empirical model that allows to test the relationship between the voter enrollment rate and a set of economic and social variables can be stated as follows:

$$(23)\quad \ln \nu_i = \beta_1 + \beta_2 \ln(\omega)_i + \beta_3 \ln(\kappa)_i + \beta_4 \ln(\delta)_i + \varepsilon_i$$

Where the betas are the parameters to be estimated, $\varepsilon$ the error term, and $i$ can be a county, region, state, country, or any geographical division.

Finally, it is remarkable that the empirical model defined in equation (23) is consistent with those proposed in the initial empirical work on voting behavior, such as Barzel and Silberberg (1973), Silberman and Durden (1975), or Settle and Abrams (1976).

## 5. Concluding remarks

This paper aimed to make a contribution to empirical literature on voting behavior by modeling the voter enrollment decision of rational citizens. Under this framework, citizens must voluntary decide about their voter enrollment status based on how relevant is democracy for them. Thus, the model was solved and analyzed from the citizen and policy maker's perspectives, which permits to identify the following remarks.

From the private solution of the model, a null and a full-enrollment Nash equilibria were derived, where the relative cost for voter enrollment and appreciation for democracy constituted the key variables. These solutions are standard under a public good framework because they depend on how much the public good (democracy) is valued by agents (citizens). Nevertheless, a third kind of Nash equilibrium was obtained where there was a threshold for democracy valuation that encourages a free-rider behavior. As a consequence, free-rider behavior will be more likely if voter registration rate is close to one and will be discarded if it is near to zero.

From the social solution of the model, a policy analysis was made and a macro-level empirical model was obtained. The optimal policy analysis suggests to set a differentiated enrollment cost only if citizen's democracy appreciation is known by the policy maker. On the contrary, if democracy appreciation is unknown and the potential electorate is fairly large, then the optimal enrollment cost has to be close to zero in order to avoid a massive free-rider behavior that triggers a representativeness crisis.

On the other hand, the resulting empirical model is coherent with the classical literature on voting behavior by setting a structural relationship between voter participation and several social and economic variables.

Finally, the empirical model is the outcome from aggregating the optimal responses of rational citizens. Therefore, future empirical research that considers the empirical model outlined in this paper and uses aggregated electoral data will be well-specified in the sense of Lichtman (1974, p. 432).

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