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Mean-variance versus stochastic dominance: Consistency in investment performance indicators for the Chilean mutual funds market

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Abstract

In this paper we analyze the consistency of financial investment ordering based on mean-variance and stochastic dominance (SD) approaches in the context of an emerging financial market. We take 47 Chilean mutual funds and compute Sharpe index and the algorithms to verify first (FSD), second (SSD), and third degree (TSD) stochastic dominance relationships. We find evidence that both approaches generate similar sets of efficient investments. However, there are important dissimilarities between the rankings elaborated according to mean-variance and TSD criteria. TSD criterion presents itself as a complete method for evaluating the risk profile of an investment, as it takes into consideration risk-relevant characteristics of the return probability distribution that are not visible in mean-variance indicators.

JEL classification: G10, G11

Keywords: investment performance, mean-variance, risk, Sharpe index, stochastic dominance

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1. Introduction

Since its appearance, the stochastic dominance (SD) concept has been an attractive theoretical framework in financial performance analysis and risk evaluation. The above statement is based on the idea that the method considers the structure and behavior of the whole investment return distribution, and not only the first two moments, (i.e., mean and variance) like traditional financial indexes. Nevertheless, the practical use of this framework in financial analysis both academic and industrial has been very limited, due probably to the lack of simplicity of its interpretations and also the complexities of calculations.

On the other side, investment return mean-variance criteria, based on the efficient portfolio theory (Markowitz, 1952), are widely used in the construction of easy-to-read performance indicators. This approach, however, have been criticized for the assumptions imposed on the investor's utility function or the expected returns distribution. According to Tobin (cited by Hanoch and Levy, 1969), the mean-variance analysis of investment returns is valid only when the utility function is quadratic and the probability distribution of returns belongs to a "two-parameter family" (i.e., normal, log-normal). Aumann and Serrano (2008), in their new risk index proposal, criticize the mean-standard deviation indexes because they do not accomplish the monotonicity assumption (regarding to first-degree stochastic dominance) and are restricted to the normal games order.

The aim of this paper is compare the two traditional investment evaluation criteria. Specifically, the mean-variance approach and the stochastic dominance approach. We intend to determine if there are any differences between the efficient sets of investments provided by both approaches and if there are dissimilarities in the investment rankings. We also explore an explanation for these dissimilarities based on empirical findings.

The structure of the paper is as follows. Section II discusses the different views for risk evaluation and investment performance. Section III describes the methodology used in the empirical study. Section IV shows the results regarding the efficient sets obtained both from mean variance and stochastic dominance approaches. Section V discusses the dissimilarities in the investments rankings derived under both approaches. Finally, section VI concludes.

2. Criteria for risk evaluation and investment performance

Investment performance requires introducing the investment portfolio concept, a topic that has been covered by finance literature since the 1950's. Markowitz (1952) formulates the portfolio decision as a problem to be solved by an investor, who has to optimally allocate its resources in order to maximize the expected return of its portfolio subject to a particular risk, or minimize the portfolio risk subject to a given expected return. The solution of this problem determines the efficient frontier, which is a theoretical construct that includes every portfolio representing an optimal choice of return and risk (i.e., those that maximize the agent's profits).

This theoretical mean-variance approach characterizes the returns distribution based on its first two moments. Also, it considers the co-movements between several assets that conform the optimal portfolio. Moreover, the mean-variance approach gave birth to the so-called "index models", such as the one proposed by Sharpe (1963). In this kind of models, the asset return reacts to market fluctuations, which sensitivity is captured by the beta coefficient and has to be estimated from financial time series.[1]

After modeling a portfolio based on economic rationality criteria, the next step was evaluating its performance. This topic was widely developed in the 1960's, a prolific decade in proposals about criteria and techniques that facilitate such evaluation. In this field, we can remark Sharpe (1966) index, Treynor (1965) ratio, and Jensen (1968)'s alpha, a toolkit that is still used today by financial brokers.

In the 1960's, a new tool for performance evaluation was proposed by Treynor and Mazuy (1966). Under their methodological approach, the investor has to move between two characteristic lines[2] (a low volatility line and a high volatility line) in order to anticipate the market return, to adjust its portfolio, and to obtain an extraordinary revenue. However, the authors were unable to find evidence pointing to the investor's ability to anticipate market returns.

Another remarkable aspect in risky investment analysis is the possibility that investors might predict future returns. In that case, we can expect to find persistence in the asset performance. That would imply we are not observing an efficient asset market[3] (i.e., performance is not a random time variable). The search for persistence has encouraged the use of more sophisticated and robust techniques to evaluate asset performance.[4]

[1] Moreover, the mean-variance approach addresses the investor's problem as a one-period problem, which may seem unrealistic. Thus, theoretical developments proposed by Fama and French (1989), or Campbell and Shiller (1988), in the 1980's helped to define the analysis as multi-period, assuming that financial returns are period-by-period independent. However, it emerges the time dependency problem for asset returns and their variance (Elton and Gruber, 1997).

[2] The characteristic line of an asset is obtained plotting the set of ordered pairs conformed by the asset and market returns, which can be measured by some stock index, and then drawing a line between these points that represents the best fit. If the resulting line has some curvature, then we are observing an investor that has some knowledge about market timing.

[3] See Malkiel (2003) for a review of the main approaches on market efficiency.

[4] See Grinblatt and Titman (1989) for a discussion on the traditional methodologies developed in the 1970's that

During the 1970's, stochastic dominance (SD) appears as a new tool to evaluate investment performance. In simple words, stochastic dominance enables ordering two assets based on their financial return probability distribution. An asset A dominates an asset B if and only if the probability that asset A reports a return that is equal or less than x per cent is lower than the probability that asset B does it. Then, asset A is more attractive for the investor than asset B.[5]

Following this line of thought, Hanoch and Levy (1969) are the first to propose algorithms for checking the existence of stochastic dominance relationships in its first (FSD) and second degree (SSD); Levy (1973) expands the investment horizon and evaluates a set of efficient portfolios under a multi-period approach. The empirical test for these new approaches and their predecessors was performed by Porter and Gaumnitz (1972). They used monthly data of 140 shares for the period 1960-1963, and concluded that there were not significant differences between the mean-variance and SSD approaches when an investor constructs an efficient portfolio.

On the other hand, given the interest of private risk managers, new techniques for performance evaluation were proposed. Riskmetrics (Morgan and Reuters, 1996) introduced the concept of value at risk (VaR). Essentially, this technique requires calibrating the set of parameters included in a GARCH model in order to determine the maximum loss of an asset, conditional to its expected return. See Christoffersen et al. (2001) for a comparison of several VaR measures.

Finally, a number of empirical studies, mainly focusing on US financial markets, assessed the above techniques, such as Jensen (1968), Grinblatt et al. (1995), Chen and Knez (1996), Meyer et al. (2005).

The branch of the literature looking at Chilean financial market data has also addressed the evaluation of financial performance. However, this literature has mainly addressed mean-variance indicators (Maturana and Walker, 1999; Quezada et al., 2007), the use of VaR (Johnson, 2005), and the detection of persistence in financial returns (Umaña et al., 2008). Zurita and Jara (1999) mention stochastic dominance as an evaluation approach, but they do not apply it when evaluating the Chilean pension funds management companies (known as AFP) during the period 1987-1998.

were used to evaluate asset performance.

[5]See Levy (2006, Ch. 3) for more details about stochastic dominance degree.

3. Material and methods

In order to assess possible discrepancies between the mean-variance and stochastic dominance approaches and compare them in terms of the guidance they give to investors, we elaborated two performance rankings for a set of Chilean mutual funds (CMF) using a different criterion for each one.

The selection of mutual funds was based on a variety of diversification policies, foreign investment markets, and portfolio composition, which enabled us to capture the different aspects that determine its behavior. Thus, it is possible to maximize the comparison chances between several evaluation methods for this emerging market.

In order to protect data integrity, we selected only those mutual funds that exhibit a share quote for a large period. Therefore, out of a total 452 registered mutual funds, we chose only 47.[6] The sample period used in this empirical approach spans between June 2004 and March 2011.

Monthly real return series were constructed using the nominal return of each CMF, adju sted by the Chilean consumer price index (CPI) variation. 30-day yield from Chile's Central Bank bonds (known as PDBC) were included as a *proxy* risk-free asset return. The mutual funds were numbered from 1 to 47 and their ID are described in Appendix A.

The mean-variance criterion considered in the analysis is the Sharpe index (SI), which is computed for each CMF according to the following expression:

$$(1) \quad SI_i = \frac{E[r_i - r_f]}{\sigma_i}$$

Where SI_i is the Sharpe index for CMF i, $E[\cdot]$ the expected value operator, r_i the monthly real return of investment i, r_f the monthly real return of the *proxy* risk free asset, and σ_i the risk associated to investment i which is measured by its standard deviation.

The stochastic dominance criteria in their first (FSD), second (SSD), and third (TSD) degree were applied using the statistics already computed for each CMF real return (i.e., mean and variance). The results do not imply a complete ordering but a partial one (i.e., by groups), since the criteria may not deliver a categorical SD relationship between two assets (Zurita and Jara, 1999). Hence, we verified FSD, SSD, and TSD relationships between each possible pair of assets following the algorithms proposed by Levy (2006), which are described as follows.[7]

Let x and y be vectors of real returns, which probability density functions are F and G, respectively. Each vector has n elements, which are ordered from the lowest to the highest value in the

[6] The mutual funds data was extracted from the Chilean Securities and Insurance Superintendency (SVS) web site (http://www.svs.cl).

[7] Furthermore, see Meyer et al. (2005) and Porter et al. (1973)

following manner:

$x \epsilon F : x_1 \leq x_2 \leq \ldots \leq x_n$

$y \epsilon G : y_1 \leq y_2 \leq \ldots \leq y_n$

Then, assume that every asset return has a uniform distribution. Thus, every element of x (or y) has a probability of occurrence equal to $1/n$. In order to verify if distribution F dominates distribution G by FSD we have to jointly check the following conditions:

FSD condition 1: $x_i \geq y_i \; \forall i = 1, 2, \ldots, n$, and exists at least one strict inequality $x_m > y_m$ for some $m = 1, 2, \ldots, n$.

FSD condition 2: $x_1 \geq y_1$ (left-tail condition).

In order to verify if distribution F dominates distribution G by SSD we have to jointly check the following conditions:

SSD condition 1: $X_i \geq Y_i \; \forall i = 1, 2, \ldots, n$, and exists at least one strict inequality $X_m > Y_m$ for some $m = 1, 2, \ldots, n$. Where $X_i = \sum_{k=1}^{i} x_k$ and $Y_i = \sum_{k=1}^{i} y_k$.

SSD condition 2: $X_1 \geq Y_1$ (left-tail condition).

In order to verify the TSD criterion, let z be a vector that unifies the elements from x and y. Therefore, $z = z_1, z_2, \ldots, z_{n-1}, z_n, z_{n+1}, \ldots, z_{2n}$, where $z_k = x_i$ for some $i = 1, 2, \ldots, n$, or, $z_k = y_j$ for some $j = 1, 2, \ldots, n$. Finally, it is necessary to build the following piecewise functions for each pair of assets considering their respective cumulative density function:

$$(2) \quad F_2(x) = \int_{-\infty}^{x} F(u)du = \begin{cases} 0 & x \leq x_1 \\ \frac{k}{n}x - \frac{1}{n}\left(\sum_{i=1}^{k} x_i\right) & x_k \leq x \leq x_{k+1} \text{ for } 1 \leq k \leq n-1 \\ x - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) & x \geq x_n \end{cases}$$

$$(3) \quad F_3(x) = \int_{-\infty}^{x} F_2(u)du = \begin{cases} 0 & x \leq x_1 \\ \frac{1}{2n}(x - x_1)^2 & x_1 \leq x \leq x_2 \\ \vdots & \\ F_3(x_k) + \frac{k}{2n}(x^2 - x_k^2) - \frac{1}{n}\left(\sum_{i=1}^{k} x_i\right)(x - x_k) & x_k \leq x \leq x_{k+1} \\ \vdots & \\ F_3(x_n) + \frac{1}{2}(x^2 - x_n^2) - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)(x - x_n) & x \geq x_n \end{cases}$$

Using the above expressions we have to compute $F_2(x_i)$ and $G_2(y_i)$ for $i = 1, 2, \ldots, n$, jointly with $F_3(z_k)$ and $G_3(z_k)$ for $k = 1, 2, \ldots, 2n$. Once done, it is possible to verify if distribution F dominates distribution G by TSD just checking the following conditions:

TSD condition 1: To verify if $min(F) \geq min(G)$

TSD condition 2: To compute $H(z_k) = G_3(z_k) - F_3(z_k)$ for all $k = 1, 2, \ldots, 2n$ and verify if $H(z_k) \geq 0$ for all k.

Finally, we note that the degree of stochastic dominance between F and G (or x and y) do not alter the results obtained in the immediately previous degree. That is, if F dominates G by FSD, then it will also dominates by SSD and TSD. However, when the SD degree is increased, it is possible that appears a new SD relationship, which was not identified in the previous degree. Therefore, we present only the results related to the third-degree stochastic dominance criterion.

4. Efficient portfolios according to mean-variance and TSD criteria

An efficient set of portfolios can be seen as a number of investments upon which no other investment is preferred according to certain criterion. Two efficient sets were constructed for both mean-variance (Markowitz, 1952) and stochastic dominance (SD) criteria, considering first (FSD), second (SSD), and third degree (TSD).

In order to see this mean-variance "efficient frontier" we plot all the points of mean-standard deviation combinations for the 47 mutual funds (CMF) in our sample, along with the risk free proxy investment (PDBC) (see figure 1).

The plot shows several CMF investment options falling below the zero-return line (i.e., average/expected returns are negative), therefore, their SI values will also be negative. Rankings based on negative values of Sharpe index should be assessed carefully, especially when prices show high volatility (Meyer et al., 2005). This has been the case in this particular sample period, when investments were shocked by international financial turbulences following the global financial crisis in 2008.

Starting from the return-volatility plot, we chose a few "efficient" portfolios among those securities showing the best return for each significant volatility level (the upper contour of the plot showed in figure 1). Every linear combination of these investments (i.e., a portfolio containing them) will have an expected return equal or greater than the expected return of any single security for each possible volatility level for the portfolio. The result is an efficient subset of four dominant CMF which are listed in table 1.

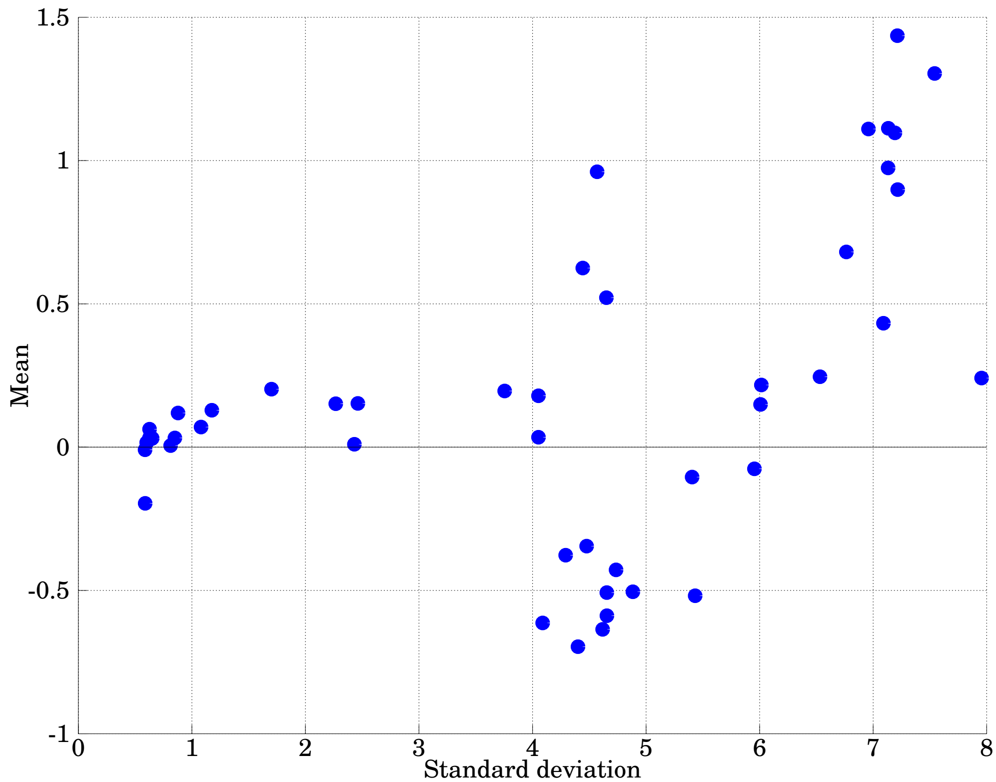
Table 1: Investments included in efficient set according to mean-variance criterion

No.	CMF	Mean	Sd. Deviation	Sharpe index	SI ranking	SI percentile
37	8100I	0.063%	0.627	0.074	15th	68.1%
34	8141A	0.119%	0.878	0.117	10th	78.7%
4	8076EJ	0.961%	4.569	0.207	1st	97.9%
26	8098B	1.436%	7.213	0.197	2nd	95.7%

Note that the proxy risk free asset (PDBC) is not included in this group, although it is in the region of the plot where is expected to be (low volatility, expected return close to zero). Nevertheless, apparently is dominated by other securities with the same low volatility but higher expected/average returns.

The efficient set according to third degree stochastic dominance criterion (TSD) has been selected once the SD algorithms described in the previous sub-section have been run for every pair of securities in the sample. The result is a matrix that shows the found/not-found third order stochastic dominance relation for each pair of investments. From this matrix we selected those

Figure 1: Standard deviation and mean return for the CMF included in the sample



securities that are not TSD dominated by another security. The set is shown in table 2.

Comparing both mean-variance and TSD criteria depicted in tables 1 and 2, we note they generate a very similar set of "efficient" investments, including in their sets funds number 4, 34 y 37. This result suggests consistency between the approaches. However, we also note that fund number 26 is present in mean-variance efficient set, but is not in TSD's. Fund 26 is the security with the highest volatility and expected return of the mean-variance efficient set.

Table 2: Investments included in efficient set according to third degree stochastic dominance criterion

No.	CMF	ND+ TSD	ND- TSD	TSD ranking
37	8100I	45	0	1^{st}
34	8141A	36	0	2^{nd}
4	8076EJ	28	0	3^{rd}

ND+: No. of investments dominated by the security

ND-: No. of investments that dominate the security

5. Investments rankings according to mean-variance and TSD criteria.

As stated earlier, empirical analysis in this article consists on contrasting performance measures for the Chilean mutual fund market, both from the mean-variance and stochastic dominance perspective. In order to do this, we constructed rankings based upon performance indicators.

For the mean-variance approach, we used the Sharpe index (SI). Securities were ranked according to their SI value, in strict magnitude order. The bigger the SI value, the upper its (ordinal) position in the ranking. Table 3 shows the top 20 securities in the SI ranking.

Table 3: Sharpe index ranking

No.	CMF	SI	SI ranking	No.	CMF	SI	SI ranking
4	8076EJ	0.207	1$^{\text{st}}$	45	8245A	0.110	11$^{\text{th}}$
26	8098B	0.197	2$^{\text{nd}}$	3	8030A	0.109	12$^{\text{th}}$
27	8160EJ	0.171	3$^{\text{rd}}$	18	8247A	0.098	13$^{\text{th}}$
14	8206A	0.157	4$^{\text{th}}$	29	8119A	0.096	14$^{\text{th}}$
15	8086A	0.154	5$^{\text{th}}$	37	8100I	0.074	15$^{\text{th}}$
25	8098A	0.150	6$^{\text{th}}$	42	8032A	0.060	16$^{\text{th}}$
1	8278A	0.137	7$^{\text{th}}$	22	8054A	0.059	17$^{\text{th}}$
23	8136A	0.134	8$^{\text{th}}$	2	8290A	0.055	18$^{\text{th}}$
24	8133A	0.122	9$^{\text{th}}$	33	8287PE	0.050	19$^{\text{th}}$
34	8141A	0.117	10$^{\text{th}}$	7	8252C	0.048	20$^{\text{th}}$

As for the case of stochastic dominance approach, the ranking was constructed following a two-step ordering procedure. First, we ordered investments according to the number of funds that dominate that particular investment (ND-). The lower this number the higher is the position of the investment in the ranking (see table 4). In the case that $ND-$ is the same for a group of assets then we order them according to the number of investments dominated by each asset (ND+).

In order to compare the resulting SI and TSD rankings, we analyzed the percentile distribution for each fund in both rankings, extracting a number of securities with strikingly different positions. Table 5 shows statistics and indicators for those securities. The last two columns show the relative position for each investment according to mean-variance (SI) and stochastic dominance (TSD) criteria. This table also include risk free asset investment No. 48.[8]

Note that investments 14, 15, 25, 26 and 27 are in high (ordinal) position in SI ranking (respectively 4th, 5th, 6th, 2nd and 3rd) and low (ordinal) position in TSD ranking (27th, 30th, 31st, 25th and 41st). It is particularly striking the case of fund No. 27, third place of the Sharpe index and 41st in TSD. Similarly, we can identify another group composed of funds No. 29, 34, 35, 37 and 48 that are in high positions in TSD ranking (8th, 2nd, 10th, 1st and 5th, respectively) and

[8] See Appendix B for the SI and TSD relationships computed for the whole sample.

Table 4: Third degree stochastic dominance ranking

No.	ND+ TSD	ND- TSD	TSD ranking	No.	ND+ TSD	ND- TSD	TSD ranking
37	45	0	1^{st}	41	31	6	11^{th}
34	36	0	2^{nd}	31	35	7	12^{th}
4	28	0	3^{rd}	30	34	7	13^{th}
36	41	1	4^{th}	33	34	10	14^{th}
48	39	1	5^{th}	45	33	12	15^{th}
40	38	1	6^{th}	2	31	14	16^{th}
38	40	2	7^{th}	42	31	14	16^{th}
29	35	2	8^{th}	1	28	15	18^{th}
39	37	3	9^{th}	6	28	16	19^{th}
35	38	4	10^{th}	7	24	17	20^{th}

low (ordinal) positions in Sharpe Index (14th, 10th, 37th, 15th and 31st). We call the first group (high in SI and low in TSD) group A, and the second (low in SI, and high in TSD) as group B.

Table 5: Statistics and indicators for those CMF with dissimilar position in TSD and SI rankings

No.	CMF	Mean	Sd. deviation	SI	ND+	ND-	TSD ranking	SI ranking
4	8076EJ	0.9609	4.5686	0.207	28	0	3^{rd}	1^{st}
14	8206A	1.1100	6.9590	0.157	9	22	27^{th}	4^{th}
15	8086A	1.1125	7.1336	0.154	6	24	30^{th}	5^{th}
25	8098A	1.0965	7.1922	0.150	5	25	31^{st}	6^{th}
26	8098B	1.4356	7.2132	0.197	9	17	25^{th}	2^{nd}
27	8160EJ	1.3039	7.5415	0.171	1	23	41^{st}	3^{rd}
29	8119A	0.1288	1.1756	0.096	35	2	8^{th}	14^{th}
34	8141A	0.1191	0.8780	0.117	36	0	2^{nd}	10^{th}
35	8187A	-0.0091	0.5877	-0.043	38	4	10^{th}	37^{th}
37	8100I	0.0625	0.6269	0.074	45	0	1^{st}	15^{th}
45	8245A	0.2025	1.7010	0.110	33	12	15^{th}	11^{th}
48	PDBC30	0.0161	0.6014	0.000	39	1	5^{th}	31^{st}

In order to illustrate only these two groups we show in figure 2 the scatter plot for their mean return and standard deviation (volatility). We can see that group A is located in the area of high volatility and high expected return. On the contrary, group B is located in the area of low volatility and low expected return. We can also note from table 5 that each of the investments in group B dominates under TSD criterion the investments in group A. This domination can be illustrated in figure 3, which shows the cumulative distribution function for each fund of these two groups.

In order to characterize both groups of investments, we decomposed the first four moments of the return distribution for each fund. The information is compiled and shown in table 6. Additionally, figure 4 presents histograms for the investment returns for dissimilar groups A and B.

Clearly, group A outperforms group B in terms of expected return. On the other side, group B presents lower return variability. Taking both moments jointly, as in the Sharpe index or the

Figure 2: Mean return and standard deviation for groups of investments with dissimilar position in SI and TSD rankings

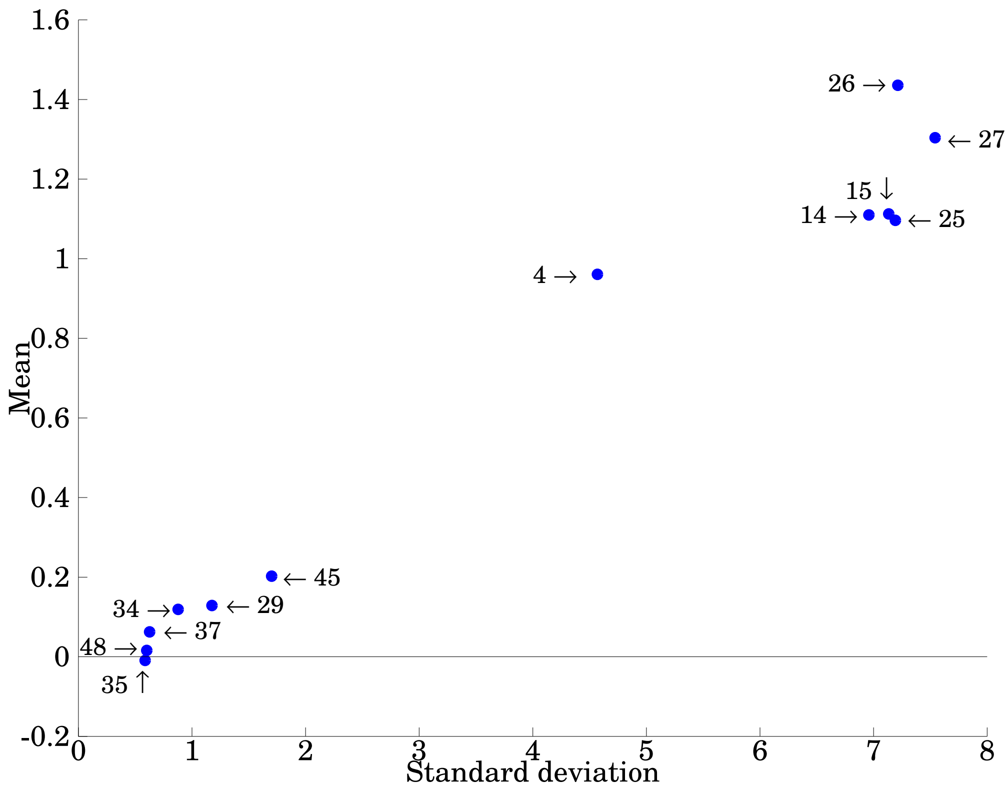


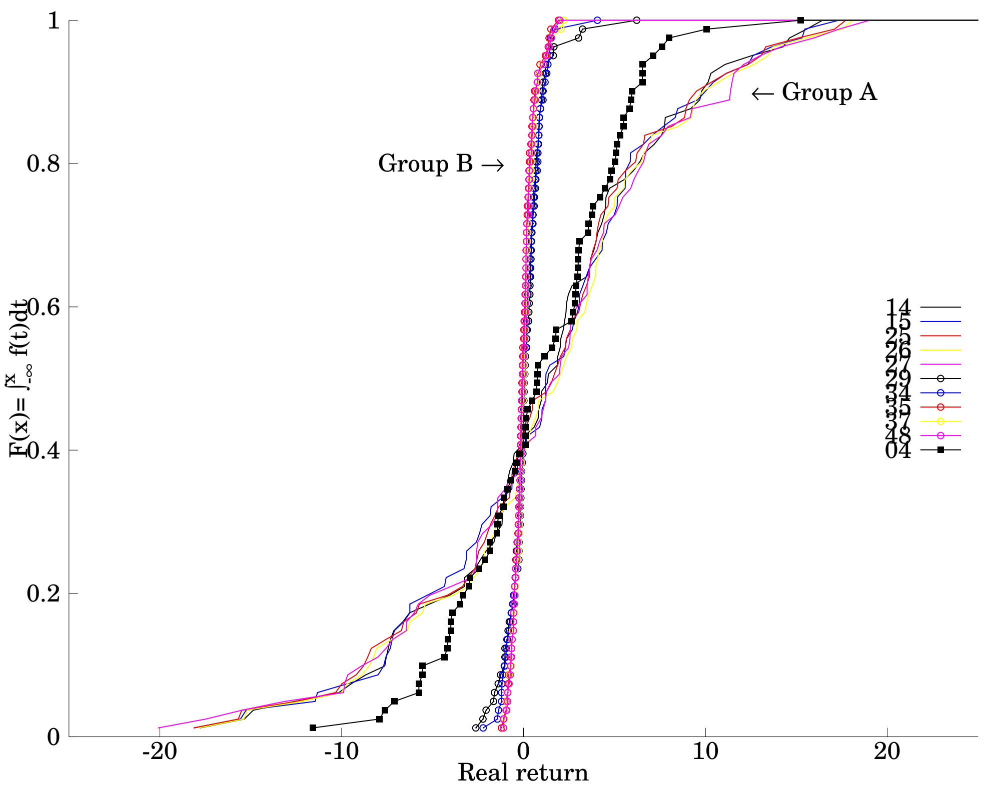
Table 6: First four sample moments for discrepant investments

Group A: High SI, low TSD						Group B: Low SI, high TSD					
No.	CMF	μ_i	σ_i	Sk_i	κ_i	No.	CMF	μ_i	σ_i	Sk_i	κ_i
14	8206A	1,1100	6,9590	-0,2576	0,1880	29	8119A	0,1288	1,1756	1,7028	8,0682
15	8086A	1,1125	7,1336	-0,2369	0,0976	34	8141A	0,1191	0,8780	0,8227	3,8005
25	8098A	1,0965	7,1922	-0,2224	0,2176	35	8187A	-0,0091	0,5877	0,7342	1,0540
26	8098B	1,4356	7,2132	-0,2203	0,2178	37	8100I	0,0625	0,6269	0,9503	1,7176
27	8160EJ	1,3039	7,5415	-0,2686	0,3551	48	PDBC30	0,0161	0,6014	0,8745	1,2364

μ_i: Mean σ_i: Sd. deviation Sk_i: Skewness κ_i: Kurtosis

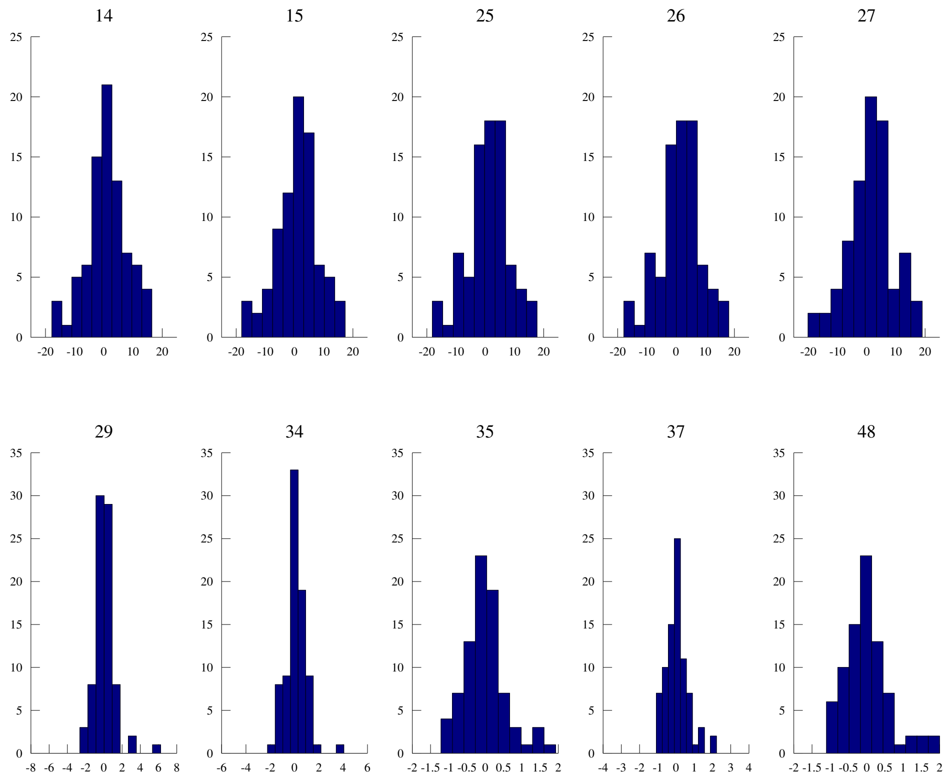
coefficient of variation, Group A outperforms B due to its lower relative dispersion. Nevertheless, when we include higher moments of distribution, we observe new elements for comparison between both groups (see table 6). Firstly, in terms of asymmetry, we observe that return distribution for group A has a negative coefficient of asymmetry around -0.24 (bias to the left towards the negative return zone). And secondly, investments in group B present positive asymmetry coefficient of around 1.02, i.e., a right-hand bias towards positive return zone.

Figure 3: Cumulative distribution function ($F(x)$) for discrepant investments



As for the forth moment, return distribution for group A has a kurtosis coefficient significantly lower than that of group B (ten times lower or more). This indicates a greater density in the tails of the distribution. If we evaluate jointly these third and fourth moments, group A investments are even riskier than what suggested by its volatility, or by the Sharpe index. In addition to its high volatility, the distribution is biased towards losses and the tails are bigger. Empirically, TSD appears to be capturing better the aspects of the return distribution associated with risk that are not present in the two-moments-only approach, and that is why group A does not perform well in TSD.

Figure 4: Histograms for discrepant investments



6. Concluding remarks

The empirical facts presented in this paper are focused on contrasting two different approaches for analysis, measurement and ordering of risk and investments performance in an emerging market context. Mean-variance approach, originated in the classic portfolio optimization theory, has generated popular indicators that use first and second moment statistics for the returns probability distribution. On the other side, stochastic dominance approach compares whole probability distributions between a pair of investments. From this comparison we can extract the following conclusions.

Both mean-variance and stochastic dominance generate a similar set of efficient investments. However, third order stochastic dominance criterion does not include high volatility investments in this category.

There exist important dissimilarities in the ordering position for some investments in rankings elaborated under mean-variance and those elaborated under TSD (third order stochastic dominance). Therefore, the two approaches do not generate a consistent criterion for evaluating and ordering investments profiles. The analysis of this dissimilarity suggests that TSD penalizes investments with high return volatility that also present negative asymmetry and low kurtosis.

TSD criterion presents itself as a complete method for evaluating the risk profile of an investment, as it takes into consideration risk relevant characteristics of the return probability distribution that are not visible in mean-variance indicators.

Finally, we see new topics for further research, pointing mainly to perfecting the methods for generating more precise quantitative indicators of risk that comprise more features of the returns probability distribution, just as the TSD criterion does.

A Mutual funds sample used in the empirical analysis.

No.	Manager	Mutual fund name	Code
1	Euroamerica	Ventaja Local	8278A
2	Itaú	Itaú Mix	8290A
3	Corpcapital	Corp Acciones	8030A
4	Santander	Acciones Chilenas	8076EJ
5	Corpcapital	Corp USA	8233A
6	Cruz del Sur	Diversificacion	8298AF
7	Principal	Lifetime 2030	8252C
8	Banchile	Europe Fund	8129A
9	Santander	Santander Europeo	8158NOEJ
10	Santander	Global Desarrollado	8090EJ
11	Itaú	Itaú World Equity	8237A
12	Banchile	USA Accionario	8189A
13	BICE	Best Asia	8178A
14	Celfin	Acc. Latinoamericana	8206A
15	Banchile	Latina Accionario	8086A
16	Principal	USA	8113C
17	Larrain Vial (ex Consorcio)	Emerging Equity	8198A
18	Euroamerica	Euroamerica Capital	8247A
19	Santander	Multinac. Emergente	8058UNEJ
20	Santander	Asiático	8158EJ
21	Santander	Asiatico	8159APV
22	Banchile	Emerging Fund	8054A
23	Banchile	Latin America Fund	8136A
24	Corpcapital	Emerging Markets	8133A
25	Principal	Andes	8098A
26	Principal	Andes	8098B
27	Santander	Latinoamericano	8160EJ
28	BBVA	Renta Mixta 50	8116A
29	Corpcapital	Más Futuro	8119A
30	BICE	BICE Beneficio	8029A
31	BBVA	BBVA Familia	8106A
32	BBVA	BBVA Familia	8106E
33	Santander	Bonos y Letras	8287PE
34	BICE	BICE Extra	8141A
35	Scotia	Proximidad	8187A
36	BICE	BICE Manager	8100A
37	BICE	BICE Manager	8100I
38	Scotia	Prioridad	8255A
39	Scotia	Prioridad	8255B
40	Banchile	Liquidez 2000	8115U
41	Banchile	Euro Money Market	8272U
42	BICE	Target	8032A
43	LarrainVial	Multi Estrategico	8303A
44	LarrainVial	Multi Estrategico	8303E
45	LarrainVial	Portfolio Lider	8245A
46	LarrainVial	Global Equity	8173A
47	LarrainVial	Global Equity	8173E

B Sharpe index and third-degree stochastic dominance relationships by asset included in the sample.

No.	Mutual fund code	SI	Times dominated by TSD	Times that dominates by TSD
1	8278A	0,137	15	28
2	8290A	0,055	14	31
3	8030A	0,109	19	24
4	8076EJ	0,207	0	28
5	8233A	-0,141	27	2
6	8298AF	-0,003	16	28
7	8252C	0,048	17	24
8	8129A	-0,098	31	1
9	8158NOEJ	-0,022	24	7
10	8090EJ	-0,081	24	10
11	8237A	-0,094	25	4
12	8189A	-0,162	24	4
13	8178A	-0,015	30	1
14	8206A	0,157	22	9
15	8086A	0,154	24	6
16	8113C	-0,092	23	14
17	8198A	0,035	31	1
18	8247A	0,098	25	3
19	8058UNEJ	0,028	46	0
20	8158EJ	0,022	27	2
21	8159APV	0,033	25	3
22	8054A	0,059	32	1
23	8136A	0,134	27	1
24	8133A	0,122	27	2
25	8098A	0,150	25	5
26	8098B	0,197	17	9
27	8160EJ	0,171	23	1
28	8116A	-0,154	25	2
29	8119A	0,096	2	35
30	8029A	-0,013	7	34
31	8106A	0,019	7	35
32	8106E	-0,107	30	0
33	8287PE	0,050	10	34
34	8141A	0,117	0	36
35	8187A	-0,043	4	38
36	8100A	0,027	1	41
37	8100I	0,074	0	45
38	8255A	0,017	2	40
39	8255B	0,023	3	7
40	8115U	0,022	1	38
41	8272U	-0,361	6	31
42	8032A	0,060	14	31
43	8303A	0,005	20	20
44	8303E	0,040	19	24
45	8245A	0,110	12	33
46	8173A	-0,130	24	3
47	8173E	-0,113	23	6
48	PDBC30	0,000	1	39

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