Essays on Nonlinear Pricing and Welfare

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Chapter 1

An Introduction to Nonlinear Pricing and Welfare

1.1 Introduction and Outline of the Chapter

Problems associated with monopoly power have received considerable attention in economic literature. It is well known that a firm exercising monopoly power over a given market can raise its price above the competitive price (i.e. the marginal cost). This leads to a dead-weight welfare loss for society. In order to reduce the welfare loss that is caused by the pricing behavior of the monopoly, several alternative suggestions have been presented in the literature. One recommendation is to regulate the monopoly in the sense that a social planner (regulator) specifies the price that the firm is allowed to charge the consumers. A second proposal is to let the government finance the deficit that is associated with marginal cost pricing through taxation. These dividing lines are discussed in Section 1.2, where we start by investigating under what circumstances a (natural) monopoly arises.

One of the main conclusions in Section 1.2 is that the welfare loss can be reduced by allowing the monopoly to charge the consumers a nonlinear tariff. However, there are other arguments in favor of nonlinear pricing. The two main motivations are: nonlinear tariffs typically Pareto dominate linear pricing rules and nonlinear pricing can be used as an instrument to redistribute income among consumers. Hence, nonlinear pricing can be motivated both by efficiency arguments and by redistributive objectives. Nonlinear
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Pricing and related topics are discussed in Section 1.3. In order to investigate and analyze nonlinear pricing schedules, a theoretical model is required. In Section 1.4, we introduce the "standard" framework for analyzing nonlinear pricing schedules in a finite economy with a (single product) natural monopoly. We state, discuss and motivate the "standard" assumptions in the literature. This framework forms the base on which this thesis is built. In Section 1.5, we specify the purpose of this thesis and summarize the main findings.

Note, finally, that this introductory chapter should by no means be regarded as a complete introduction to the natural monopoly and the nonlinear pricing literature. Many interesting contributions have not been included and some important topics will only be discussed briefly in footnotes. In the latter case, we refer to at least one seminal article or a general survey of the topic without going into details. The material in this chapter borrows from the work of Baumol et al. (1982), Braeutigam (1989), Tirole (1988), Varian (1989) and Wilson (1993).

1.2 The Natural Monopoly

In this section, we describe circumstances under which a natural monopoly arises. We also state some of the problems that are associated with monopoly power and briefly discuss a few proposals that have been suggested in order to, at least partially, overcome these problems.

1.2.1 The Definition of a Natural Monopoly

In the traditional view, a natural monopoly arises when there are increasing returns to scale or, more generally, non-convexities in the production set of the firm.\footnote{For a thorough description of these concepts see e.g. Kahn (1971) or Quinzii (1992) among others.} If the firm produces a single product\footnote{In this chapter, as well as in the rest of the thesis, we shall only consider a single product natural monopoly. For the multiproduct case, see e.g. Baumol (1977) or Baumol et al. (1982).} and faces constant factor prices, the notation of economies of scale means that the average cost schedule for the firm declines as market output increases. In this case, the market is said to be characterized by a natural monopoly since competition within the market is not possible. The traditional definition of a natural monopoly has been challenged by a definition that rests on the concept of subadditivity.
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of the cost function, see e.g. Baumol et al. (1977,1982) or Sharkey (1982). Subadditivity of the cost function refers to a situation where one firm can produce the outputs at least as cheaply as two separate firms. The definition of a natural monopoly then becomes (Baumol et al., 1982, p.17):

"An industry is said to be a natural monopoly if, over the entire relevant range of outputs, the firms’ cost function is subadditive".

These two definitions are, of course, related to each other but they are not identical. If, for example, a firm produces a single good, faces constant factor prices and there are (global) economies of scale, the average cost is (globally) declining and, as a consequence, the cost function is subadditive. Hence, (global) increasing returns to scale imply subadditivity in the single product case.\(^3\) The converse is, however, false. To see this, consider the following example that we have illustrated in Figure 1.1.\(^4\) Assume that a single firm produces a single good and that all firms that might like to supply the good on the market have identical cost functions. Suppose also that all consumers face the same marginal price, \(p\), and that the firm receives no subsidy from the government, i.e. total cost must be covered solely by the price that the consumers are charged. The aggregated demand curve \(D(q)\) intersects the average cost curve \(AC(q)\) when consumption is \(q^0\). The output level that is associated with minimum average cost \(q^1\) is, however, smaller than \(q^0\). Hence, there are only economies of scale in the range of output \(0 < q < q^1\). The cost function is, however, subadditive in the interval \(0 < q < q^0\) since it not is possible for two separate firms to produce output \(q^0\) at a lower price (recall that all firms have identical cost functions). Thus, subadditivity does not imply global economies of scale.

By applying the above arguments, we conclude that in the case of a single product monopoly that faces constant factor prices, the subadditivity definition of a natural monopoly is more appropriate than the traditional definition since a monopoly does not need to exhibit economies of scale in the whole range of output and, moreover, global increasing returns imply subadditivity.

\(^3\)In the multiproduct case this need not to be true, see Baumol et al. (1982,pp.173) for an example.

\(^4\)The example is from Braeutigam (1989,pp.1295).
1.2.2 The Control of the Natural Monopoly

As established earlier in this chapter, there is a dead-weight welfare loss associated with the monopolist’s pricing behavior since a firm with monopoly power can raise its price above the competitive price. However, forcing the monopolist firm to sell its product at the marginal cost is not feasible due to negative profits. So, on the one hand, we have the problem of distortions due to the pricing behavior of the monopoly and, on the other hand, we have the feasibility (break-even) problem. Several different ways have been suggested in the literature in order to, at least partially, overcome both these problems. One stream of economists has advocated a marginal cost pricing rule, where the deficit is financed through taxation. An alternative suggestion is to regulate the monopoly in the sense that a social planner specifies the price that the firm is allowed to charge the consumers. In both cases, total revenue for the monopoly firm will be larger than or equal to total costs, i.e. the monopoly will make a non-negative profit. In this section, we discuss these two dividing lines in the literature and a few other interesting contributions that are related to auction theory.

For example Lerner (1937) and Hotelling (1938,1939) advocated marginal cost pricing
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in combination with taxation (the "MC rule", henceforth) for industries with increasing returns. The advantage of the MC rule is that the firm breaks-even and the socially optimal quantity is supplied on the market. This suggestion is, however, controversial and has been criticized on several grounds. One of the main criticisms, pointed out by Meade (1944), is that in the absence of a lump-sum tax, the social planner must impose a distortionary tax on the tax payers. Coase (1945,1946) has criticized the MC pricing rule from a different perspective. He argues that if the consumers are charged the marginal cost and if the deficit is financed by the government, then the government decides for the consumers that they are willing to finance the deficit. It is likely that the government lacks information of the social valuation of the good (i.e. the consumers' willingness-to-pay) and it is therefore possible that the MC rule is implemented, even though it may not be worth incurring the fixed cost for society.

As argued above, the obvious alternative to the MC rule is to allow the monopoly to operate on or above the average cost curve. In this case, negative profits are eliminated. However, in order to reduce the distortions that are caused by the pricing behavior of the monopoly, the monopoly must be regulated. If a system of regulation has been devised by the government (the social planner) for the natural monopoly, we refer to this system as a regulatory contract. This contract specifies the price that the firm is allowed to charge the consumers. The "optimal" design of the regulatory contract depends on several factors, among them the information that is available for the social planner, legal constraints and dynamics. Different types of regulatory contracts in various economic environments are investigated and analyzed in the magisterial book *A Theory of Incentives in Procurement and Regulation* written by J.J. Laffont and J. Tirole (1993).

If the monopoly is subject to regulation, the pricing rule is specified in the regulatory contract as stated above. One example of a pricing rule is the Ramsey-Boiteux rule, where the prices are given by the Ramsey-Boiteux formula, i.e. the price-marginal cost ratio of the good is inversely proportional to the elasticity of demand for the good. Depending on the elasticity of demand for the good, on the given market, prices will deviate from the socially preferred price. This enables the firm to break-even and the distortions are reduced at the same time. For a rigorous analysis, see for instance Vogelsang and

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5The Boiteux (1956) contribution is closely related to Ramsey's (1927) optimal tax problem, which explains the name.
Finsinger (1979), Brown and Sibley (1985) or Wilson (1993, ch. 5). A second example is the so-called two-part tariff, where the consumers pay a fixed charge and a constant marginal price. The pricing rules in both these examples have one thing in common: they are both nonlinear. Nonlinear tariffs are appealing for several reasons. One reason is the possibility for the monopoly to break-even. A second reason is that nonlinear tariffs typically Pareto dominate linear pricing rules (e.g. the average cost pricing rule). Section 1.3, as well as the whole of this thesis, is devoted to nonlinear pricing and we, therefore, content ourselves, for the moment, with the fact that nonlinear price schedules have some desirable characteristics, among them the possibility for the monopoly to break-even.

So far we have focused on a situation where there is no competition within the market due to the monopoly power. An alternative approach to the regulatory process is to investigate the possibilities for firms to compete for the market. Demsetz (1968) suggests in his, by now, classical article that even if competition within the market is not possible, competition can be introduced for the right to operate on the market by means of an auction. This requires that inputs are available at competitive prices and that the cost of collusion among bidding firms is “sufficiently high”. In a proposal that is related to the Demsetz suggestion, which applies in the case when the monopoly is publicly owned, Williamson (1976) suggests that the social planner can auction off the right to operate the publicly owned monopoly to one of several bidding firms. In the case where there are no sunk costs, Baumol et al. (1982) demonstrate that competition for the market can be introduced without the supervision of a social planner. In all the above cases, the competition forces the firms to reduce prices and, as a consequence, the dead-weight loss that is associated with monopoly power decreases. Note also that the firm will not compete for the market unless the profit is non-negative, which solves the feasibility problem.

For a thorough description of the various suggestions in this section, see for example Braeutigam (1989) or Viscusi et al. (2000).

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6For more on collusion, see e.g. Laffont (2000).
1.3 Nonlinear Pricing

As concluded in the previous section, nonlinear pricing can be motivated from a feasibility (i.e. break-even) perspective. There are, however, other arguments in favor of nonlinear pricing. In this section, we state a few of these. We also define the concept of nonlinear pricing and relate it to the concept of (second-degree) price discrimination.

1.3.1 What is Nonlinear Pricing?

The price of a good is said to be nonlinear if the unit price is not constant but depends on how much the consumer buys\(^7\), i.e. the producer sells different units of output at different prices, but every consumer who buys the same amount of the good pays the same price. This form of pricing is commonly used in many regulated\(^8\) industries such as the electricity, water and gas industries. To illustrate the principle, consider Table 1.1 where the price of water in the city of Lund, Sweden, is displayed for a sample of different quantities.\(^9\)

<table>
<thead>
<tr>
<th>Quantity (cubic metres)</th>
<th>Outlay (SEK)</th>
<th>Price per cubic metre</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>14000</td>
<td>14</td>
</tr>
<tr>
<td>2000</td>
<td>27375</td>
<td>13.68</td>
</tr>
<tr>
<td>3000</td>
<td>40750</td>
<td>13.58</td>
</tr>
<tr>
<td>4000</td>
<td>54125</td>
<td>13.53</td>
</tr>
</tbody>
</table>

By investigating the table, it is clear that the price per cubic metre depends on how much water that is purchased. Hence, the price of water is nonlinear in the above example. A nonlinear tariff or, equivalently, a nonlinear outlay schedule specifies the total charge

\(^7\)In this section, we focus on nonlinear prices that are dependent on the quantity purchased. For the case of quality-dependent nonlinear prices, the classical reference is Mussa and Rosen (1978).

\(^8\)Note here that nonlinear tariffs are common in many unregulated industries. Examples of this include the price of advertising in magazines that depends on the size of the advertisement, and railroad tariffs that specify charges based on e.g. weight, volume and distance.

\(^9\)Source: Lunds Kommun, Tekniska Förvaltningen VA-verket (www.lund.se). Prices include taxes and are valid for the year 2003 for water meters with a diameter of 20 millimetres.
payable by the consumers for their chosen consumptions of the good. The perhaps simplest example of a nonlinear tariff is the so-called *two-part tariff*, in which the consumer pays an initial fixed fee for the first unit plus a smaller constant price for each unit after the first. This is the case in the above example, where the consumers pay an access charge of 625 SEK and a constant unit price of 13.375 SEK per cubic metre of water, i.e. the tariff, $t$, is given by: $t = 625 + 13.375 \cdot q$, where $q$ represents the chosen consumption. Another example of a nonlinear tariff is the so-called *blocking-tariff*. In this case, the marginal prices of successive units decline in steps. If, for example, a motorist in Sweden (Denmark) takes the Öresund Bridge to Denmark (Sweden), the marginal price is dependent on how many times the motorist crosses the bridge. This is illustrated in Table 1.2.

<table>
<thead>
<tr>
<th>Quantity (number of crossings)</th>
<th>Outlay (SEK)</th>
<th>Price per crossing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>275</td>
<td>275</td>
</tr>
<tr>
<td>2</td>
<td>$275 \cdot 2 = 550$</td>
<td>275</td>
</tr>
<tr>
<td>$n &gt; 2$</td>
<td>$750 + 137 \cdot (n - 2)$</td>
<td>$\frac{476}{n} + 137$</td>
</tr>
</tbody>
</table>

Two-part tariffs and blocking-tariffs are just two examples of nonlinear tariffs. Obviously, there are several other nonlinear tariffs e.g. fixed-fee tariffs, three-part tariffs and Ramsey prices. The interested reader is referred to Wilson (1993) for a thorough overview. The various types of nonlinear tariffs do, however, have one thing in common: they can all be completely described by a *menu of consumption-outlay pairs*. A consumption-outlay pair or, equivalently, a *bundle* consists of a chosen consumption (e.g. 1000 cubic metres of water) and an outlay that is associated with the chosen consumption (e.g. 14000 SEK). The menu is a list of consumption-outlay pairs, which is offered to the consumers and every consumer who chooses to buy the good picks a preferred consumption and pays the associated charge. Since consumers typically have different valuations of the good, the menu consists of a number of differing bundles. Note, finally, that in the case when the firm is regulated, the social planner designs the menu that the firm is allowed to offer the consumers, see e.g. Laffont and Tirole (1993, pp.175).

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10Source: Öresundskonsortiet (www.oresundsbron.com). Prices are valid for the year 2003 (including taxes) when the motorist has a BroBizz, i.e. a small transmitter that automatically registers the journey at the payment station. The cost of the BroBizz is 200 SEK.
1.3.2 Price Discrimination versus Nonlinear Pricing

It is hard to come up with a satisfactory definition of price discrimination. Roughly, price discrimination is said to exist when the same physical good is sold to different consumers at different prices. This definition is, however, unsatisfactory for several reasons. For example, if a monopoly serves a geographic area (such as Sweden) and if the monopoly provides the transportation of the good, a uniform delivered price is discriminatory, whereas delivered prices that respond to transportation-cost differentials are not. Since there is no consensus in the literature on what price discrimination actually is, economists have come up with two competing definitions. The first is the price-cost-ratio (see e.g. Stigler, 1987) and the second is the so-called price-cost-margin (see e.g. Philips, 1983). Clerides (2001) compares these two definitions and concludes that “the margin definition is the more appropriate one to use in most situations and corresponds more closely to our usual notation of price discrimination” (ibid., p.1).

Following Pigou (1920), it is customary to distinguish between three types of price discrimination. First-degree (or perfect) price discrimination refers to a situation where the firm has perfect information about each consumer’s demand for the good and therefore each consumer is being charged his exact willingness-to-pay. Third-degree price discrimination refers to a situation where consumer characteristics such as age or geographical location are correlated with willingness-to-pay. In this case, discrimination is based on these characteristics. Examples of this include senior citizen discounts and differentiated prices of pharmaceuticals in geographically separated markets. What is of larger interest for this thesis is the so-called second-degree price discrimination\footnote{Second-degree price discrimination is also known as versioning, see Shapiro and Varian (1998) and Varian (2000).}, in which discrimination occurs on the basis of unobservable consumer characteristics, e.g. willingness-to-pay. The fact that consumer characteristics are unobservable does not mean that the good is sold at a single price on the market. Instead the firm offers the consumers a menu of consumption-outlay bundles to choose from. This schedule involves different unit prices for different amounts of the good purchased. Hence, nonlinear pricing is equivalent to second-degree price discrimination.

However, in order for second-degree price discrimination (in fact, any type of price discrimination) to be a viable pricing strategy for the firm, the following three conditions
must be satisfied:\footnote{See e.g. Varian (1989,p.599) or Wilson (1993,pp.10)}

- The firm must have monopoly power. If the market is competitive, the firm will have difficulties charging different prices for different consumers.

- The firm must possess information about different types of consumers and their demand curves. If this information is unknown, it is not possible to design consumer (alternatively, market or household) specific prices.

- The firm must be able to prevent resale of the good. If resale is possible, some consumers can profit from arbitrage.

If the above three conditions are satisfied, it is possible to design a menu of differing consumption-outlay bundles, i.e. nonlinear pricing is feasible. Note, finally, that there may be legal constraints that forbid the monopoly to discriminate between consumers. For a discussion of the legal aspects of price discrimination, see e.g. Neale and Goyder (1980), Varian (1989,pp.643) or Viscusi et al. (2000).

\subsection*{1.3.3 Motivations for Nonlinear Pricing}

In Section 1.2, we concluded that a monopoly can cover its cost by offering the consumers a nonlinear tariff. There are, however, other motivations in favor of nonlinear tariffs. The main objective of this section is to investigate two of these motivations. In particular, we argue that nonlinear tariffs typically Pareto dominate linear pricing rules and that nonlinear pricing can be used as an instrument to redistribute income among consumers.

In a well known article, Willing (1978) demonstrated that any uniform price not equal to marginal cost can be Pareto dominated by a nonlinear tariff (see also Seade, 1977). To see this, consider the following simple example.\footnote{A similar example can be found in Braeutigam (1989,pp.1329).} Suppose that there are two consumers in the economy, one with a low demand for the good, \( D_L \), and one with a high demand for the good, \( D_H \). The cost function is assumed to be affine, i.e. the production involves a fixed cost, \( F \), and a constant marginal cost, \( \beta \). Moreover, the monopoly is publicly owned.
Figure 1.2. A Pareto Improving Tariff

and restricted by a balanced-budget requirement\(^\text{14}\). Consider next Figure 1.2, where the lowest uniform price that allows the firm to break-even is given by \(p^0\). At this price, the profit is zero, implying that \(p^0\) is equal to the average cost and strictly larger than the marginal cost. Moreover, \(q_L^0\) and \(q_H^0\) are consumption for the two consumers, so the deadweight welfare loss for society is given by the area \(A + B\). Consider next a nonlinear tariff of the type:

\[
 t = \begin{cases} 
 p^0 q_i & \text{if } q_i \leq q_H^0 \\
 p^0 q_H^0 + \beta(q_i - q_H^0) & \text{if } q_i > q_H^0 
\end{cases}
\]

With this new schedule, consumer \(H\) increases consumption to the first-best level \(q_H^*\). Consumer \(L\) still consumes \(q_L^0\) units of the good since \(q_L^* < q_H^0\) by assumption (see Figure 1.2). Hence, the deadweight loss \(B\) is eliminated. Moreover, the above nonlinear schedule is Pareto improving since consumer \(H\) is strictly better off, consumer \(L\) is not worse-off and the firm still breaks-even. The main conclusion from the above example is that nonlinear pricing can be motivated from an efficiency perspective.

A second argument in favor of nonlinear pricing is that nonlinear schedules can be used as a tool to redistribute income among consumers. This point is also illustrated with the aid of a simple example. The example is based on the above premises and we shall also assume that the income elasticity is zero, i.e. that the preferences of the consumers can be represented by a quasi-linear utility function. Suppose first that the consumers are

\[^{14}\text{This is known as } \text{cost-of-service} \text{ pricing. For theoretical contributions, see e.g. Averch and Johnson (1962), Breyer (1982), Khan (1971) and Schmalensee (1979) among others.}\]
offered a uniform two-part tariff\textsuperscript{15} of the type \( t = A + pq_i \), where \( A \) is a fixed access charge and \( p \) is the marginal price. Since the firm is restricted by a balanced-budget requirement, we set \( p = \beta \) and \( A = \frac{F}{2} \). Facing the two-part tariff \( t = \frac{F}{2} + \beta q_i \), both consumers choose to consume the first-best quantity, \( q_i^* \).\textsuperscript{16} In Figure 1.3, consumer \( i \)'s indifference curves, \( \bar{u}_i \), are represented in the quantity-outlay space. The utility for consumer \( i \in \{L, H\} \) is increasing in the south-east direction. Note also that the indifference curves are parallel-shifted since the income elasticity is zero. Say now that the firm offers the consumers a nonlinear tariff of the type:

\[
t^d = \begin{cases} 
A_L + \beta q_i & \text{if } q_i \leq q_i^* \\
A_H + \beta q_i & \text{if } q_i > q_i^*
\end{cases}
\]

where \( A_L < A_H \) and \( \frac{A_L + A_H}{2} = \frac{F}{2} = A \). From the figure, it is clear that both consumers consume their first-best quantity. But in this case, consumer \( L \) ends up with a higher utility and consumer \( H \) with a lower utility. Hence, the price schedule redistributes

\textsuperscript{15}This is also known as Coase two-part pricing.

\textsuperscript{16}This rests on the assumption that the surplus for consumer \( L \) is so high that he can afford to pay half of the fixed cost.
income from consumer $H$ to consumer $L$. Note also that it is easy to construct tariffs that redistribute income from type $L$ to type $H$.\footnote{Again, this requires that net utility for type $L$ is non-negative.}

The above examples illustrate two appealing characteristics of nonlinear pricing. Namely, nonlinear pricing can be used as a tool to achieve efficiency, and nonlinear pricing can be used as an instrument to redistribute income.

\section{1.4 The Standard Framework}

The objective of this section is to introduce a framework for analyzing nonlinear outlay schedules. This framework is based on the theory of optimal nonlinear taxation, which was formulated first by Mirrlees (1971).\footnote{See also Mirrlees (1976). Alternative formulations are developed by e.g. Brito et al. (1990), Guesnerie and Seade (1982) and Roberts (1979).} Nonlinear pricing has, however, been investigated and analyzed in a large number of different economic environments, so in order to make our presentation of the "standard framework" manageable, we shall limit our description to a finite static economy with a single product natural monopoly.\footnote{Note that this does not mean that we will only refer to work that is based on a finite economy in the subsequent sections.} This is also the type of economy that is under consideration in Chapters 2-4 of this thesis. The reader who is interested in nonlinear pricing in an economy with a continuum of consumers is referred to Goldman et al. (1984), Littlechild (1975), Roberts (1979), Spence (1977) or Srinagesh (1986). Much of the work on dynamic nonlinear pricing is based on the seminal article by Coase (1972). Multiproduct nonlinear pricing is investigated by e.g. Armstrong (1996) and Spence (1980).

\subsection{1.4.1 The Consumers}

Consumers have preferences over consumption-outlay bundles $x_i = (q_i, t_i)$, where $q_i$ represents consumption and $t_i$ represents the outlay that is associated with consumption $q_i$. The economy is finite and the consumers are gathered in the set $\{1, ..., n\} = N$. This type of finite economy is considered in e.g. Guesnerie and Seade (1982) and Sharkey and Sibley (1993). Consumer preferences are represented by a utility function: $u_i(q_i, t_i)$ for
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$i \in N$. If we set $t_i$ equal to zero, it is natural to interpret $u_i(q_i, 0)$ as the consumer’s maximum willingness-to-pay for a purchase of size $q_i$. A standard assumption is that willingness-to-pay for the good is measured by the Marshallian consumer surplus. It is well known that the use of the (Marshallian) demand to measure the welfare change that is associated with a price change will be an exact measure if there are zero income effects. Willing (1976) demonstrated that even if there are non-zero income effects, the (Marshallian) consumer surplus may serve as a good measure to approximate the actual welfare change. This result is often used to motivate the assumption that preferences can be represented by a quasi-linear utility function, i.e. a utility function with zero income effects. Utility functions with non-zero income effects has been investigated by e.g. Guesnerie and Laffont (1984), Mirrlees (1976), Spence (1977) and Wilson (1993, ch.7).

Differences in willingness-to-pay between consumers are often supposed to captured with a single parameter, $\theta_i \in \mathbb{R}_{++}$. This assumption, in combination with quasi-linearity, enables us to write the net utility for consumer $i \in N$ as: $u(q_i, t, \theta_i) = \phi(q_i, \theta_i) - t_i$, where $\phi(q_i, \theta_i)$ represents the consumer’s maximum willingness-to-pay for a purchase of size $q_i$. One motivation for this representation of net utility is identical preferences for the good and net income, combined with variations in gross income. In this case, differences in $\theta_i$ are entirely due to differences in (gross) incomes. By normalization, a higher value of $\theta_i$ is associated with a higher willingness-to-pay. Since willingness-to-pay is increasing in $\theta_i$, it is natural to assume that the marginal utility of the good, for any given quantity, is also increasing in $\theta_i$. If this is the case, the so-called single-crossing condition \(^{21}\) is satisfied. The interpretation of this condition is that for the given quantity where the indifference curves of two types cross (e.g. $q^*_L$ in Figure 1.3), the indifference curve of the type with the highest willingness-to-pay is steeper. A second interpretation is that the demand curves of different consumers never cross. This condition enables us to sort the consumers in a natural way (recall from Section 1.3.2 that a necessary condition for nonlinear pricing is that it is possible to sort the consumers). It should be noted here that not all work in the literature is based on preferences that satisfy the single-crossing property, see e.g. Araujo.

\(^{20}\)This is for example the case in Cooper (1984), Goldman et al. (1984), Leyland and Meyer (1976), Maskin and Reily (1984), Spence (1977), Srinagesh (1986) and Thomas (2002).

\(^{21}\)This property is also known as the Spence-Mirrlees condition and is attributed to Mirrlees (1971) and Spence (1973).
and Moreira (2000) or Brito et al. (1990). We conclude that in a large proportion of the studies in the nonlinear pricing literature, consumers are supposed to be completely described by a single parameter and, moreover, the preferences of the consumers are assumed to be represented by a quasi-linear utility function, which satisfies the single-crossing property.

1.4.2 The Determination of the Optimal Outlay Schedule

Suppose that all nonlinear outlay schedules of type $x = \{x_i\}_{i \in N}$ are collected in the set $X$. Since this set typically consists of a large number of schedules, the problem is to identify the "optimal" schedule. In the "standard" framework, the optimal schedule is defined as the schedule that maximizes a weighted combination of consumers’ net utility, $u(x_i, \theta_i)$, and producer profit, $\pi(x)$. Hence, the optimal schedule is found by maximizing:

$$\max_{x \in X} \left\{ \sum_{i \in N} \alpha_i u(x_i, \theta_i) + \gamma \pi(x) \right\}. \quad (1.1)$$

In the above specification, the parameter $\alpha_i$ and $\gamma$ represents the welfare weights. The way in which "optimal" nonlinear price schedules vary, depending on these weights, has been investigated in the literature by e.g. Sharkey and Sibley (1993). Note also that by defining $\gamma = 1 + \lambda$, the parameter $\lambda > 0$ can be interpreted as the shadow cost of public funds, see e.g. Vogelsang (1990) or Laffont and Tirole (1993). There are three benchmark cases in the literature:

(i) The monopoly is privately owned and unregulated. In this case, all $\alpha_i$ are set to zero. This type of environment has been investigated by e.g. Cooper (1984), Maskin and Reily (1984) and Oi (1971).

(ii) The monopoly is privately owned and regulated. In this case $\alpha_i \geq 0$ and $\gamma \geq 0$. For
example, Laffont and Tirole (1993,ch.3) and Wilson (1993,ch.15) provide a model for analyzing nonlinear pricing schedules in this type of environment.

(iii) The monopoly is *publicly owned* and operates under a balanced-budget requirement. In this case, $\gamma$ is set to zero. Nonlinear outlay schedules have been analyzed in this type of environment by e.g. Laffont (1997) and Sharkey and Sibley (1993).

Note here that in Cases (ii) and (iii), a social planner determines the nonlinear schedule that the firm is allowed to offer the consumers. If the monopoly has compete information, it is a straightforward exercise to find the solution to the above maximization problem. It is, however, not unreasonable to assume that there are asymmetries of information in the economy. For example, a motorist (i.e. a consumer) in Sweden knows, at least approximately, how many times he is going to cross the Öresund Bridge between Sweden and Denmark during the next month. The ticket-collector (i.e. the monopolist), on the other hand, can not, by inspecting the motorist, find out the motorist’s demand for crossings over the bridge during the next month. Hence, the consumer knows something that the monopolist does not, so we have an asymmetry of information in the economy. It is, however, not unlikely that the ticket-collector has some information about the number of motorists that cross the bridge $y$ times every month, where $y$ is some non-negative integer, i.e. the ticket-collector has some information about the disaggregated demand for the number of crossings. Formally, this means that, if the consumers are characterized by a single parameter (as described above), consumer $i \in N$ knows his characteristic $\theta_i$ but the monopolist only knows the vector of characteristics $\theta = (\theta_1, \ldots, \theta_n)$.

Asymmetries of information can occur in a variety of different forms in the economy. In a nonlinear pricing environment, the above demand asymmetry is, however, always present. To see this, note first that, if there is an exogenous signal that reveals each consumer’s demand function, the monopolist will practice first-degree price discrimination (see Section 1.3.2). Moreover, if the monopolist does not know $\theta$ then it is not possible for the firm to sort the consumers and, therefore, it is not possible for the firm to design a nonlinear outlay schedule (again, see Section 1.3.2). Additional asymmetries of information may, however, be present in the economy. In Case (ii) above, it may well be the case that only the monopolist knows the true demand profile $\theta$ and that the social planner only knows that $\theta$ belongs to some set of demand profiles $\Theta$, see e.g. Lewis and
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Sappington (1988). It can also be the case that only the monopolist knows the true cost function and the social planner only knows that the cost function belongs to some class of cost functions, see e.g. Laffont and Tirole (1993, ch.3). In the most complicated scenario, the social planner neither knows the true demand profile nor the true cost function, see e.g. Laffont et al. (1987) or McAfee and McMillian (1988).

The presence of asymmetric information in the economy leads to that the above maximization problem (1.1) must be restricted by a set of incentive compatibility constraints. These constraints guarantee that the best choice for the agent that possesses the private information is to reveal the information to the uninformed principal. That is, if a consumer is free to choose among the (incentive compatible) quantity-outlay pairs in the menu, then his best choice is to pick the bundle that is directed to him. The requirement of incentive compatibility often results in marginal prices that are different from the marginal cost. This result can, for example, be found in all papers cited in Cases (i)-(iii) above. Note also that the number of relevant incentive compatibility constraints are drastically reduced if the single-crossing condition (see Section 1.4.1) is satisfied. In this case, consumer \( i \in N \) prefers the bundle that is designed for him over every bundle in the menu if he prefers the bundle that is intended for him over the bundles that are designed for his adjacent neighbors (i.e. type \( i-1 \) and type \( i+1 \)). This result is attributed to Cooper (1984).

In the search for the "optimal" nonlinear schedule, a set of additional restrictions must be imposed on top of the incentive compatibility constraints. One such restriction is the non-negative profit constraint. The reason for this is that if the monopoly is privately owned, as in Cases (i) and (ii) above, the firm will not supply the good on the market unless profits are non-negative. This constraint can also be motivated in Case (iii) when it not is possible or desirable to cover deficits through taxation.

In the process of finding the "optimal" nonlinear schedule, the reservation utility of the consumers must also be taken into account. The reason for this is that when a consumer faces the menu, he will only choose to purchase the good if there exists a quantity-outlay pair in the menu that gives him a non-negative net utility. Otherwise, he is better off by not consuming the good. As a consequence, individual rationality constraints must restrict the above maximization problem. These constraints guarantee that consumers receive a non-negative net utility from consumption. If the single crossing condition is
satisfied, it suffices that net utility for the consumer with the lowest valuation of the good is non-negative, since this implies that the other consumers in the economy obtain a positive net utility, see e.g. Cooper (1984). Laffont (1997) assumes that the utility from consumption is so high that the individual rationality constraints can be ignored.

We conclude that in the "standard framework", the "optimal" nonlinear outlay schedule is identified by maximizing a social welfare function that is represented by a weighted summation of consumers surplus and producer profit. This maximization problem is restricted by a set of incentive compatibility constraints, a non-negative profit constraint and a set of individual rationality constraints.

1.5 Summary of the Thesis

In this section we summarize the main findings of this thesis and state the main purpose of the work.

1.5.1 Aim and Scope

This thesis consists of three theoretical essays. The economy under consideration is finite and the natural monopoly is assumed to be publicly owned and restricted by a balanced-budget requirement. Given these premises, we characterize and analyze optimal nonlinear outlay schedules. As established earlier in this chapter, several factors are important in the process of finding the optimal outlay schedule. In this thesis, we investigate explicitly two of these factors. Namely:

(i) The set of restrictions that the bundles in the menu are constrained by, e.g. the incentive compatibility constraints and the non-negative profit constraint.

(ii) The distributional preferences of the social planner. These preferences can, for example, be captured by the selection of the welfare weights.

The first aim of this thesis is to characterize the set of optimal nonlinear outlay schedules for various sets of restrictions (Chapter 2 and the first half of Chapter 3). The second aim of this thesis is to investigate how the characteristics of the optimal nonlinear outlay schedule are affected by various distributional values (the second half of Chapter
Section 1.5  Summary of the Thesis

3 and Chapter 4). In both these respects, this thesis contributes to the existing nonlinear pricing literature.

1.5.2 The Model and Basic Assumptions in the Thesis

In this section, we specify the model that will be used throughout this thesis. We assume the following about consumers. (i) There is a finite set of consumers. (ii) Consumers have preferences over one perfectly divisible good and money. The preferences can be represented by a quasi-linear utility function. (iii) The marginal utility from consuming a given quantity of the good is increasing in type.

We assume the following about the natural monopoly. (i) The natural monopoly is publicly owned. (ii) The monopoly produces a single good with an affine technology. (iii) The social planner knows the distribution of consumer types but he is unable to tell consumers apart. (iv) The objective for the social planner is to design a nonlinear outlay schedule.

1.5.3 Overview of the Thesis

In the second chapter, Nonlinear Pricing under a Balanced-Budget Requirement, we consider a finite economy that is based on the premises in Section 1.5.2 with the additional assumption that the net utility from consumption is so high that we do not have to consider the individual rationality constraints (see also Laffont, 1997). In this economy, we characterize the set that consists of all envy-free (i.e. incentive compatible), budget-balanced and Pareto efficient nonlinear outlay schedules. In our characterization process, we develop a procedure that is based on the concept of fairness, which was first explored by Foley (1967). Hence, our approach is somewhat different from the "standard" procedure of maximizing a weighted linear-in-utility social welfare function (as described in Section 1.4.2) and has, to the best of my knowledge, not been presented in the literature before. Apart from introducing a new procedure to investigate and analyze nonlinear outlay schedules, we contribute to the existing literature in the sense that we provide a complete characterization of the set of envy-free, budget-balanced and Pareto efficient nonlinear pricing schedules in a finite economy that is based on the above premises. This characterization has not been presented earlier in the literature. Our results generalize
most of the findings in Sharkey and Sibley (1993) and the second-degree price discrimination results in Laffont (1997). We also reproduce some well known results contained in the literature.

In the third chapter, *Nonlinear Pricing as a Cooperative Game*, we investigate, as e.g. in Sorenson et al. (1976,1978) and Young (1998), nonlinear outlay schedules that are based on a cooperative surplus game with transferable utility. We first characterize the budget-balanced core of the cooperative pricing game and demonstrate that it is non-empty. This result is well known. The budget-balanced core consists of all budget-balanced nonlinear outlay schedules that satisfy the stand-alone principle, i.e. the realized net surplus for any consumer (or coalition of consumers) should not be lower than the consumer (or the coalition of consumers) can achieve on his (their) own. However, since there is an asymmetry of information in the economy (see Section 1.4.2), the schedules in the budget-balanced core need not be envy-free (i.e. incentive compatible). We, therefore, investigate the nonlinear schedules in the budget-balanced core that satisfy the envy-freeness criterion. A nonlinear schedule that belongs to this set is said to be in the envy-free core. We provide necessary and sufficient conditions for the envy-free core to be non-empty and demonstrate that, if the envy-free core is non-empty, it typically consists of a large number of nonlinear outlay schedules. Suppose, however, that the social planner would like to implement one specific schedule. For this purpose, we consider three solution concepts for the cooperative pricing game, each of them having the property that they single out a unique price schedule. The solution concepts under consideration are the Shapley value, the Lorentz criterion and the equal-share rule. These solution concepts reflect the social planners distributional preferences. Under a restrictive assumption, we demonstrate that the only one of the above three nonlinear schedules that is always in the envy-free core is the schedule that is based on the Shapley value. The chapter contributes to the existing literature in two senses. Firstly, we characterize the envy-free core and provide necessary and sufficient conditions for the envy-free core to be non-empty. Secondly, we implement nonlinear outlay schedules that are based on a cooperative surplus pricing game with transferable utility in a standard nonlinear pricing environment with asymmetric information. Our results generalize the work by Sorenson et al. (1978).

In the last chapter, *Nonlinear Pricing and Equality of Opportunity*, we implement the equality of opportunity (EOp, henceforth) criterion in a nonlinear pricing environment
that is based on the assumptions in Section 1.5.2, and investigate the welfare properties of the optimal nonlinear EOp outlay schedule. In our analysis, the maximin and the utilitarian nonlinear schedules serve as benchmarks. We demonstrate that the optimal EOp policy is a reasonable compromise between the optimal maximin and the optimal utilitarian policy in that the EOp policy is more (less) efficient, from a first-best perspective, than the maximin (utilitarian) policy and, at the same time, more (less) egalitarian than the utilitarian (maximin) policy. The main contribution to the literature is that we present a framework under which the EOp criterion (in the sense of Roemer, 1998) can be applied in order to derive optimal EOp nonlinear pricing schedules. As far as I know, the EOp criterion has never been considered in the nonlinear pricing literature before.

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