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Abstract:
Using an over-lapping generations (OLG) model, we show how small open economies can enhance their growth through educational subsidies financed via public debt and reduce their fertility rate. We show that subsidizing education through public debt leads to a Pareto improvement of all generations. Even if a country is a net borrower in the international capital market, we show that this subsidy-policy can help, under certain conditions, to improve its net borrowing position. Especially, our analysis can be applied to less-developed countries.

Keywords: economic growth; fertility; human capital; education subsidy; public debt.

JEL: E62, H23, O15, O41

1 Introduction

It is well-known in the economic literature that human capital accumulation creates positive externalities for future generations. This externality is due to the fact that parents who invest in the human capital of their children do not internalize the increase of the overall efficiency of the human capital accumulation process. Therefore, many economists (c.f. Peters (1995), Becker and Murphy (1988), and more recently Rangel (2003), Boldrin and Montes (2005), Kaganovich and Meier (2012)) interpret a pay-as-you-go pension system as a mechanism through which the intergenerational externality can be internalized. The basic idea in these models is simple; if a pay-as-you-go (PAYG) pension system exists then the pensions depend on the labor income of the working generation. The labor income depends on the human capital stock of the young generation and therefore on the investments of their parents in education. Consequently, the parents invest more in education, because they benefit from it through higher pensions.

However, the majority of papers, which focus on the question of how to internalize the externality created by human capital building assume a constant population or a constant fertility rate.

In contrast to these models, we endogenize the fertility behavior and instead of analyzing a pay-as-you-go system, we propose an education subsidy which is financed by a public debt to internalize the intertemporal externality.

Even though a public debt is very similar to a pay-as-you-go pension system, there are some differences which make an investigation worthwhile. The difference between a pension system and a public debt is that the introduction of pension system redistributes immediately income from the working population to the older non-working generation. Unfortunately, a PAYG system reduces the private savings. If the government sells government bonds on the international capital market or borrow alternatively from an institution like the World Bank or donor country in the case of a less-developed country (LDC), the country will experience an inflow of income from abroad. Additionally, in the case of LDCs, the introduction of a PAYG pension system seems to be unrealistic because of the existing public budget constraints in these countries. Moreover, even though the argument that parents benefit from educational investments in the presence of a PAYG pension system is absolutely correct from an economic point of view, it is not clear if this coherence is recognized by parents in reality. In contrast, a subsidy is obvious for everyone and maybe that is the reason, why Boldrin and Montes (2004, p.22) call the proposal '*in our view more compelling*', although that they did not work out this idea in their paper in detail. Furthermore, their model differs from the approach here also in some respects.

The literature on endogenous fertility and human capital in growth model goes back to Becker et al. (1990) and Becker and Barro (1988). Using an intertemporal utility function, they assume that that a higher fertility rate of the present generation raises the discount factor on per capita future consumption. Consequently, a higher fertility rate discourages investments in human and physical capital. On the other hand, higher stocks of physical and human capital (education) reduce the number of children because of the high cost of raising and caring for children.

One strand of the literature is based on the work of Kolmar (1997), who integrated only endogenous fertility behavior in a standard OLG framework. His idea was further extended by Fenge and Meier (2005, 2009) and Groezen et al. (2003). However, these scholars did not concentrate on public debts but on PAYG pension systems. They show that for a small country under certain conditions, it is possible to increase the fertility rate and the welfare simultaneously. They assumed that the pensions depend on the number of children and that the growth in population attracts higher capital inflows so that all individuals are better off. These results hold only for small open economies. On the contrary, Stauvermann et al. (2013) argues that, except for increase in fertility rates, these results do not hold in an OLG-model with endogenous growth because reduced savings tend to reduce the per-capita growth rates and hence the welfare of future generations. Furthermore, the authors show that an increase in fertility is accompanied by high costs for future generations, which further decrease the rate of capital accumulation, per capita growth and per capita capital, and hence the labor income.

Another strand of literature goes back to Bental (1989), Raut (1992), Cigno (1993), Stauvermann (1996), Zhang and Nishimura (1993), Zhang and Zhang (1995) and Zhang (1995). These authors interpret children as insurance for the old age. In their models, the introduction of a PAYG pension scheme decreases the importance of children and therefore the number of them declines. As a consequence, the per capita income growth rises and the fertility-reducing effect of a PAYG pension system offset the savings-reducing effect, which is usually the outcome of the introduction of a pay-as-you-go pension system.

Moreover, Zhang (1997, 2003, and 2006) and Li and Zhang (2008) provide another dimension on fertility and human capital accumulation. Their main assumptions are that individuals are perfectly altruistic in the sense of Barro (1974) with respect to their offspring and that the human capital accumulation is associated with external economies of scale. The

latter idea introduced by Lucas (1988) show that the private rate of return of human capital building is lower than the social rate of return. Subsequently, a government subsidy as an incentive to invest in human capital is able to equalize the private and social rate of return.

Although we acknowledge the role of external economies of scale, we make our arguments and the model even stronger without explicitly factoring it. Notably, all models which include human capital accumulation, but not perfect altruism in the sense of Barro (1974) exhibit intergenerational externalities because parents, who invest in the human capital of their off-spring do not take into account the positive effects of their investments for their grandchildren and great-grandchildren.

Another approach was developed by Wigger (2001), who argues that an education subsidy financed by a lump-sum tax can be welfare-enhancing in a closed economy with a constant population, if the cross derivative of the production function between human capital and physical capital is positive and sufficiently large.

As many others in the literature, we assume that parents derive a benefit from investing in the number of children and from investments in their children's education. In such a framework, we want to show that it is possible to internalize the intergenerational externality in a small open economy with the help of a public debt without harming any generation. We argue that the generation that gains from a human capital investment (children) should pay for it instead of putting the entire burden of responsibility on the current generation (parents). Under these conditions, we show that it is possible to increase the growth of the economy by decreasing the number of children and increasing the growth rate of human capital accumulation. Probably, the model which is most closest to ours, is the model of Fanti and Gori (2008) who introduce a child allowance financed by a lump-sum tax and analyze the effects on human capital accumulation and fertility. In summary, their study show that a child allowance increases the investments in human capital and raises the fertility rate in the long run, while it lowers the fertility rate in the short run provided the preference for the quality of children exceeds the preference for the number of children. In our model, this is not the case; a child allowance would lead to the opposite regarding human capital building. We argue that by subsidizing education of children and not the number of them will have a growth and welfare enhancing effect and a negative effect on fertility if the preference for the quantity of children exceeds the preference for the quality for children.

The rest of the paper is outlined as follows. In section 2, we introduce the basic model. In section 3, we derive the short-run effects caused by the introduction of an education subsidy followed by an analysis of the long-run effects. Finally, in Section 5, we conclude.

2 The model

To model the production side of a small open economy, we use the approach of Lucas (1988) and Uzawa (1965). The production depends on physical and human capital and the production function has the following form:

$$Y_t = F(K_t, H_t) = F(K_t, L_t h_t). \tag{1}$$

Here Y_t is the production, $K_t$is the capital stock, and $H_t = h_t L_t$, is the human capital stock which results from the product of the average human capital per head (h_t) times the aggregate labor time (L_t). The production function exhibits the usual diminishing marginal productivities in each input factor, fulfills the Inada conditions and is linear homogenous.[1] The subscript *t* indicates the period of time.

[1]Expressed in per human capital unit the production function becomes to $f(\tilde{k}_t) = F\left(\frac{K_t}{H_t}, 1\right)$. We assume that the corresponding Inada conditions hold: $f(0) = 0$; $f(\infty) = \infty$; $f'(\infty) = 0$; and $f'(0) = \infty$.

Regarding the creation of human capital, for our purposes a very simple formulation of the human capital building process is sufficient, because we assume that all individuals are identical:

$$h_t = h_{t-1}(1 + \varphi u_t). \tag{2}$$

Therefore the average human capital stock equals the individual human capital stock. This means the average human capital in *t*, depends on the parental average human capital stock, the parental investments in education u_t like schooling and the effectiveness of education $\varphi > 0$. The investments can be measured in monetary units,[2] or in time units like parental time of home-schooling. However, we measure the human capital investments without loss of generality in time units the parents spend for teaching their children. These investments can be expressed in monetary terms, if we use the wage rate as the value of time at the margin. For simplicity, we define the total available time of an adult to be equal one. The efforts of the parents or investments in education are represented by u_t, where $0 \leq u_t < 1$. It is easy to see that the human capital production function is one specification of the more general function established by de la Croix and Doepke (2003) or the one introduced by Azariadis and Drazen (1990). However, the resulting growth rate of human capital g_t^h becomes then to:

$$g_t^h = \varphi u_t. \tag{3}$$

We should mention that there is a positive externality caused by human capital building, because subsequent generations benefit from educational investments of former generations.

The wage rate per human capital unit $\widetilde{w}_t$ and the interest factor R_t are determined on the world capital market, because we consider a small country. The capital stock used in this economy adjusts according to the factor prices on the world market. Assuming that the physical capital is totally depreciated within one period, we get the following equations:

$$\widetilde{w}_t = f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t \tag{4}$$
$$R_t = f'(\tilde{k}_t) \tag{5}$$

The function $f(\tilde{k}_t) = F\left(\frac{K_t}{H_t}, 1\right)$ represents the production function per human capital unit. Then the resulting wage rate per capita is given by $w_t = \widetilde{w}_t h_t$. For the rest of the paper we assume that world economy is in the long-run equilibrium and therefore, $\widetilde{w}_t = \widetilde{w}, \forall t$ and $R_t = R, \forall t$.

To model the consumption side of the economy, we use a three period overlapping generation (OLG) model, which is based on Allais (1947), Samuelson (1958) and Diamond (1965). This approach is extended by the introduction of human capital and endogenous fertility. Even that the fertility behavior is based on similar considerations like in Becker (1960) and Becker and Lewis (1973),[3] who introduced the quantity-quality trade-off of children, we use a different definition of a child's quality. While for example Fanti and Gori (2008) use the definition of Becker (1960), where quality is defined as all expenditures for children, we define quality as time spent for the education of a child. The use of this narrower definition is not new (see for example Galor and Weil (1999) or de la Croix and Doepke (2003)). It should be noted that the results of the model depend on this definition. For example, the model of Fanti and Gori (2008), which used Becker's broader definition of

[2] Then u_t must be interpreted as the share of income which is spent for education.

[3] Liu, Zhang, and Yi (2008), Rosenzweig and Wolpin (1980) and Rosenzweig and Zhang (2009) confirm the existence of the quality-quantity tradeoff of children econometrically.

child's quality, has the disadvantage, that parents never invest in the quality of children if they do not get a child allowance. That means in a world without child allowances parents never invest in human capital building. If the total costs of rearing and educating a child matter as an argument in the utility function, then the pure child rearing costs have a utility enhancing effect and they are a perfect substitute to investments in education. In our model these costs do not enter the utility function directly and an increase of them decreases the utility of the representative individual indirectly. One good reason, why we decided to use human capital as an argument in the utility function is caused by the fact that it coincides with the assumptions of the Unified Growth Theory (Galor (2005)).

In our model, in the first period of life, an individual is relatively young, not yet prepared to work, and/or participate in economic decision making, and hence undergoes education funded by her parents. In the second period of life, the individual supplies labor which is considered to be offered inelastically; gives birth to N_t children, rears and educates her off-spring and consumes c_t^1 and saves s_t amounts respectively. In the third period of life the individual is unable to work and lives from her savings and interest income, that is,$c_{t+1}^2 = Rs_t$. Subsequently, the utility function of the individual over the three periods is defined as the following log-linear equation. This form of the utility function is a variation of those used by de la Croix and Doepke (2003). The idea is that parents are not altruistic but enjoy directly the human capital of their children. However in contrast to the approach of Fanti and Gori (2008), Strulik (2003, 2004) or Galor and Weil (1996) we explicitly consider human capital of the children as an argument in the utility function. Therefore:

$$U(c_t^1, c_{t+1}^2, N_t, h_{t+1}) = \ln c_t^1 + q\ln c_{t+1}^2 + \mu \ln N_t + \beta \ln(h_{t+1}). \tag{6}$$

The parameter q reflects the subjective discount factor. Moreover, we make the natural assumption that parents have a stronger preference for the quantity than for the quality of children: $\mu > \beta$. This assumption guarantees that the law of demand holds for the quality and quantity of children. In addition, it should be noted, that we assume that the parent treats all her children equally with respect to their education. A representative parent is constrained by the following budget:

$$c_t^1 \leq w_t(1 - (1-\tau)u_t N_t - bN_t - T_t) - s_t \tag{7}$$

$$c_{t+1}^2 \leq R_{t+1} s_t \tag{8}$$

In the second period of life each individual has to allocate her available time between working time, time to educate the children $(1-\tau)u_t N_t$ and time to rear the children bN_t. The variable τ represents the subsidy rate. Assuming that it makes no difference, if parents educate their children or if they are educated in public or private schools, an alternative interpretation would be that parents pay for the schooling the amount $(1-\tau)u_t w_t$.

Additionally, the individual has to pay a payroll tax, where the tax rate is T_t. If we normalize the available time to one and define l_t as labor time, then $l_t = (1 - (1-\tau)u_t N_t - bN_t - T_t)$ and the net income becomes to $l_t w_t = l_t \tilde{w} h_t$. A second interpretation is that the individual earns the gross labor income w_t and spends $w_t(1-\tau)u_t N_t$ units of her income for the education, $w_t b N_t$ units for child-rearing, where the parameter $0 < b < 1$ represents the constant time share or constant income share which is needed to rear a child. Furthermore, she pays $w_t T_t$ units as taxes. Equation (8) states that the individual lives in the third period of life from her savings and interest. Combining (7) and (8) gives us a single budget constraint:

$$w_t(1 - (1-\tau)u_t N_t - bN_t - T_t) - c_t^1 - \frac{c_{t+1}^2}{R_{t+1}} = 0. \tag{9}$$

We assume that the government finances the education subsidy by issuing government bonds B_t with a term of one period. Additionally the government collects the income share of T_t as a payroll tax to finance its debt and interest payments. This means that the public debt per worker at the beginning of the current period D_t equals the subsidy per child of the previous period times the interest factor, R. This implies, that a worker pays back the subsidy her parents received to market conditions. That is:

$$D_t = RB_{t-1} = R\tau u_{t-1} w_{t-1}. \tag{10}$$

It should be noted that parents are unable to get an analogous loan contract on the capital market, because of legal restrictions and moral hazard problems. To guarantee a balanced government budget, the tax revenue per worker in period t has to be:

$$T_t = \frac{RB_{t-1}}{w_t} = R\frac{\tau u_{t-1} h_{t-1}}{h_t} = \frac{R\tau u_{t-1}}{1+\varphi u_{t-1}}. \tag{11}$$

The government is financing the education subsidies by a public debt, which will be covered by the tax revenue in the future. Using (2), we maximize (6) subject to (9) by constructing the following Lagrange function:

$$\begin{aligned} L(c_t^1, c_{t+1}^2, \lambda) = & \ln c_t^1 + q \ln c_{t+1}^2 + \mu \ln N_t + \beta \ln\big(h_{t-1}(1+\varphi u_t)\big) \\ & -\lambda\left(w_t(1-(1-\tau)u_t N_t - bN_t - T_t) - c_t^1 - \frac{c_{t+1}^2}{R_{t+1}}\right). \end{aligned} \tag{12}$$

Accordingly, the first order conditions are:

$$\frac{1}{c_t^1} + \lambda = 0, \tag{13}$$

$$\frac{q}{c_{t+1}^2} + \frac{\lambda}{R} = 0, \tag{14}$$

$$\frac{\mu}{N_t} + \lambda w_t[(1-\tau)u_t + b] = 0, \tag{15}$$

$$\frac{\varphi\beta}{1+\varphi u_t} + \lambda w_t(1-\tau)N_t = 0, \tag{16}$$

$$w_t(1-(1-\tau)u_t N_t - bN_t - T_t) - c_t^1 - \frac{c_{t+1}^2}{R} = 0. \tag{17}$$

Substituting (15) into (16) and solving for the investment in education gives us:

$$u^* = \frac{\varphi\beta b - (1-\tau)\mu}{(1-\tau)(\mu-\beta)\varphi}. \tag{18}$$

From equation (18) we can derive the necessary condition for the existence of an interior solution for $u^* \in [0,1]$:

$$\frac{\mu}{\varphi\beta} \leq \frac{b}{(1-\tau)} < \left(\frac{\varphi+1}{\varphi}\right)\left(\frac{\mu}{\beta}\right) - 1. \tag{19}$$

Unfortunately, inequality (19) will not hold for all $\tau \in [0,1]$, because the expression in middle will strive to infinity if the subsidy rate converges to one. Therefore, we must restrict our analysis with respect to the subsidy rate. Especially, we assume throughout the paper that:

$$\frac{\mu}{\varphi\beta} \leq b < \left(\frac{\varphi+1}{\varphi}\right)\left(\frac{\mu}{\beta}\right) - 1. \tag{20}$$

Given this condition the existence of an interior equilibrium is guaranteed in this economy without any government intervention. Consequently, we analyze the effects of a subsidy rate $\tau \in [0, \bar{\tau}]$, because all $\tau > \bar{\tau}$ lead into an undesirable corner solution in the equilibrium.

Due to the fact that b has to lie between zero and one, we can derive from (20), that $\varphi\beta > \mu$ must hold in the case of no subsidy $(\tau = 0)$. In combination with the assumption that $\mu > \beta$, we can conclude that $\varphi b > \frac{\mu}{\beta} > 1$.

Subsequently**,** the growth rate of human capital is given by:

$$g^{h*} = \frac{\varphi\beta b-(1-\tau)\mu}{(1-\tau)(\mu-\beta)}. \tag{21}$$

Solving for the remaining independent variables, we get the following equilibrium quantities:

$$c_t^{1*} = \frac{h_{t-1}\tilde{w}[\varphi\beta(b(\varphi-\tau R)-(1-\tau))+(1-\tau)\mu\tau R]}{\varphi(1-\tau)(1+q+\mu)(\mu-\beta)}, \tag{22}$$

$$c_{t+1}^{2*} = Rq\left(\frac{h_{t-1}\tilde{w}[\varphi\beta(b(\varphi-\tau R)-(1-\tau))+(1-\tau)\mu\tau R]}{\varphi(1-\tau)(1+q+\mu)(\mu-\beta)}\right), \tag{23}$$

$$N_t^* = \frac{(\mu-\beta)[\varphi\beta(b(\varphi-\tau R)\ -(1-\tau))+(1-\tau)\mu\tau R]}{(1+q+\mu)\beta(1-\varphi b-\tau)^2}. \tag{24}$$

Here we restrict our analysis to interior equilibria, which are characterized by a non-negative equilibrium fertility rate ($N_t^* \geq 0$), a non-negative and feasible investment in human capital ($0 \leq u^* \leq 1$) and non-negative consumption levels in both periods of life.

The reader may wonder, why we relate the equilibrium values to h_{t-1} instead of h_t, the reason is that we need this later to calculate the long-run effects caused by a change of τ. To secure that the tax rate T_t is smaller than one, the maximum subsidy rate $\bar{\tau}$ is given by the value of:

$$\bar{\tau} = \frac{\beta\varphi+R(\mu-b\beta\varphi)+\sqrt{\varphi^2\beta^2(bR-1)^2-2R\mu\varphi\beta(1+bR-2b\varphi)+\mu^2R^2}}{2\mu R}. \tag{25}$$

In addition, it can be shown, that $N_t^* \geq 0$, as long as $\tau \in [0, \bar{\tau}]$.

Proposition 1: The existence of a unique interior equilibrium without government intervention is guaranteed if condition (20) is fulfilled.

Proof:

In the absence of a government intervention (18), and (21)-(24) becomes:

$$u^* = \frac{\varphi\beta b-\mu}{(\mu-\beta)\varphi}. \tag{18'}$$

$$g^{h*} = \frac{\varphi\beta b-\mu}{(\mu-\beta)}. \tag{21'}$$

$$c_t^{1*} = \frac{h_{t-1}\tilde{w}\beta(\varphi b-1)}{(1+q+\mu)(\mu-\beta)}, \tag{22'}$$

$$c_{t+1}^{2*} = Rq\left(\frac{h_{t-1}\tilde{w}\beta(\varphi b-1)}{(1+q+\mu)(\mu-\beta)}\right), \tag{23'}$$

$$N_t^* = \frac{(\mu-\beta)\varphi}{(1+q+\mu)(\varphi b-1)}. \tag{24'}$$

As seen above, condition (20) guarantees, that $0 < u^* < 1$. Furthermore, the condition ensures that $\varphi b > 1$. This, combined with the assumption $\mu > \beta$ ascertains that all other independent variables have positive values.

Proposition 2: A unique interior equilibrium described by equations (18) and (21)-(24) exists, if $\tau \in [0, \bar{\tau}]$ and condition (20) is fulfilled.

Because of our small country assumption, the capital intensity per human capital unit is determined on the international capital market. The private savings per capita are given by:

$$s_t^* = qh_{t-1}\widetilde{w}\left(\frac{[\varphi\beta(b(\varphi-\tau R)-(1-\tau))+(1-\tau)\mu\tau R]}{\varphi(1-\tau)(1+q+\mu)(\mu-\beta)}\right). \tag{26}$$

To calculate the amount of resources available to invest in physical capital, we have to subtract the amount of government bonds per head that are issued in the corresponding period. The government bonds per capita issued in the current period accumulate to:

$$B_t = \tau h_{t-1}\widetilde{w}u_t^* N_t^* = \tau h_{t-1}\widetilde{w}\frac{[\varphi\beta(b(\varphi-\tau R)-(1-\tau))+(1-\tau)\mu\tau R]((1-\tau)\mu-\varphi\beta b)}{(1-\tau)(1+q+\mu)\beta\varphi(\beta\varphi-1+\tau)^2}. \tag{27}$$

The net wealth per capita in the beginning of period t+1, a_{t+1}, is the savings per capita of period t minus the government bonds per capita issued in period t and the result divided by the number of children born in period t:

$$a_{t+1}^* = h_{t-1}\widetilde{w}\frac{[\tau\beta^2\varphi b+\beta(\tau^2(q+\mu)+\tau(\varphi b-1)(2q-\mu)+q(\varphi b-1)^2)+(1-\tau)\tau\mu^2]}{(1-\tau)\varphi(\mu-\beta)^2}. \tag{28}$$

However, if the wealth per capita is smaller than the capital per capita, the country is a net borrower on the international capital market. Now we can analyze the short- and long-run effects of an increasing subsidy rate τ on the equilibrium values of this economy.

3 Short run effects

If the government introduces an education subsidy in period 0, the parents are offered a subsidy for education, but they do not have to pay a tax like all following parent generations and their human capital stock is given. As a consequence, the budget constraint is somewhat different from (9), because $T_0 = 0$. Repeating the maximization procedure presented in the previous section with adjusted budget restriction leads to the following results:

$$u_0 = \frac{\varphi\beta b-(1-\tau)\mu}{(1-\tau)(\mu-\beta)\varphi}. \tag{29}$$

$$g_0^h = \frac{\varphi\beta b-(1-\tau)\mu}{(1-\tau)(\mu-\beta)}. \tag{30}$$

$$c_0^1 = \frac{h_0\widetilde{w}}{(1+q+\mu)}, \tag{31}$$

$$c_1^2 = Rq\frac{h_0\widetilde{w}}{(1+q+\mu)}, \tag{32}$$

$$N_0 = \frac{(\mu-\beta)\varphi}{(1+q+\mu)(\varphi b-1+\tau)}. \tag{33}$$

Obviously, the consumption behavior, and therefore the savings behavior is not influenced by the introduction of the subsidy. However, the subsidy influences the level of education, the number of children and the per capita growth rate. Differentiating (29), (30) and (31) gives us:

$$\frac{\partial u_0}{\partial\tau} = \frac{\beta b}{(1-\tau)^2(\mu-\beta)} > 0, \tag{34}$$

$$\frac{\partial g_0^h}{\partial \tau} = \frac{\varphi \beta b}{(1-\tau)^2(\mu-\beta)} > 0, \tag{35}$$

$$\frac{\partial c_0^1}{\partial \tau} = \frac{\partial c_1^2}{\partial \tau} = 0. \tag{36}$$

$$\frac{\partial N_0}{\partial \tau} = -\frac{(\mu-\beta)\varphi}{(1+q+\mu)(1-\varphi b-\tau)^2} < 0. \tag{37}$$

As expected, the subsidy enhances the level of education and therefore the growth rate of human capital accumulation. Simultaneously, the number of children declines. The result, that the aggregate consumption expenditures and the share of income spent for consumption remain unchanged is due to the additive separability of the utility function. This characteristic of the utility function leads to the result that only a substitution effect between the quantity and quality of children occurs. The subsidy decreases the relative price of the child quality and raises the relative price of the quantity of children. The outcome is a lower number of children, who receive a higher level of education. These effects lead to a change of the parents' welfare:

$$\frac{\partial U\left(c_0^1, c_1^2, N_0, h_1\right)}{\partial \tau} = \frac{\varphi \beta b-(1-\tau)\mu}{(1-\tau)(\varphi b-1+\tau)} > 0. \tag{38}$$

The less surprising result is an increase of parents' utility.

Proposition 3*:* Undoubtedly, in the short run, the introduction of an education subsidy leads to an increase in the growth rate of human capital accumulation, the utility of the parents, and a decline of the fertility rate.

Before entering the analysis of the long-run effects, we should note that the aggregated non-financial wealth of the economy declines, because a part of the savings will be invested in the government bonds, which are needed to finance the subsidies. However, the change of the non-financial wealth per capita does not necessarily decrease, because the number of children is also lower than in the previous periods. The non-financial wealth in period 1 is defined by:

$$a_1 = \frac{s_0-\tau h_0 \tilde{w} u_0 N_0}{N_0} = \frac{h_0 \tilde{w}\left(q(\varphi b-1)-\tau^2(q+\mu)-\tau\left(b\beta\varphi-\mu-q(2-\varphi b)\right)\right)}{(1-\tau)(\mu-\beta)\varphi}. \tag{39}$$

If an education subsidy will be introduced, the wealth per capita can be determined by differentiating (39) with respect to the subsidy rate and evaluating the derivative at $\tau = 0$:

$$\left.\frac{\partial a_1}{\partial \tau}\right|_{\tau=0} = \frac{h_0 \tilde{w}(q+\mu-b\beta\varphi)}{\varphi(\mu-\beta)} \lesseqgtr 0. \tag{40}$$

Proposition 4: The introduction of an education subsidy results in an increase of the non-financial wealth per capita in the following period, only if the subjective time preference for the future is sufficiently large ($q > b\beta\varphi - \mu$).

This also means that the net borrowing position of the country will improve, if the condition of proposition 4 holds.

4 Long-run effects

This part of the analysis is relevant for all generations born in period zero and thereafter. In difference to the short-run analysis we have to take into account that each individual has to pay a tax, which is used to cover the public debt and the corresponding interest. Differentiating (18) and (21) leads to the following results:

$$\frac{\partial u^*}{\partial \tau} = \frac{\beta b}{(1-\tau)^2(\mu-\beta)} > 0. \tag{41}$$

$$\frac{\partial g_h^*}{\partial \tau} = \frac{\varphi \beta b}{(1-\tau)^2(\mu-\beta)} > 0. \tag{42}$$

The long-run effects regarding the educational level and the growth rate of human capital are identical to the corresponding effects in the short-run.

Because of the fact that the subsidy rate has to be sufficiently small, as pointed out in proposition 2, we evaluate the following derivatives at $\tau = 0$. As opposed to the educational time and growth rate, the number of children will decline more than in the short run. This is induced by the payroll tax which reduces the net income, and the assumption that the costs of rearing children develop proportionally to the gross income.

$$\left.\frac{\partial N^*}{\partial \tau}\right|_{\tau=0} = -\frac{(\mu-\beta)\big(\varphi\beta+R(\varphi b\beta-\mu)\big)}{(\varphi b-1)^2(1+q+\mu)\beta} < 0. \tag{43}$$

As distinct from the short-run effects, the consumption in the first and second period of life will change, therefore the individual savings.

$$\left.\frac{\partial c_t^{1*}}{\partial \tau}\right|_{\tau=0} = \frac{\tilde{w}\big(\varphi^2 b\beta-R(\varphi b\beta-\mu)\big)}{(1+q+\mu)(\mu-\beta)\varphi} \lesseqgtr 0, \tag{43}$$

$$\left.\frac{\partial c_{t+1}^{2*}}{\partial \tau}\right|_{\tau=0} = \frac{\tilde{w}Rq\big(\varphi^2 b\beta-R(\varphi b\beta-\mu)\big)}{(1+q+\mu)(\mu-\beta)\varphi} \lesseqgtr 0. \tag{44}$$

The sign of the derivatives (43) and (44) depends, on among other parameter values, on the ratio between the education coefficient φ and the interest factor R.

Proposition 5: The introduction of an education subsidy financed by a public debt increases the consumption in both periods of life and the private savings only if, $\frac{\varphi}{R} > 1 - \frac{\mu}{\varphi b\beta}$.

As we know, the RHS of the condition in proposition 5 is by assumption less than one, and therefore the consumption can increase, even though the interest factor exceeds the education coefficient. This is caused by the reduction of the number of children.

Furthermore, it is interesting to examine how the education subsidy affects the non-financial wealth per capita. Differentiating (28) subject to the subsidy rate and evaluating it at $\tau = 0$, gives:

$$\left.\frac{\partial a_{t+1}^*}{\partial \tau}\right|_{\tau=0} = \frac{\tilde{w}\Big(\beta\big(\varphi\beta b+q(\varphi^2 b^2-1)-\mu(1+\varphi b)\big)+\mu^2\Big)}{\varphi(\mu-\beta)^2} \lesseqgtr 0. \tag{45}$$

Whether the non-financial wealth is increasing or decreasing depends strongly on the time preference for future consumption. Solving for the critical value of q leads to:

$$\overline{q} = \frac{(\mu-\beta)(\varphi b\beta-\mu)}{\beta(\varphi b-1)^2}. \tag{46}$$

Proposition 6: The introduction of an education subsidy raises the non-financial wealth per capita, only if the subjective time preference factor q exceeds the critical value $\overline{q}$.

If the condition of proposition 6 holds, the net borrowing position of the country will improve and maybe if the country was a debtor country it will become a lending country after introduction of the education subsidy in the long run.

What remains is to analyze the long-run welfare effects. Accordingly, we have to consider whether all the generations in the (very) long-run are better-off because of the increased growth rate. Therefore, the relevant generation to consider is the one, which receives a minimal gain from the growth rate increase but has to bear a higher tax burden caused by the increased subsidy.

We insert the corresponding equilibrium values (18) and (21)-(24) in the utility function (6), differentiate it with respect to the subsidy rate and evaluate the expression at $\tau = 0$. Then, the result is:

$$\left.\frac{\partial U\left(c_t^{1*}, c_{t+1}^{2*}, N_t^*, h_{t+1}\right)}{\partial \tau}\right|_{\tau=0} = \frac{(1+\mu+q)\left[\varphi\beta b(\varphi-R)+\mu\left(R-\frac{\varphi\beta}{1+\mu+q}\right)\right]}{\varphi\beta(\varphi b-1)} \lesseqgtr 0. \tag{47}$$

Setting the RHS of the derivative to zero and solving for R gives us:

$$\overline{R} = \frac{\varphi\beta(\varphi b(1+\mu+q)-\mu)}{(1+\mu+q)(\varphi b\beta-\mu)}. \tag{48}$$

Proposition 7: The introduction of an education subsidy financed by a public debt generates a Pareto-improvement only if the market interest factor is sufficiently low ($R < \overline{R}$).

If the interest rate is too high, the tax burden outweighs the positive effect of increased growth and the additional utility generated by the extra human capital. Of course, the more time has elapsed since the introduction of the education subsidy, the more important is the growth effect. However, when the interest factor is not too large, a Pareto improvement can be realized.

5 Conclusions

In this paper, we show that building human capital financed through a public debt can provide a beneficial outcome both on the current and future generations, even in the absence of positive externalities and altruism in the sense of Barro (1974). Specifically, we have shown that the proposed finance mechanism reduces the fertility rate and enhance the human capital accumulation, and hence the per capita growth rate of income. It should be noted, that it is not necessary with respect to this result that the rate of return on human capital φ exceeds the rate of return of physical capital R. Additionally, it was shown that the net borrowing position of the economy improves and the welfare in the sense of Pareto increases under certain conditions.

If the propensity to save is not too small, the education subsidy will improve the net borrowing position of the country. In a world where altruistic motivations dominates parents preferences for their children's education as future investment, the argument of subsidizing education becomes even stronger and the net savings tend to increase under certain conditions (as indicated above). Nevertheless, we have shown that when education is financed via a subsidy, the outcome is unambiguously welfare enhancing, if the real rate of return of physical capital is not too high. Therefore, from development aid perspective, it would make sense to give concessional loans to small and developing countries to finance education and

human capital development in areas of resource deficit identified both in the developing and the developed economies.

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