Strategic Subsidy Policies with Endogenous Choice of Competition Mode

Seonyoung Lim and Kangsik Choi

Graduate School of International Studies, Pusan National University,
Graduate School of International Studies, Pusan National University

24. October 2014

Online at http://mpra.ub.uni-muenchen.de/59462/
MPRA Paper No. 59462, posted 25. October 2014 06:01 UTC
Strategic Subsidy Policies with Endogenous Choice of Competition Mode

Seonyoung Lim* and Kangsik Choi†

This version: 24 October 2014

Abstract
We investigate government subsidy policies in which a home firm and a foreign firm choose to strategically set prices or quantities in a third market. We show that even though each firm can earn higher profits under Cournot competition than under Bertrand competition regardless of the nature of goods, choosing Bertrand competition is the dominant strategy for both firms. This leads each firm to face a prisoners’ dilemma in equilibrium. We also show that from the aspects of governments under subsidy regime, Cournot competition is more efficient than Bertrand competition when the goods are substitutes, and vice versa when the goods are complements. For this, from the aspects of firms, the Cournot equilibrium could be Pareto superior (inferior) with government’s intervention of subsidy policy when the goods are substitutes (complements). Thus, the conflict of interests between governments and firms occur when goods are complements. Hence, our result may justify that when the goods are substitutes, a general principle is that the incentive to intervene in the international trade is greater under Cournot competition than under Bertrand competition.

JEL Classification: F12, F13, L13.
Keywords: Subsidy, Cournot, Bertrand, Social Welfare, Prisoners’ Dilemma.

1 Introduction
The analysis of strategic trade policy has attracted much attention since the beginning of the 1980s. As is often the case in an international trade, the theory of strategic export policy for oligopolies started with a pioneering work by Brander and Spencer (1985). In their model, a domestic government first decides upon an export subsidy, and then a domestic firm and a foreign firm compete in a third market. Brander and Spencer (1985) showed that an export subsidy was optimal under Cournot competition, whereas Eaton and Grossman (1986) demonstrated that an export tax was optimal under Bertrand competition on the third market. Recently, Clarke and Collie (2003) analysed the welfare effects of free trade in the Bertrand competition with product differentiation. The main stream focuses on extensions and generalizations of Brander and Spencer (1985) and Eaton and Grossman (1986). For example, de Meza (1986), Bandyopadhyay (1997), Neary and Leahy (2000), Collie and de Meza (2003), Clarke and Collie (2006), among others, have analyzed counter-examples based on the original framework by allowing for a wider range of cost and demand asymmetries.

Although previous works considered strategic export policy, the existing literature on international trade paid relatively little attention to the endogenous choice of strategic variables for prices or quantities with subsidy or tax regime. In fact, since our issue was addressed in the industrial organization context, the potential impact of government subsidy policy was not theoretically incorporated. Key
paper in this area includes Singh and Vives (1984). They were the first to analyze this issue and to show, from the standpoint of consumer surplus and social welfare, that Bertrand competition is more efficient than Cournot competition regardless of the nature of goods. They also show that when goods are substitutes, Cournot equilibrium profits are higher than Bertrand equilibrium profits, and vice versa, when goods are complements. In the industrial organization context, many strands of the literature have produced an array of extensions and generalizations of the analysis in Singh and Vives (1984). For example, one strand that focuses on extensions and generalizations of their study, Dastidar (1997), Qiu (1997), Lambertini (1997), Hackner (2000), and Zanchettin (2006) reveals counter-results based on the original framework by allowing for a wider range of cost and demand asymmetries.

Under the framework of the strategic trade policies, with comparisons of Bertrand and Cournot competition, the only exceptions, to the best of the authors’ knowledge, are Cheng (1988), Bagwell and Staiger (1994), Maggi (1996), Schroeder and Tremblay (2014) and Ghosh and Pal (2014) where the endogenous choice of strategic variables is not provided\(^2\). Cheng (1988) derived the optimal tariff and production subsidy under Cournot and Bertrand competition with differentiated products and showed that the optimal tariff is lower under Bertrand competition than under Cournot competition. Moreover, Maggi (1996) showed that capacity subsidy is generally a welfare improving policy regardless of the competition mode and Bagwell and Staiger (1994) indicated that R&D subsidies might also be the best policy in both Cournot and Bertrand setups. Schroeder and Tremblay (2014) investigated the welfare effect of an export subsidy/tax in the “third market trade model, where the home government chooses subsidy and other countries are assumed to be policy inactive by considering all strategic possibilities (Cournot versus Bertrand versus Bertrand-Cournot versus Cournot-Bertrand). Finally, Ghosh and Pal (2014) analyzed strategic trade policy in differentiated network goods oligopoly only comparing Cournot versus Bertrand competition. The present paper fills this gap. Thus, we address how the endogenous choice of strategic variables for prices or quantities affects social welfare and firm’s profit when a home firm and a foreign firm compete in a third market. Notably, the present study differs from previous ones that do not consider the endogenous choice of strategic variables for prices or quantities in a third market with strategic export policy.

The main result of our paper is that regardless of the nature of goods, even though each firm can earn higher profits under Cournot competition than under Bertrand competition, choosing Bertrand competition is the dominant strategy for a home firm and a foreign firm when both firms export to a third-country market with strategic trade policies. A higher (less) export subsidy (tax) forces both firms to be aggressive in determining the output, which leads to be higher output and lower price under choosing price variable regardless of what the rival firm chooses competition mode. Thus, each firm prefers choosing price variable to choosing quantity variable regardless of the nature of goods. However, this leads, in equilibrium, each firm to face a prisoners’ dilemma regardless of the nature of goods. This intuition is as follows. Since the effect on a higher price with lower output under Cournot competition dominates the effect on a lower price with a higher output under Bertrand competition, each firm could obtain higher profit under Cournot competition than under Bertrand competition.

We also show that from the aspects of governments under subsidy or tax regime, Cournot competition is more efficient than Bertrand competition when the goods are substitutes, and vice versa\(^2\)Kikuchi (1998) explored how optimal export policies are affected by the nature of competition mode (Cournot or Bertrand) with a home firm only under subsidy regime.
when the goods are complements. For this, from the aspects of firms, the equilibrium could be Pareto superior (inferior) with government’s intervention of subsidy policy when the goods are substitutes (complements). Thus, if each government could choose Cournot competition in equilibrium when the goods are substitutes, there would be no a conflict of interests between firms and governments. This can be better because each firm’s prisoners’ dilemma problem is resolved by the government, while choosing the Bertrand competition is the dominant strategy for both firms regardless of the nature of goods. However, this conflict of interest between firms and government can occur if each government chooses Bertrand competition in equilibrium when the goods are complements. Finally, we provide comparisons of social welfare and firm’s profit between free trade and subsidy regime, which depends on both the competition mode, and the degree of product differentiation when the goods are complements.

The paper is organized as follows. Section 2 outlines the third-market model. Section 3 analyzes market equilibrium with competition mode under either free trade or subsidy regime. Section 4 determines choice of competition mode under subsidy regime. Section 5 concludes.

2 The Third-Market Model

Consider the third-market model of Brander and Spencer (1985), in which a home and a foreign firm both export to a third-country market. Let 1 and 2 also represent two countries, firm \( i \) belonging to country \( i \). The firms compete in a third market. The inverse and direct demands are:

\[
p_i = 1 - x_i - bx_j, \quad x_i = \frac{1 - b - p_i + bp_j}{1 - b^2}; \quad i, j = 1, 2, i \neq j,
\]

where \( p_i \) is the price, \( x_i \) is the quantity, and the parameter \( b \) denotes the degree of product differentiation between \( x_i \) and \( x_j \) : \( b \in (-1, 1) \). If \( b > (\leqslant)0 \), the products are substitutes (complements).

Let the foreign and home government offer the foreign and home firm’s export a per unit subsidy \( s_i \), respectively. Moreover, we assume that for each firm, the marginal cost is constant \( c_i = 0 \) and the two exporting firms’ profit are given by

\[
\pi_i = (p_i + s_i)x_i; \quad i = 1, 2,
\]

where each government finances the export subsidies \( s_i \) for each firm. Thus, each social welfare \( SW_i \) is given by

\[
SW_i = \pi_i - s_i x_i, \quad i = 1, 2,
\]

This study considers that each firm can make two types of binding contracts with consumers, as described by Singh and Vives (1984). Thus, we posit a three-stage game. In the first stage, exporting firm \( i \) simultaneously commit to choosing strategic variable, i.e., either price or quantity (which determines the type of contract), to set in the international duopoly. In the second stage, the exporting
countries decide on the optimal subsidy \( s_i \) to maximize its welfare. In the third stage, each exporting firm \( i \) chooses its quantity or price simultaneously, in order to maximize its objective knowledge of the strategic variable.

3 Equilibrium Outcomes

3.1 The Choice of Competition Mode under Free Trade

In this subsection, we briefly present the solution of the Cournot- and Bertrand-Nash games in order to illustrate free trade issue. As stated in Introduction, since this issue was addressed in the industrial organization context, the potential impact of government subsidy policy was incorporated by Singh and Vives (1984). Given the basic model, it is straightforward to verify that these relationships hold for our model with quantity and price strategies, and we present the results here only for purposes of comparison with what is to come; the model and results are well known in industrial organization.

Consider free trade where government \( i \)'s social welfare is the same as firm \( i \)'s profit by setting \( s_i = 0 \). Since four different cases of contract games are explained, so under free trade, determining endogenous choice of strategic variables is the same as in Singh and Vives (1984), except for the comparisons of social welfare. Table 1 below provides a summary of the Cournot and Bertrand equilibrium output levels, firm’s profits, and government welfare.

<table>
<thead>
<tr>
<th></th>
<th>Cournot</th>
<th>Bertrand</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{x}_i^C ) = ( \hat{p}_i^C ) = ( \frac{1}{2+b} )</td>
<td>( \hat{x}_i^B ) = ( \frac{1-b}{2-b} ), ( \hat{p}_i^B = \frac{1}{(1+b)(2-b)} )</td>
<td></td>
</tr>
<tr>
<td>( \hat{s}_{i}^C = S\hat{W}_i^C = \frac{1}{(2+b)^3} )</td>
<td>( \hat{s}_{i}^B = S\hat{W}_i^B = \frac{1-b}{(1+b)(2-b)^2} )</td>
<td></td>
</tr>
</tbody>
</table>

It is straightforward to verify that under free trade, (1) Cournot competition is more efficient than Bertrand competition when the goods are substitutes, and vice versa when the goods are complements\(^4\); (2) Cournot equilibrium profits are greater than Bertrand equilibrium when the goods are substitutes, and vice versa when the goods are complements; (3) the dominant strategy for firms is to choose Cournot competition when the goods are substitutes, and vice versa when the goods are complements.

3.2 Equilibrium Outcomes under Subsidy Regime

Before the type of contract is applied under subsidy regime in the international model to identify the point of equilibrium, four different cases of contract games are explained. In Bertrand competition, firms set prices, whereas in Cournot competition, firms set quantities. In mixed cases, firm \( i \) sets the quantity and firm \( j \) sets the price and vice versa. Such a game is solved by backward induction, i.e. the solution concept used is the subgame perfect Nash equilibrium (SPNE).

[Cournot Competition]: At third stage, taking arbitrary subsidy rates \((s_1, s_2)\) and using inverse demand functions, \( p_i = 1 - x_i - bx_j \), we obtain that the firm \( i \)'s best response function under Cournot

\(^4\)Note that \( S\hat{W}_i^C - S\hat{W}_i^B = 2b^3 > (\leq)0 \) when the goods are substitutes(complements).
competition is given by \( BR_i(x_j, s_i, s_j) = (1 - bx_j + s_i)/2 \), which is downward-sloping. Solving the system of response functions, we obtain price and quantity under Cournot competition:

\[
x_i^C = \frac{2 - b - bs_j + 2s_i}{4 - b^2}, \quad p_i^C = \frac{2 - b - (2 - b^2)s_i - bs_j}{4 - b^2}.
\]

Given the output and price at the third stage and competition mode, each government simultaneously chooses to subsidy in order to maximize each social welfare at the first stage

\[
\max_{s_i} SW_i^C = \frac{(2 - b - bs_j + 2s_i)^2 - s_i(4 - b^2)(2 - b - bs_j + 2s_i)}{(4 - b^2)^2}.
\]

The first-order conditions for each government are given by \( s_i = [b^2(2 - b) - b^3s_j]/4(2 - b^2) \). Hence, straightforward calculation yields

\[
s_i^C = \frac{b^2}{4 + 2b - b^2}.
\]

These optimal subsidies lead to the following expression for the equilibrium values, welfare and firm \( i \)'s profit.

**Lemma 1:** Suppose that the goods are substitutes under Cournot competition with subsidy regime. Then, the equilibrium output, price, firm’s profit and social welfare are given by

\[
x_i^C = \frac{2}{4 + 2b - b^2}, \quad p_i^C = \frac{2 - b^2}{4 + 2b - b^2}, \quad \pi_i^C = \frac{4}{(4 + 2b - b^2)^2}, \quad SW_i^C = \frac{2(2 - b^2)}{(4 + 2b - b^2)^2}.
\]

[Bertrand Competition]: Consider that firm \( i \) faces the direct demand function as in equation (2). At stage three, taking \( s_i \), firm \( i \)'s best response function under Bertrand competition with arbitrary subsidy rates \( (s_1, s_2) \) is given by \( BR_i(p_j, s_i, s_j) = (1 + bp_j - b - s_i)/2 \). Solving the system of response functions, we obtain price and quantity under Bertrand competition:

\[
p_i^B = \frac{(2 + b)(1 - b) - bs_j - 2s_i}{4 - b^2}, \quad x_i^B = \frac{(2 + b)(1 - b) + s_j(2 - b^2) - bs_i}{(1 - b^2)(4 - b^2)}.
\]

Given the price and quantity at third stage, each government simultaneously chooses to subsidy in order to maximize each social welfare at the first stage

\[
\max_{s_i} SW_i^B = \frac{[(2 + b)(1 - b) - bs_j + s_i (2 - b^2)]^2 - s_i(4 - b^2)[(2 + b)(1 - b) - bs_j + s_i (2 - b^2)]}{(1 - b^2)(4 - b^2)^2}.
\]

The first-order conditions for both governments are given by \( s_i = -[b^2(1 - b)(2 + b) + b^3s_j]/4(2 - b^2) \). Hence, solving the response subsidy functions yields

\[
s_i^B = \frac{-b^2(1 - b)}{4 - 2b - b^2},
\]

which has the same effect as in Eaton and Grossman (1986). That is, governments want to tax when the firms use price strategies and to subsidize when the firms use quantity strategies. It is desirable to force the firm to set a higher price under the international trade by imposing export taxes when the firms use price strategies. The reason why imposing export taxes is welfare-enhancing when the firms
use price strategies. Hence, this export taxes, \( s^B_i \) lead to the following expression for the equilibrium values, welfare and firm \( i \)'s profit.

**Lemma 2:** Suppose that the goods are substitutes under Bertrand competition with subsidy regime. Then, the equilibrium output, price, firm’s profit and social welfare are given by

\[
x^B_i = \frac{2 - b^2}{(1 + b)(4 - 2b - b^2)}, \quad p^B_i = \frac{2(1 - b)}{4 - 2b - b^2}, \quad \pi^B_i = \frac{(1 - b)(2 - b^2)^2}{(1 + b)(4 - 2b - b^2)^2}, \quad SW^B_i = \frac{2(1 - b)(2 - b^2)}{(1 + b)(4 - 2b - b^2)^2}.
\]

**[Asymmetric Competition]:** Let firm \( i \) optimally choose its quantity as a best response to any price chosen by firm \( j \), and let the firm \( j \) optimally choose its price as a best response to any quantity chosen by firm \( i \). Both demand functions that firms \( i \) and \( j \) face are given by \( p_i = 1 - b + bp_j - (1 - b^2)x_i \) and \( x_j = 1 - bx_i - p_j \), respectively.

At the third stage, taking arbitrary subsidy rates \((s_1, s_2)\), we obtain that the each firm’s best response function under asymmetric competition is given by \( BR_i(p_j, s_i, s_j) = (1 - b + bp_j + s_i)/(2(1 - b^2)) \) and \( BR_j(x_i, s_i, s_j) = (1 - bx_i - s_j)/2 \), which are upward- and downward-sloping, respectively. Solving the system of response functions, we obtain the price and quantity as follows:

\[
x^Q_i = \frac{2 - b + 2s_i - bs_j}{4 - 3b^2}, \quad p^Q_i = \frac{(2 - b)(1 - b^2) - bs_j(1 - b^2) - s_i(2 - b^2)}{(4 - 3b^2)}, \\
x^Q_j = \frac{(2 + b)(1 - b) - bs_i + s_j(2 - b^2)}{4 - 3b^2}, \quad p^Q_j = \frac{(2 + b)(1 - b) - 2s_j(1 - b^2) - bs_i}{4 - 3b^2},
\]

where denotes firm \( i \)'s \((j \)'s\) equilibrium values with superscript “\(Q(P)\)” when firm \( i \) \((j)\) sets quantity \((price)\) as a best response to any price \((quantity)\) chosen by firm \( j(i) \).

Given the equilibrium output and price at the third stage, each government simultaneously chooses to subsidize in order to maximize each social welfare at the first stage

\[
\max_{s_i} SW^Q_i = \frac{(1 - b^2)(2 - b + 2s_i - bs_j)^2 - s_i(4 - 3b^2)(2 - b + 2s_i - bs_j)}{(4 - 3b^2)^2}, \\
\max_{s_j} SW^Q_j = \frac{[(2 + b)(1 - b) - bs_i + s_j(2 - b^2)]^2 - s_j(4 - 3b^2)[(2 + b)(1 - b) - bs_i + s_j(2 - b^2)]}{(4 - 3b^2)^2}.
\]

As analyzed before, using first-order conditions, \( s_i = -[b^2(2 - b) + b^3s_j]/(4(2 - b^2)), s_j = [b^2(2 + b)(1 - b) + b^3s_i]/4(1 - b^2)(2 - b^2) \) yields the optimal subsidies as follows:

\[
s^Q_i = \frac{-b^2(1 - b)(4 + 2b - b^2)}{16 - 20b^2 + 5b^4}, \quad s^Q_j = \frac{b^2(4 - 2b - b^2)}{16 - 20b^2 + 5b^4}.
\]

As in previous competition modes, the optimal subsidies, \( s^Q_i \) and \( s^Q_j \) imply that governments want to tax when the firms use price strategies and to subsidize when the firms use quantity strategies. It is desirable to force the firm to set a higher price by imposing an export tax when the firm \( j \) optimally choose its price as a best response to any quantity chosen by firm \( i \), while to force the firm to produce more output by imposing an export subsidy when firm \( i \) optimally choose its quantity as a best response to any price chosen by firm \( j \). Thus, the reason why imposing an export tax and an export subsidy is simultaneously welfare-enhancing in the case of asymmetric competition. These optimal
subsidies lead to the following Lemma 3.

**Lemma 3:** Suppose that the goods are substitutes under the case of asymmetric competition with subsidy regime. Then, the equilibrium price, output, and each firm’s profit and social welfare are respectively given by

\[ x^Q_i = \frac{2(1 - b)(4 + 2b - b^2)}{16 - 20b^2 + 5b^4}, \quad x^P_j = \frac{(2 - b^2)(4 - 2b - b^2)}{16 - 20b^2 + 5b^4}, \]
\[ p^Q_i = \frac{(1 - b)(2 - b^2)(4 + 2b - b^2)}{16 - 20b^2 + 5b^4}, \quad p^P_j = \frac{2(1 - b^2)(4 - 2b - b^2)}{16 - 20b^2 + 5b^4}, \]
\[ \pi^Q_i = \frac{4(1 - b)^2(1 - b^2)(4 + 2b - b^2)^2}{(16 - 20b^2 + 5b^4)^2}, \quad \pi^P_j = \frac{(2 - b^2)^2(4 - 2b - b^2)^2}{(16 - 20b^2 + 5b^4)^2}, \]
\[ SW^Q_i = \frac{2(1 - b)^2(2 - b^2)(4 + 2b - b^2)^2}{(16 - 20b^2 + 5b^4)^2}, \quad SW^P_j = \frac{2(1 - b^2)(4 - 2b - b^2)^2}{(16 - 20b^2 + 5b^4)^2}. \]

4 The Choice of Competition Mode under Subsidy Regime

Once the equilibria for four fixed types of contract and social-welfare levels are derived per the preceding section, the type of contract can be determined endogenously by taking each social welfare level and firm’s profit as given. Therefore, we will consider the cases of substitutes and complements at the same time.

To employ the three-stage game, let “C” and “B” represent, respectively, Cournot and Bertrand competition with regard to each firm’s choice. In this section of firm’s choice of competition mode under subsidy regime, the SPNE will be found in the second stage for any given pair of competition types. Thus, the payoff matrix for the competition mode between firms can be represented by the following table 2.

<table>
<thead>
<tr>
<th>i \ j</th>
<th>C</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(\pi^C_i, \pi^C_j)</td>
<td>(\pi^Q_i, \pi^P_j)</td>
</tr>
<tr>
<td>B</td>
<td>(\pi^B_i, \pi^Q_j)</td>
<td>(\pi^B_i, \pi^B_j)</td>
</tr>
</tbody>
</table>

Comparing each firm’s profit shows that

\[ \pi^C_i - \pi^P_i = -256b^4 + 384b^6 - 176b^8 + 28b^{10} - b^{12} < 0, \]
\[ \pi^Q_i - \pi^B_i = -256b^4(1 - b) + 640b^6(1 - b) - 560b^8(1 - b) + 196b^{10}(1 - b) - 21b^{12}(1 - b) < 0. \]

Hence, choosing Bertrand competition is the best firm can do, regardless of whether the goods are substitutes or complements\(^5\). The following proposition can be stated.

**Proposition 1:** Suppose that a home and a foreign firm both export to a third-country market under subsidy or tax regime. Then, choosing Bertrand competition is the dominant strategy for both firms

---

\(^5\)Another way of straightforward calculations is in the Appendix.
Proposition 1 suggests that even in practice export taxes are rarely used, choosing Bertrand competition is the dominant strategy for both firms regardless of the nature of goods. The intuition is as follows. The firm receives greater subsidy under choosing price variable than under choosing quantity variable, $s_i^C < s_i^P$, which leads to be higher output and lower price, i.e., $x_i^C < x_i^P$, and $p_i^B < p_i^C$. A higher export subsidy forces both firms to be aggressive in determining the output. On the other hand, from $s_i^Q < s_i^B < 0$, each firm is levied less export tax under choosing price variable than under choosing quantity variable. This effect also leads to be higher output and lower price, i.e., $x_i^Q < x_i^B$, and $p_i^B < p_i^Q$. A less export tax also forces both firms to be aggressive in determining the output. That is, by either imposing export subsidy or levying tax, each firm’s output under Bertrand competition are higher than under Cournot competition. Thus, each firm prefers choosing price variable to choosing quantity variable regardless of the nature of goods. Under subsidy regime, there exists a dominant strategy only for the firm that chooses Bertrand competition, regardless of the nature of goods. Hence, our result differs from Singh and Vives (1984), who showed that a dominant strategy exists for both firms that choose Cournot (Bertrand) competition if the goods are substitutes (complements).

Moreover, from relationships, $\pi_i^Q < \pi_i^B < \pi_i^C < \pi_i^P$ under the subsidy regime, we understand that even though each firm can earn higher profits under Cournot competition than under Bertrand competition, choosing Bertrand competition is a dominant strategy for both firms. Consequently, from the aspects of both firms, the endogenous choice of contract might be Pareto inferior regardless of the nature of goods. Hence, each firm faces a prisoners’ dilemma regardless of the nature of goods under the subsidy regime. This observation leads to the result.

Proposition 2: Suppose that a home and a foreign firm both export to a third-country market under subsidy or tax regime. Then, each firm faces a prisoners’ dilemma regardless of the nature of goods.

Proposition 2 suggests that even though each firm could obtain higher profit choosing Cournot competition, the endogenous choice of strategic variable is Bertrand competition regardless of the nature of goods. The intuition behind Proposition 2 is as follows. By straightforward comparisons, we obtain that $x_i^C < x_i^B \iff p_i^B < p_i^C \iff \pi_i^B < \pi_i^C$. This implies that since the effect on a higher price with a lower output under Cournot competition dominates the effect on a lower price with a higher output under Bertrand competition, each firm could obtain higher profit under Cournot competition than under Bertrand competition.

Similar to the choice of strategic variables among firms, comparing each government’s social welfare shows that

$$SW_i^C - SW_i^P > 0 \iff 4b^{10} - 2b^{12} > 0, \quad SW_i^Q - SW_i^B < 0 \iff -4b^{10} + 4b^{11} + 2b^{12} - 2b^{13} < 0,$$

regardless of nature of goods. Clearly, governments prefer subsidy (tax) regime under Cournot (Bertrand) competition, while SPNE can be sustained with Bertrand competition when firms consider to choose the strategic variables in equilibrium.

Next, with the endogenous choice of strategic variables and the equilibrium of subsidies or taxes levels, we are ready to assess the impacts on social welfare. By comparing social welfare obtained un-
under subsidy or tax regime, we summarize government’s preference orderings over roles (subsidy versus tax) in Proposition 3 (straightforward calculations and numerical examples with table A-2 and A-3 are in the Appendix).

**Proposition 3**: Suppose that a home and a foreign firm both export to a third-country market under subsidy or tax regime. Then, government’s preference orderings over roles are as follows:

- If the goods are substitutes, \( SW_i^C > SW_i^P > SW_i^B > SW_i^Q \)
- If the goods are complements, \( SW_i^B > SW_i^Q > SW_i^C > SW_i^P \)

Contrast to Proposition 2, Proposition 3 suggests that when governments consider the level of welfare, they do not face a prisoners’ dilemma regardless of the nature of goods under the subsidy regime. As already explained with Proposition 1, the intuition behind Proposition 3 may be partly reversed from intuition behind Proposition 1. Since social welfare consists of \( SW_i = \pi_i - s_i x_i = p_i x_i \), from the government’s point of view, the higher both price and quantity are, the greater social welfare becomes. Instead, suppose that each government faces the choice of strategic problem for welfare, this may lead the firms to increase price (quantity), \( p_i^P < p_i^C \) \( x_i^Q < x_i^P \) with the export subsidy (tax) under Cournot (Bertrand) competition. Hence, regardless of what other firm chooses the strategic variable, the governments want to be chosen Cournot (Bertrand) competition with a higher price (quantity) by both firms.

Moreover, Proposition 3 suggests that from the aspects of governments under subsidy regime (i.e., \( SW_i^C - SW_i^B = 8b^5 - 4b^7 \)), Cournot competition is more efficient than Bertrand competition when the goods are substitutes, and vice versa when the goods are complements. For this, from the aspects of firms, the equilibrium could be Pareto superior (inferior) with government’s intervention of subsidy policy when the goods are substitutes (complements). Thus, the conflict of interests between governments and firms occur when goods are complements. In other words, depending on the nature of goods, the comparison of social welfare in equilibrium thus points out a channel through which imposing a subsidy would actually increase firm’s outputs or levying a tax would actually increase firm’s prices and enhance welfare. Consequently, if each government chooses Cournot competition in equilibrium when the goods are substitutes, there is no a conflict of interest between firms and government. This can be better because each firm’s prisoners’ dilemma problem is resolved by the government, while choosing the Bertrand competition is the dominant strategy for both firms regardless of the nature of goods. However, this conflict of interest between firms and government can occur if each government chooses Bertrand competition in equilibrium when the goods are complements. Hence, from the aspects of firms, the equilibrium could be Pareto superior with government’s intervention of subsidy policy when goods are substitutes.

Next, to address the government preference orderings over free trade and subsidy regimes, we compare social welfare under free trade with that under subsidy regime, respectively. Straightforward calculation yields as follows:

\[
\hat{SW}_i^C - SW_i^C = 4b^3 + 3b^4, \quad \hat{SW}_i^B - SW_i^B = -4b^3 + 7b^4 - 3b^5.
\]

This may justify that a general principle is that the incentive to intervene in international trade is greater under Cournot competition than under Bertrand competition.
Thus, we obtained the following results.

**Proposition 4:** Suppose that a home and a foreign firm both export to a third-country market. Then,

1. both governments prefer free trade when the goods are substitutes under Cournot competition, while both governments prefer subsidy regime when the goods are substitutes under Bertrand competition.
2. both governments prefer subsidy regime when the goods are complements under Cournot competition, while both governments prefer free trade when the goods are complements under Bertrand competition.\(^7\)

Finally, we turn to compare each firm’s profit under either free trade or subsidy regime. When comparing firms’ profit between free trade and subsidy regime, each firm prefers Cournot competition to Bertrand when the goods are substitutes, i.e., \(\pi_C^i > \hat{\pi}_C^i > \hat{\pi}_B^i > \pi_B^i\) (Another way of straightforward calculations is in the Appendix.). However, the more degree of imperfect complementarity is, the higher the firm’s profit becomes under free trade with Bertrand competition\(^8\). That is, if the degree of imperfect complementarity falls into range \(b \in (0, -0.81]\) \(b \in [-0.82, -0.87]\), then \(\pi_C^i > \hat{\pi}_B^i > \hat{\pi}_C^i > \pi_B^i\) \((\pi_C^i > \hat{\pi}_B^i > \hat{\pi}_C^i > \pi_C^i\)). Moreover, if the degree of imperfect complementarity falls into range \(b \in [-0.88, -0.97]\) \(b \in [-0.98, -0.99]\), then \(\hat{\pi}_B^i > \pi_C^i > \pi_B^i > \hat{\pi}_C^i\) \((\hat{\pi}_B^i > \pi_B^i > \pi_C^i > \hat{\pi}_C^i\)). Hence, when the goods are complements, whether bilateral export intervention is profit increasing compared with free trade, depends on the degree of product differentiation. This observation leads to the result.

**Proposition 5:** Suppose that a home and a foreign firm both export to a third-country market when each government finances the export subsidies for the each firm or each government exists under free trade. Then, regardless of government’s trade policy, each firm’s profit is greater under Cournot competition than under Bertrand competition when the goods are substitutes. However, when the goods are complements, the profit of firms depends on both the competition mode and the degree of imperfect complementarity.

5 Concluding Remarks

Incorporating the third-market model into strategic export policy, we have demonstrated the endogenous choice of strategic variables for prices or quantities. Unlike the industrial organization context, we have suggested that choosing Bertrand competition is the dominant strategy for both firms regardless of the nature of goods, which faces a prisoners’ dilemma where both countries are worse off in Bertrand competition in subsidy regime than under Cournot competition. However, from the perspective of the government, Cournot competition is more efficient than Bertrand competition when the goods are substitutes, and vice versa when the goods are complements. These results may provide economic implications that from the aspects of firms, the equilibrium could be Pareto superior (inferior) with government’s intervention of subsidy policy when the goods are substitutes (complements). Thus, the

\(^7\)Clarke and Collie (2008) showed that focusing on only substitutes, social welfare in export taxes is always higher than that under free trade.

\(^8\)Straightforward calculations are in the Appendix.
conflict of interests between governments and firms occur when goods are complements. Moreover, our result may justify that when the goods are substitutes, a general principle might be that the incentive to intervene in international trade is greater under Cournot duopoly than under Bertrand duopoly.

We conclude by discussing the limitations of our paper. We have used the simplifying assumption that one home and one foreign firm are symmetric. By making this assumption, we do not take into account any cost or demand difference that may arise from the subsidy regime that occurs between one home firm and one foreign firm. Moreover, in this paper, it is assumed that symmetric subsidies or taxes occur in equilibrium. However, there can be existed in the international trade that the optimal domestic response to a foreign export subsidy is to retaliate with (partial) countervailing duties. If countervailing duties and import tariffs are set in different ways and for different purposes, we need to re-examine the relationship between countervailing duties, foreign export subsidies and import tariffs under imperfect competition (e.g. Collie, 1991; Wang, 2004). Finally, we did not extend our results by considering nonlinear demand structures. The extension of our model in these directions is left for future research.

References


Appendix

*Table A-1*: Comparison of Profits under Subsidy Regime

<table>
<thead>
<tr>
<th>Substitutes</th>
<th>Complements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>$\pi^c_i - \pi^t_i$</td>
</tr>
<tr>
<td>0.01</td>
<td>-2.55962E-06</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.025217757</td>
</tr>
<tr>
<td>0.3</td>
<td>-1.803046554</td>
</tr>
<tr>
<td>0.5</td>
<td>-10.6604901035</td>
</tr>
<tr>
<td>0.7</td>
<td>-25.65734435</td>
</tr>
<tr>
<td>0.9</td>
<td>-30.16991817</td>
</tr>
<tr>
<td>0.99</td>
<td>-22.35095418</td>
</tr>
</tbody>
</table>

*Table A-2*: Comparison of Social Welfare under Subsidy Regime with $b \in (0, 1)$
<table>
<thead>
<tr>
<th>$b$</th>
<th>$SW_i^C$</th>
<th>$SW_i^P$</th>
<th>$SW_i^B$</th>
<th>$SW_i^Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.247518564</td>
<td>0.247518564</td>
<td>0.247518564</td>
<td>0.247518564</td>
</tr>
<tr>
<td>0.1</td>
<td>0.226701830</td>
<td>0.226701830</td>
<td>0.226701543</td>
<td>0.226701543</td>
</tr>
<tr>
<td>0.3</td>
<td>0.187806353</td>
<td>0.187806347</td>
<td>0.187742269</td>
<td>0.187742264</td>
</tr>
<tr>
<td>0.5</td>
<td>0.155124654</td>
<td>0.155123470</td>
<td>0.154269972</td>
<td>0.154268795</td>
</tr>
<tr>
<td>0.7</td>
<td>0.125269100</td>
<td>0.125204490</td>
<td>0.119705572</td>
<td>0.119643831</td>
</tr>
<tr>
<td>0.9</td>
<td>0.095581945</td>
<td>0.092069913</td>
<td>0.064832647</td>
<td>0.062450457</td>
</tr>
<tr>
<td>0.99</td>
<td>0.081595264</td>
<td>0.030433523</td>
<td>0.009478756</td>
<td>0.003535401</td>
</tr>
</tbody>
</table>

Table A-3: Comparison of Social Welfare under Subsidy Regime with $b \in (-1, 0)$

<table>
<thead>
<tr>
<th>$b$</th>
<th>$SW_i^B$</th>
<th>$SW_i^Q$</th>
<th>$SW_i^C$</th>
<th>$SW_i^P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.01</td>
<td>0.252518939</td>
<td>0.252518939</td>
<td>0.252518939</td>
<td>0.252518939</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.277080015</td>
<td>0.277080015</td>
<td>0.277079664</td>
<td>0.277079664</td>
</tr>
<tr>
<td>-0.3</td>
<td>0.348783227</td>
<td>0.348783217</td>
<td>0.348664214</td>
<td>0.348664204</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.465379611</td>
<td>0.465370410</td>
<td>0.462809917</td>
<td>0.462806386</td>
</tr>
<tr>
<td>-0.7</td>
<td>0.709858236</td>
<td>0.709492110</td>
<td>0.678331574</td>
<td>0.677981709</td>
</tr>
<tr>
<td>-0.9</td>
<td>1.816056964</td>
<td>1.749328345</td>
<td>1.231820299</td>
<td>1.186558686</td>
</tr>
<tr>
<td>-0.99</td>
<td>16.23745749</td>
<td>6.056271139</td>
<td>1.886272491</td>
<td>0.70354473</td>
</tr>
</tbody>
</table>

Table A-4: Comparison of Profits under Subsidy Regime and Free Trade with $b \in (0, 1)$

<table>
<thead>
<tr>
<th>$b$</th>
<th>$\pi_i^C$</th>
<th>$\hat{\pi}_i^C$</th>
<th>$\pi_i^B$</th>
<th>$\pi_i^P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.247531</td>
<td>0.247519</td>
<td>0.247519</td>
<td>0.247506</td>
</tr>
<tr>
<td>0.1</td>
<td>0.227841</td>
<td>0.226757</td>
<td>0.226643</td>
<td>0.225568</td>
</tr>
<tr>
<td>0.3</td>
<td>0.196656</td>
<td>0.189036</td>
<td>0.186319</td>
<td>0.179294</td>
</tr>
<tr>
<td>0.5</td>
<td>0.177285</td>
<td>0.160000</td>
<td>0.148148</td>
<td>0.134986</td>
</tr>
<tr>
<td>0.7</td>
<td>0.165919</td>
<td>0.137174</td>
<td>0.104420</td>
<td>0.090378</td>
</tr>
<tr>
<td>0.9</td>
<td>0.160642</td>
<td>0.118906</td>
<td>0.043497</td>
<td>0.038575</td>
</tr>
<tr>
<td>0.99</td>
<td>0.160006</td>
<td>0.11856</td>
<td>0.004926</td>
<td>0.004834</td>
</tr>
</tbody>
</table>

Table A-5: Comparison of Profits under Subsidy Regime and Free Trade with $b \in (-1, 0)$
### Complements

<table>
<thead>
<tr>
<th></th>
<th>$\pi_i^C$</th>
<th>$\tilde{\pi}_i^B$</th>
<th>$\tilde{\pi}_i^C$</th>
<th>$\hat{\pi}_i^B$</th>
<th>$\hat{\pi}_i^C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.01</td>
<td>0.252532</td>
<td>0.252519</td>
<td>0.252519</td>
<td>0.252506</td>
<td></td>
</tr>
<tr>
<td>-0.1</td>
<td>0.278472</td>
<td>0.277148</td>
<td>0.277008</td>
<td>0.275695</td>
<td></td>
</tr>
<tr>
<td>-0.3</td>
<td>0.365093</td>
<td>0.351067</td>
<td>0.346021</td>
<td>0.333088</td>
<td></td>
</tr>
<tr>
<td>-0.5</td>
<td>0.528926</td>
<td>0.480000</td>
<td>0.444444</td>
<td>0.407202</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>-0.81</td>
<td>1.345971</td>
<td>1.206458</td>
<td>0.706165</td>
<td>0.698253</td>
<td></td>
</tr>
<tr>
<td>-0.82</td>
<td>1.404497</td>
<td>1.271454</td>
<td>0.722171</td>
<td>0.718184</td>
<td></td>
</tr>
<tr>
<td>-0.83</td>
<td>1.467281</td>
<td>1.344093</td>
<td>0.748805</td>
<td>0.730514</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>-0.87</td>
<td>1.770452</td>
<td>1.746363</td>
<td>0.895182</td>
<td>0.783147</td>
<td></td>
</tr>
<tr>
<td>-0.88</td>
<td>1.888825</td>
<td>1.862212</td>
<td>0.946759</td>
<td>0.797194</td>
<td></td>
</tr>
<tr>
<td>-0.89</td>
<td>2.057185</td>
<td>1.961844</td>
<td>1.007617</td>
<td>0.811622</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>-0.97</td>
<td>7.444441</td>
<td>3.193907</td>
<td>2.947374</td>
<td>0.942596</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>-0.98</td>
<td>11.14815</td>
<td>4.280527</td>
<td>3.431897</td>
<td>0.961169</td>
<td></td>
</tr>
<tr>
<td>-0.99</td>
<td>22.25926</td>
<td>8.280291</td>
<td>3.698936</td>
<td>0.980296</td>
<td></td>
</tr>
</tbody>
</table>

14