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Abstract
Relying on Pigou's view, environmental taxes increase the costs of polluting activities reflecting in this way the true social cost imposed to society by the caused environmental damage by these activities. The total pollution cost (TPC) is defined by adding up the marginal abatement (MAC) and the marginal damage (MD) costs. That is the random variable TPC includes the social costs associated with pollution. We relate this with contaminated locations and propose a weighted location differentiated tax and a corresponding index that adjusts taxation to the damages caused. It is clear that the value of the expected total pollution (social) cost, E(TPC), would be of interest and therefore we proceed to the evaluation through the use of the γ-order Generalized Normal. The value of the variance, Var(TPC), is also evaluated and we provide a generalized form of the E(TPC) as far (i) the form of TPC and (ii) the probability density function.

Keywords: Weighted-location adjusted differential tax; pollution related social cost; expected value; technology; probability density function.

Keywords: C02; C60; Q50; Q53; Q58.

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1. Introduction

In the past, command and control regulations (like limiting the use of specific fuels or demanding certain pollution sources to use specific methods) dominated environmental policies with market based instruments (like taxes and tradable permits) to dominate over the last decades. Environmental taxation relies on Pigou's concept of increasing polluters' private costs to a level that includes the associated true social costs imposed to the society by their activities and the resulting related environmental damages.

Economic theory indicates that the optimal tax rate is determined where marginal abatement cost (MAC) equals to marginal damage cost (MD) of pollution to be abated. In a first best policy taxes should be differentiated between pollution sources according to the size of their resulting damage costs. A second best policy relies on the imposition of a high uniform tax rate. Halkos (1993) showed that moving from the first best optimum to a uniform tax rate does make a difference. Specifically and in the case of the acid rain problem in Europe it was shown that the costs of moving from the first-best to the imposition of a high uniform tax rate may not differ so much across countries but may be quite different within countries.

Pollution control and damage cost functions are non-linear and their exact shapes are usually unknown (Halkos and Kitsos, 2005; Halkos and Kitsou, 2014). At the same time, environmental effects are associated with significant irreversibilities interacting often in a very complex way with uncertainty. This complexity becomes even worse when taking into account the very long-run character of many environmental problems. Uncertainties in abatement and damage cost functions affect policy design in various ways. When marginal abatement costs are known and constant, the policy maker of the environmental issues (for instance the local authorities) can minimize social cost by introducing a pollution tax that equalizes marginal abatement and damage costs.

But firms do not have always an incentive to reveal their true abatement costs.[1] A weighted location adjusted differentiated taxation is introduced, based on the principle that when pollution is above "an optimal and accepted level" more taxation has to be imposed, while if it is below there is a chance of less taxation. In this way a new index to adjust taxation to the damage caused is proposed.

In the absence of information about costs, the level of emissions taxes needed to achieve a target level of pollution abatement is unknown. This problem can be overcome by using an iterative procedure in which tax is adjusted. The tax that its tax system results in the social optimal pollution level is the differential tax. With differential taxation, the marginal emission tax paid by firm i is always equal to marginal damage costs and thereby minimizing social costs. The reason why this tax system results in the social optimal pollution level is that the firms -faced with a tax level that depends on emissions of firms- have an incentive to share information with respect to their abatement cost.

The analysis becomes more complicated when the abatement costs are stochastic, i.e. developed around it a probabilistic randomness. In this case or when we have changes in the marginal abatement costs, specific environmental policies are required, because the results from the changes of the Pigouvian taxation may be

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[1] Estimation of damage cost functions is much more complicated compared to abatement costs, as the influences of pollution cannot be identified with accuracy and there are many cases where it takes a long time to realize the effects of the damage imposed. To extract damage estimates in the case of acidification and the related transboundary pollution nature, a model taking account of the distribution of the externality among various countries (victims) is needed. As it is difficult to have a direct estimate the damage function its parameters may instead be inferred assuming countries equate national MD with national MAC and where restrictions on the derivatives of the damage cost function are significant. In this way the damage function may be "calibrated" assuming that national authorities act as Nash partners in a non-cooperative game with the rest of the world, taking as given deposits originating in the rest of the world (Hutton and Halkos, 1995; Halkos, 1996).

considered obsolete. Marginal abatement costs may change over time, by changing the innovative standards in the industry and by adopting the rapidly evolving new technologies. It is worth mentioning that adoption of new technologies reduces or aims to reduce emissions.

Given innovation outcome (X), the Total Pollution Cost (TPC) is defined by the sum of the marginal abatement (MAC) and the marginal damage (MD) costs. That is the random variable TPC includes the social costs associated with pollution. In this paper we evaluate the expected value of TPC and introduce the estimation of its variance. Specifically, choosing as TPC the general form TPC= $(\kappa X+\lambda)^2$ (with κ, λ constants and X the introduced technology) coming from the γ-order generalized normal distribution[2] we provide a generalization of the E(TPC) both in the form of TPC and the probability density function. The introduced technology represented by X is related with the distribution described by a *shape* parameter, a *location* parameter (the center of the pollution) and a *scale* parameter (the variance of pollution concentration around the center of pollution). In this way we propose a weighted location differentiated taxation to existing tax systems and a corresponding ratio to provide us with an index adjusting taxation to the damage imposed.

The structure of the paper is as follows. Section 2 reviews the relative existing literature while section 3 explains and defines the proposed weighted location adjusted differential taxation. Section 4 discusses the use of the appropriate distribution and the evaluation of the expected value and the variance of the total pollution social cost. The last section concludes the paper and refers to the associated policy implications of the proposed tax differentiation.

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[2] For more information on the γ-order generalized normal distribution see Appendix 1 and Kitsos and Tavoularis (2009), Kitsos and Toulias (2010) and Kitsos et al. (2012).

2. A brief review of existing relative literature

As mentioned, economic efficiency demands that the marginal cost of emissions reduction equals to the marginal damage costs imposed. It is obvious that the problem of relating taxation and pollution has been considered by many researchers. The target is an equitable sharing of charges on polluters. Such a model could be used for example in harbors with heavy traffic, where the entrance or exit of ships pollutes the environment corresponding to the quality of the vessel. Therefore, the technology used, will be associated with the taxation system. A new tax, which depends on innovation and at the same time, is above the expectations of a Pigouvian analysis was proposed by Requate (2004). The proposal of Requate under stochastic innovation has the same importance as the analysis of the other environmental measures.

As the distance and the location of GHG emissions' sources are not related to the location of the environmental damages and degradation, they are considered as uniformly mixing pollutants[3] with their concentration levels to be invariant from place to place. In the case of uniformly mixing pollutants the pollution levels depend on their total emissions levels. Similarly, in the case of non-uniformly mixing pollutants locations of their emission sources are significant in determining the spatial distribution of ambient levels of pollution (Perman et al., 2003).

In the case of non-uniformly mixed damage efficiency demands that the marginal costs of emissions control should be different across pollution sources and should be determined by the damage caused (Tietenberg, 2006). This may be accomplished by taking into consideration the associated marginal damages imposed

[3] Uniformly mixing pollutants take place when physical processes function to disperse them to the point where their spatial distribution is uniform (Perman et al. 2003, p. 178).

across sources. Coping with this issue we extend the existing results and propositions and introduce the weighted location adjusted tax**.**

In cases of existing regulations implementations are performed as spatially uniform with undifferentiated policies and with emissions being penalized at the same tax rate and permit prices (Fowlie and Muller, 2013). Theoretically, market based policies may tackle non-uniformly mixed pollutants (like $NO_{X,}$ SO_2) with the optimal tax to be calculated by the marginal damage imposed. Taxes are different by pollution source for different levels of damage imposed. Differentiation will be profitable depending on the variation in damages caused across sources as well as the slopes of MACs (Mendelsohm, 1986; Halkos, 1993, 1994; Fowlie and Muller, 2013).

In general, almost all tax systems involve differentiated tax rates among the various sectors (industry, commerce, households etc). In the case of uniform taxation the same marginal abatement costs are assumed with the economy in total to use the cheapest pollutant control methods in each sector. Reducing the tax rate in a sector may impose increases in the taxes imposed in the other sectors to attain the imposed environmental target. This implies that any deviation from uniform taxation may impose excess costs. Thus differentiated taxation among different sectors of an economy is optimal due to, among others, initial tax distortions, distributional concerns, trade terms and leakage motives (Böhringer and Rutherford, 2002).

That is why we adopt the generalized γ-order Normal distribution for the analysis bellow. This distribution is based on an extra, shape parameter γ, which under different values of γ coincides with a number of well known distributions. Among them, and as it will be shown in the next section, with γ=1 is the Uniform distribution, with γ=2 is the well known Normal distribution and with γ=infinity, practically very large (or very small) coincides with the Laplace distribution.

Pre-existing tax distortions influence the efficiency effects of newly imposed environmental taxes. Among others, Bovenberg and van der Ploeg (1994), Bovenberg and Goulder (1996) and Goulder et al. (1997) propose that tax interaction leads to higher efficiency costs (net of environmental benefits) of environmental taxation compared to a first-best case leading to optimal second-best environmental tax rates lower than the Pigouvian rate. At the same time, revenues raised by the imposition of environmental taxes may be used to reduce the distortions of the existing taxes (Terkla 1984; Oates 1995) offsetting in this way part of potentially negative tax interaction effects (Goulder 1995).

3. The weighed location adjusted differential tax

The way we move on is by defining the "weighted location differential tax". Theoretically this tax will be non-linear (since high pollutants should face appropriate taxes i.e. exponential greater and not linear) and non–time consistent (as pollution is not time constant depending for instance on weather conditions, amount of production, etc). This new indicator for environmental policy is based on a generalization of the differential taxation (Halkos, 1993, 1994; Kim and Chang, 1993; Mc Kitrick, 1999) and provides another look of differentiation in taxation, based on the location and the assumed distribution the new introduced technologies follow (see the definition of TPC above).

Our argument is that around the pollution center (*source of pollution*) the pollution is distributed according to a (possible) statistical model, related with the actual situation. In such a case it may be uniformly distributed i.e. in a distance, left or right from the pollution center the pollution to remain constant. That might be a helpful, mathematically, assumption, but it is difficult to be true. Another approach is to consider a normally distributed pollution, with the mean being at the pollution center, so plus or minus it one standard deviation concentrates approximately the 0.68 of the pollution. In cases of 0.99 levels of pollution concentration we may consider a $\pm 3\sigma$ confidence interval (or $L=6\sigma$) as essential. This is near to be true, as the tails contain a very small probability level to allow a pollution influence.

Similarly, the Laplace distribution offers a solution to provide a "strong" pollution center and fat tails. All these three distributions are special cases of the γ-order Generalized Normal distribution[4]. In this particular distribution, the third involved parameter, the shape one, called γ, taking all real values, but not within [0, 1], offers a number of different distributions with fat tails mainly. With the value of $\gamma=1$, it is reduced to Uniform; with the value $\gamma=2$ is reduced to Normal; with the value of γ "infinity" practically very large is Laplace. In Proposition 1, in section 4, we obtain the appropriate evaluations for the total pollution (social) cost (TPC).

Now, having the expected value and the variance of the total pollution cost, E(TPC) and Var(TPC), approximate 95% confidence intervals (CI) can be obtained – which are precise only in the Normal case of the form

$$CI(TPC) = (E(TPC) -2\, (Var(TPC))^{0.5} , E(TPC) + 2(Var(TPC)^{0.5}) \quad (1)$$

The length of this .95 confidence interval is $L=4[Var(TPC)]^{0.5}$. Similarly and in the case of a 99% CI, as mentioned, we work with the "distance" D of the end points of $\pm 3\sigma$ (or 6σ) CI with $D=6[Var(TPC)]^{0.5}$ a kind of *Quality Control criterion* of the pollution. That is how far from the center of the pollution the area is contaminated with a 99% probability.

[4] See Appendix 1 and Kitsos and Tavoularis (2009), Kitsos and Toulias (2010) and Kitsos et al. (2012).

When pollution is at the optimal level the optimal length as above is L^* or D^*. Therefore the ratio

$$\Delta(Tax) = \frac{L}{L^*} \qquad \textbf{or} \qquad \Delta(Tax) = \frac{D}{D^*} \tag{2}$$

is essential and can be a fair index to provide a weighted location differentiated taxation, as the case $\Delta(\mathrm{Tax})>1$ is expected to be faced in existing tax system. The tax burden will be determined using expression (2) which depends on the optimal level of pollution L^* or D^* based on the choice of the appropriate new technology X and the corresponding TPC. More simply, with L^* and D^* we denote the optimal cases where the variance of TPC that is the variability of pollution is as expected and as a consequence the confidence limits are also expected. L and D may be the real length of the confidence intervals for 95% and 99% respectively. That is the corresponding ratio as in (2) provides researchers with an index adjusting taxation to contamination caused. If the evaluated in each case L and D are less that the optimal then the tax burden will be less. In such a way a source of pollution (industry, firm, etc) has an incentive to look for more efficient control methods.

This idea can be also adopted when the pollution centre (that is the pollution source point) might be moving, as an aeroplane or a boat. In such a case around the pollution centre a “sphere” of pollution is created of the form

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2 \tag{3}$$

With R being the radius of the sphere and K(a,b,c) the pollution centre. If $R>R^*$, with R^* being the optimal pollution level radius, a weighted location differentiated taxation is needed, in the sense that a radius of pollution R is accepted, based on the adopted technologies, but beyond that, there is a problem. In section 4 we proceed with the evaluation of the expected value and the variance of the total pollution cost.

4. Adopting the appropriate distribution

The easiest way, as far as the mathematical calculations are concerned, despite its unrealistic character, is to assume that the stochastic variable X –as a result of the R&D procedure, is uniformly distributed in the interval $\left[\frac{1}{2}-\delta, \frac{1}{2}+\delta\right]$, say, recalling the definition of the Uniform distribution. This means that in this research we suppose eventually the variable TPC is derived from the Uniform distribution, i.e. $u(\frac{1}{2}-\delta, \frac{1}{2}+\delta)$ implying a uniform density function for X of the form

$$f(X)=\frac{1}{2\delta} \quad \text{for} \quad X\in\left[\frac{1}{2}-\delta, \frac{1}{2}+\delta\right] \tag{4}$$

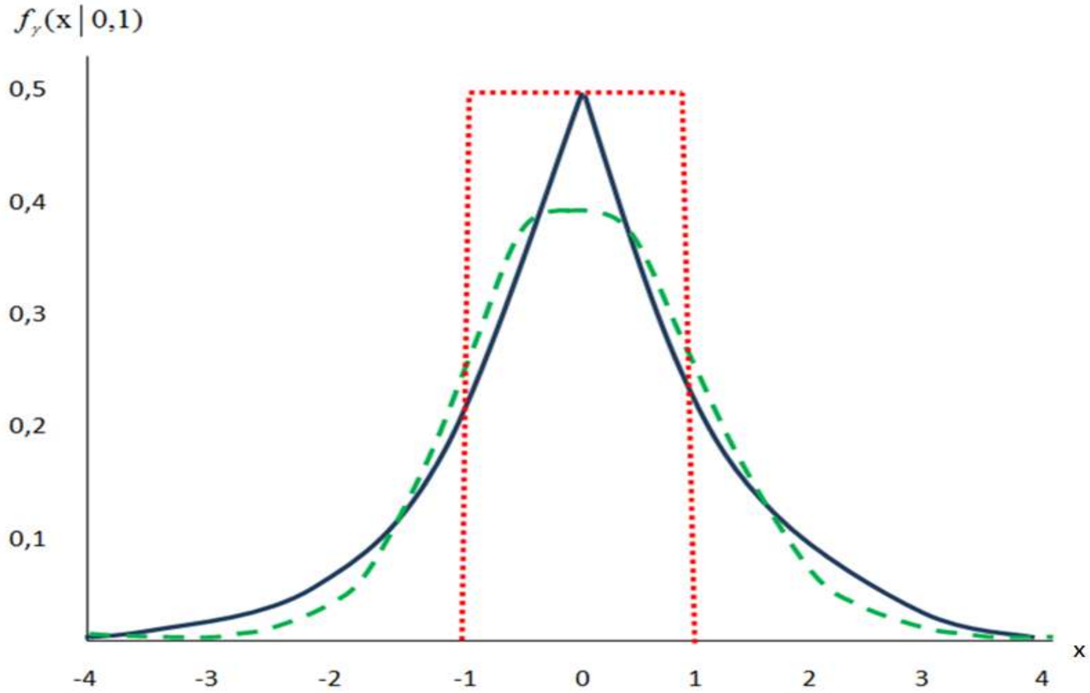
From the definition of the expected value the pollution related t- social cost for the linear tax, $\mathrm{E}\left[TPC_{t_l}\right]$, is equal to $\int_{\frac{1}{2}-\delta}^{\frac{1}{2}+\delta} TPC\ f(x)dx$. Any general form of TPC=(κX+λ)^2 is presenting the appropriate area for TPC.

An extension of the calculation of expected value is needed as it can be either normal with the known tails or a "sharp" one around 'center' with 'heavy tails', a Laplace distribution among others. Therefore the γ-order generalized Normal distribution was adopted[5] as the extension of the Uniform distribution. The expected value of TPC can be evaluated and it can be seen that that the distribution is not only the Uniform but the $\mathrm{N}_\gamma(\mu,\sigma^2)$.[6] Figure 1 clarifies the generalization and represents the relation between Uniform, Normal and Laplace. This distribution regards a number of other distributions which are with 'fat tails'[7] and can be used in various

[5] See Kitsos and Tavoularis (2009), Kitsos et al. (2012), Halkos and Kitsou (2014).
[6] See appendix 1.
[7] This is why we are referring to "a family of distributions".

economic analyses like for instance in stock markets. So, the following results are proposed for the form $(\kappa g+\lambda)^2$ and $f_\gamma(x;\mu,\Sigma)$ letting X represent a random variable describing innovation.

Figure 1: Graphical presentation of the relationship between Uniform, Normal and Laplace



Proposition 1*:* If $X \sim N_\gamma(\mu,\sigma^2)$ it holds that:

$$E\left[(\kappa X+\lambda)^2\right]=\left(\frac{\gamma}{\gamma-1}\right)^{2\frac{\gamma-1}{\gamma}}\frac{\Gamma(3\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})}(\kappa\delta)^2+\kappa\mu(\kappa\mu+2\lambda)+\lambda^2 \quad (5)$$

$$Var((\kappa X+\lambda^2))=\left(\frac{\gamma}{\gamma-1}\right)^{4\frac{\gamma-1}{\gamma}}(\kappa\delta)^4\left[\frac{\Gamma(5\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})}-4\frac{\Gamma^2(3\frac{\gamma-1}{\gamma})}{\Gamma^2(\frac{\gamma-1}{\gamma})}\right]-(\kappa\mu)^3(\kappa\mu+4\lambda)$$

$$+2(\kappa\delta)^2\left[2\lambda^2-(\kappa\mu)^2-2\kappa\lambda\mu\right]\left(\frac{\gamma}{\gamma-1}\right)\frac{2\gamma-1}{\gamma}\frac{\Gamma(3\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})} \quad (6)$$

Proof in Appendix 2.

With different values of κ and λ a number of calculations for the corresponding TPC can be obtained. Next we present a number of examples.

***Example 1**:* Let us assume that TPC = $(1/4-3/8X)^2$. Then it holds:

$$\mathrm{E}[\mathrm{T}PC_{\hat{t}_\ell;\gamma}]=\tfrac{1}{4}+9(\tfrac{\gamma}{\gamma-1})^{2\frac{\gamma-1}{\gamma}}\frac{\Gamma(3\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})}\delta \tag{7}$$

$$\mathrm{Var}\left(TPC_{\hat{t}_\ell;\gamma}\right)=\tfrac{1}{4}(\tfrac{\gamma}{\gamma-1})^{4\frac{\gamma-1}{\gamma}}(6\delta)^4\left[\frac{\Gamma(5\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})}-4\frac{\Gamma^2(3\frac{\gamma-1}{\gamma})}{\Gamma^2(\frac{\gamma-1}{\gamma})}\right]+954\delta^2(\tfrac{\gamma}{\gamma-1})^{2\frac{\gamma-1}{\gamma}}\frac{\Gamma(3\frac{\gamma-1}{\gamma})}{\Gamma(\frac{\gamma-1}{\gamma})}+13\tfrac{27}{4} \tag{8}$$

Example 2: Based on Example 1, for this particular TPC, it holds that the expected value and variance of TPC can be evaluated for the Uniform, Normal and Laplace distributions as:

$$\mathrm{E}[\mathrm{T}PC_{\hat{t}_\ell;\gamma}]=\begin{cases}\tfrac{1}{4}+3\delta, & Uniform, \gamma=1,\\ \tfrac{1}{4}+9\delta, & Normal, \gamma=2,\\ \tfrac{1}{4}+18\delta, & Laplace, \gamma=\pm\infty,\end{cases} \tag{9}$$

$$\mathrm{Var}\left(TPC_{\hat{t}_\ell;\gamma}\right)=\begin{cases}13\tfrac{27}{4}+318\delta-\tfrac{11}{45}(36\delta)^2, & \gamma=1,\ Uniform\\ 13\tfrac{27}{4}+954\delta-(18\delta)^2, & \gamma=2,\ Normal\\ 13\tfrac{27}{4}+1908\delta+2(6\delta)^2, & \gamma=\pm\infty,\ Laplace\end{cases} \tag{10}$$

***Example 3*:** From (9) it obviously holds that the quantity $\mathrm{E}\left[TPC_{t_l},_{\gamma}\right]$ in the case of Uniform distribution is less than the corresponding Normal distribution, which is less that the corresponding Laplace distribution. That is:

$$\mathrm{E}\left[TPC_{t_l},_1\right]<\mathrm{E}\left[TPC_{t_l,2}\right]<\mathrm{E}\left[TPC_{t_l},_{\pm\infty}\right]$$

For (10) and for $0<\delta<49.074$ it holds that:[8]

$$Var^U(TPC)<Var^N(TPC)<Var^L(TPC)$$

Figure 2 shows that with γ=1 (the case of uniform) the expected value is less than in the case of γ=2 (the case of normal) and flatter compared to the other two cases. Similarly the results for the comparison between γ=2 (Normal) and γ=±∞ (the case of Laplace) show that Laplace is sharper among them. This implies that changing D the expected value of TPC in the case of the uniform distribution is more stable compared to the other cases. With Laplace being the most sensitive in changing parameter δ, a small change in δ causes a sharp change in the expected value.

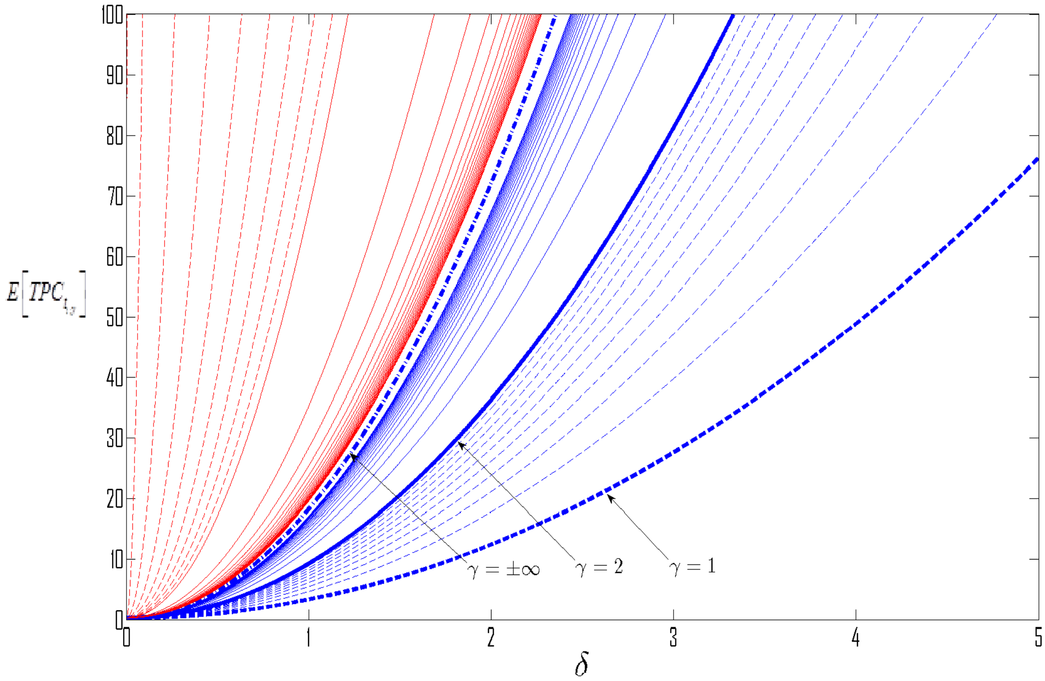


Figure 2: Graphical presentations of $\mathrm{E}[\mathrm{TPC}_{t_\ell;\gamma}] = \mathrm{E}(1/4 - 3/8X)^2$ with $X_\gamma \sim \mathcal{N}_\gamma(\mu, \delta^2)$ as function of the scale parameter δ, for different values of the parameter γ (blue is for $\gamma \geq 1$ while red is for $\gamma < 0$).

5. Discussion and policy implications

Environmental taxes should be targeted to the pollutants and should be related to the environmental damage caused. Without any government intervention countries (firms) will not take into consideration any environmental damage caused as this may be either spread across different regions or countries (as in the case of transfrontier pollution) or may be accumulated (stock pollution). For instance GHG emissions from one location may play an important role in global climatic changes. The way to cope with the problem is to tax directly the environmental damage costs due to the damages imposed.

Specifically, in this research we have considered a general distribution that the random variable X of the new technologies adopted by a firm, and therefore the total pollution cost (TPC) follows it covering 3 different lines of thought: a uniform approach of pollution around the center of pollution, adopting the new technologies; a normal that is most of the pollution around the centre; and a ‘sharp’ portion of pollution around the centre, i.e. the Laplace distribution.

Due to this general distribution a weighted location differential tax was introduced. The proposed tax is differentiated according to the “level of distance” from the centre of pollution i.e. how far from it has the area being contaminated due to this particular source of pollution. As a conclusion it is very clear that we are depending on the assumption of the distribution for the (stochastic) TPC variable.

In this paper as shown the application of the γ-ordered generalized Normal provides to the researcher the option to choose among three distributions: The Uniform, Normal and Laplace. That is among no-tails, normal tails, and fat tails. The decision is also based on the value of δ we choose at the first step – ‘how far’ from the ‘origin of pollution’ we go.

The question of what is the shape of the distribution to be followed is important. That is why the expected value of the total pollution cost, E(TPC), can be related to the appropriately calculated variance, Var(TPC), so that approximate .95 confidence interval of the form $\mathrm{E}(TSC) \pm 2\sqrt{Var(TPC)}$ to be evaluated, while for a .99 approximate confidence interval the factor 2 is replaced by 3. As the TPC includes the social cost related to pollution the greater expected value has to be associated with higher taxation under the weighted location tax while the larger the variance the larger the area polluted and affected socially. Therefore the taxation system should take into consideration these issues.

Due to difficulties in having available reliable direct cost estimates this approach may be used with various sensitivity scenarios and existing sensitivity maps of ecosystems applied to various indirect effects of depositions (see for instance Kämäri et al., 1992). It is feasible for every country to estimate the area in a number of sensitivity classes with values determined by ecological criteria like geology, vegetation, soil type, rainfall amounts etc. For instance acidic depositions vary significantly with time and location.

If the relationship between source and receptor locations is not considered then the externality imposed will not be taken into examination. The externality is considered by the appropriate consideration of the transfer coefficients as provided by the co-operative programme for monitoring and evaluation of the long range transmission of air pollutants in Europe (European Monitoring and Evaluation Program, EMEP). Then mathematical models may be used by policy makers to define the optimal necessary emissions reductions for each pollution source (country) i and under the ecosystem sensitivity thresholds (see among others Halkos, 1994).

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Appendix 1
The γ-ordered Normal Distribution

We remind that the normal distribution $\mathrm{N}(\mu,\sigma^2)$, with mean μ and variance σ^2, is defined as:

$$f(x)=\frac{1}{(2\pi)^{\frac{1}{2}}\sigma}\exp\left\{-\frac{1}{2\sigma^2}(\chi-\mu)^2\right\} \tag{i}$$

The multivariate generalization for a multivariate random variable with p-conditions, μ mean and matrix covariance Σ is compared with (i) resulting to:

$$\varphi(\chi)=\frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{\frac{1}{2}}}\exp\left\{-\frac{1}{2}(\chi-\mu)^{\mathrm{T}}\Sigma^{-1}(\chi-\mu)\right\} \tag{ii}$$

We denote this with $N_p(\mu,\Sigma)$, $|\Sigma|$=det(Σ).

A more general form of the multivariate distribution was investigated with an extra shape parameter. Indeed Kitsos and Tavoularis (2009) introduced through Logarithm Sobolev Inequalities (LSI) a new family of univariate γ-ordered Normal distribution the $\mathrm{N}_\gamma^\rho(\mu,\Sigma)$, which generalizes the Normal Distribution $\mathrm{N}^\rho(\mu,\Sigma)$, through an additional parameter $\gamma\in\mathbb{R}-[0,1]$. The new generalized Normal distribution commonly referred as *γ-ordered Normal distribution*.

When $f(x)$ is the probability density function of a random variable $\mathrm{X}\sim\mathrm{N}_\gamma^\rho(\mu,\Sigma)$ then, compared with (ii) above, $f(x)$ is defined as:

$$f_\gamma(x;\mu\Sigma)=C_\gamma^p\,|\det\Sigma|^{-\frac{1}{2}}\exp\left\{-\frac{\gamma-1}{\gamma}[Q(x)]^{\frac{\gamma}{2(\gamma-1)}}\right\}\text{ with } x\in\mathbb{R}^\rho \tag{iii}$$

Where $Q(x)=(x-\mu)\Sigma^{-1}(x-\mu)^{\mathrm{T}}$ as in (ii) with the normality factor

$$C_\gamma^p=\pi^{-\frac{p}{2}}\frac{\Gamma(\frac{p}{2}+1)}{\Gamma(p\frac{\gamma-1}{\gamma}+1)}\left(\frac{\gamma-1}{\gamma}\right)^{p\frac{\gamma-1}{\gamma}} \tag{iv}$$

Where if we set γ=2, i.e. $\mathrm{N}_2^\rho(\mu,\Sigma)$ it follows that:

$$C_2^p=\pi^{-\frac{p}{2}}\frac{\Gamma(\frac{p}{2}+1)}{\Gamma(\frac{p}{2}+1)}\left(\frac{1}{2}\right)^p=(2\pi)^{-\frac{p}{2}}=\frac{1}{2\pi^{\frac{p}{2}}} \tag{v}$$

Theorem 1: It holds that the multivariate γ-ordered Normal distribution $\mathrm{N}_\gamma^\rho(\mu,\Sigma)$ for order values of γ=1,2 ±∞ coincides with

$$\mathrm{N}_\gamma^\rho(\mu,\Sigma)=\begin{cases}D^\rho(\mu) & \gamma=0\quad p=1,2\quad \textit{Dirac distribution}\\ U^\rho(\mu,\Sigma) & \gamma=1\quad \textit{Uniform distibution}\\ \mathrm{N}^\rho(\mu,\Sigma) & \gamma=2\quad \textit{Normal distibution}\\ L^\rho(\mu,\Sigma) & \gamma=\pm\infty\quad \textit{Laplace distibution}\end{cases}$$

Proof: In Kitsos et al. (2012, page 52).

Appendix 2
Proof of Proposition 1

We have seen that:

$$\mathrm{E}[(\kappa X+\lambda)^2]=\kappa^2\mathrm{E}[X^2]+2\kappa\lambda\mathrm{E}[X]+\lambda^2=\kappa^2\left(\mathrm{Var}(X)+\mathrm{E}^2[X]\right)+2\kappa\lambda\mathrm{E}[X]+\lambda^2,$$

Assuming $X\sim\mathcal{N}_\gamma(\mu,\delta^2)$ and using the variance of the γ-order Normal distribution, (see Kitsos and Toulias, 2010) we have that

$$\mathrm{E}[(\kappa X+\lambda)^2]=\kappa^2\left(\mathrm{Var}(X)+\mu^2\right)+2\kappa\lambda\mu+\lambda^2. \quad (7)$$

For the variance of $Y=(\kappa X+\lambda)^2$ we have

$$\begin{aligned}\mathrm{Var}(Y)&=\mathrm{E}[Y^2]-\mathrm{E}^2[Y]=\mathrm{E}[(\kappa X+\lambda)^4]-\mathrm{E}^2[(\kappa X+\lambda)^2]=\\&=\kappa^4\mathrm{E}[X^4]+4\kappa^3\lambda\mathrm{E}[X^3]+6(\kappa\lambda)^2\mathrm{E}[X^2]+4\kappa\lambda^3\mathrm{E}[X]+\lambda^4-\mathrm{E}^2[(\kappa X+\lambda)^2]=\\&=\kappa^4\mathrm{Kurt}(X)\mathrm{Var}^2(X)+6(\kappa\lambda)^2\mathrm{Var}(X)+6(\kappa\lambda\mu)^2+4\kappa\lambda^3\mu+\lambda^4-\mathrm{E}^2[(\kappa X+\lambda)^2]\end{aligned}$$

As $\mathrm{E}[X^3]=0$ ($\mathcal{N}_\gamma(\mu,\delta^2)$ is symmetric distribution i.e. has zero obliquity), thus eventually, the above equation can be written sequentially as

$$\begin{aligned}\mathrm{Var}(Y)=&\ \kappa^4\mathrm{Kurt}(X)\mathrm{Var}^2(X)+6(\kappa\lambda)^2\mathrm{Var}(X)+6(\kappa\lambda\mu)^2+4\kappa\lambda^3\mu+\lambda^4\\&-\left[\kappa^2\left(\mathrm{Var}(X)+\mu^2\right)+2\kappa\lambda\mu+\lambda^2\right]^2=\\&=\kappa^4\mathrm{Kurt}(X)\mathrm{Var}^2(X)+6(\kappa\lambda)^2\mathrm{Var}(X)+6(\kappa\lambda\mu)^2+4\kappa\lambda^3\mu+\lambda^4\\&-\kappa^4\left(\mathrm{Var}(X)+\mu^2\right)^2-4(\kappa\lambda\mu)^2-\lambda^4\\&-4\kappa^3\lambda\mu\left(\mathrm{Var}(X)+\mu^2\right)-2(\kappa\lambda)^2\left(\mathrm{Var}(X)+\mu^2\right)-4\kappa\lambda^3\mu=\\&=\kappa^4\mathrm{Kurt}(X)\mathrm{Var}^2(X)+6(\kappa\lambda)^2\mathrm{Var}(X)+6(\kappa\lambda\mu)^2+4\kappa\lambda^3\mu+\lambda^4\\&-\kappa^4\mathrm{Var}^2(X)-(\kappa\mu)^4-2\kappa^4\mu^2\mathrm{Var}(X)-4(\kappa\lambda\mu)^2-\lambda^4\\&-4(\kappa\mu)^3\lambda-4\kappa^3\lambda\mu\mathrm{Var}(X)-2(\kappa\lambda)^2\mathrm{Var}(X)-2(\kappa\lambda\mu)^2-4\kappa\lambda^3\mu\end{aligned}$$

And so

$$\mathrm{Var}(Y)=\kappa^4[\mathrm{Kurt}(X)-1]\mathrm{Var}^2(X)+[4(\kappa\lambda)^2-2\kappa^4\mu^2-4\kappa^3\lambda\mu]\mathrm{Var}(X)-(\kappa\mu)^3(\kappa\mu+4\lambda)$$

Since $X\sim\mathcal{N}_\gamma(\mu,\delta^2)$, the above equation can be written finally in the form of (6) using the variance and the convexity of the γ-order Normal distribution (For more details, see Kitsos and Toulias, 2010).

Appendix 3
Checking the differences in variances

Evaluating the differences between the variances of Uniform, Normal and Laplace distributions we may see which are positive or negative, so that to rearrange the order among them.

(A) Between Uniform and Normal (take the difference from (10))

$$12.96\delta^2 - 636\delta = 0 \Leftrightarrow$$

$$\delta(12.96\delta - 636) = 0 \Leftrightarrow \begin{cases} \delta = 0 \\ \delta = \dfrac{636}{12.96} = 49.074 \end{cases}$$

$$U - N = \begin{cases} > 0 & \delta < 0, \delta > 49.074 \\ < 0 & 0 < \delta < 49.074 \end{cases}$$

The Normal distribution is greater than the Uniform distribution when $\delta \in (0, 49.074)$ Otherwise, when $\delta > 49.074$, the Uniform distribution of TPC is greater than its Normal distribution.

$$Var^U(TPC) < Var^N(TPC)$$
$$0 < \delta < 49.074$$

(B) Between Laplace and Normal (take the difference from (10))

$$-954\delta - 396\delta^2 = 0$$

$$\delta(396\delta + 954) = 0 \Leftrightarrow \begin{cases} \delta = 0 \\ \delta = -\dfrac{954}{396} = -2.40 \end{cases}$$

$$N - L = \begin{cases} < 0 & \delta \in (-2.40, 0) \\ > 0 & \delta < -2.40 \quad or\ \delta > 0 \end{cases}$$

Because δ is taken always positive, we are interested in the case where δ>0, so the Normal distribution is grater that the Laplace distribution.

$$Var^N(TPC) < Var^L(TPC)$$

(C) Between Uniform and Laplace (take the difference from (10))

$$-1590\delta - 383.04\delta^2$$

$$= -\delta(383.04\delta + 1590) \Leftrightarrow \begin{cases} \delta = 0 \\ \delta = -\dfrac{1590}{383.04} = -4.15 \end{cases}$$

$$U - L = \begin{cases} > 0 & \delta \in (-4.15, 0) \\ < 0 & \delta < -4.15 \quad or\ \delta > 0 \end{cases}$$

So, for δ>0, the PSC Uniform distribution is less than the Laplace distribution.

$$Var^U(TPC) < Var^L(TPC)$$

From (A), (B) and (C) we have that for $0 < \delta < 49.074$ holds:

$$Var^U(TPC) < Var^N(TPC) < Var^L(TPC)$$