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# Consumer search behavior and willingness to pay for insurance under price dispersion 

Malakhov, Sergey

Pierre-Mendès France University

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Consumer search behavior and willingness to pay for insurance under price dispersion.


#### Abstract

When the increase in income reduces the time of search and increases prices of purchases, the increase in price can be presented as the increase in the willingness to pay for insurance. The optimal consumer decision represents the trade-off between the propensity to search for proficient insurance and marginal savings on insurance policy. Under price dispersion the indirect utility function takes the form of cubic parabola, where the saddle point represents the comprehensive insurance. The comparative static analysis of the saddle point of the utility function discovers the ambiguity of the departure from risk-neutrality. This ambiguity can produce the ordinary risk seeking behavior as well as mathematical catastrophes of Vebleneffect's imprudence and over prudence of family altruism. The comeback to risk aversion is also ambiguous and it results either in increasing or in decreasing relative risk aversion. The paper argues that the decreasing risk aversion results in the optimum quantity of money.


Keywords: consumer search, risk, insurance, real balances, Veblen effect, family altruism, mathematical catastrophe
JEL Classification: D11, D81.

## Introduction to indirect utility function of satisficing optimal decision

The analysis of consumption-leisure choice $U=U(Q, H)$ with respect to the wage rate $w$ and to the purchase price reduction and marginal savings got from the search, or to the value $\partial P / \partial S$, has discovered many interesting phenomena. The satisficing consumer decision becomes optimal in the sense that it equalizes marginal costs of search with its marginal benefit and that equality provides the maximization of the utility function (Malakhov 2014). The use of the truly relative price, i.e., purchase price $P$ with regard to the time of search $S$ or to the place of purchase, enables the explanation of some anomalies of economic behavior like endowment effect, sunk costs sensitivity, little pre-purchase search of big ticket items, and, finally, Veblen effect and money illusion. From the point of view of the utility function $U(Q, H)$ subject to $w / \partial P /\left.\partial S\right|_{\text {const }}=Q / \partial L / \partial S$, where the constraint represents the equality of marginal values of search $w \times \partial L / \partial S=Q \times \partial P / \partial S$, and the value $\partial L / \partial S$ represents the propensity to search, i.e., propensity to substitute labor $L$ by search $S$, the equilibrium price $P_{e}$ becomes equal to the total of consumers'
labor costs $w L$ and transaction cost $w S$, or, if we consider the household activity as a specific form of search, decreasing the final price, the total of labor costs and transformation costs $P_{e}=w(L+S)$ :

$$
\begin{equation*}
\frac{\partial U / \partial H}{\partial U / \partial Q}=-\frac{w}{\partial P / \partial S} \partial^{2} L / \partial S \partial H=-\frac{w}{T \partial P / \partial S}=\frac{w}{w(L+S)}=\frac{w}{P_{e}} \tag{1}
\end{equation*}
$$

where the value $T=1 / \partial^{2} L / \partial S \partial H$ represent the time horizon until the same purchase, or the commodity lifecycle.

Although the original values of the model $\partial P / \partial S$ and $\partial L / \partial S$ look unusual, their modeling tries not to forget the testament of A.Marshall, who told that "when a great many symbols have to be used, they become very laborious to any one but the writer himself" (Marshall 1920[1890], p.12). Sometimes such relative values are indispensable, especially when the original G.Stigler's assumption is used ( $\partial P / \partial S<0 ; \partial^{2} P / \partial S^{2}>0$ ) or when the behavior of the propensity to search is derived $\left(\left(\partial L / \partial S<0 ; \partial^{2} L / \partial S^{2}<0\right)\right.$. However, the understanding of these relative values can be simplified by the graphical illustration of the interrelation between implicit optimal decision and explicit satisficing decision (Fig.1):



Fig.1. Implicit optimal decision and explicit satisficing decision
The presentation of relatives values in absolute terms, $|\partial P / \partial S|$ and $|\partial L / \partial S|$ also facilitates their mathematical treatment without logical losses. This tactic enables to derive the marginal utility of money income and the marginal utility of money expenditures and to present marginal utilities of consumption and leisure with the respect to the given place of purchase ( $|\partial P / \partial S|$ const ) in more habitual manner (Malakhov 2013):

$$
\begin{align*}
& M U_{w}=\lambda ;  \tag{2.1}\\
& M U_{|\partial P / \partial S|}=-\lambda \frac{w}{|\partial P / \partial S|} \tag{2.2}
\end{align*}
$$

$$
\begin{align*}
& M U_{Q}=\lambda \frac{|\partial P / \partial S|}{|\partial L / \partial S|}=\lambda T \frac{|\partial P / \partial S|}{L+S}=\lambda \frac{P_{e}}{L+S}=\frac{\partial v}{\partial w} \frac{P_{e}}{T-H}  \tag{3.1}\\
& M U_{H}=\lambda \frac{w}{|\partial L / \partial S|} \partial^{2} L / \partial S \partial H=\lambda \frac{w}{|\partial L / \partial S|} \frac{1}{T}=\lambda \frac{w}{L+S}=\frac{\partial v}{\partial w} \frac{w}{T-H} \tag{3.2}
\end{align*}
$$

The analysis of the second order cross partial derivatives, i.e, the change in the marginal utility of received money income with the change of the place of purchase, or $\partial M U_{w} / \partial|\partial P / \partial S|$, and the change in the marginal utility (disutility) of the habitual place of purchase with the change in money income, or $\partial M U_{|\partial P / \partial S|} / \partial w$, results in the following equation:

$$
\begin{equation*}
e_{\lambda,|\partial P / \partial S|}+e_{\lambda ; w}=e_{|\partial P / \partial S| \cdot w}-1 \tag{4}
\end{equation*}
$$

Under the assumption of the diminishing efficiency of search the elasticity of price reduction $e_{|\partial P / \partial S| \mid w}$ illustrates both the increase in the willingness to overpay and the decrease in time of search after the increase in the wage rate $\left(\left|\partial P_{i} / \partial S_{i}\right|>\left|\partial P_{j} / \partial S_{j}\right| \rightarrow P_{i}>P_{j} ; S_{i}<S_{j}\right)$. Hence, it is always positive. When the value of the elasticity of price reduction $e_{|\partial P / \partial S| . w}$ is equal to one, we have

$$
\begin{equation*}
e_{\lambda,|\partial P / \partial S|}+e_{\lambda, w}=0 \tag{5}
\end{equation*}
$$

The Equation (4) also enlightened the way for the comparative static analysis of the utility function. Indeed, it shows us that the indirect utility function depends on two variables in the following manner:

$$
\begin{equation*}
v(w,|\partial P / \partial S|)=v(w,|\partial P / \partial S|(w)) \tag{6}
\end{equation*}
$$

The total derivative of this utility function gives us the following:

$$
\begin{align*}
& d v(w,|\partial P / \partial S|(w))=d w\left(\left.\frac{\partial v}{\partial w}\right|_{\partial P / \partial S \text { Sonst }}+\frac{\partial v}{\partial|\partial P / \partial S|} \frac{\partial|\partial P / \partial S|}{\partial w}\right) ;  \tag{7}\\
& \frac{d v}{d w}=\lambda-\lambda \frac{w}{|\partial P / \partial S|} \frac{\partial|\partial P / \partial S|}{\partial w}=\lambda\left(1-e_{|\partial P / \partial S|, w}\right)
\end{align*}
$$

We see that when the price reduction is unit elastic ( $e_{|\partial P / \partial S| w}=1$ ), the Equation (5) takes place and the utility stays constant, or $d v / d w=0$. And the following choice of the purchase price which is accompanied by a greater price reduction $\left(e_{|\partial P P \Delta S| w}>1\right)$ decreases the utility of consumptionleisure choice (Fig.2):


Fig.2. Utility function under price dispersion

## Willingness to overpay as insurance premium

The appearance of the saddle point in the utility function gives an answer to the question what the consumer should do in order to avoid the decrease in utility. Obviously, he should decrease relative price reduction i.e., to be... not more modest, but less ambitious with regard to purchase prices after the following increase in the wage rate $\left(|\partial P / \partial S| / \partial w>0\right.$, but $\left.\mid e_{|\partial P / \partial S| . w}<1\right)$. We see that it is really possible. However, this change represents the change in the model of behavior - from risk aversion to risk seeking. Indeed, the prospect theory tells us that facing the inevitable loss, here the decrease in utility, the consumer should take risk (Kahneman and Tversky 1979). Hence, the utility function changes its shape and becomes close to the cubic parabola.

Usually, guarantees and insurance contracts increase both prices and price dispersion and we can await that guarantees and insurance contracts increase the equilibrium price reduction $|\partial P / \partial S|$ that equalizes marginal costs of search with its marginal benefit.
We can assume that the increase in the wage rate results not in the simple increase in the purchase price with respect to the increased income but in the increase in the insurance premium, accompanied by the increase in price reduction. The consumer details his insurance policy and increases the insurance premium with every increase in the wage rate. Other words, the consumer behaves like a homeowner who raises progressively the fence with any subsequent increase in income. And more insurance policies are detailed, the more efficient is the search, i.e., the greater is the absolute value of the equilibrium price reduction.

## Unwillingness to overpay for insurance as driver of risk behavior

When we determine the second derivative of the utility function, we should keep in mind the marginal utility of money income $\lambda$ as well as the unwillingness to overpay ( $1-e_{|O P / \partial S| \cdot w) \text { also }}$ represent functions of two variables. However, we can omit labor-intensive intermediate
calculations and to present the second derivative directly in its total form and in its elasticity form:

$$
\begin{gather*}
\frac{d^{2} v}{d w^{2}}=\frac{d \lambda}{d w}\left(1-e_{|\partial P / \partial S|, w}\right)+\lambda \frac{d\left(1-e_{|\partial P / \partial S| w}\right)}{d w} \\
\frac{d^{2} v}{d w^{2}}=\frac{\lambda}{w}\left(1-e_{|\partial P / \partial S| w}\right)\left(e_{\lambda, w}+e_{\lambda,|P P / \partial S|} e_{|\partial P / \partial S| w}+e_{(1-e|\partial P / \partial S|, w), w}\right) \tag{9}
\end{gather*}
$$

The form of the total second derivative is very useful for the step-by-step analysis of changes in the model of behavior. The elasticity form, although its use is limited by critical points, is helpful in the derivation of the relative measure of risk aversion and in following optional high-order derivations of measures of prudence, which are omitted from the present analysis and left for analysts who are not afraid to work with relative values of the model. Thus, the relative ArrowPratt measure takes the following form:

$$
\begin{equation*}
\eta=-\left(e_{\lambda, w}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S|, w}+e_{(1-e|\partial P / \partial S|, w), w}\right) \tag{10}
\end{equation*}
$$

Although we get here the second order elasticity, it is rather simple to understand it. We can denote the value ( $1-e_{|\partial P / o S| w)}$ ) as the unwillingness to overpay and consider its elasticity with respect to the wage rate. When the increase in wage rate decreases the unwillingness to overpay, the second derivative $d^{2} v / d w^{2}$ is strictly negative. Moreover, while the unwillingness to overpay is decreasing, the absolute value of its elasticity $e_{(1-e|\partial P / \partial S| w), w}$ is increasing. And with the increase in absolute value of the elasticity of the unwillingness to overpay the relative risk aversion is increasing, i.e., the share of risky assets, i.e., unsecured consumption, is decreasing. Of course, it certainly happens if the subsequent increase in the wage rate and in the equilibrium value of price reduction always results in the increase in real balances, which follow the optimal consumption path, or $\left(e_{\lambda, w}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S| w}\right)<0$. The last assumption can be verified by the following transformation of the relative increase in real balances with the help of the Equation (4):

$$
\begin{equation*}
e_{\lambda, w}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S|, w}=e_{\lambda, w}+e_{\lambda, \mid \partial P / \partial S S} e_{|\partial P / \partial S|, w}+e_{\lambda,|\partial P / \partial S|}-e_{\lambda,|\partial P / \partial S|}=\left(e_{|\partial P / \partial S|, w}-1\right)\left(1+e_{\lambda,|\partial P / \partial S|}\right) \tag{11}
\end{equation*}
$$

The Equation (11) tells us that when the price reduction elasticity of the marginal utility of money is positive, or $e_{\lambda,|\partial P / \partial S|}>0$, the value $\left(e_{\lambda, w}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S| w}\right)$ is non-positive, or ( $e_{\lambda, w}$ $+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S| w} \leq 0$ for the values $e_{|\partial P / \partial S| . w} \leq 1$. Hence, when $e_{|\partial P / \partial S| . w}<1$, any increase in wage rate raises real balances and decreases the marginal utility of money.

The behavior of the utility function at this stage is described by the following expressions:

$$
\begin{equation*}
1-e_{|\partial P / \partial S| . w}>0 ; \lambda>0 ; d \lambda / d w<0 ; d e_{(1-e|\partial P / \partial S| . w), w} / d w<0 \rightarrow d^{2} v / d w^{2} \ll 0 \tag{12}
\end{equation*}
$$

Here, the relative risk aversion is increasing because the consumer increases the overpayments or, in the case of insurance, makes the latter more and more detailed. The homeowner begins
with insurance for the house and he details it with furniture and paintings. Once there is no object to be insured except coffer. And the consumer insures it by the following increase in the wage rate and he spends on the coffer's insurance the total increase in income. This action means that neither consumption nor real balances kept in the coffer are changed. The insurance policy becomes full or comprehensive. The elasticity of price reduction becomes equal to one $\left(e_{|\partial P / \partial S| \cdot w}=1\right)$, the unwillingness to overpay becomes equal to zero $\left(e_{(1-e|\partial P / \partial S|, w), w}=0\right)$, and, according to the Equation (5) the increasing marginal utility of money expenditures completely offsets the decreasing marginal utility of money income:

$$
\begin{equation*}
e_{\lambda, w}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S| w}=e_{\lambda,|\partial P / \partial S|}+e_{\lambda, w}=0 \tag{13}
\end{equation*}
$$

This stationary point B also represents the decision node (Fig.2). If the consumer decides to reinsure his comprehensive insurance $\left(e_{|\partial P / O S| w}>1\right)$ for the given level of consumption, he will decrease his money balances. Thus, the utility function will go down $(d v / d w<0)$. The only way to increase both consumption and money balances is not to reduce absolute overpayments (the value $\partial|\partial P / \partial S| / \partial w$ is always positive) but to reduce relative overpayments, or to make them less income elastic, i.e., $e_{|\partial P / \partial S| . w}=0,9 ; 0,8 ; 0,7 \ldots$ etc.

This decision increases the unwillingness to overpay. However, when the increase in the wage rate raises the unwillingness to overpay, the second derivative $d^{2} v / d w^{2}$ becomes positive. The consumer begins to seek risk:

$$
\begin{equation*}
1-e_{|\partial P / \partial S| . w}>0 ; \lambda>0 ; d \lambda / d w<0 ; d e_{(1-e|\partial P / \partial S| . w), w} / d w \gg 0 ; d^{2} v / d w^{2}>0 \tag{14}
\end{equation*}
$$

It happens because at the beginning the relative increase in the unwillingness to overpay is greater than the absolute value of the relative decrease in money balances, or

$$
\left(\left(e_{\lambda, w}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S| w}\right)+e_{(1-e|\partial P / \partial S| \cdot w), w}\right)>0 .
$$

Here, we need some comments on the relationship between money balances and overpayments. The risk-seeking behavior means that the increase in consumption is not well secured. However, the insurance is provided not only by insurance policy but also by real balances, which could represent the precautionary savings. The risk-seeking model of behavior means that the total of precautionary savings and insurance policy is insufficient for the optimal level of consumption. It happens because here the relative increase in money balances is followed by the relative decrease in overpayments. Real balances as the tool of protection of consumption, i.e., of wealth, begin to substitute overpayments.

Here we come to the question whether precautionary savings and insurance are substitutes or complements. In spite of some analytical solutions of this problem (Ehrlich and Becker (1972)), this question is still open in the general economic analysis. Moreover, when this issue is studied, the attention is usually paid to health and social insurance (Hubbard, Skinner and Zeldes (1995), Guariglia and Rossi (2004)). Here we can only assume the substitutability between money
balances and overpayments. The only reason for this assumption is the response of relative overpayments to the continuous decrease in the value of $\lambda$, i.e., in the marginal utility of increasing real balances. Moreover, we cannot use here without doubt the direct cross-price elasticity that is presented in the model by the price reduction elasticity of the marginal utility of money, or by the value $e_{\lambda,|\partial P / \partial S|}$. While at this stage the consumer continues to increase overpayments but he makes it more modestly, the derivative $\partial \lambda / \partial|\partial P / \partial S|$ is still positive. However, the economic sense of the decrease in the relative overpayments with respect to the decrease in the marginal utility of money, i.e., in the "price" of money, presumes the substitutability. In addition, the increase in relative overpayments with respect to the decrease in the marginal utility of money presumes that when the consumer is risk-averse, money balances and overpayments becomes complements from the standpoint of the protection of wealth. In any way, the rather harmonic assumption that precautionary savings and insurance are complements in the risk-aversion model and they are substitutes in the risk-seeking model needs, and we are going to see it, more profound analysis.

The comeback from risk seeking to risk aversion is quite ambiguous. While the positive elasticity of the unwillingness to overpay $e_{(I-e|\partial P / \partial S| w) w}$ is decreasing, once it certainly matches the negative elasticity of the marginal utility of money:

$$
\begin{equation*}
e_{\lambda, w}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S| \cdot w}+e_{(1-e|\partial P / \partial S| \cdot w), w}=0 \tag{14}
\end{equation*}
$$

The analysis of the second derivative of the utility function discovers two possible outcomes from the risk neutrality While the first part of the Equation (14) $\left(e_{\lambda_{, w}}+e_{\lambda,|\partial P / \partial S|} e_{|\partial P / \partial S|, w}\right)$ is always negative, or the absolute value of the decrease in the marginal utility of money with the increase in the wage rate is greater than its increase with the price reduction, the model of behavior depends here on the decision whether to continue to decrease relative overpayment and to increase the unwillingness to overpay ( $e_{(1-e|\partial P / \partial S| \cdot w), w}>0$ ), or to increase relative overpayments and to decrease, as it was done on low levels of income, the unwillingness to overpay ( $e_{(1-}$ ${ }_{e|\partial P / \partial S|, w), w}<0$ ). The continuous increase in real balances with the decreasing marginal utility of money provides the negative second derivative $d^{2} v / d w^{2}$ in both outcomes. However, the continuous increase in the unwillingness to overpay, i.e., in the unwillingness to detail and to enlarge insurance, results in the "steeper" sortie from the risk neutrality. We can verify this fact without laborious calculations of high-order derivatives but with simple back-on-the envelope reasoning. The continuous increase in the unwillingness to overpay simply states the fact that the consumer relies more on precautionary savings than on insurance and he increases the share of risky assets, i.e., the share of uninsured commodities. Hence, his relative risk aversion becomes decreasing. On the other hand, if he chooses the extension of insurance policy, he increases his risk aversion. The option to restart the insurance activity and to detail insurance policies results
in the flat transformation of the utility curve. And with the increasing risk aversion the consumer should come again to new saddle point with the unit elasticity of the price reduction $e_{|\partial P / \partial S| \mid w}=1$ on the lower level of utility (Fig.3):


Fig.3.Decreasing vs. increasing relative risk aversion
The path of the decreasing relative risk aversion is more intriguing. There, the consumer can continue to decrease relative overpayments until the moment when the value of price reduction $|\partial P / \partial S|$ becomes constant. At this moment the elasticity of the unwillingness to pay $e_{(1-e|\partial P / \partial S| \cdot w), w}$ as well as the elasticity of price reduction $e_{|\partial P / \partial S| \cdot w}$ becomes equal to zero, and the second derivative of the utility function in the Equation (8) gets its "true" value, or $d^{2} v / d w^{2}=d \lambda / d w$, i.e., the marginal utility of income becomes unit elastic. It looks like the signaling that now the consumer could activate his insurance activity. However, the maximum value of real balances, accompanied with the constant overpayments, gives an idea that at the saddle point M on the decreasing risk aversion curve the consumer gets the optimum quantity of money, where the "true" value of real becomes equal to zero because the marginal utility of money becomes equal to zero. The constant $|\partial P / \partial S|$ value simply represents the prolongation of insurance policy for the coffer where the optimum quantity of money is held. ${ }^{1}$ This assumption corresponds to M.Friedman's reasoning on the optimum quantity of money:
"The amount held will, at the margin, reduce utility - because of concern about the safety of the cash, perhaps, or because of pecuniary costs of storing and guarding the cash." (Friedman 2005 [1969], p.18).

There, the ratio of real balances, i.e., of precautionary savings, to consumption, is so important that it protects against any disaster.

[^0]This assumption raises the question why the consumer cannot change the manner of risk aversion at low levels of income, i.e., why the shift from the increasing to the decreasing risk aversion cannot take place at low values of relative overpayments $e_{|\partial P / \partial S| . w} \ll 1$. Moreover, it seems that in this case the consumer could avoid saddle points and he could reproduce the exact contour of the Friedman-Savage's "snake". However, in this case high values of the marginal utility of money balances of low income levels could hardly be offset by the marginal decrease in the unwillingness to overpay and the consumer will meet at the saddle point "catastrophic" consequences of both imprudence and over prudence.

## Economic and mathematical catastrophes: Veblen effect and family altruism

When G.Becker issued his famous rationalization of family altruism, he stressed the importance of the role of security:

Therefore, altruism helps families insure their members against disasters and other consequences of uncertainty: each member of an altruistic family is partly insured because all other members are induced to bear some of the burden through changes in contributions from the altruist (Becker 1981, pp.3-4).

Hence, the family altruism can be introduced in our model as an additional insurance. There are two possible descriptions of this extra insurance.

We can reproduce the decrease in the individual utility function of the head of the family when relative overpayments really become disproportionate to his individual security, or $e_{|\partial P / \partial S|, w}>1$. The extra insurance is provided by the decrease in money balances ( $\partial \lambda / \partial w>0$ ). However, the following set of equations demonstrates that the decrease in utility ( $\partial v / \partial w<0$ ) is accompanied there not by the risk-seeking behavior but by risk-aversion ( $\left.\partial^{2} v / \partial w^{2}<0\right)$. The utility function takes the form of parabola:

$$
\begin{equation*}
1-e_{|\partial P / \partial S| . w}>0 ; \lambda>0 ; d \lambda / d w>0 ; d e_{(1-e|\partial P / \partial S| . w), w} / d w \ll 0 \rightarrow d^{2} v / d w^{2}<0 \tag{15}
\end{equation*}
$$

Here we could wait for the moment when money balances become equal to zero and the family changes her model of behavior. Unfortunately, in the absence of budget constraints the family could borrow. Moreover, in this case the marginal utility of money $\lambda$ becomes negative and the head of the family can increase his utility if he continues to increase overpayments ( $\lambda<0$; ( 1 $\left.e_{|\partial P / \partial S|, w}\right)<0 ; d v / d w>0$ ).

Here, the head of the family reproduces the Veblen effect. The previous analysis discovered the correspondence between negative marginal utility of money and the extra overpayments (Malakhov 2013). This is the first "pitfall" the stationary point B prepares for imprudent consumers. Moreover, from the individual point of view the Veblen-effect-like leaving of the saddle point looks more positive than the increase in the unwillingness to overpay. This way can
provide more utility until the moment when real balances will be exhausted or the borrowing will be closed and the comeback to ordinary risk-seeking behavior will take place (Fig.4):


Fig.4.The option of Veblen effect in risk-seeking behavior
The occurrence of Veblen effect with regard to the previous reasoning on the optimum quantity of money tells us that Veblen effect can take place at rather modest levels of income where consumption is far from satiation. However, although this scenario can take place, it does not seem well compatible with the description of the individual utility function within the family. There is another possibility to present family altruism. We can pretend the head of the family to be more "economic man" and to separate altruism from the individual utility function. If we take the factor of giving as the share of the individual wage rate, we get the following utility function $v^{g}(w)=v(w)-g w$. However, there we automatically get the other "pitfall" or the mathematical "bold"-type catastrophe due to the existence of the saddle point B in the original utility function (Fig.5):


Fig.5."Bold" catastrophe of family altruism
In this case the decrease in the utility function starts at point A when the consumer, the head of the family, is still risk averse and he continues to make protection of his wealth by the increasing real balances and by increasing overpayments. The continuous increase in overpayments discovers the unwillingness of the head of the family to economize. Here, the behavior looks like
"pure" altruism. However, once the head of the family changes the model of his behavior and he begins to make risky decisions. It happens at point B when he passes the saddle point of the original utility function with the unit elastic price reduction ( $e_{|\partial P / \partial S| \cdot w}=1$ ). The following increasing unwillingness to overpay gives an idea that the nature of his altruism has been changed. The head of the family becomes more "pragmatic". Although his generosity does not exhausted, his purchase decisions become more prudent. They begin to look like investments. The investments in family reach its peak at point C. Finally, the head of the family begins to feel again the increase in his utility function and at point D he no longer suffers from his altruism, or he finally gets returns on investments:
"Altruistic parents might not have more children than selfish parents, but they invest more in the human capital or quality of children because the utility of altruistic parents is raised by investment returns that accrue to their children." (Becker 1981, p.12).
Indeed, the movement of the utility curve from point $A$ to point $D$ reminds the parental behavior from the birth of a child till the go-out of a young man from the nest. At the beginning parents do not economize on purchases for babies. They are trying to buy everything of high quality and with guarantees. Once, at point B, these purchases take the form of investments, which even in prudent manner lead to point C in the bottom due to their importance. However, the earlier decision at point B to reduce relative overpayments continues to work and finally it pulls out the head of the family from the "pitfall".
When G.Becker cited King Lear's Fool in order to illustrate the Rotten Kid Theorem by the parental willingness to delay contributions until last stage of life he did not take into account the possibility of saddle points in the parental utility function. We have seen that if the consumer continues to increase overpayments without change in the model of behavior his utility can go down infinitely. Once upon a time King Lear simply missed that point. And from the literature point of view it would be better here to remember d'Artagnan-father, who contributed to his son (only or the whole?) quinze écus, his horse, and some parental advices.

## Conclusion

The analysis of consumer behavior presented in this paper discovers the methodological power of relative values, which are produced by the process of search. The common question addressed to the model presented here - what about interest rate - can be answered in the same manner, if we envisage the risk of delay of consumption, i.e., the risk of unexpected rise in prices, and explain overpayments as payments for consumer credit. In this case the comprehensive insurance is transformed in the comprehensive consumer credit and the extra comprehensive insurance $\left(e_{|\partial P / \partial S| . w}>1\right)$ is transformed into the refinancing of existing debt, a mortgage, for example. There
the risk seeking starts when the borrowing is closed and consumers increase risk of the delay in consumption. In addition, this approach can revive the discussion on the optimum quantity of money with an interesting argument. Indeed, when overpayments become constant they could represent not direct interest payments but some fixed expenditures the consumer pays to the government to finance the interest payments on money. Hence, residual constant overpayments show that real balances even on their optimal level are not costless.

The question of the limp-sum taxation leads to the understanding that the model presented here could be useful in the analysis of the optimal taxation. If we substitute in the individual utility function the factor of giving by income tax we also get the "bold"-type catastrophe. However, if one tries to go further and to explain overpayments by VAT or excise tax, the coming trade-off between income taxes and overpayments should be examined with prudence.

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[^0]:    ${ }^{1}$ In the original model of search the constant $|\partial P / \partial S|$ value means that consumer continues to buy commodities in the same place or in the same store for the same price.

