# Efficiency Wage and Endogenous Job Destruction in the DMP Model—— an Extension

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**Abstract:** This paper introduces efficiency wage relation and assumes endogenous job destruction in the benchmark DMP model. A comparative static analysis has also been made which shows that most of the results obtained in the DMP model get altered in the extended model.

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**Keywords:** Efficiency wage, job destruction.

**Introduction:** The path-breaking work in the labour market is the DMP model. This model provides a useful framework where different labour market policies can be analyzed in the presence of matching friction. In the benchmark model of DMP we find that a positive productivity shock raises Nash wage rate and the market tightness but reduces unemployment rate. A high discount rate reduces both the wage rate and the market tightness but raises the equilibrium rate of unemployment. An increase in the unemployment benefit and / worker's bargaining power raises both wage rate and unemployment rate and lower market tightness. In the DMP model we also find that paradox appears if the worker's bargaining power vanishes. This is usually known as the Diamond Paradox.

The existing literature shows that the DMP model has been extended by introducing cyclicality, endogenous job destruction and efficiency wage relation. These includes the works of Albrechet et al. (1984), Cole and Rogerson (1989), Mortensen and Pissarides (1994), Abraham et al. (1995), Zenou and Smith (1995), Andolfatto (1996), Hagedorn and Manovskii (2005), Shimmer(2005), Zenou (2005), Arozamena and Ceneteno (2007), Elsby et al. (2007), Pissarides(2007) etc. The efficiency wage adjusted extension of the DMP model considers Shapiro and Stiglitz (1984) type efficiency wage relation where worker's efficiency depends on the wage paid to the worker.

In this paper we introduce efficiency wage relation in the DMP model where worker's efficiency depends on the wage rate and the market tightness. We also assume that higher efficiency implies lower job destruction rate and so job destruction is endogenized through the worker's efficiency. We examine the parametric effects on the nash-wage rate, market tightness and on the unemployment rate at steady state equilibrium. In most cases we observe that the results obtained in benchmark model of DMP are changed under the Solow elasticity condition. Further, if the worker's bargaining power is nil the so-called Diamond paradox can be solved if worker's efficiency is greater than one. In this special case market tightness does not respond to changes in parameters.

Our paper is organized as follows. In section 2 we describe the model. Section 3 embraces the comparative static exercises. Section 4 throws light on the Diamond paradox and section 5 concludes.

**2 The Model:**

We introduce efficiency wage relation in the benchmark DMP model. We assume that worker's efficiency ($h$) depends on the wage rate ($w$) and the market tightness ($\theta$). Thus the efficiency function of the worker is

$$h = h(w,\theta) \tag{1}$$

Where $h_w, h_\theta > 0$; $h_{ww}, h_{\theta\theta} < 0$; $h_{w\theta} = h_{\theta w} = 0$.

We assume that the job destruction rate ($\lambda$) is endogenous and is inversely associated with the worker's efficiency.[1] Thus, we may write

[1] We find a strong negative relation between wage rate and separation rate in the works of Leonard (1987), Anderson and Meyer (1994), Galizzy and Lang (1998) and Christensen et al. (2005). Capelli and Chauvin (1991).

$$\lambda = \lambda(h), \lambda' < 0. \tag{2}$$

The Bellman equations for unemployment ($U$), employment ($W$), vacancy ($C$) and jobs filled in ($J$) are

$$rU = b + \theta q(\theta)(W - U) \tag{3}$$

$$rW = \frac{w}{h(w,\theta)} - \lambda(h)(W - U) \tag{4}$$

$$rV = -C + q(\theta)(J - V) \tag{5}$$

$$rJ = y - \frac{w}{h(w,\theta)} - rk - \lambda(h)J \tag{6}$$

$r$ is the discount rate, $b$ is the unemployment benefit, $C$ is the cost of maintaining vacancy, $y$ is the constant match productivity and $q$ is the job offer rate.

A firm creates jobs up to the point where $V = 0$. Using this condition from (5) and (6) we get job creation condition at steady state as

$$y = \frac{w}{h(w,\theta)} + rk + \left[\frac{r + \lambda(h)}{q(\theta)}\right]C \tag{7}$$

This is the zero-profit condition of a Firm since value of job is exactly matched by the wage cost plus rental cost plus recruitment cost of labour.

Following DMP we also assume Nash-wage equation which can be derived from the following exercise:

$$\underset{(w)}{Max.}\ \Omega = (W - U)^{\beta}(J - V)^{1-\beta} \tag{8}$$

where $\beta$ is the bargaining strength of the worker and $1 > \beta > 0$

Assuming interior solutions exist the first order conditions are[2]

[2] See Appendix A.

$e_{h,w} = 1$

and /

$$\frac{w}{h(w,\theta)} = (1-\beta)b + \beta(r + C\theta - rk) \tag{9}$$

The unemployment rate at steady state is

$$u = \frac{\lambda(h)}{\lambda(h) + \theta q(\theta)} \tag{10}$$

Now, using (7) and (9) we get equilibrium steady state values of $w,\theta$ . Then, we get $h,\lambda$ from (1) and (2). Finally, $u$ can be obtained from Equation (10).

3. **Comparative Static Effects:**

Taking total differentials of Equations (7), (9) and (10) and using (1), (2), (12) and the Solow elasticity condition one can get[3]

$$\left.\begin{array}{l}
\frac{\hat{w}}{\hat{y}} < 0 \text{ and } \frac{\hat{w}}{\hat{r}} > 0 \text{ iff } \left(e_{q,\theta} \le e_{\lambda,h}.e_{h,w}\right); \\
\frac{\hat{w}}{\hat{b}} > 0,\ \frac{\hat{w}}{\hat{\beta}} > 0; \\
\frac{\hat{\theta}}{\hat{y}} < 0, \frac{\hat{\theta}}{\hat{r}} > 0, \frac{\hat{\theta}}{\hat{b}} < 0, \frac{\hat{\theta}}{\hat{\beta}} < 0; \\
\frac{\hat{u}}{\hat{y}} > 0, \frac{\hat{u}}{\hat{r}} < 0 \text{ and} \\
\frac{\hat{u}}{\hat{b}} < 0, \frac{\hat{u}}{\hat{\beta}} < 0 \text{ iff } \frac{w}{h} > J\left(\frac{\lambda - re_{q,\theta}}{e_{h,\theta}}\right)
\end{array}\right\} \tag{11}$$

The above results lead to the following propositions:

**Proposition1:** Under the Solow elasticity condition, a positive productivity shock dampens Nash wage rate and market tightness but raises unemployment rate.

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[3] See Appendix B and C.

However, in the benchmark DMP model we get the opposite results where worker's efficiency was not considered.

**Proposition 2:** In the presence of efficiency wage relation a high discount rate raises both the wage rate and the market tightness but lower unemployment rate if Solow elasticity condition holds.

In the benchmark DMP model where worker's efficiency was absent we get absolutely different results.

**Proposition 3:** In the presence of efficiency wage relation and the Solow elasticity condition an increase in the unemployment benefit raises Nash wage rate but lowers both market tightness and unemployment rate under some reasonable condition. The same outcomes are obtained if worker's bargaining power improves.

However, in the benchmark DMP model without efficiency wage relation the parametric effects of unemployment benefits and worker's bargaining power on unemployment rate are different.

## 4. The DMP Paradox:

Now, we assume that $\beta = 0$. In this case, $\frac{w}{h(w,\theta)} = b.$ If $h \leq 1,\ w \leq b.$ In this situation, a worker can get $b$ if he remains unemployed but if he is employed he gets $w$ which is less than $b$. So, he has no incentive to search jobs and this leads to Diamond paradox. However, if $h > 1,\ w > b$ and so Diamond paradox does not arise here. Thus, the efficiency wage relation can resolve the Diamond Paradox if $h > 1$.

We now examine the comparative static results if $\beta = 0$. From Equations (7), (9) and (10) and using $e_{h,w} = 1, \beta = 0$ we get[4]

$$\left.\begin{array}{l}\frac{\hat{w}}{\hat{y}} < 0, \frac{\hat{w}}{\hat{r}} > 0, \ \frac{\hat{w}}{\hat{b}} > 0; \\ \frac{\hat{\theta}}{\hat{y}} = 0, \frac{\hat{\theta}}{\hat{r}} = 0, \frac{\hat{\theta}}{\hat{b}} < 0; \\ \frac{\hat{u}}{\hat{y}} > 0, \frac{\hat{u}}{\hat{r}} < 0 \text{ and} \\ \frac{\hat{u}}{\hat{b}} < 0 \text{ iff } \frac{w}{h} > J\left(\frac{\lambda - re_{q,\theta}}{e_{h,\theta}}\right)\end{array}\right\} \tag{12}$$

From (12) it is evident that if $\beta = 0$ market tightness does not respond to changes in $y, r$.

**5. Conclusions:**

In this paper we extend the DMP model by incorporating efficiency wage relation in the benchmark DMP model. We also assume endogenous job destruction where it is inversely associated to the worker's efficiency. The presence of wage efficiency relation modifies the basic equations of the original DMP model. These changes are also reflected in the comparative static results. In most cases, the parametric effects on the wage rate, market tightness and the equilibrium rate of unemployment are opposite to those usually obtained in the benchmark DMP model.

Further, we also find that even if $\beta = 0$, the Diamond Paradox can be resolved if $h > 1$. In this case market tightness is purely passive to changes in match productivity and discount rate.

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[4] See Appendix B and C.

## Appendix A: Derivation of Nash bargaining solutions

The Nash –bargaining problem is

$$\underset{w}{Max}\ \Omega = (W - U)^{\beta}\,(J - V)^{(1-\beta)} \tag{A.1}$$

The first order condition for maximization is

$$\beta\,(J - V)\frac{\partial}{\partial w_2}(W - U) + (1-\beta)(W - U)\frac{\partial}{\partial w_2}(J - V) = 0 \tag{A.2}$$

Using (4), (6) and the zero-profit condition $V = 0$ into (A.2) one gets

$$\frac{\left(1 - e_{h,w}\right)}{h(r+\lambda)}\left[\beta\,(J - V) - (1-\beta)(W - U)\right] = 0$$

or

$$\left(1 - e_{h,w}\right)\left[\frac{w}{h(w,\theta)} - (1-\beta)\,b - \beta\,(y + C\theta - rk)\right] = 0 \tag{A.3}$$

## Appendix B: Effects of changes in $y, r, b, \beta$ on $w, \theta$ :

Total Differentials of Equations (7) and (9) yields

$$\left[\frac{w}{h}\left(1 - e_{h,w}\right) + e_{\lambda,h}.e_{h,w}.\lambda J\right]\hat{w} - \left[e_{h,\theta}\left(\frac{w}{h} - e_{\lambda,h}.\lambda J\right) + (r+\lambda)\,Je_{q,\theta}\right]\hat{\theta} = y\hat{y} - (J + k)\,r\hat{r} \tag{A.4}$$

$$\frac{w}{h}\left(1 - e_{h,w}\right)\hat{w} - \left(\frac{w}{h}e_{h,\theta} + \beta C\theta\right)\hat{\theta} = (1-\beta)\,b\hat{b} + \left(\frac{w}{h} - b\right)\hat{\beta} + \beta\,y\hat{y} - \beta\,rk\hat{r} \tag{A.5}$$

Using Cramer’s rule from (A.4) and (A.5) we get

$$\hat{w} = \frac{1}{\Delta}\left[\begin{array}{r}\left\{(1-\beta)b\hat{b} + \left(\frac{w}{h} - b\right)\hat{\beta} + \beta y\hat{y} - \beta rk\hat{r}\right\}\left\{e_{h,\theta}\left(\frac{w}{h} - e_{\lambda,h}.\lambda J\right) + (r+\lambda)Je_{q,\theta}\right\} - \\ \left(y\hat{y} - (J+k)r\hat{r}\right)\left(\frac{w}{h}e_{h,\theta} + \beta C\theta\right)\end{array}\right] \tag{A.6}$$

$$\hat{\theta} = \frac{1}{\Delta}\left[\begin{array}{r}\left\{(1-\beta)b\hat{b} + \left(\frac{w}{h} - b\right)\hat{\beta} + \beta y\hat{y} - \beta rk\hat{r}\right\}\left\{\frac{w}{h}\left(1 - e_{h,w}\right) + e_{\lambda,h}.e_{h,w}.\lambda J\right\} - \\ \left(y\hat{y} - (J+k)r\hat{r}\right)\frac{w}{h}\left(1 - e_{h,w}\right)\end{array}\right] \tag{A.7}$$

where

$$\begin{aligned}\Delta = \frac{w}{h}\left(1 - e_{h,w}\right)\left\{e_{h,\theta}\left(\frac{w}{h} - e_{\lambda,h}.\lambda J\right) + (r+\lambda)Je_{q,\theta}\right\} - \\ \left(\frac{w}{h}e_{h,\theta} + \beta C\theta\right)\left\{\frac{w}{h}\left(1 - e_{h,w}\right) + e_{\lambda,h}.e_{h,w}.\lambda J\right\}\end{aligned} \tag{A.8}$$

Under the Solow elasticity condition,

$$\Delta = -\underbrace{\left(\frac{w}{h}e_{h,\theta} + \beta C\theta\right)}_{(+)}\underbrace{e_{\lambda,h}}_{(-)}.\lambda J > 0 \tag{A.8.1}$$

From (A.8.1) it follows that under stability

$$\left[e_{h,\theta}\left(\frac{w}{h} - e_{\lambda,h}.\lambda J\right) + (r+\lambda)Je_{q,\theta}\right] > 0 \tag{A.9}$$

From (A.6) and (A.7) we get

$$\frac{\hat{w}}{\hat{y}} = \frac{y}{\Delta}\left[\begin{array}{r}\beta\left\{e_{h,\theta}\left(\frac{w}{h} - e_{\lambda,h}.\lambda J\right) + (r+\lambda)Je_{q,\theta}\right\} - \\ \left(\frac{w}{h}e_{h,\theta} + \beta C\theta\right)\end{array}\right] < 0 \tag{A.6.1}$$

$$\text{iff}\left(e_{q,\theta} \le e_{\lambda,h}.e_{h,\theta}\right)$$

$$\frac{\hat{w}}{\hat{r}} = -\frac{r}{\Delta}\left[\begin{array}{r}\beta k\left\{e_{h,\theta}\left(\frac{w}{h}-e_{\lambda,h}.\lambda J\right)+(r+\lambda)Je_{q,\theta}\right\}- \\ (J+k)\left(\frac{w}{h}e_{h,\theta}+\beta C\theta\right)\end{array}\right]>0 \tag{A.6.2}$$

$$\text{iff } \left(e_{q,\theta} \le e_{\lambda,h}.e_{h,\theta}\right)$$

$$\frac{\hat{w}}{\hat{b}} = \underset{(+)}{\frac{(1-\beta)b}{\Delta}}\left[\underset{(+)}{e_{h,\theta}\left(\frac{w}{h}-e_{\lambda,h}.\lambda J\right)+(r+\lambda)Je_{q,\theta}}\right]>0 \tag{A.6.3}$$

$$\frac{\hat{w}}{\hat{\beta}} = \underset{(+)}{\frac{1}{\Delta}}\left(\underset{(+)}{\frac{w}{h}-b}\right)\left[\underset{(+)}{e_{h,\theta}\left(\frac{w}{h}-e_{\lambda,h}.\lambda J\right)+(r+\lambda)Je_{q,\theta}}\right]>0 \tag{A.6.5}$$

$$\frac{\hat{\theta}}{\hat{y}} = \underset{(+)}{\frac{y}{\Delta}}\beta \underset{(-)}{e_{\lambda,h}}.\lambda J < 0 \tag{A.7.1}$$

$$\frac{\hat{\theta}}{\hat{r}} = -\underset{(+)}{\frac{r}{\Delta}}.\beta k\underset{(-)}{e_{\lambda,h}}.\lambda J > 0 \tag{A.7.2}$$

$$\frac{\hat{\theta}}{\hat{b}} = \underset{(+)}{\frac{(1-\beta)b}{\Delta}}.\underset{(-)}{e_{\lambda,h}}.\lambda J < 0 \tag{A.7.3}$$

$$\frac{\hat{\theta}}{\hat{\beta}} = \underset{(+)}{\frac{1}{\Delta}}\underset{(+)}{\left(\frac{w}{h}-b\right)}.\underset{(-)}{e_{\lambda,h}}.\lambda J < 0 \tag{A.7.4}$$

**Appendix C: Effects of changes in $y, r, b, \beta$ on $u$ :**

Total differentials of Equation (10) give

$$\hat{u} = \left[(1-u)e_{\lambda,h}.e_{h,w}\right]\hat{w} + (1-u)\left[e_{\lambda,h}.e_{h,\theta} - \left(1+e_{q,\theta}\right)\right]\hat{\theta} \tag{A.10}$$

Using (A.6.1) - (A.7.4) from (A.10) we get

$$\frac{\hat{u}}{\hat{y}} = \left[\underset{(-)}{(1-u)e_{\lambda,h}}\right]\underset{(-)}{\left(\frac{\hat{w}}{\hat{y}}\right)} + (1-u)\left[\underset{(-)}{e_{\lambda,h}.e_{h,\theta}} - \left(1+\underset{(+)}{e_{q,\theta}}\right)\right]\underset{(-)}{\left(\frac{\hat{\theta}}{\hat{y}}\right)} > 0 \tag{A.9.1}$$

$$\frac{\hat{u}}{\hat{r}} = \left[\underset{(-)}{(1-u)e_{\lambda,h}}\right]\underset{(+)}{\left(\frac{\hat{w}}{\hat{r}}\right)} + (1-u)\left[\underset{(-)}{e_{\lambda,h}.e_{h,\theta}} - \left(1+\underset{(+)}{e_{q,\theta}}\right)\right]\underset{(+)}{\left(\frac{\hat{\theta}}{\hat{r}}\right)} < 0 \tag{A.9.2}$$

$$\frac{\hat{u}}{\hat{b}} = \left[\underset{(-)}{(1-u)e_{\lambda,h}}\right]\underset{(+)}{\left(\frac{\hat{w}}{\hat{b}}\right)} + (1-u)\left[\underset{(-)}{e_{\lambda,h}.e_{h,\theta}} - \left(1+\underset{(+)}{e_{q,\theta}}\right)\right]\underset{(-)}{\left(\frac{\hat{\theta}}{\hat{b}}\right)} < 0 \text{ iff } \frac{w}{h} > \frac{J\left(\lambda - re_{q,\theta}\right)}{e_{h,\theta}} \tag{A.9.3}$$

$$\frac{\hat{u}}{\hat{\beta}} = \left[\underset{(-)}{(1-u)e_{\lambda,h}}\right]\underset{(+)}{\left(\frac{\hat{w}}{\hat{\beta}}\right)} + (1-u)\left[\underset{(-)}{e_{\lambda,h}.e_{h,\theta}} - \left(1+\underset{(+)}{e_{q,\theta}}\right)\right]\underset{(-)}{\left(\frac{\hat{\theta}}{\hat{\beta}}\right)} < 0 \text{ iff } \frac{w}{h} > \frac{J\left(\lambda - re_{q,\theta}\right)}{e_{h,\theta}} \tag{A.9.4}$$

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