Is Bitcoin business income or speculative bubble? Unconditional vs. conditional frequency domain analysis

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September 2014
Is Bitcoin Business Income or Speculative Bubble?
Unconditional vs. Conditional frequency domain analysis

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Abstract: The present study addresses one of the most problematic phenomena: Bitcoin price. We explore the Granger causality for two relationships (Bitcoin price and transactions; Bitcoin price and investors’ attractiveness) from a frequency domain perspective using Breitung and Candelon’s (2006) approach. Intuitively, this research gauges empirically the causal links between these variables unconditionally on the one hand and conditionally to the Chinese stock market and the processing power of Bitcoin network on the other hand. The observed outcomes reveal some differences with respect to the frequencies involved, highlighting the complexity of assessing what Bitcoin looks like and the difficulty to gain clearer insights into this nascent crypto-currency. Beyond the nuances of short-, medium- and long-run frequencies, this paper confirms the extremely speculative nature of Bitcoin without neglecting its usefulness in economic reasons (trade transactions). The consideration of the Chinese market index and the hash rate has led to solid and unambiguous findings connecting further Bitcoin to speculation.

Keywords: Bitcoin price; transactions; investors’ attractiveness; unconditional frequency domain; conditional frequency domain.
1. Introduction

Over the last years, various digital currencies have emerged including Litecoin, Auroracoin, Dogecoin and Bitcoin. The latter has attracted the most substantial number of users since its creation. From 2009, Bitcoin has succeeded to win an increasingly popularity in few times. Every passing day, the increase in the number of companies which accept Bitcoin is making the perceived value of this crypto-currency real. The excessive ups and downs of Bitcoin price have led finance professionals and many interviewees to suppose that it is a speculative trap more than future currency or business income.

Running freely, without any control from the central body, the famous money Bitcoin facilitates trade transactions. With this new crypto-currency, businesses may transact much more easily and people may execute everyday transactions. Institutions accept it continuously as an effective exchange tool, especially because it allows to better coping with the vagaries of financial markets and inflationary pressures. Nevertheless, the existing literature on the field suggests that Bitcoin is likely to be a “speculative bubble” than business income for at least five main reasons. First, it remains far from being certain because of its sizeable price volatility. The inelastic money supply coded by mathematic formula leading to inherent deflationary bias and other risks stemming from the lack of legal security. There is no guarantee of repayment at any time. Second, Bitcoin seems a digital currency in a nascent stage that may be highly linked to multiple risks of Bitcoin system. This new money is not yet substantially accepted as a payment system across wide markets. Third, this virtual money does not have an underlying value derived neither from consumption nor production process such as the precious metals including gold. Fourth, being a digital currency, Bitcoin seems vulnerable to excessive cyber-attacks that may aggravate the short-run disturbances of prices and play a destabilizing role in the Bitcoin system. Finally, the Bitcoin’s price is also driven by users’ attractiveness to this digital money. Kristoufek (2014) tries then to tackle accurately the impact of investors’ interest on Bitcoin price dynamic. Using wavelet coherence, the author shows that the relationship appears clearer over the period of price contraction rather than bubble build-up period. More precisely, during the explosive prices period, the interest drives this currency price up while it drives it down during the period of rapid decline. Ciaian et al. (2014) confirm this evidence by indicating that Bitcoin users may affect significantly the focal variable in the short- and in the long-run. This highlights that investors in this crypto-currency should worry about the future Bitcoin’s evolution closely linked to financial risk mainly owing to the speculative behavior of people who hold it.
It is therefore crucial to be cognizant that the nascent crypto-currency has many challenges still to overcome especially its excessive fluctuations. Even though Bitcoin has been frequently criticized as a risky investment, the extremely speculative nature of this virtual money needs a deeper assessment to reach better paths. To properly understand the main characteristics of Bitcoin well known by wide price swings and declines, this study seeks to address the following question: Is Bitcoin business income, log-term promise or “a speculative bubble”? While trying to effectively answer this question, none of the few papers on the issue (Yermack (2011), Buchholz et al. (2012), Kristoufek (2013, 2014), Ciaiaian et al. (2014), Glaser et al. (2014)) have yet attempted to accurately assess short-, medium- and long-run interaction dynamic between Bitcoin and trade transactions on the one hand and this digital currency and investors’ attractiveness on the other hand. The present article focuses on analyzing the link between these variables more closely by carrying out the frequency domain Granger causality approach proposed by Breitung and Candelon (2006). This method may allow reaching insightful evidences because tests are performed for well-defined frequencies. It distinguishes between long-run trends, business cycles or short-run dynamics, which may have important implications especially for the case of new money. This technique effectively captures causal links between each two variables even if the interconnection between them is non-linear. This test seems not sensitive to the presence of “volatility clustering” (Bodart and Candelon, 2009), a common characteristic of financial variables.

The intuition here is that more and more people are getting into Bitcoin for speculative reasons. Indeed, few merchants accept it today even if their number increases continuously. Our goal here is twofold: Firstly, to verify empirically to what extent this assertion is correct. Secondly, to go beyond and assess whether the relationships transactions-Bitcoin price and users’ interest-Bitcoin are more complex than they may appear, since they depend substantially on frequency horizons (i.e. short-, medium and long-term cyclical components). Similarly, it should be noted that from an economic point of view, the speculation is not always considered harmful. It can contribute to the thinning of the market. More interestingly, the causal links between the focal variables may be driven by various factors including the Chinese market index, the total Bitcoin in circulation, monetary velocity, the Dow Jones index and the hash rate (Kristoufek, 2014). Arguably, Bouoiyour and Selmi (2014) show that Bitcoin behaves heavily as a “speculative bubble”, short-term hedge and risky investment and partially as business income, depending to the situation of Chinese market (China is the largest Bitcoin market). Hence the interest to conduct a conditional analysis to appropriately
highlight the substantial role of the Chinese market performance in explaining the causal interaction between Bitcoin and transactions on the one hand and the focal digital money and investors’ attractiveness on the other hand. We can also add other factor that may play a determinant role on the Bitcoin price dynamic as the processing power of Bitcoin network (Kristoufek, 2014).

Our main findings are summarized as follows: Bitcoin price (BPI) Granger-causes exchange-trade ratio (ETR) in the short- and the medium-run cyclical components (the medium and the long-run time scales). The reverse link is not verified. We also found that TTR Granger-cause BPI in the long-run frequencies (unconditional causality). This link changes when accounting for SI and HASH (conditional causality), i.e. TTR causes BPI at both lower (the long-run time scales) and higher frequencies (short-run time scales). The reverse causal relationship that runs from BPI to TTR seems visible in lower frequencies. These outcomes do not change when moving from unconditional to conditional causality. Beyond the nuances of short- and long-term frequencies, we clearly show that as well as its speculative nature, Bitcoin may be also used for economic reasons (particularly in trade transactions). The consideration of the Chinese market index and the hash rate has confirmed the above findings and has led thus to “one sided” outcomes connecting further Bitcoin price to users’ interest (speculation). Interestingly, nobody knows whether this crypto-currency will still or disappear. It is crucial to mention that the different obtained results highlight: (i) the complexity of assessing what does crypto-currency look like. It seems therefore highly difficult to Gain robust and unambiguous insights into Bitcoin Phenomenon; (ii) the need to account for additional variables that may have potential influence on the focal links. The conditional frequency domain analysis seems more parsimonious than unconditional standard Granger causality. Unsurprisingly, this approach accounts for the cyclical components conditioning upon other control variables that may play determinant role in explaining these causalities and then may help practitioners and investors in their decisions-making at well-defined frequency horizons and elucidate readers’ information.

The remainder of the article proceeds as follows: Section 2 presents a brief literature survey. Section 3 describes the data, presents frequency domain method and the possible contributions of conditional causality compared to the unconditional ones. Section 4 reports the main findings (unconditional and conditional outcomes) and discusses them. Section 5 offers concluding remarks and some economic implications that may be very useful for Bitcoin’ investors, practitioners and regulators.
2. Brief literature survey

Bitcoin has attracted a substantial number of users since its creation. From 2009, Bitcoin has succeeded to win an increasingly popularity in few times. The global financial crisis has sustained the investors’ attractiveness towards this currency. However, the existing studies remain mainly interested to technical and legal issues (Grinberg (2011), Barber et al. (2012), Kroll et al. (2013), Moore and Christin (2013) and Bornholdt and Sneppen (2014)) compared to works focused on economic and financial aspects which appear very scarce (Buchholz et al. (2012), Ciaian et al. (2014), Kristoufek (2014) and Bouoiyour and Selmi (2014)).

The study of Palombizio and Morris (2012) relies on the possible impacts of global macroeconomic and financial development indicators on Bitcoin price formation. They show that oil price and exchange rate may be considered as main Bitcoin’s drivers, since they lead to cost pressures and provides a potential indicator of inflation development and thus impact positively Bitcoin price. Conversely, using VECM model, Ciaian et al. (2014) find that oil price has no influence on Bitcoin price, while the demand and supply play the most important role in explaining it. The last outcome is consistent with Buchholz et al. (2012). These latter add that the Bitcoin’ attractiveness of investors affects intensely the focal time series. This variable may detect the short-run speculative behavior of Bitcoin’users.

Kristoufek (2013) investigates the relationship between Bitcoin price and the users’interest measured through the search queries on Google Trends and Wikipedia. He shows a strong bidirectional causal relationship. More precisely, he finds that if Bitcoin prices increase, the investors’ attractiveness in response will grow. Obviously, people who buy Bitcoin today do so for speculative reasons (i.e. to win money). Few of them use this money to make purchases. Therefore, it is highly expected that when the prices increase, the attractiveness of people to Bitcoin increases simultaneously. The reverse link that runs from users’interest to Bitcoin price is also highly supported. This outcome may be explained by the fact that when businesses and investors deem that there are too risky markets including that of Bitcoin and as response seeks to secure their money by withdrawing from these uncertain markets, Bitcoin prices will decrease as a result.

Glaser et al. (2014) explore whether Bitcoin may be considered as an asset or a currency. They try to answer “what intentions are users following when changing a domestic currency into a crypto-currency like Bitcoin?” This study relies on Autoregressive Conditional Heteroskedasticity (ARCH) model to evaluate if Bitcoin changes within the
network and exchange. They show that the motivation of investors to Bitcoin and their intention to gather proper and additional information about this virtual currency has a significant and substantial impact on the Bitcoin exchange volume, while this influence seems insignificant when considering the volume within the Bitcoin system. They attribute these findings to the fact that exchange users prefer usually to keep their Bitcoins in their exchange wallet to avoid possible speculative effects without any intention to use them in transaction goals or in paying goods or services.

Furthermore, Kristoufek (2014) tries to tackle accurately the potential sources of Bitcoin price fluctuations including supply-demand fundamentals, speculative and technical drivers and assesses whether there are significant interconnections between them at different time frequencies during the period between 14/09/2011 and 28/02/2014. A continuous wavelet approach (in particular wavelet coherence) has been applied in order to analyze effectively the evolution of correlations between series under consideration over time and frequencies. The obtained results reveal that the fundamental factors such as trade-exchange ratio and supply play substantial roles in the long-run (i.e. high time scales). The Chinese index seems an important source of Bitcoin price evolution, while the contribution of gold price dynamics appears minor. He finds also that Bitcoin prices are mainly influenced by investors’ interest, as it captures the information about the demand of the currency. This interconnection is most dominant at low time frequencies (in the long terms). There is a quick impact of users ‘interest on Bitcoin price dynamic over the period of price contraction rather than bubble build-up period. More precisely, during the explosive prices period, the interest drives this currency price up, while it drives it down during the period of rapid declines. The author adds that technical drivers proxied via a hash rate have no great effect on Bitcoin price over time. This unexpected outcome has been attributed to the inability of Bitcoin to offset the computational power costs. Nevertheless, the followed technique allows it to consider the interconnection between each two variables without considering the possible interaction with other time series, which may be incomplete. More accurately, wavelet coherence may not be considered usually as perfect technique. On the one hand, it may lead to confuse outcomes since the occurrence of noise cannot be heavily neglected, disrupting then the studied relationship (Ng and Chan, 2012). On the other hand, wavelet decomposition is generally applied to assess the periodicity and the multiple signals that happen over time. Moreover, when we consider only two variables in wavelet analysis, we generally fall on the problem of simple regression without control variable which is unable to capture appropriately proper results with regard to the concerned relationship since it may distort the estimate. In that
context, Aguiar-Conraria and Soares (2011) and Bouoiyour et al. (2014) argue that the findings may change intensely when we move from wavelet investigation with two variables for conditional wavelet estimation (with more than two variables or by adding other explanatory time series). This highlights an insightful evidence that the use of large-scale parameters of each two variables as the case of Kristoufek (2014)’s study may prompt inconclusive results in terms of the interaction dynamic between Bitcoin price and its main drivers. This reinforces the need to further assessment of the focal link.

The contribution of the present study is to check the robustness of the above results using unconditional frequency causality and to find new insights by applying conditional causality through two main factors (Chinese market index and hash rate).

3. Data and Methodology

3.1. Data

The main goal of this study is to measure the causalities between Bitcoin price and business transactions on the one hand and Bitcoin and investors’ attractiveness on the other hand unconditionally and conditioning on Chinese market performance and the hash rate (technical driver of Bitcoin). The variables under consideration are therefore as follows:

- **Bitcoin price**: Bitcoin is a nascent crypto-currency not yet substantially accepted as a payment system across wide markets. It is an alternative currency to the fiat currencies (the US dollar, the Euro) with several advantages such as low fees and informational transparency of all transactions and various drawbacks including the lack of legal security and the speculative behavior (Kristoufek, 2014). Bitcoin does not have an underlying value derived from consumption or production process such as the precious metals such as gold. Accordingly, Bouoiyour and Selmi (2014) show that there is any sign of Bitcoin being a “safe store”.

- **Business transactions**: Theoretically, the price of Bitcoin is positively associated to transactions, as it increases the utility of holding the currency leading to an increase in Bitcoin price. The exchange-trade ratio (ETR) is measured as a ratio between volumes on the currency exchange market and trade, and can be considered as measure of transactions (Kristoufek, 2014).

- **Investors’ attractiveness**: Kristoufek (2013, 2014) show that an increased users’ interest searching for information about Bitcoin leads to an increase in Bitcoin prices.
To measure the investors’ interest to Bitcoin, we can use daily Bitcoin views from Google\(^1\) as it allows us to capture the speculative of Bitcoin’ investors. More accurately, we use the number of times a key search word search term in relation to Bitcoin is entered into the Google engine. Several researches have employed Google search volume data as proxy of users’ attractiveness. Some of them have showed a great correlation between Google search and employment (Askitas and Zimmermann, 2009). Others have linked Google views to the consumption expenditures (Vosen and Schmidt, 2011), the housing prices (Kulkarni et al. 2009), the stock trading volume (Vlastakis and Markellos, 2010).

- **Chinese market index:** The Chinese market index is considered as a major player in explaining the evolution or the volatility of digital currencies and particularly the nascent crypto-currency (Bitcoin). Ciaian et al. (2014) discuss this viewpoint by depicting the sharp coincidence between some decreases and price increases in the Bitcoin exchange rate and the dramatic events that have undergone the Chinese market and the Chinese regulation problems. Arguably, Kristoufek (2014) takes an important example which is the development around Baidu that may be considered as a potential determinant of the Chinese online shopping. The announcement that Baidu is accepting bitcoins in October 2013 has affected significantly and positively the price dynamic of Bitcoin.

- **Hash rate:** The emergence of the famous virtual money has provided new approaches concerning Bitcoin payments. Hence, some new words have emerged such as the hash rate. It may be considered as an indicator or measure of the processing power of the Bitcoin network. For security goal, Bitcoin network must make intensive mathematical operations, leading to an increase in the hash rate itself heavily connected with an increase in cost demands for hardware. This may affect widely Bitcoin purchasers and thus expands the demand of this new currency and in turn their prices. Theoretically, the hash rate is associated positively to Bitcoin price. According to Kristoufek (2014), an increase in Bitcoin price generates the intention of market participants to invest and to mine, which prompts a higher hash rate.

\(^1\) It corresponds to the frequency of Google searches (i.e. the online Google search queries) related to this new crypto-currency. Ciaian et al. (2014) and Piskorec et al. (2014) put in evidence the effectiveness of this proxy potential investors’ interest measure enables to properly capture the information’s demand about the focal digital money and to appropriately detect the speculative behavior of Bitcoin users.
The daily data of Bitcoin price index (BPI) is collected from Blockchain (https://blockchain.info/) has been used in our investigation, and the data for the exchange-trade ratio (ETR) and users ‘attractiveness to Bitcoin (TTR) is accessed from quandl (http://www.quandl.com/) and via daily Bitcoin views from Wikipedia, respectively. All the time series under consideration are converted to natural logarithms in order to smooth them. The long time range of our time series data (from December 2010 to June 2014) may help practitioners and investors to reach more information about the price evolution of this new crypto-currency and assess it accurately over time.

Figure 1 depicts the evolution of BPI, ETR and TTR and the sharp correlation between them. The excessive swings of Bitcoin price have engaged the attention of Medias and researchers. This great attention to this nascent crypto-currency may be mainly attributed to the volatile behavior that heavily characterize Bitcoin price dynamic since its creation. Figure 1 confirms the sizeable volatility closely associated to this digital money. It was 1.14 dollars on 01/06/2011 and becomes more than 900 dollars on 11/01/2014. We can argue here that GARCH effects might be present in Bitcoin process evolution. Fortunately, frequency domain analysis is not sensitive to volatility clusters or the succession of tranquil and turbulent periods (Bodart and Candelon, 2009).

Figure 1. Bitcoin price, transactions and investors attractiveness

Notes: The variables of interest are taken in black, while the conditional variables dynamics are in gray.
To investigate whether there is a short- or long-run causal relationship between Bitcoin price and transactions and between this digital money and investors’ attractiveness, we start by reporting the descriptive statistics (see Table 1) in order analyze the nature of data. We clearly show a substantial data variability, highlighting the very prime need to use robust models which may incorporate this nature of the data. The coefficient of kurtosis appears inferior to 3 for all the variables in question (except ETR), indicating that the distribution is less flattened than the normal distribution. The Skewness coefficient is positive for all the time series in question. This implies that the asymmetrical distribution is more plausible. The Jarque-Bera test revealed high and significant values (except for TTR), leading to reject the assumption of normality for the concerned variables.

<table>
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<tr>
<th>Table 1. Summary of statistics</th>
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<td><strong>BPI</strong></td>
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<td>Jarque-Bera</td>
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<td>Probability</td>
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3.2. Methodological framework

3.2.1. Unconditional versus conditional causality

Various researches have criticized the unconditional causality or more accurately the causality analysis between two series without accounting for other control variables that may have great influence on the studied linkage. When we take into account only two variables in the analysis, it will be difficult to capture solid findings with regard to the focal interaction dynamic since it may distort the estimate. This highlights the importance of deeper assessment of each causal relationship while considering the factors that may play a substantial role in explaining the dynamic of the studied nexus.

Considering X and Y the variables of interest and Z the conditional variable. Bi-variate and trivariate equations allow us to rely the focal variables as mentioned below:
\[ X_t = \sum_{i=1}^{\infty} a_i X_{t-i} + \sum_{i=1}^{\infty} b_i Z_{t-i} + \varepsilon_{0t} \quad (3.1) \]
\[ Z_t = \sum_{i=1}^{\infty} a_i X_{t-i} + \sum_{i=1}^{\infty} h_i Z_{t-i} + \mu_{vt} \quad (3.2) \]

where \( \varepsilon_{0t} \) and \( \mu_{vt} \) are independent and identically distributed with zero mean and variance-covariance matrix written as follows:

\[ \Sigma 1 = \left( \begin{array}{cc} \sum \varepsilon_0, \mu_0 \\ \sum \mu_0, \varepsilon_0 \end{array} \right) \quad (3.3) \]

\[ Y_t = \sum_{i=1}^{\infty} \delta_i Y_{t-i} + \sum_{i=1}^{\infty} \gamma_i Z_{t-i} + \nu_{vt} \quad (3.4) \]
\[ Z_t = \sum_{i=1}^{\infty} \delta_i Y_{t-i} + \sum_{i=1}^{\infty} \gamma_i Z_{t-i} + \mu_{vt} \quad (3.5) \]

where \( \nu_{vt} \) and \( \mu_{vt} \) are independent and identically distributed with zero mean and variance-covariance matrix written as follows:

\[ \Sigma 2 = \left( \begin{array}{cc} \sum \nu_0, \mu_1 \\ \sum \mu_1, \nu_0 \end{array} \right) \quad (3.6) \]

Then, we can explore the link that runs from \( X \) to \( Y \) and inversely by accounting for the conditional variable by estimating trivariate equations expressed as follows:

\[ X_t = \sum_{i=1}^{\infty} \alpha_i X_{t-i} + \sum \beta_i Y_{t-i} + \sum \delta_i Z_{t-i} + \xi_{0t} \quad (3.7) \]
\[ Y_t = \sum_{i=1}^{\infty} \alpha_i X_{t-i} + \sum \beta_i Y_{t-i} + \sum \delta_i Z_{t-i} + \xi_{0t} \quad (3.8) \]
\[ Z_t = \sum_{i=1}^{\infty} \alpha_i X_{t-i} + \sum \beta_i Y_{t-i} + \sum \delta_i Z_{t-i} + \eta_{it} \quad (3.9) \]

Where \( \xi_{0t} \), \( \zeta_{it} \) and \( \eta_{it} \) are independent and identically distributed with zero mean and variance-covariance matrix can be expressed as following:
The causalities between $X$ and $Y$ conditionally to $Z$ are measured based on Geweke (1984)’s study by:

$$F_{X\rightarrow Y|Z} = \ln \left( \sum_{i} \xi_{i} \right)$$  \hspace{1cm} (3.11)

And

$$F_{Y\rightarrow X|Z} = \ln \left( \sum_{i} \xi_{i} \right)$$  \hspace{1cm} (3.12)

The effect of the additional control variables (i.e. the conditional variable) on the relationship between $X$ and $Y$ may be detected properly from the above equations (3.11) and (3.12). If $Z$ has no influence on the focal linkage (i.e. there is no remarkable changes in the relationship in question), this means that $\sum_{i} \xi_{i} = \sum_{i} \xi_{i}$ and $\sum_{i} \mu_{i} = \sum_{i} \xi_{i}$ and therefore $F_{Y\rightarrow X|Z}$ and $F_{X\rightarrow Y|Z}$ are insignificant. However, if the conditional variables under consideration lead to a modification (either slightly or heavily) on the studied relationships, this implies that $F_{Y\rightarrow X|Z}$ and $F_{X\rightarrow Y|Z}$ are significant.

Both conditional and conditional causalities are estimated within frequency domain framework. This technique is well explained throughout the rest of this section.

**3.2.2. Frequency domain approach**

The frequency domain analysis has been widely used recently to evaluate lead-lag relationships between macroeconomic variables with respect to frequency rather than time. It tests the causality between two series as the Granger causality tests. However, the latter seems not suited to distinguish between the short and long-run effects. To get a new look at the relationship and more proper analysis depending to frequency transformations, we carry out a frequency domain Granger causality test (Breitung and Candelon, 2006). Hence, the starting
point of this last testing procedure is the Granger causality test. “A variable $Y_t$ is said to Granger cause $X_t$, if $Y_t$ contains information to predict $X_t$ that is not available otherwise” (Lütkepohl 2005, pp.41). Obviously, this technique is restrictive since it cannot capture the studied links at different paths. It can be written as matrix notation as following:

$$\Theta(L) = \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \theta_11(L) & \theta_{12}(L) \\ \theta_{21}(L) & \theta_{22}(L) \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \varepsilon_t \quad (3.13)$$

$Y_t$ does not Granger cause $X_t$ if ($\Theta_{12}(L)=0$), indicating that the past values of $Y_t$ seem not closely related to $X_t$. This can be tested by using an F-Test for the coefficients $\Theta_{12,i}$ for $i = 1 \ldots p$. To overcome the Granger (1969)’s test restrictions, we choose to apply frequency-domain Granger causality as a decomposition of the total spectral interdependence between each macroeconomic variables into a sum of instantaneous and bidirectional causality terms (Tiwari et al. 2014). Geweke (1982) put in evidence that the causality test can be performed under different frequencies without loss of explanatory power, implying that causality measure ($F_{y \rightarrow x}$) can be decomposed as follows:

$$F_{y \rightarrow x} = \int_0^\pi f_{y \rightarrow x}(\omega)d\omega \quad (3.14)$$

Or with a consideration of conditional variable, meaning that causality measure ($F_{y \rightarrow x}$) can be decomposed as follows:

$$F_{y \rightarrow x/z} = \int_0^\pi f_{y \rightarrow x/z}(\omega)d\omega \quad (3.15)$$

Various tests have been proposed to analyze the frequency domain nexus between two variables (Geweke (1982), Breitung and Candelon (2006) and Lemmens et al. 2008). In the present research, we refer to Breitung and Candelon’s (2006) study. They construct an F-test for the coefficients $\Theta(L)$ at different frequencies to overcome the gaps of standard Granger causality test. More precisely, to measure the causality under different frequencies, the above bivariate and trivariate equations should be re-written as following (matrix form):

$$\Psi(L)\nu = \begin{bmatrix} \psi_{11}(L) & \psi_{12}(L) \\ \psi_{21}(L) & \psi_{22}(L) \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} \quad (3.16)$$
where \( \psi_{11}(0) = 1; \psi_{22} = 1; \psi_{12} = 0; \psi_{21} = 0; \text{cov}(\tau_h, \tau_{12}) = 0; \), \( \Psi' = [\Theta(L)G]^{-1}; \) \( G \) denotes the lower triangular matrix of the Cholesky decomposition; \( \tau_l = G^{\frac{1}{2}}. \)

\[
\Gamma(L) \kappa = \begin{pmatrix}
\tau_{11} (L) \tau_{12} (L) \tau_{13} (L) \\
\tau_{21} (L) \tau_{22} (L) \tau_{13} (L) \\
\tau_{31} (L) \tau_{32} (L) \tau_{33} (L)
\end{pmatrix}
\begin{pmatrix}
X_l \\
Y_l \\
Z_l
\end{pmatrix}
= \begin{pmatrix}
\kappa_{1r} \\
\kappa_{2r} \\
\kappa_{3r}
\end{pmatrix}
\tag{3.17}
\]

where \( \tau_{11}(0) = 1; \tau_{22} = 1; \tau_{33} = 1; \tau_{12} = 0; \tau_{21} = 0; \tau_{13} = 0; \tau_{31} = 0; \tau_{32} = 0; \tau_{23} = 0; \), \( \Gamma_l = [\Phi(L)G]^{-1}; \) \( G \) denotes the lower triangular matrix of the Cholesky decomposition.

Ultimately, the null hypothesis of no Granger Causality at frequency \( \omega \) can be written based on matrix notations (3.16) and (3.17) respectively as follows:

\[
H_0 : R(\omega)\Theta(L) = 0
\tag{3.18}
\]

\[
H_0 : R(\omega)\Phi(L) = 0
\tag{3.19}
\]

where \( R(\omega) = \begin{bmatrix}
\cos(\omega) \cos(2\omega) \ldots \cos(p\omega) \\
\sin(\omega) \sin(2\omega) \ldots \sin(p\omega)
\end{bmatrix} \)

Based on the above equations, the obtained time-domain graph depicts a signal changes over time, whereas a frequency-domain graph indicates how the signal moves among given frequency bands. Basically, the time scale is the ability to highlight precisely when a variation happens by identifying well-defined time horizons, while the frequency band is seemingly a component that able to determine effectively the degree of a certain variation. The presence of volatility clusters (i.e. succession of higher and lower volatilities periods) that may greatly characterize Bitcoin price dynamic has no substantial effect on the interdependence between each two variables when applying frequency domain causality (Bodart and Candelon, 2009). This technique may allow us to condition upon a set of control variables that may have great influence on the directional causality. For our case of study, we test in the first case the interconnection between business transactions and Bitcoin price and then the interdependence between users 'interest to the nascent digital money and Bitcoin price (unconditional causality). Then, we re-assess the frequency causality test but conditional to the performance of Chinese market proxied by the Shangai index and the hash rate.
4. Main findings and discussion

The main objective of this study is to investigate the main causes of Bitcoin price dynamic under different frequencies (the short, medium and long terms). Firstly, a bi-variate model is estimated for the nexus between exchange-trade ratio and Bitcoin to address whether Bitcoin behaves as business income (Figure 2). Secondly, the model is performed to explore the link between investors ’attractiveness and Bitcoin price (Figure 3) to find whether this new virtual currency is likely to be a “speculative bubble” rather than business transactions tool. These tests are performed within unconditional and conditional causality frameworks. The control or the conditional variables added here are the Chinese stock market index as proxy of Chinese market performance and the hash rate as indicator of Bitcoin technical difficulty.

4.1. The lead-lag relationship between exchange-trade ratio and Bitcoin

Figure 2 disentangles the short and long terms business cycle causality among two variables of interest (transactions proxied by the exchange-trade ratio and Bitcoin price). Our results clearly reveal that both time series Granger-cause each other at different frequency horizons. The frequencies range from 0 to 2 may be translated into periodicities of $T$ days by $T = 2\pi/\omega$. More accurately, we show that Bitcoin price (BPI) Granger-causes exchange-trade ratio (ETR) in the short- and the medium-run cyclical components particularly in the frequency range of 0.01 to 1.73, indicating the business cycle of 3.7 to 628.3 days. A potential explanation for this finding is the substantial role that plays the level of price in explaining exchange and trade transactions. The usage of this new crypto-currency in trade transactions and the evolution of Bitcoin prices may be emerged due to speculative attacks. It is highly expected that the greater is the number of the coins used the stronger the demand for them and the higher will be the prices. Nevertheless, if Bitcoin prices are driven by speculation, the uncertainty about the price may lead to unexpected nexus. Admittedly, the increase of prices prompts a drop in demand for the currency at exchanges. When price closely linked to one currency decreases with respect to the price of other one, the first currency will appreciate and exchange rate will increase. This implies that there is a causality running from the price to exchange rate, affecting then trade transactions especially in extreme price periods. In addition, the null hypothesis of no Granger causality that runs from ETR to BPI is not rejected at any frequency. This result is inconsistent with Kristoufek (2014) providing evidence that

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2 By definition the exchange rate is the relative price of basket of traded and non-traded goods.
the increasing use of Bitcoin in ETR expands Bitcoin in the long-run and then there is a significant link that runs from ETR to BPI, which becomes stronger in the long term. But, this author adds that, this nexus changes substantially among frequency horizons to give us any solid and unambiguous conclusion. This goes without saying that these results should be treated with caution.

These results do not change conditionally to the situation of Chinese market or the hash rate and the technical difficulty. We clearly show from Figure 2 that there is any change in the causal relationships in question when moving from unconditional causality to conditional causality analysis. This implies that neither the Chinese market environment nor the processing power of Bitcoin network have any influence on the causal interaction between business transactions and Bitcoin price. The first outcome may be attributed to the fact that the investors in the Chinese stock market are considerably regarded as being heavily speculative. The second finding may be owing to the fact that despite the usefulness of Bitcoin technology in settling Bitcoin transactions, miners who mine this new crypto-currency in order to certify the transactions in blocks may prompt a sharp inflow of new Bitcoins into circulation. In the case of higher demand and then when there is an intense increase in trade volume, mining may be unable to solve appropriately and computationally the problem associated to great demand.
Figure 2. Frequency domain analysis of the link between transactions and Bitcoin price

![Frequency domain analysis graph]

Unconditional causality

Values of F-statistics

Conditional causality

Notes: BPI and ETR denote respectively Bitcoin price index and exchange-trade ratio; Frequency \((\omega) = 2\pi/cycle length (T)\); the dark gray shaded area corresponds to the significance area of the null hypothesis (BPI does not Granger cause ETR).
4.2. The lead-lag relationship between investors’ attractiveness and Bitcoin

Figure 3 depicts the nexus between investors-attractiveness to the focal cryptocurrency and Bitcoin price. It’s worthy to notice that TTR Granger-cause BPI in the medium- and the long-run. TTR Granger-causes BPI at higher frequencies (Graph 1, Figure 3) or more precisely when $\omega \in [2.2\pi - 3.0\pi]$. This finding appears logical since it seems obvious that if the number of persons interested in BPI will increase gradually, the Bitcoin prices will as response increase. The great media attention to this new digital money substantially pushes bitcoin to an all-time increase. Similar dynamic is worthy notable when Zynga\(^3\) and Overstock\(^4\) begin accepting Bitcoin from January 2014.

Besides, the reverse causal relationship that runs from BPI to TTR seems visible in lower frequencies (the long-run time scales) or accurately when $\omega \in [0.01\pi - 0.1\pi]$. One of the most substantial possible explanations of this last outcome is that people who buy Bitcoin today do so for speculative reasons (i.e. to win money). Few ones use Bitcoin to make purchases (real economy). So, we should expect that when the price increases, the TTR increases considerably. For instance, the first quarter of 2013 was marked by an extreme bubble when Bitcoin price increases intensely, moving from $13 to above $200. Bitcoin users believe that they will be able to easily reach a lot of money, in particular when prices hit a record high of 900 dollars for instance in 11/01/2014.

These outcomes change remarkably when considering HASH and SI. More precisely, we find that the causal relationship running from BPI to TTR becomes insignificant, while the reverse interaction dynamic that runs from TTR to BPI changes among the different frequencies involved. It appears significant when $\omega \in [0.01\pi - 0.2\pi] \cup [2.4\pi - 3.0\pi]$. A great connection is therefore sharply observed at the nascent stages and under the bubble-building up periods after hitting high price levels. This implies that the Chinese market and the Bitcoin technology play determinant roles in explaining these causalities. Unsurprisingly, the Chinese market has experienced several events that coincide with great changes in Bitcoin prices. For example, the announcement that Baidu is accepting Bitcoins has impacted significantly the behavior of businesses and users in general, affecting then positively the Bitcoin prices. Unfortunately, investors deem that these markets are too risky and seek to secure their money

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\(^3\)Zynga is a recent company of social games, founded in 2007 in California. It works specifically on mobile phone platforms such as AppleiOS and Android.

\(^4\)Overstock is an American online retailer launched in 1999. It sales products from individuals or businesses to the end-user.
by withdrawing from these uncertain markets, especially when darken events occur such as the great observable drop in Bitcoin after the closing of the trading platform aimed at Chinese investors after the Central Bank clampdown\(^5\), touching a record low of 120 dollars. These events and uncertainties obviously generate the volatility. Furthermore, the fact that Bitcoin’ users invest into the hardware may prompt an increase in the hash rate and in higher technical difficulty. This obviously will increase the inability of Bitcoin to offset the computational power costs, expanding as response the speculative behavior of investors. More precisely, several investors have hardly acknowledged about mathematical programs, and hence it is unknown for them how far it can go. This uncertainty creates therefore unpleasantly a great speculation.

Summing up, we have verified the speculative nature of Bitcoin, which is already obvious. We have showed also that Bitcoin may be used for economic reasons. Nevertheless, these outcomes appear counter-intuitive since it is well expected that speculation causes Bitcoin in short- and medium-run, while the usage of Bitcoin in trade transactions is likely to be observed in the long term. More importantly, nobody accurately knows the future of this nascent money, i.e. if it is risky investment or long-term promise. It seems uncertain; it can remain as it can disappear. In our case of study and as mentioned above, we have found unexpected results, highlighting the complexity of assessing what does crypto-currency look like. It seems therefore highly difficult to Gain clearer insights into Bitcoin Phenomenon.

\(^5\) It is important to mention here that China’s biggest bank (ICBC) and 10 other banks have intensely banned activities related to trading in Bitcoin under the Central Bank crashdown.
Figure 3. Frequency domain analysis of the link between attractiveness and Bitcoin price

Unconditional causality

Functional (ω) = 2π/cycle length (T);
the light gray shaded area corresponds to the significance area of the null hypothesis (TTR does not Granger cause BPI); the dark gray shaded area corresponds to the significance area of the null hypothesis (BPI does not Granger cause TTR).

Conditional causality

Notes: BPI and TTR denote respectively Bitcoin price and investors' attractiveness; Frequency (ω) = 2π/cycle length (T);
5. Conclusions and policy implications

Due to the growing acceptance of Bitcoin as a nascent digital currency and as a new payment tool, users need to better understand this phenomenon. The present research tries to see whether Bitcoin seems a good driver of business transactions, a future promise or a “speculative bubble”. This study contributes to the existing literature on the field by assessing the lead-lag relationship between Bitcoin price and exchange-trade ratio on the one hand and BPI and investors’ attractiveness on the other hand, conditionally to the performance of Chinese market and the processing power of Bitcoin network. To this end, we decompose the causal connections between these variables into different frequency components by applying Breitung and Candelon’s (2006) method using daily data covering the period from December 2010 to June 2014. Our findings provide insightful evidences that may have important policy implications.

The aforementioned causality results appear far from stable and unambiguous. Bitcoin price Granger-causes exchange-trade ratio in lower and medium frequencies. We do not support the reverse nexus. There is any change in the obtained results when moving from unconditional to conditional causality. Besides, the investors’ attractiveness Granger-cause Bitcoin price at higher frequencies. This link changes substantially when considering the Shangai index (proxy of the performance of Chinese market) and the hash rate. When applying conditional causality test, we provide evidence that TTR causes BPI at short- and long-run frequencies, while the reverse link running from BPI to TTR is clearly observed at lower frequencies. These results are unexpected. Normally, the speculation should cause Bitcoin in short- and medium-run, while the Bitcoin price may be used in economic reasons including business transactions in the long term. At any case, nobody have up to now any clear idea about the predictive content of Bitcoin, acknowledging the complexity of assessing what does crypto-currency look like (Bouoiyour and Selmi (2014) and Bouoiyour et al. (2014)).

To sum, the two studied relationships (BPI and ETR; BPI and TTR) are bidirectional and cyclical. These cycles can be short, medium or long depending to the studied direction of causality. The first causal relationship (BPI and ETR) does not change conditionally on SI and HASH, while the second ones (BPI and TTR) changes remarkably when considering the performance of Chinese market and the technical difficulty of Bitcoin Network. This sharply acknowledges the great complexity characterizing the focal new digital money and highlights the utmost importance to consider control variables when investigating the potential sources.
of Bitcoin price dynamic in general and the causal relationships (BPI and ETR) and (BPI and TTR) in particular. The conditional frequency domain analysis appears fruitful and valuable to reach better paths since it allows us to explore the focal links over time and under different frequencies conditioning upon additional variables that may drive significantly the obtained results and then may improve substantially the decisions-making by businesses and practitioners.

The originality of this research can be viewed on the nature of link between (BTP and ETR) and (BTP and TTR), which appear bidirectional and conditional on other control variables including SI and HASH (only for the linkage between Bitcoin price and investors’ attractiveness). We can also conclude that beyond the speculative behavior. So, should businesses and investors have confidence in a currency that wins or loses more the half of its value in very few days?

Since its creation, Bitcoin has attracted a huge number of users due to its lower transactions fees and deflationary bias. Unfortunately, the excessive volatility of Bitcoin and the rampant hacking attacks are likely to discourage users of this digital money. Even though Bitcoin’s dependency to computer algorithms, the majority of users has very little experience in mathematical programs, and hence it is unknown for them how far it can go. This structural problem may create uncertainty and then speculation, which may be aggravated by the absence of effective financial tools such as forward contracts and swaps that are routinely used to prevent external shocks for official currencies and to properly manage the risk arising from the great volatile behavior of Bitcoin.

In a nutshell, to be bidirectional and conditional on Chinese stock market and the hash rate clearly indicates that the linkage between Bitcoin price and investors’ attractiveness is more complex than it may appears. Unsurprisingly, this relationship is dominant by speculation aspect and the road seems therefore long, winding and frightening before shoring up fully confidence in this new crypto-currency.

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6The purchasing power of bitcoins does not affect inflation dynamic, since the release schedule seems predictable.
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DOI: 10.1175/JTECH-D-12-00056.1


