Labour Policies In The DMP Model—A Theoretical Analysis

Bandopadhyay, Titas Kumar

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LABOUR POLICIES IN THE DMP MODEL—A THEORETICAL ANALYSIS

By
Titaskumar Bandyopadhyay*

Abstract: In this paper we examine different types of labour policies in the benchmark model of DMP in both cases where job-destruction rate is exogenous and endogenous. Our theoretical results show that the labour market tightness and the unemployment rate would be more volatile if job-destruction is endogenous.

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Introductions: In the labour market workers search good jobs and firms also search good workers. The unemployed workers are absorbed in the vacant positions through matching with the firms. Production starts only when labour and firm are matched. Matching is not an instantaneous process, rather it is time-consuming and costly. Once there is a match in the labour market, both parties need not search for other and so search costs are saved and this gives birth to the surplus which is shared between the labour and the matched firm. The most commonly used surplus sharing rule is the Nash-bargaining rule. These features of the labour market have been conceptualized in the benchmark model of Diamond-Mortensen-Pissarides (called DMP hereafter).

The DMP model has been extended by introducing endogenous job-destruction, efficiency wage relation and business cycles. We find these extensions in the models of Albrechet et al. (1989), Cole and Regerson (1989), Mortensen and Pissarides (1994), Abraham et al. (1995), Zenou and Smith (1995), Andolfatto (1996), Shimmer (2005), Zenou (2005), Arozamena and Ceneteno (2007), Pissarides (2007) etc.

* Address of correspondence: Titas Kumar Bandopadhyay; 24/1 B.B.Halder Lane, Shibpur,Howrah-711102; West Bengal, India. E-mail:titasban1@yahoo.in

Matching plays the central role in the DMP model. Matching is a function of unemployment rate and vacancy rate and is subject to CRS. The matching function has also been used by Hall (1979), Pissarides (1979), Diamond and Maskin (1979), Bowden (1980) etc. Mortensen (2011) considers two types of matching function: linear and quadratic. Hosios (2003) also shows that in efficiency worker's bargaining power is related to the elasticity of the matching function.

In this paper, we first describe the benchmark model of the DMP and then we examine the consequences of different labour market policies on the Nash-wage rate, labour market tightness and on the unemployment rate in both the cases where job-destruction is exogenous as well as endogenous. Our comparative static results show that equilibrium unemployment rate would be more volatile if job-destruction rate is endogenous. Further, labour policies like more unemployment benefit and / lower worker's bargaining power may produce different effects on unemployment rate when job-destruction is endogenous.

2. The Benchmark DMP Model:

In the DMP model, job-matching between a job-seeker and a firm is expressed by the matching function: $m = m(u,v)$, where u is the unemployment rate and v is the vacancy rate in the labour market and $m_1, m_2 > 0; m_{11}, m_{22} < 0 and m_{12}, m_{21} = 0.$ Total match flows is $m = au$ and total job flows is $m = vq.$ So, the job- arrival rate is $a = \frac{m}{u}$ and the job- offer rate is $q = \frac{m}{v}.$ Matching function is assumed to possess CRS property and we can write $a = \frac{m}{u} = \frac{m}{v}.\frac{v}{u} = q(\theta).\theta,$ where $\theta = \frac{v}{u}$ is the labour market tightness and $q'(\theta) < 0, \left|e_{\theta}\right| < 1.$

The Bellman equations for unemployment (U), employment (W), vacancy (V) and jobs filled in (J) are

$$rU = b + \theta q(\theta)(W - U) \tag{1}$$
$$rW = w - \lambda(W - U) \tag{2}$$
$$rV = -C + q(\theta)(J - V) \tag{3}$$
$$rJ = y - w - rk - \lambda J \tag{4}$$

Where r is the discount rate, b is the unemployment benefit, C is the cost of maintaining vacancy, y is the constant match productivity and q is the job offer rate, λ is the job-destruction rate, w is the wage rate and k is the capital hired per labour.

Using the zero-profit condition and using Equations (3), (4) we can write the steady state job creation condition as

$$y = w + rk + \frac{(r+\lambda)}{q(\theta)}C \tag{5}$$

The Nash-wage equation in the DMP model is

$$w = (1-\beta)b + \beta\left(y + C\theta - rk\right) \tag{6}$$

Solving (5) and (6) one gets the equilibrium values of w, θ.

The Beverage curve is given by

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{7}$$

Equilibrium u can be obtained from Equation (7) after determining equilibrium θ.

3. **Comparative Static Exercises:**

Taking total differentials of Equations (5), (6) and after simple manipulations one gets[1]

$$\left.\begin{array}{l} \dfrac{\hat{W}}{\hat{y}} > 0, \dfrac{\hat{W}}{\hat{r}} < 0, \dfrac{\hat{W}}{\hat{\beta}} > 0, \dfrac{\hat{W}}{\hat{b}} > 0, \dfrac{\hat{W}}{\hat{\lambda}} < 0, \\ \dfrac{\hat{\theta}}{\hat{y}} > 0, \dfrac{\theta}{\hat{r}} < 0, \dfrac{\hat{\theta}}{\hat{\beta}} < 0, \dfrac{\hat{\theta}}{\hat{b}} < 0, \dfrac{\hat{\theta}}{\hat{\lambda}} < 0 \end{array}\right\} \tag{8}$$

Again, taking total differentials of (7) and using (8) one gets

$$\frac{\hat{u}}{\hat{y}} < 0, \frac{\hat{u}}{\hat{r}} > 0, \frac{\hat{u}}{\hat{\beta}} > 0, \frac{\hat{u}}{\hat{b}} > 0, \frac{\hat{u}}{\hat{\lambda}} > 0 \tag{9}$$

The results obtained in (8) and (9) yield the following propositions:

Proposition 1: A positive productivity shock raises Nash-wage rate and labour market tightness but reduces unemployment rate.

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[1] See Appendix A.1.

Proposition 2: A high discount rate reduces wage rate and market tightness but raises unemployment rate.

Proposition 3: Labour market reforms (i.e. a lower β) reduces wage rate and unemployment rate but raises market tightness.

Proposotion 4: A rise in unemployment benefit raises wage rate and unemployment rate and lowers market tightness.

Proposition 5: A higher job-destruction rate lowers wage rate and market tightness but raises unemployment rate.

4. **Endogenous Job-Destruction:**

Let us now assume that the job –destruction rate is endogenous. Empirically, it has been found that there is a strong negative relation between job- destruction rate and wage rate. Thus, we may write

$$\lambda = \lambda\left(W\right), \lambda' < 0 \tag{10}$$

Now taking total differentials of (5), (6), (7), (10) and after simplifications one gets

$$\left.\begin{aligned}&\left(\frac{\hat{W}}{\hat{y}}\right)_{\lambda(W)} > 0, \left(\frac{\hat{W}}{\hat{r}}\right)_{\lambda(W)} < 0, \left(\frac{\hat{W}}{\hat{\beta}}\right)_{\lambda(W)} > 0, \left(\frac{\hat{W}}{\hat{b}}\right)_{\lambda(W)} > 0,\\ &\left(\frac{\hat{\theta}}{\hat{y}}\right)_{\lambda(W)} > \left(\frac{\hat{\theta}}{\hat{y}}\right) > 0, \left(\frac{\theta}{\hat{r}}\right)_{\lambda(W)} < \left(\frac{\theta}{\hat{r}}\right) < 0, \left(\frac{\hat{\theta}}{\hat{\beta}}\right)_{\lambda(W)} < \frac{\hat{\theta}}{\hat{\beta}} < 0, \left(\frac{\theta}{\hat{b}}\right)_{\lambda(W)} < \left(\frac{\hat{\theta}}{\hat{b}}\right) < 0\end{aligned}\right\} \tag{11}$$

and

$$\left(\frac{\hat{u}}{\hat{y}}\right)_{\lambda(W)} < \left(\frac{\hat{u}}{\hat{y}}\right) < 0, \left(\frac{\hat{u}}{\hat{r}}\right)_{\lambda(W)} > \left(\frac{\hat{u}}{\hat{r}}\right) > 0, \left(\frac{\hat{u}}{\hat{\beta}}\right)_{\lambda(W)} < 0, \left(\frac{\hat{u}}{\hat{b}}\right)_{\lambda(W)} < 0$$

$$\text{iff } e_{\lambda W} > \frac{W\left(1 + e_{q,\theta}\right)}{J\left(re_{q,\theta} - \lambda\right)} \tag{12}$$

The above results lead to the following propositions:

Proposition 6: Market tightness and unemployment rate would be more volatile to changes in productivity and / discount rate if job-destruction is endogenous.

Proposition 7: If the negative association between job-destruction and Nash-wage be very strong, unemployment rate responds differently to changes in worker's bargaining power and unemployment benefit. This is different from the case where job-destruction is exogenous.

5. Conclusions:

Search and matching are the two basic features in the labour market. The path-breaking work in the line is the DMP model. This model analyses frictional unemployment in the matching framework in the labor market. In the benchmark DMP model, we find that if job-destruction is exogenous, a positive productivity shock raises Nash-wage rate and labour market tightness but lowers equilibrium rate of unemployment. A high discount rate reduces both the wage rate and the market tightness but raises unemployment rate. Both the Nash-wage rate and unemployment rate rise and market tightness falls if unemployment benefit is increased. Further, labour market reforms through the reduction in the worker's bargaining power lowers Nash-wage and unemployment rate but raises market tightness.

However, in reality it has been observed that job-destruction rate is more flexible than job-destruction rate. Empirical works suggest a strong negative association between job-destruction rate and wage rate. Our theoretical results establish that market tightness and equilibrium unemployment rate would be more volatile if job-destruction rate is endogenous. Further, labour market reforms and / unemployment benefits may produce different effects on labour market tightness and unemployment rate in the case of endogenous job-destruction rate. These theoretical results may provide an insight to the policy makers to pursue appropriate policies in the labour market to mitigate the problem of unemployment.

Appendices

Appendix A.1: Effects on W, θ when λ is exogenous

Taking total differentials of Equations (5) and (6) and after simplifications one gets

$$W\hat{W} - (r+\lambda) Je_{q,\theta}\hat{\theta} = y\hat{y} - (J+k) r\hat{r} - \lambda J\hat{\lambda} \tag{A.1}$$

$$W\hat{W} - \beta C\theta\hat{\theta} = (1-\beta) b + \beta (y + C\theta - rk - b)\hat{\beta} + \beta y\hat{y} - \beta rk\hat{r} \tag{A.2}$$

Using Crammer's rule one may write

$$\hat{W} = \frac{1}{\Delta}\left[\begin{array}{l} \beta y\left\{(r+\lambda) Je_{q,\theta} - C\theta\right\}\hat{y} + \beta r\left\{C\theta (J+k) - k(r+\lambda) Je_{q,\theta}\right\}\hat{r} \\ \qquad + \beta C\theta\lambda J\hat{\lambda} + (r+\lambda) Je_{q,\theta} (1-\beta) b\hat{b} + (r+\lambda) Je_{q,\theta} (W-b)\hat{\beta} \end{array}\right] \tag{A.3}$$

and

$$\hat{\theta} = \frac{W}{\Delta}\left[-y(1-\beta)\hat{y} - \left\{\beta k - (J+k)\right\} r\hat{r} + (W-b)\hat{\beta} + (1-\beta) b\hat{b} + \lambda J\hat{\lambda}\right] \tag{A.4}$$

where

$$\Delta = W\left[(r+\lambda) Je_{q,\theta} - \beta C\theta\right] < 0 \tag{A.5}$$

Using (A.3), (A.4) and (A.5) we may get

$$\frac{\hat{W}}{\hat{y}} = \underset{(-)}{\frac{\beta y}{\Delta}}\left[\underset{(-)}{(r+\lambda) Je_{q,\theta} - C\theta}\right] > 0 \ \} \tag{A.6.1}$$

$$\frac{\hat{W}}{\hat{r}} = \underset{(-)}{\frac{\beta r}{\Delta}}\left[\underset{(-)}{C\theta (J+k) - k(r+\lambda) Je_{q,\theta}}\right] < 0 \tag{A.6.2}$$

$$\frac{\hat{W}}{\hat{\beta}} = \underset{(-)}{\frac{1}{\Delta}}.\underset{(-)}{(r+\lambda) Je_{q,\theta}}\underset{(+)}{(W-b)} > 0 \tag{A.6.3}$$

$$\frac{\hat{W}}{\hat{b}} = \underset{(-)}{\frac{1}{\Delta}}.\underset{(-)}{(r+\lambda) Je_{q,\theta}} (1-\beta) b > 0 \tag{A.6.4}$$

$$\frac{\hat{W}}{\hat{\lambda}} = \underset{(-)}{\frac{1}{\Delta}}\beta C\theta\lambda J < 0 \tag{A.6.5}$$

$$\frac{\hat{\theta}}{\hat{y}} = -\frac{W}{\Delta} y(1-\beta) > 0 \tag{A.7.1}$$

$$\frac{\hat{\theta}}{\hat{r}} = -\frac{Wr}{\Delta}\left\{\underset{(-)}{\beta k} - \underset{(-)}{(J+k)}\right\} < 0 \tag{A.7.2}$$

$$\frac{\hat{\theta}}{\hat{\beta}} = \frac{W}{\Delta}(W-b) < 0 \tag{A.7.3}$$

$$\frac{\hat{\theta}}{\hat{b}} = \frac{W}{\Delta}(1-\beta) b < 0 \tag{A.7.4}$$

$$\frac{\hat{\theta}}{\hat{\lambda}} = \frac{W}{\Delta}.\lambda J < 0 \tag{A.7.5}$$

Appendix A.2: Effects on u when λ is exogenous

Taking total differentials of Equation (7) and using (A.7.1) to (A.7.5) and simplifications we get

$$\frac{\hat{u}}{\hat{y}} = -\frac{1}{\Delta} Wy(1-u)\left(1+e_{q,\theta}\right)(1-\beta) < 0 \tag{A.8.1}$$

$$\frac{\hat{u}}{\hat{r}} = \frac{1}{\Delta} Wr(1-u)\left(1+e_{q,\theta}\right)(\beta k - J - k) > 0 \tag{A.8.2}$$

$$\frac{\hat{u}}{\hat{\beta}} = -\frac{1}{\Delta} W(1-u)\left(1+e_{q,\theta}\right)(W-b) > 0 \tag{A.8.3}$$

$$\frac{\hat{u}}{\hat{b}} = -\frac{1}{\Delta} Wb(1-u)\left(1+e_{q,\theta}\right)(1-\beta) < 0 \tag{A.8.4}$$

$$\frac{\hat{u}}{\hat{\lambda}} = -\frac{1}{\Delta} W(1-u)\left[\beta C\theta + \left(\lambda - re_{q,\theta}\right) J\right] > 0 \tag{A.8.5}$$

Appendix A.3: Effects on W, θ when λ is endogenous

Taking total differentials of Equations (5) and (6) and after simplifications one gets

$$W(1+\lambda' J)\hat{W} - (r+\lambda) Je_{q,\theta}\hat{\theta} = y\hat{y} - (J+k) r\hat{r} \tag{A.9}$$

$$W\hat{W} - \beta C\theta\hat{\theta} = (1-\beta) b + \beta(y + C\theta - rk - b)\hat{\beta} + \beta y\hat{y} - \beta rk\hat{r} \tag{A.10}$$

Using Crammer's rule one may write

$$\hat{W}=\frac{1}{\Delta}\left[\begin{array}{l}\beta y\left\{(r+\lambda)Je_{q,\theta}-C\theta\right\}\hat{y}+\beta r\left\{C\theta(J+k)-k(r+\lambda)Je_{q,\theta}\right\}\hat{r}\\ \qquad +\beta C\theta\lambda J\hat{\lambda}+(r+\lambda)Je_{q,\theta}(1-\beta)b\hat{b}+(r+\lambda)Je_{q,\theta}(W-b)\hat{\beta}\end{array}\right] \tag{A.11}$$

and

$$\hat{\theta}=\frac{W}{\Delta}\left[-y\hat{y}+(J+k)r\hat{r}+\left\{(W-b)\hat{\beta}+(1-\beta)b\hat{b}\,|+\beta y\hat{y}-\beta rk\hat{r}\right\}(1+\lambda' J)\right] \tag{A.12}$$

where

$$\Delta=W\left[(r+\lambda)Je_{q,\theta}-\beta C\theta(1+\lambda' J)\right]<0 \tag{A.13}$$

Using (A.11), (A.12) and (A.13) one gets

$$\frac{\hat{\theta}}{\hat{y}}=-\frac{W}{\Delta}y\left[(1-\beta)-\beta J\lambda'\right]>0 \tag{A.14.1}$$

$$\frac{\hat{\theta}}{\hat{r}}=-\frac{Wr}{\Delta}\left[\underset{(-)}{\left\{\beta k-(J+k)\right\}}+\underset{(-)}{\beta Jk\lambda'}\right]<0 \tag{A.14.2}$$

$$\frac{\hat{\theta}}{\hat{\beta}}=\frac{W}{\Delta}(W-b)(1+\lambda' J)<0 \tag{A.14.3}$$

$$\frac{\hat{\theta}}{\hat{b}}=\frac{W}{\Delta}(1-\beta)(1+\lambda' J)b<0 \tag{A.14.4}$$

Taking total differentials of (7) and using (A.6.1)- (A.6.4), (A.14.1)-(A.14.4) and after simplification we get

$$\frac{\hat{u}}{\hat{y}}=(1-u)\left[e_{\lambda,w}\left(\frac{\hat{W}}{\hat{y}}\right)-\left(1+e_{q,\theta}\right)\frac{\hat{\theta}}{\hat{y}}\right]<0 \tag{A.15.1}$$

$$\frac{\hat{u}}{\hat{r}}=(1-u)\left[e_{\lambda,w}\left(\frac{\hat{W}}{\hat{r}}\right)-\left(1+e_{q,\theta}\right)\left(\frac{\hat{\theta}}{\hat{r}}\right)\right]>0 \tag{A.15.2}$$

$$\frac{\hat{u}}{\hat{\beta}}=(1-u)\left[e_{\lambda,w}\left(\frac{\hat{W}}{\hat{\beta}}\right)-\left(1+e_{q,\theta}\right)\left(\frac{\hat{\theta}}{\hat{\beta}}\right)\right]<0 \text{ iff } e_{\lambda,W}>\frac{W\left(1+e_{q,\theta}\right)}{J\left(re_{q,\theta}-\lambda\right)} \tag{A.15.3}$$

$$\frac{\hat{u}}{\hat{b}} = (1-u)\left[e_{\lambda,w}\left(\frac{\hat{W}}{\hat{b}}\right) - \left(1+e_{q,\theta}\right)\left(\frac{\hat{\theta}}{\hat{b}}\right)\right] < 0 \text{ iff } e_{\lambda,W} > \frac{W\left(1+e_{q,\theta}\right)}{J\left(re_{q,\theta}-\lambda\right)} \qquad \text{(A.15.4)}$$

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