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A Note on Loadings and Deductibles: Can a Vicious Circle Arise?

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In a recently reprinted paper Borch wonders whether an increase in insurance loadings, together with the consequent increase in customers' deductibles, may be the start of a vicious circle, in which higher deductibles produce higher loadings and *vice versa*, *ad infinitum*. This paper rules out the possibility of a vicious circle, in a model *à la* Borch. First of all, increases in costs of the type considered by Borch are not necessarily followed by increases in loadings. Second, increases in loadings are not necessarily followed by increases in deductibles, since in equilibrium insurance may be Giffen. Last but not least, loadings do not increase with deductibles, because the only viable equilibrium is a Stackelberg one. *Key words: Deductibles, insurance, loadings, Stackelberg equilibrium.*

1. INTRODUCTION

A recently reprinted paper by Borch [5,7] poses an unresolved question, regarding direct relationships between proportional loadings and deductibles: can higher loadings, caused for instance by higher costs, act as a catalyst for a vicious circle with ever increasing deductibles and loadings? If so, can this process ever be brought to a halt?

This paper rules out the possibility of a vicious circle. It proves that with rational insurance buyers and sellers the only viable equilibrium is a Stackelberg one, in which higher costs are not necessarily followed by higher loadings, higher loadings do not always lead to higher deductibles, higher deductibles are not followed by higher loadings.

In order to discuss Borch's vicious circle, we break it into two parts: the first in which higher loadings cause higher deductibles, and the second, in which higher deductibles cause higher loadings.

The plan of the paper is as follows: in Section 2, we revise the customer's problem in the set up of Borch. In Section 3, we analyze the insurance company's problem, and point out that the insurance company can set loadings only taking into consideration their falldown on deductibles: as a consequence, the equilibrium is Stackelberg, with the insurer acting as a leader. In this equilibrium, both the first and the second part of Borch's vicious circle are ruled out. In Section 4 we conclude and outline future research. All the proofs are in the Appendices.

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2. MODEL SET-UP

Borch builds a model with two utility maximizing agents: a (representative) customer and an insurance company.

The customer's wealth has an initial level S. A risk X with non-negative values and positive expected value $E(X)$ impinges on it. Against the risk X the customer writes an insurance contract $Y = y(X)$, by paying a proportional premium $P[y]$:

$$P[y] = (1 + \lambda)E(y(X))$$

It has been demonstrated[1] that the optimal insurance contract is linear in risk, with a deductible M. If we denote with $F(x)$ the distribution function of X, with $\bar{M}$ the maximum probable loss

$$\bar{M} \overset{d}{=} \min\{x: F(x) = 1\}$$

the linearity property turns into

$$Y = y(X) = \begin{cases} 0 & X < M \\ X - M & M \leqslant X \leqslant \bar{M} \end{cases} \tag{2.1}$$

and implies that for every X the premium P is a function of M and λ: $P = P_X(M, \lambda)$.

Borch assumes that the insurance company determines $\lambda \geqslant 0$, while the customer chooses the deductible M so as to maximize his expected utility:

$$\sup_{0 \leqslant M \leqslant \bar{M}} H(M) \tag{2.2}$$

where

$$H(M) \overset{d}{=} \int_0^M u(w(x, M))\, dF(x) + u(w(x, M) + x - M)(1 - F(M))$$

u is the twice differentiable utility function of the customer, $w(x, M) = S - P_X(M, \lambda) - x$—or simply w—is the customer's final wealth when $X < M$, $w(x, M) + x - M = S - P_X(M, \lambda) - M$ is his wealth when $X > M$.

The existence of interior or boundary solutions to problem (2.2) has been extensively studied[2]. As far as we are concerned, in Appendix 1 we take quite a different approach. We start by characterizing the class of risks X endowed with the properties mentioned above, which entail positive values with positive probability. We study when they are also insurable, in the sense that they guarantee the existence of a solution to the customer's problem, i.e., of a demand for insurance:

THEOREM 2.1 (partial deductibles). *An interior maximum $0 < M^* < \bar{M}$ exists—for a given, positive λ—if*:

[1] See Arrow [2].
[2] See for instance [11].

1. *X is an absolutely continuous random variable in $(0, \bar{M})$ with (at most) a positive probability p_0 of being equal to zero;*
2. *the marginal utility is decreasing in such a way that*

$$\frac{u'(S-\bar{M})}{\int_0^{\bar{M}} u'(S-x)\,dF(x)} > 1+\lambda$$

Having done this, we demonstrate that, for the class of insurable risks[3]:

LEMMA 2.2. *$\bar{M}$ is always a stationary point for the function $H(M)$.*

THEOREM 2.3 (no full deductible). *If a maximum in $[0, \bar{M})$ exists, there is not a maximum at $M = \bar{M}$.*

Provided that X satisfies theorem 2.1, an interior solution for the customer's problem exists. In case it does not, a boundary solution can still exist. In both of these cases, the insurer's problem is well posed. We turn now to this problem. While discussing it we will demonstrate that it has no solution consistent with a boundary deductible.

3. BORCH'S VICIOUS CIRCLE[4]

The key point in analyzing the insurance company's behaviour, and consequently in studying the vicious circle, is that in the model of Borch the insurer cannot determine its loadings ignoring the customer's reaction or acting as a follower in a von Stackelberg game. Let us keep in mind that a follower in such a game acts as if his/her counterpart kept its decision variable fixed. In the insurance company—customer game, a follower company would fix loadings as if the demand for insurance were not sensitive to them. As a result, it would drive loadings to infinity.

Opposite to that, an insurance company acting as a leader in a von Stackelberg game moves from the following belief: the insured will react to the insurer's loadings assuming that they will not change further. As a consequence, the insurance company postulates that the customer's decision depends on its own one and takes into consideration the dependence of deductibles on loadings, while deciding the latter.

[3] We do not discuss the existence of a stationary boundary deductible $M^* = 0$: however, it is well known that it exists iff $\lambda = 0$ (see for instance [11]).

[4] The purpose of this section is to study how the buyer analyzed above might interact with a rational seller: in this sense, the section is very much in the spirit of [1, 4, 12]. However, since our main purpose is to discuss Borch's vicious circle, we stick to his case of fair actuarial premium plus loadings on the supply side and deductibles on the demand side, without exploring other Pareto optimal policies. For the same reason we assume fixed costs: they are the only ones in Borch's model.

This rules out the second part of Borch's vicious circle. In order to discuss the effect of increased costs and the first part of the circle, however, we must enter into the details of the insurance company's problem.

Let us assume that the insurance company has a (cardinal) utility function U, with $U' > 0$, $U'' \leqslant 0$ and consider fixed costs $C > 0$ only, because these are the ones assumed in Borch's paper.

Let us denote by $h(\lambda)$ the insurer's expected utility:

$$h(\lambda) \stackrel{d}{=} U(W(\lambda) - C)F(M^*(\lambda)) + \int_{M^*(\lambda)}^{\bar{M}} U(W(\lambda) - x + M^*(\lambda) - C)\, dF(x)$$

where $W(\lambda)$ is the sum of the initial wealth T and the premium $P_X(M^*, \lambda)$,

$$W(\lambda) = T + P_X(M^*, \lambda)$$

The insurer's problem can be formalized as:

$$\sup_{\lambda \geqslant 0} h(\lambda) \tag{3.1}$$

subject to the zero-utility principle (or participation condition)

$$\sup_{\lambda \geqslant 0} h(\lambda) \geqslant U(T) \tag{3.2}$$

and given the contract (2.1).

First of all, we notice that, as is well known and can be proven with some algebra, the constraint (3.2) is satisfied if and only if the difference between overall loadings λEY and total costs C is greater than the insurance company's risk premium at $T - C + P - Y$. Second, we remark that, with $EY > 0$, the loading $\lambda = C/EY$ satisfies the constraint (3.2) if and only if the insurance company is risk-neutral. If it is risk-averse, λ must be greater than[5] C/EY. In other words, since the insurer's risk premium is greater or equal to zero, when $EY > 0$ (i.e. $M < \bar{M}$), a necessary condition for participation is that loadings be greater or equal to the ratio of total costs to the expected outflows, $\lambda \geqslant C/EY$. Due to the fact that $C/EY > 0$ a prerequisite for participation is then that λ^*, the optimum λ, be positive. In turn, $\lambda^* > 0$ rules out the case of no deductibles ($M^* = 0$).

Also, we can rule out the existence of an optimum λ which entails $M^* = \bar{M}$, since in this case $EY = P = 0$, $h(\lambda) = U(T - C)$ and the constraint (3.2) is not satisfied:

$$U(T - C) < U(T)$$

We are then left with the problem of finding a maximum for $h(\lambda)$ which corresponds to interior deductibles: $0 < M^* < \bar{M}$.

[5] Please note that the risk-neutral case could also be studied using [12] and the competitive equilibrium results in [3]. The risk-averse case instead cannot be described as a competitive equilibrium. I thank the editor for having pointed out to me this circumstance.

The insurance company's necessary condition[6] for a maximum at $\lambda^* > 0$ when $0 < M^* < \bar{M}$ is:

$$\left.\frac{dW}{d\lambda}\right|_{\lambda=\lambda^*, M=M^*} U'(W-C)F(M^*(\lambda^*))$$

$$+\left(\left.\frac{dW}{d\lambda}\right|_{\lambda=\lambda^*, M=M^*}+\left.\frac{dM^*}{d\lambda}\right|_{\lambda=\lambda^*}\right)\int_{M^*(\lambda^*)}^{\bar{M}} U'(W-C-x+M^*(\lambda^*))\,dF(x)=0 \quad (3.3)$$

where

$$\left.\frac{dW}{d\lambda}\right|_{\lambda=\lambda^*, M=M^*}=EY|_{\lambda=\lambda^*, M=M^*}-[1-F(M^*(\lambda^*))](1+\lambda^*)\left.\frac{dM^*}{d\lambda}\right|_{\lambda=\lambda^*} \quad (3.4)$$

We could demonstrate from this first order condition that an increase in costs does not necessarily produce an increase in loadings: the derivative of λ^* with respect to C must be computed using the implicit function theorem on (3.3). This derivative is of unknown sign, so that increased fixed costs are not necessarily followed by an increase in loadings. The only sure effect they have is on the participation constraint.

In Appendix 2, two necessary conditions for (3.3) to admit a solution are given:

THEOREM 3.1 (solutions with insurance). *If $M^* < \bar{M}$, both in the risk-neutral and in the risk-averse insurance case, a solution to problem (3.1) exists provided that either*

$$\left.\frac{dM^*}{d\lambda}\right|_{\lambda=\lambda^*}<\frac{EY}{\lambda^*-F(M^*)(1+\lambda^*)} \quad (3.5)$$

or

$$\left.\frac{dM^*}{d\lambda}\right|_{\lambda=\lambda^*}>\frac{EY}{(1-F(M^*))(1+\lambda^*)} \quad (3.6)$$

Remark 1. Please notice that, in the first case, insurance is a Giffen good[7] *at*

[6] Even if the necessary condition is satisfied, no sufficient condition for the existence or the uniqueness of a maximum can be found.

[7] Please note that a Giffen good is characterized by demand increasing with price. The fact that the Giffen property is obtained through a maximization of both the insurance buyer and seller distinguishes our approach from previous results on the same property, such as [6, 8, 9]. As for the buyer-only results, according to Borch, if u'' is strictly negative, the first order condition for the customer entails that M^* increases with λ, i.e. insurance is not a Giffen good. He relates this to theorem (6) of Arrow [2], who subordinates it to a specific condition on relative risk-aversion. Also, by applying the implicit function to the customer problem only, it is easy to demonstrate that with $M^* > 0$, a sufficient condition for insurance not to be Giffen is that the customer's absolute risk aversion at the minimum wealth guaranteed by the insurance policy be smaller than

$$\frac{1-F(M^*)}{EY|_{M=M^*}[(1+\lambda)F(M^*)-\lambda]} \quad (3.7)$$

$\lambda = \lambda^*$, while in the second it is not, since the right hand sides of (3.5) and (3.6) are respectively negative and positive.

We can sum up the results obtained so far by saying that if an equilibrium between the insurance company and its customer exists, then:

1. the insurance company can act only as a von Stackelberg leader, so that the second part of the vicious circle is ruled out;
2. loadings do not necessarily increase with costs, since λ^* is given by (3.3)—instead of being equal to C/EY—and the sign of the sensitivity of loadings to costs cannot be deduced from (3.3);
3. insurance may or may not be a Giffen good. In the first case, also the first part of the circle is ruled out.

4. CONCLUSIONS AND EXTENSIONS

We have demonstrated in the previous section that, when insurance exists, because deductibles are smaller than the maximum probable loss, then the vicious circle is ruled out. This happens not only when insurance is Giffen at the equilibrium, as expected, but in every case, since it follows from the Stackelberg nature of the equilibria.

The result just mentioned has been obtained in two steps. First of all, we have re-examined the problem of the insured, as described by Borch (Section 2). We have analyzed the strategic behavior of an insurance company which faces Borch's customer (Section 3). By so doing, we have excluded the second part of the vicious circle. We have also excluded the first part, when insurance is Giffen: we have demonstrated that insurance may or may not be Giffen not only when the insured alone is considered, but also when the interaction between the insured and the insurer is taken into account. In the process, we have also ruled out the fact that higher costs always cause higher loadings.

Insurance, however, may not exist, because deductibles are equal to the maximum probable loss. We are left with the doubt that this situation can be produced by a vicious circle of the type described by Borch, in which deductibles and loadings increase for ever. We are going to argue that this can happen at most with a different cost structure.

We know that the insured's expected utility has always a stationary point at $M = \bar{M}$: however, under the hypotheses of theorem 1, this is not a maximum. Even if these hypotheses are not satisfied and $M = \bar{M}$ is a maximum for the insurance buyer, the insurance company's participation constraint is not satisfied at $M = \bar{M}$: with fixed costs, the insurance company has no incentive to offer a policy with both premium and reimbursement equal to zero.

Let us suppose, however, that in this borderline case costs too are equal to zero: formally, this means that the constant cost function C turns into a step one, $C(M)$:

$$C(M) = \begin{cases} C & M < \bar{M} \\ 0 & M = \bar{M} \end{cases}$$

With the latter costs, the insurance company is indifferent as to whether to offer the above zero-reimbursement zero-premium policy or not. It is also indifferent to any loading corresponding to it: the loading level is totally ineffective in this case, since both the premium and the reimbursement are equal to zero.

From this reasoning, we understand that the only way in which a vicious circle of the type envisaged by Borch can exist—in the sense of loadings which, for $dM/d\lambda > 0$, are high enough to produce $M = \bar{M}$—is when costs are of the step type, not constant as in the previous sections.

Apart from this step-cost case, it would be interesting to study the possibility of a vicious circle in the presence of Pareto optimal policies different from Borch's (proportional loadings and no coinsurance), such as the ones discussed in [10]. We leave this investigation in the agenda for future research.

APPENDIX 1

This appendix demonstrates the lemma and theorems of Section 2.

We demonstrate theorem 2.1 through the following lemmas:

LEMMA 5.1 (interior stationary points). *An interior stationary point M^* exists—for given, positive λ—if*

1. *X is an absolutely continuous random variable in $(0, \bar{M})$ with (at most) a positive probability p_0 of being equal to zero;*
2. *the marginal utility is decreasing in such a way that*

$$\frac{u'(S-\bar{M})}{\int_0^{\bar{M}} u'(S-x)\, dF(x)} > 1+\lambda \tag{5.1}$$

LEMMA 5.2 (concavity). *Under the hypothesis* (1) *of the previous lemma every interior stationary point for the problem* (2.2) *is a maximum.*

Proof of lemma 5.1. In order to study the existence of solutions to the customer's first order condition

$$(1+\lambda)\int_0^M u'(w)\, dF(x) - [(1+\lambda)F(M)-\lambda]u'(w+x-M) = 0 \tag{5.2}$$

we use the following shortcut for his expected, truncated marginal utility

$$V(u', M) \stackrel{d}{=} \int_0^M u'(w)\, dF(x) \tag{5.3}$$

and the two functions:

$$g(M) \stackrel{d}{=} \frac{1+\lambda}{(1+\lambda)F(M)-\lambda}$$

$$G(M) \stackrel{d}{=} \frac{u'(w+x-M)}{V(u', M)}$$

In fact, we can state condition (5.2) as an equality between g and G at M^*.

We study the properties of these functions, in order to establish whether and under which conditions on X there exists at least one value M^* of X which equates their expressions.

- g) The function g is right-continuous at 0 with

$$g(0) = \frac{1}{p_0 - \dfrac{\lambda}{1+\lambda}}$$

since $F(0) = p_0$. As a consequence, $g(0)$ is negative whenever $p_0 < \lambda/(1+\lambda)$. If X is continuous in $(0, M)$ and $p_0 < \lambda(1+\lambda)$, then[8] there is a value $\ddot{M} > 0$ such that

$$F(\ddot{M}) = \lambda/(1+\lambda) \tag{5.4}$$

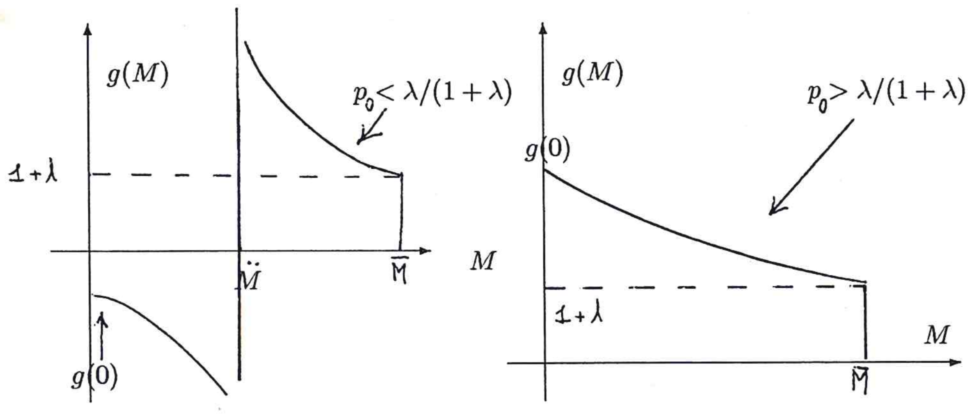
Corresponding to this value, g has a vertical asymptote with

$$\lim_{M \to \ddot{M}\pm} g(M) = \pm\infty$$

On the contrary, g has no jumps if X is continuous in $(0, M)$ and $p_0 > \lambda/(1+\lambda)$. In addition, g is monotone non-increasing in M, due to the monotonicity properties of F as a distribution function, and

$$g(\bar{M}) = 1+\lambda$$

We draw the two possible behaviours of g in the following figures:



[8] Please note that the case of X continuous at zero is included in this one, since if $p_0 = 0$ then $p_0 < \lambda/(1+\lambda)$.

- G) is always non-negative, being a ratio of marginal utilities. It is greater than one, since $u'' < 0$ implies $V(u', M) < u'(w + x - M)$ when $M > 0$. We have

$$\lim_{M \to 0+} G(M) = +\infty$$

if $p_0 = 0$, $G(0) = 1/p_0$ otherwise, and

$$G(\bar{M}) = \frac{u'(S - \bar{M})}{\displaystyle\int_0^{\bar{M}} u'(S - x)\, dF(x)}$$

G is continuous in $(0, M)$ whenever X is absolutely continuous in that interval. The derivative of G, which is well defined if X is absolutely continuous and our hypotheses on u hold, can have any sign.

From our study of g and G it follows that:

1. if X is not continuous when it is positive, g and G have jumps and no sufficient condition for g to be equal to G can be given;
2. if X has at most a discontinuity at 0 and $p_0 < \lambda/(1 + \lambda)$, then g is continuous and monotonically decreasing in $(\ddot{M}, \bar{M})$. As a consequence, it takes all the real values in $[g(\bar{M}), +\infty)$. Let us suppose now that $G(\bar{M}) > g(\bar{M})$, and observe that this implies

   $$G(\bar{M}) \in [g(\bar{M}), +\infty)$$

   From this it follows that there exists (at least) a value $\ddot{M} < M^* < \bar{M}$ such that $g(M^*) = G(M^*)$.
3. if X has at most a discontinuity at 0 and $p_0 > \lambda/(1 + \lambda)$, then G has a finite value at 0. We have $g(0) > G(0)$, since this inequality can be written

   $$\frac{1}{p_0 - \dfrac{\lambda}{1 + \lambda}} > \frac{1}{p_0}$$

   and the latter can be easily transformed into $1 > -\lambda/(1 + \lambda)$.
   The monotonicity of g in $[0, \bar{M}]$ implies that g takes all the real values in $[g(\bar{M}), g(0)]$.
   The continuity of G implies that G takes at least all the real values in

   $$[\min\{G(0), G(\bar{M})\}, \max\{G(0), G(\bar{M})\}]$$

   If we assume $G(\bar{M}) > g(\bar{M})$, then the intersection of the intervals $[g(\bar{M}), g(0)]$ and $[G(0), G(\bar{M})]$ or $[G(\bar{M}), G(0)]$ is not empty (keep in mind that $g(0) > G(0)$). Actually, if we are in the case $G(\bar{M}) < G(0)$, the interval $[G(\bar{M}), G(0)]$ is contained in $[g(\bar{M}), g(0)]$. Both in the case $G(\bar{M}) < G(0)$ and in the opposite one (because of the non-empty intersection) there exists (at least) one value $0 < M^* < \bar{M}$ such that $g(M^*) = G(M^*)$.

We obtain the lemma by observing that (i) the condition $G(\bar{M}) > g(\bar{M})$ can be rewritten as (5.1); (ii) at point 2 above we assumed (5.1), X absolutely continuous in $(0, \bar{M})$ and $p_0 < \lambda/(1+\lambda)$; (iii) at point 3 above we assumed (5.1), X absolutely continuous in $(0, \bar{M})$ and $p_0 > \lambda/(1+\lambda)$; (iv) both in 2 and 3 we proved the existence of a stationary point.□

Proof of lemma 5.2. A sufficient condition for an interior stationary point to be a maximum, if the distribution function is differentiable at it (as stated in the hypothesis) and the corresponding density is denoted by $f(M^*)$, is that

$$-f(M^*)\mathscr{G}(M^*, \lambda) + (1 - F(M^*))^2(1+\lambda)^2 \int_0^{M^*} u''(w(x, M^*))\, dF(x)$$

$$+ (1 - F(M^*))[(1+\lambda)F(M^*) - \lambda]^2 u''(w + x - M^*) < 0$$

where

$$\mathscr{G}(M, \lambda) \stackrel{d}{=} (1+\lambda) \int_0^M u'(w)\, dF(x) - [(1+\lambda)F(M) - \lambda]u'(w + x - M)$$

It is evident from the latter that risk-aversion of the insurance buyer (concavity of u) guarantees that every interior solution to the first order conditions is a local maximum.[9]□

Theorem 2.1 easily follows from the two previous lemmas.
We can then turn to the

Proof of lemma 2.2. First of all, let us notice that the first order conditions for a maximum of the buyer's problem at $M = M^*$ in $(0, \bar{M}]$ are

$$\left.\frac{\partial Eu'}{\partial M}\right|_{M=M^*} \geqslant 0 \tag{5.5}$$

$$(M^* - \bar{M})\left.\frac{\partial Eu'}{\partial M}\right|_{M=M^*} = 0 \tag{5.6}$$

Since

$$\left.\frac{\partial Eu'}{\partial M}\right|_{M=\bar{M}} = (1 - F(\bar{M}))\mathscr{G}(\bar{M}, \lambda)$$

and $F(\bar{M}) = 1$ conditions (5.5, 5.6) are always satisfied as equalities when $M^* = \bar{M}$: $\bar{M}$ is always a stationary point.□

In regard to theorem 2.3, we are going to argue that $\bar{M}$ is always a minimum, if an interior stationary point exists.

[9] If u is strictly concave, the maximum is unique, as usual.

Proof of theorem 2.3. The fact that every interior stationary point is a local maximum (lemma 5.2) implies that in a right neighborhood of the greatest interior maximum

$$\frac{\partial Eu'}{\partial M} < 0$$

The partial derivative under exam cannot become positive without going through zero. By so doing, however, it would generate a stationary point: this cannot happen, since we assumed to be at the right of the greatest stationary value. The right neighborhood above then is a left neighborhood of $\bar{M}$. This implies that $\bar{M}$ is a minimum and demonstrates the theorem. □

APPENDIX 2

Proof of theorem 3.1. We study the existence of a solution to condition (3.3) by distinguishing the cases of risk-neutral and risk-averse insurer.[10]

In the first case, with U' constant for every X, condition (3.3) can be re-written, by dropping the arguments of the derivatives, as

$$\frac{dW}{d\lambda} F(M^*(\lambda^*)) + \left(\frac{dW}{d\lambda} + \frac{dM^*}{d\lambda}\right)[1 - F(M^*(\lambda^*))] = 0 \tag{6.1}$$

With $M^* < \bar{M}$ the distribution function in (6.1) is smaller than one: $1 - F > 0$. In addition, we have to analyze only the case $F \neq 0$, since when $F = 0$ condition (5.2) collapses into the one for $M^* = 0$. In turn, the latter is known to provide no equilibrium solution, due to the incompatibility with $\lambda > 0$. With $F \neq 0$, if $dW/d\lambda \neq 0$, condition (6.1) becomes

$$\frac{\dfrac{dW}{d\lambda}}{\dfrac{dW}{d\lambda} + \dfrac{dM^*}{d\lambda}} = -\frac{1 - F(M^*(\lambda^*))}{F(M^*(\lambda^*))} < 0$$

which can be true only if the numerator and the denominator of the left hand side have different signs. We are going to study this possibility after having analyzed the risk-averse case.

In this second case, with $M^* < \bar{M}$, since U' and its integral are positive, we immediately notice that a necessary condition for (3.3) to admit a solution is that $dW/d\lambda$ and $dW/d\lambda + dM^*/d\lambda$ have opposite signs.

Let us turn to this question and write F for $F(M^*(\lambda^*))$. We know from (3.4) that

[10] Please note that, since the effects of loadings on deductibles are taken into account, the optimal choice for a risk-neutral insurance company is not $M^* = \bar{M}$.

$$\frac{dW}{d\lambda} = EY - (1-F)(1+\lambda)\frac{dM^*}{d\lambda}$$

Also, we deduce from it that

$$\frac{dW}{d\lambda} + \frac{dM^*}{d\lambda} = EY - [\lambda - F(1+\lambda)]\frac{dM^*}{d\lambda} \tag{6.2}$$

The derivatives $dW/d\lambda$ and $dW/d\lambda + dM^*/d\lambda$ then have opposite sign whenever

$$\begin{cases} EY - (1-F)(1+\lambda)\dfrac{dM^*}{d\lambda} > 0 \\ EY - [\lambda - F(1+\lambda)]\dfrac{dM^*}{d\lambda} < 0 \end{cases}$$

or the opposite inequalities hold. Let us posit

$$b_1 \stackrel{d}{=} \frac{EY}{(1-F)(1+\lambda)}$$

$$b_2 \stackrel{d}{=} \frac{EY}{\lambda - F(1+\lambda)}$$

and notice that $b_1 > 0$, $b_2 < 0$, since, at the maximum of the customer's problem, $\lambda - F(1+\lambda) < 0$: in turn, this depends on the fact that $g = -(1+\lambda)/[\lambda - F(1+\lambda)]$ is positive when it is equal to G, i.e. when the customer's first order condition is satisfied.

Using the definitions of b_1 and b_2, and the fact that $b_2 < 0 < b_1$, the previous system collapses into

$$\frac{dM^*}{d\lambda} < b_2 < 0$$

Analogously, the system with opposite inequalities collapses into

$$\frac{dM}{d\lambda} > b_1 > 0$$

By substituting back the values of b_1 and b_2 you get the theorem.□

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