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COPULAS AND DEPENDENCE MODELS IN CREDIT RISK: DIFFUSIONS VERSUS JUMP

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Abstract

The most common approach for default dependence modelling is at present copula functions. Within this framework, the paper examines factor copulas, which are the industry standard, together with their latest development, namely the incorporation of sudden jumps to default instead of a pure diffusive behavior.

The impact of jumps on default dependence – through factor copulas – has not been fully explored yet. Our novel contribution consists in showing that modelling default arrival through a pure jump asset process does matter, even when the copula choice is the standard, factor one, and the correlation is calibrated so as to match the diffusive and non diffusive case. An example from the credit derivative market is discussed.

Keywords: *Credit risk, correlated defaults, structural models, Lévy processes, copula functions, factor copula*

1. INTRODUCTION

Modelling credit risk poses extreme challenges, since one has to care at the same time about providing an adequate description of default arrival at the single firm level and about dependence modelling. Research on both topics, but especially on default dependence, has been very active in the last years.

This paper starts by reviewing the most common approach for dependence modelling, namely copula functions. It examines factor copulas, which are the industry standard, together with their latest development, namely the incorporation of sudden jumps to default. Indeed, jumps in the asset value, whose deterioration is assumed to trigger default, are economically sound and statistically helpful in model calibration. On the one side they give leptokurtic asset distributions, on the other they can correct for lack of totally unpredictability of default of diffusive models, as discussed in Fiorani Luciano and Semeraro (2007).

The impact of jumps on default dependence - through factor copulas - has not been fully explored. Earlier works include Baxter (2006), Albrecher, Ladoucette and Schoutens (2006), Moosbrucker (2006), who focus on CDO pricing. We examine joint default prediction instead. Our aim is to show that the process choice does matter, even when the copula choice is the standard, factor one, and default correlation - through asset correlation - is matched across models.

The present paper is structured as follows: section 1 reviews the copula notion, section 2 justifies their application to credit risk, by introducing structural credit risk models. It presents also the notion of factor copula. Section 3 extends structural models to the case of pure jumps in asset values and represents them through factor copulas. Section 4 shows the joint default probability evaluations which diffusions and pure jumps provide, when calibrated to the same names and equalizing asset correlation. The conclusions follow.

2. COPULA FUNCTIONS

Copula functions are joint distribution functions of standard uniform margins. They have been introduced by Sklar (1959), but are closely related to Hoeffding's "standardized distributions" and have been studied under different names (t norms, dependence functions, uniform representations). The standard introductory textbook for copulas is Nelsen (1999), while applications in Finance are presented in Cherubini, Luciano, Vecchiato (2004).

Let us consider, for the sake of simplicity, the bivariate case[2]. Define as I

2 *For the definition and properties in the n-dimensional case see for instance Nelsen (1999).*

the unit interval, $I = [0,1]$.

Definition 1 *A two-dimensional copula $C(v,z)$ is a real function defined on I^2,*

$$C : I^2 \to I$$

such that, for every $v, z \in I$
i)

$$C(0,z) = C(v,0) = 0$$

ii)

$$C(v,1) = v, \qquad C(1,z) = z$$

iii) for every rectangle $[v_1, v_2] \times [z_1, z_2]$ whose vertices lie in I^2, and such that $v_1 \leq v_2$, $z_1 \leq z_2$

$$C(v_2,z_2) - C(v_2,z_1) - C(v_1,z_2) + C(v_1,z_1) \geq 0$$

Example 2 *The functions* $\max(u+v-1,0), uv, \min(u,v)$ *are copula functions. They are called respectively the minimum, product and maximum copula, and are denoted as $C^-, C^\perp, C^+$.*

Example 3 *Consider the function*

$$C^{Ga}(v,z) = \Phi_\rho\left(\Phi^{-1}(v), \Phi^{-1}(z)\right)$$

where Φ_ρ is the joint distribution of a bi-dimensional standard normal, with linear correlation coefficient ρ, while Φ^{-1} is the inverse of the univariate standard normal distribution Φ. C^{Ga} is a copula: it is called the Gaussian copula[3].

Example 4 *Consider the function*

$$C^t(v,z) = t_{\rho,v}\left(t_v^{-1}(v), t_v^{-1}(z)\right)$$

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[3] *Recalling the definition of a bivariate normal, Φp, C^{Ga} can be re-written as*

$$C^{Ga}(v,z) = \int_{-\infty}^{\Phi^{-1}(v)} \int_{-\infty}^{\Phi^{-1}(z)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{2\rho st - s^2 - t^2}{2(1-\rho^2)}\right) ds dt$$

where $t_\nu : \mathbb{R} \to \mathbb{R}$ is the (central) univariate Student's t distribution function, with ν degrees of freedom (dof) :

$$t_\nu(x) = \int_{-\infty}^{x} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{s^2}{\nu}\right)^{-\frac{\nu+1}{2}} ds$$

Γ is the usual Euler function, while $t_{\rho,\nu}$ the bivariate Student's t distribution with correlation ρ and ν degrees of freedom (dof):

$$t_{\rho,\nu}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} \frac{1}{2\pi\sqrt{1-\rho}}\left(1+\frac{s^2+t^2-2\rho st}{\nu\left(1-\rho^2\right)}\right)^{-\frac{\nu+2}{2}} dsdt$$

The function C^t is called the Student or t copula.

Copulas are satisfy the so-called Fréchet inequality, which states that each copula function is bounded by the minimum and maximum one:

$$C^-(v,z) = \max(v+z-1,0) \le C(v,z) \le \min(v,z) = C^+(v,z) \tag{1}$$

The usefulness of copula functions in applications relies on the following theorem, which clarifies the link between copulas and joint distribution functions of non uniform variables. Let X Y be two continuous random variables (rv s)

Theorem 5 (Sklar (1959)) *Let $F_1(x) = \Pr(X \le x)$, $F_2(y) = \Pr(Y \le y)$ be (given) continuous marginal distribution functions. Then, for every $(x,y) \in \mathbb{R}^2$,*
i) if C is any copula,

$$C(F_1(x), F_2(y))$$

is a joint distribution function with margins $F_1(x)$, $F_2(y)$;
ii) Conversely, if $F(x,y) = \Pr(X \le x, Y \le y)$ is a joint distribution function with margins $F_1(x)$, $F_2(y)$, there exists a unique copula C such that

$$F(x,y) = C(F_1(x), F_2(y)) \tag{2}$$

Based on Sklar's theorem, the copula C - which we will call the copula of X and Y - represents only the dependence between the rv s . While writing

$$F(x,y) = C(F_1(x), F_2(y))$$

one splits the joint probability into the marginals and dependence. From this modelling separation it follows that also in the estimation one can identify the marginals and, at a second stage, specify the copula function. Current methodologies for copula identification indeed include the joint estimate of the copula and margin parameters: this is the case for instance of the traditional maximum likelihood approach. However, they allow also for separate identification of the margins, at a first stage, and the copula, at a second stage (inference function for margins). Or for the use of empirical margins in the likelihood function for the copula (canonical maximum likelihood). In addition, a number of identification approaches for copulas relies on dependence measures.

Copula functions are related to non parametric measures of association or dependence, such as Kendall's tau, τ, or Spearman's rho, ρ_S, which generalize the notion of linear dependence of the usual linear correlation coefficient, ρ. They represent the difference between the probability of concordance of X and Y and the one of discordance (respectively per se and with respect to the independence case) and can be written as

$$\tau = 4 \int\int_{I^2} C(v,z)dC(v,z) - 1 \tag{3}$$

$$\rho_S = 12 \int\int_{I^2} C(v,z)dvdz - 3 = 12 \int\int_{I^2} vzdC(v,z) - 3$$

They range between -1 and +1, and are equal to zero when X and Y are independent. It follows from the above expressions that the copula parameters can be written (although not always in closed form) in terms of the dependence measures.

Example 6 *The minimum copula corresponds to $\tau = \rho_S = -1$, the product one to $\tau = \rho_S = 0$, the maximum one to $\tau = \rho_S = 1$. These copulas represents therefore perfect (non linear) negative dependence, independence and perfect (non linear) positive dependence.*

The representation in Sklar's theorem on one side, and the correspondence between copulas (or their parameters) and association on the other are extremely fruitful for applications. Indeed, Sklar's theorem becomes a very powerful tool when, as in credit risk modelling, the usual joint normality assumption can be questioned.

3. COPULAS AND CREDIT RISK

Two very preliminary probabilistic problems can be identified in credit risk: assessing the default probability for single obligors and determining their corresponding joint default likelihood. Usually, when working with copulas at the multivariate level, one starts from marginal assessments based on the so called structural models, which date back to Merton (1974). We will briefly introduce the approach in section 2.1, and come back to the copula representation of joint probabilities in section 2.2

3.1 SINGLE DEFAULTS

In the seminal model of Merton (1974), each firm i, $i = 1, 2, ..n$, is assumed to have a single zero coupon debt, with face value K_i, which expires at maturity t.

Default of firm i can therefore occur only at debt maturity. It is triggered by the fact that the firm's asset value $V_i(t)$ falls at the liability one, $K_i(t)$. The distribution of the time to default, τ_i, is

$$\tau_i = \begin{cases} t & P(V_i(t) \leq K_i(t)) \\ +\infty & P(V_i(t) > K_i(t)) \end{cases}$$

while the default probability at maturity t is

$$F_i(t) = P(V_i(t) \leq K_i(t)) \tag{4}$$

Merton assumes that asset returns are normal, or, equivalently, that the asset value follows a geometric Brownian motion. Under his assumptions the (marginal) default probability of firm i is

$$F_i(t) = \Phi(-d_{2i}(t))$$

where

$$d_{2i} := \frac{\ln(V_i(0)/K_i(t)) + (\mu_i - \sigma_i^2/2)t}{\sigma_i\sqrt{t}}$$

and μ_i and σ_i are respectively the instantaneous mean return on assets (r under the risk neutral measure) and its standard deviation.

Merton's model has been extended in a number of ways, including the existence of coupons on debt or debt covenants, and therefore the possibility of default before expiry. The extensions are well beyond the scope of this article. However the extensions, once compared with the actual data, provide credit spreads [4] well below the actual ones for short maturities and high ratings. This happens because of the lack of total unpredictability of diffusive processes.

3.2 JOINT DEFAULTS

As in section 1, consider the bivariate case to start with.

In Merton's model, if one assumes that log assets are jointly normally distributed, the joint default probability of two names with common expiry of debt is

$$F(t) = P(V_1(t) \leq K_1(t), V_2(t) \leq K_2(t)) = \Phi_\rho\left(-d_{21}(t), -d_{22}(t)\right)$$

Since $-d_{2i}(t) = \Phi^{-1}(F_i(t))$, the copula representation of the joint default probability follows:

$$F(t) = \Phi_\rho\left(\Phi^{-1}(F_1(t)), \Phi^{-1}(F_2(t))\right)$$

The copula which we have obtained is the Gaussian copula presented above. Analogously, for the n names, the joint default probability is

$$F(t) = P(V_1(t) \leq K_1(t), \ldots V_n(t) \leq K_n(t)) = \Phi_R\left(-d_{21}(t), \ldots, -d_{2n}(t)\right)$$

which can be written using the corresponding n dimensional Gaussian copula, with correlation matrix R:

$$F(t) = \Phi_R\left(\Phi^{-1}(F_1(t)), \ldots, \Phi^{-1}(F_n(t))\right) \tag{5}$$

Instead of joint normality, one can assume a Student t copula among asset values, so that the joint default probability is

$$F(t) = t_{R,v}(t_v^{-1}(F_1(t)), \ldots, t_v^{-1}(F_n(t))) \tag{6}$$

3.3 FACTOR COUPULAS

If the number of obligors increases, the representations (5) or (6), in spite of their conceptual simplicity, can become cumbersome. Therefore, it is common

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4 *Credit spreads are differences between the required rate of return on risky debt and the riskless rate (r in Merton's model).*

practice, especially for pricing and hedging applications, which can involve more than a hundred names, to substitute the actual copula with the so called corresponding factor one, as follows.

Let us assume that the asset value has unit value at time zero and normalize the log asset value (or asset return) of firm i. The latter is:

$$V_i^t = \frac{\ln V_i - \left(\mu_i - \sigma_i^2/2\right)t}{\sigma_i\sqrt{t}}$$

An analogous expression holds under the risk neutral, pricing measure. Assume that each log asset value in the portfolio can be factorized in a common component Z and an idiosyncratic one, ε_i, as follows:

$$V_i^t = \rho_i Z + \sqrt{1-\rho_i^2}\,\varepsilon_i \tag{7}$$

where $\rho_i \in \mathbb{R}$, Z and $\varepsilon_i, i = 1,\dots,n$ are independent standard Gaussian. The weighting coefficients ρ_i and $\sqrt{1-\rho_i^2}$ are chosen so that V_i^t is standard normal. The assumptions on the factorization are such that not only the log asset values V_i^ts are independent, conditionally on the common factor Z, but also that the unconditional linear correlation coefficient between two log asset values V_i^t and V_j^t is $\rho_i\rho_j$. The conditional marginal default probabilities, $p_i^t(z)$, if K_i^t is the properly normalized log liability,

$$K_i^t(t) = \frac{\ln K_i - \left(\mu_i - \sigma_i^2/2\right)}{\sigma_i\sqrt{t}}$$

are easily calculated:

$$p_i^t(z) = P\left(V_i^t \leq K_i^t \mid z\right) = \Phi\left(\frac{K_i^t - \rho_i z}{\sqrt{1-\rho_i^2}}\right)$$

The expressions for the unconditional ones follow by simple integration over the support of the factor:

$$F_i(t) = \int_{\mathbb{R}} p_i^t(z)\varphi(z)dz \tag{8}$$

where φ is the standard Gaussian density.

Taking into consideration that asset values - and therefore defaults - are conditionally independent, the conditional joint default probability is simply the product of the marginal ones:

$$F(t \mid z) = \prod_{i=1}^{n} p_i^t(z)$$

It can be written in copula terms using the product copula $C^{\perp}$:

$$F(t \mid z) = C^{\perp}\left(p_1^t(z), \dots, p_n^t(z)\right)$$

The corresponding expressions for the joint unconditional probability easily follow:

$$F(t) = \int_{\mathbb{R}} \prod_{i=1}^{n} p_i^t(z)\varphi(z)dz = \int_{\mathbb{R}} C^{\perp}\left(p_1^t(z), \dots, p_n^t(z)\right)\varphi(z)dz \tag{9}$$

The technique above can be extended beyond the Gaussian case. In general, if the common factor Z has a density $f(z)$ on the real line $\mathbb{R}$, it follows from the definition of conditional probability that the marginal (unconditional) default probabilities can be written as

$$F_i(t) = \int_{\mathbb{R}} p_i^t(z)f(z)dz \tag{10}$$

The joint unconditional probabilities can be represented through the (conditional) product copula $C^{\perp}$, as desired:

$$F(t) = \int_{\mathbb{R}} \prod_{i=1}^{n} p_i^t(z)f(z)dz = \int_{\mathbb{R}} C^{\perp}\left(p_1^t(z), \dots, p_n^t(z)\right)f(z)dz \tag{11}$$

We will indeed examine in the next section Lévy models in which the conditional asset value is still lognormal, while the factor is gamma distributed instead of being normal. These models allow a factor representation and therefore can be easily handled.

Summing up, both in the original Merton model and in non Gaussian structural models, factorization permits to substitute the original copulas with the product one, according to (11). This makes computational handling of default and survival probabilities - and therefore pricing and hedging results - much easier to obtain. In section 4 below we will examine the impact of the factor copulas when the margins correspond to the most traditional Merton model and when they correspond to the most advanced jump ones, but the factor copula is maintained. In order to do this, we need a brief introduction to jump models for asset values: this is provided in the next section.

4. LÉVY MODELS

The very high pace of research in risk management in the last decade spurred the interest in asset models able to describe skewness, kurtosis, and other deviations from normality. Lévy models, which include the diffusive Brownian motion adopted by Merton on the one side, pure jump processes on the other, seem to be the general environment in which asset processes for the 21 century can be studied. Given the observed deviations from normality, the interest has been concentrated on non diffusive and even pure jump Lévy models, which easily incorporate fat tails and leptokurtosis.

In turn the possibility of writing pure jump models as time changed Brownian motions provides the following intuition for them: even tough prices are diffusions in business or transaction time, they are not in calendar time. The pace at which business time runs constitutes a sort of stochastic clock, and asset prices are not any more diffusions, once the stochastic time change has been accounted for. This intuition, which is discussed at length in Geman et alii (2001), has been pursued and related to empirical trade evidence by Geman and Ané (1996). It constitutes a well established support for the adoption of pure jump models.

As an alternative, the use of such models in credit risk is advocated on the ground that only sudden jumps to default can overcome the lack of total unpredictability of default of diffusive models (see for instance Fiorani, Luciano, Semeraro (2007)).

As a main consequence, it is by now well understood that Lévy models such as the Variance Gamma can be appropriate for describing - one by one - risk neutral asset prices.

Let us assume then that log returns on asset i, $i = 1, \ldots n$, are Geometric Brownian motions in transaction time, namely before any time change (and that they have unit value at time zero, as above):

$$V_i'(t) = \theta_i t + \sigma_i W_t^{(i)}, \qquad t \geq 0.$$

with θ_i and $\sigma_i > 0$ constant.

If the time change has value G_t at time t, the calendar time log return is:

$$V_i'(t) = \theta_i G_t + \sigma_i W_{G_t}^{(i)}, \qquad t \geq 0. \tag{12}$$

If one specifies the time change to be of the gamma type with parameter ν, as we will do in the sequel for illustrative purposes, one gets the so called Variance Gamma (VG) process for log asset prices or asset returns. This process has been introduced by Madan and Seneta (1990), and developed in a number of publications[5].

[5] *Non biasedness implies that G_t must be gamma with parameters $(\nu t; \nu)$, where $\nu > 0$.*

In the VG case, the return process has the following features, for simplicity considered at time 1:

- mean:

$$E[V_i'(1)] = \theta_i$$

- a variance which decomposes as:

$$Var[V_i'(1)] = \sigma_i^2 + \nu\theta_i^2$$

- a level of asymmetry equal to

$$\theta_i\nu(3\sigma_i^2 + 2\nu\theta_i^2)/(\sigma_i^2 + \nu\theta_i^2)^{3/2}$$

- a coefficient of kurtosis equal to

$$3(1 + 2\nu - \nu\sigma_i^4(\sigma_i^2 + \nu\theta_i^2)^{-2}).$$

Under the risk neutral measure, the value process in calendar time becomes

$$V_i'(t) = \theta_i G_t + \sigma_i W_{G_t}^{(i)} + m_i t, \qquad t \geq 0, \tag{13}$$

where

$$m_i = r + \nu^{-1}\log\left(1 - \frac{1}{2}\sigma_i^2\nu - \theta_i\nu\right).$$

If two VG, V_1' and V_2', are driven by the same time change, as in Luciano and Schoutens (2006), their correlation coefficient is

$$\rho = \frac{\theta_1\theta_2\nu}{\sqrt{\sigma_1^2 + \theta_1^2\nu}\sqrt{\sigma_2^2 + \theta_2^2\nu}}. \tag{14}$$

4.1 FACTOR COPULAS AND LÉVY MODELS

Conditional normality of log-returns allows us to write joint default probabilities of Lévy models - and of the VG case in particular - in a straightforward way. The idea is the same used for factor models, so that the similarities will be evident.

The distribution of each single asset return at time 1, conditional on a realization z of the (Gamma) time change, is Normal, with risk neutral mean $m_i + \theta_i z$

and variance $z\sigma_i^2$. Let us denote the corresponding marginal default probability as $p_i^1(z)$:

$$p_i^1(z) = \Phi\left(\frac{v_i - m_i - \theta_i z}{\sigma_i\sqrt{z}}\right), \tag{15}$$

where $v_i = \ln K_i$. As for the unconditional distribution, $F_i(1)$, we have

$$\begin{aligned} F_i(1) &= \int_0^{+\infty} p_i^1(z)\frac{\nu^{-1/\nu}}{\Gamma(1/\nu)}z^{\frac{1}{\nu}-1}\exp(-z/\nu)\mathrm{d}z \\ &= \int_0^{+\infty}\Phi\left(\frac{v_i - m_i - \theta_i z}{\sigma_i\sqrt{z}}\right)\frac{\nu^{-1/\nu}}{\Gamma(1/\nu)}z^{\frac{1}{\nu}-1}\exp(-z/\nu)\mathrm{d}z. \end{aligned}$$

A closed form expression for this integral has been given in Madan, Carr, Chang (?), in terms of Hypergeometric functions and the modified Bessel functions. We note that the integral can also be computed very fast using the inverse Fourier transform approach or via Partial-Differential Integral Equations (PDIEs).

Once more, starting from the marginal conditional distributions (15) also the joint unconditional one can be obtained, since conditional independency holds:

$$F(t) = \int_0^{+\infty}\prod_{i=1}^{2}\Phi\left(\frac{v_i - m_i - \theta_i z}{\sigma_i\sqrt{z}}\right)\frac{\nu^{-1/\nu}}{\Gamma(1/\nu)}z^{\frac{1}{\nu}-1}\exp(-z/\nu)\mathrm{d}z.$$

As in the Merton's case, the previous expression can be written via the factor copula $C^{\perp}$:

$$F(1) = \int_0^{+\infty} C^{\perp}\left(p_1^1(z), p_2^1(z)\right)\frac{\nu^{-1/\nu}}{\Gamma(1/\nu)}z^{\frac{1}{\nu}-1}\exp(-z/\nu)\mathrm{d}z. \tag{16}$$

5. CALIBRATION AND COOMPARIONS OF FACTOR DEFAULT PROBABILITIES

This section calibrates to the same data set the factor copulas (9) and (16), so as to compare them in dependence terms.

5.1 CALIBRATION

We select a sample of five names: Autozone, Ford, Kraft, Walt Disney, Whirlpool, which are active in the credit derivative market. The parameters of their asset values under the VG hypothesis have been derived (from credit default swap prices) in Luciano and Schoutens (2006). For the present purpose, we take the riskless rate to be zero and we derive the corresponding parameters under the Merton's diffusion hypothesis by the first two moment matching. Table 1 presents the marginal parameters for both models.

Tab. 1;

	variance gamma			Merton	
	θ	σ	ν	μ	σ
Autozone	-0.02500	0.2025	0.7068	-0.02500	0.20359
Ford	-0.02500	0.25616	0.7068	-0.02500	0.25702
Kraft	-0.02957	0.15096	0.7068	-0.02957	0.15299
Walt Disney	-0.03299	0.15429	0.7068	-0.03299	0.15676
Whirlpool	-0.03957	0.17445	0.7068	-0.03957	0.17759

The correlation coefficients in the jump case, computed according to (14), are collected in the next table:

Tab. 2:

	Autozone	Ford	Kraft	Walt Disney	Whirlpool
Autozone					
Ford	0.701				
Kraft	0.694	0.695			
Walt Disney	0.692	0.693	0.686		
Whirlpool	0.691	0.692	0.685	0.683	

The parameters ρ_i in the Gaussian case are calibrated so as to match as closely as possible the VG ones: indeed, they are chosen so as to minimize the distance with respect to those of table 2. This is done in order for the comparison not to be affected by the parameters level, but only by the processes behavior and characteristics of dependence (instead of its level).

5.2 DEFAULT PROBABILITIES

The one year pairwise joint default probabilities, under the Merton's diffusive hypothesis and the corresponding VG ones, are represented in basis points and collected in tables 3 and 4, respectively .

Evidently, the probabilities in both tables are very low, since they are over one year only and refer to companies with a good credit standard. We are interested not in their absolute level, but in their (percentage) differences, which are visualized in figure 1. From the figure one can argue that even with reference to

Tab. 3:

Autozone	Ford	Kraft	Walt	Disney	Whirlpool
Autozone		6.065	0.030	0.053	0.508
Ford	6.065		0.080	0.143	1.356
Kraft	0.030	0.080		0.001	0.007
Walt Disney	0.053	0.143	0.001		0.012
Whirlpool	0.508	1.356	0.007	0.012	

Tab. 4:

Autozone	Ford	Kraft	Walt	Disney	Whirlpool
Autozone	2.421	0.599	0.712	1.215	
Ford	2.421		0.966	1.149	2.035
Kraft	0.599	0.966		0.350	0.565
Walt Disney	0.712	1.149	0.350		0.667
Whirlpool	1.215	2.035	0.565	0.667	

this sample of five names, adopting the product copula approach and considering, as in the VG case, that asset values jump or neglecting this, produces extremely different evaluations. Some of the differences are positive, other negatives. It is then incorrect to conclude that neglecting jumps, once the model, as in our case, is properly calibrated to data, leads to underestimating joint default probabilities.

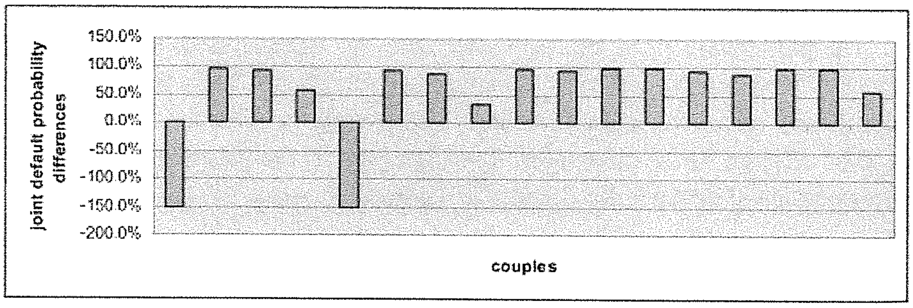

Fig. 1.

6. CONCLUSIONS

This paper has examined default dependence modelling through factor copulas, both when the underlying processes driving default are continuous and when they are pure jump. The aim of study was that of showing that, even under the same parametrization, the modelling choice has a relevant impact on default prediction. We have shown, through a calibration example, that factor copulas based on diffusive assets and factor copulas based on pure jump asset provide very different evaluations, even when the correlation is matched across models. This poses new challenges both to the users of factor copulas. Taking into consideration that pricing, hedging and reserving in the credit derivative market are most of time done via factor copulas, this seems to be a major issue for credit risk assessment of bank portfolios.

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