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Abstract

We consider a Cournot duopoly with strategic delegation, where quantities of firms are chosen by their managers. A firm can offer its manager one of the two incentive contracts: the profit incentive or the revenue incentive. We show that in this setting there are Nash equilibria in which an inefficient firm obtains higher profit than its efficient rival. This result continues to hold under a robust set of correlated equilibria.

Keywords: duopoly; managerial contract; Nash equilibrium; correlated equilibrium; anticoordination games

1 Introduction

In a duopoly where two firms compete under different marginal costs of production, the standard result is that the firm with the lower marginal cost obtains a higher profit than its rival. In this paper we show that there are situations where this result is reversed. To this end, we build on the strategic delegation framework developed by Vickers [12], Fershtman and Judd [2] and Sklivas [10].

The theory of strategic delegation studies the functioning of firms that are characterized by ownership-management separation. The production and pricing decisions within these firms are taken by their managers. The owners of firms (called simply "firms" from now onwards) strategically manipulate incentives of managers via the terms of the contracts offered to them. These contracts are designed to induce the managers to behave "aggressively" in the market that results in higher profits for firms.

This paper considers the problem of strategic delegation in a Cournot duopoly with two firms A and B. Both have constant marginal costs, A having the lower cost of the two. The interaction among firms and their managers is modeled as a two-stage game. In the first stage (delegation stage), firms offer the terms of managerial contracts. In the second stage

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(competition stage), managers choose quantities of their firms based on these contracts.[1] It is usually assumed in the literature that firms ask managers to maximize mixtures of profits and revenues, the weights of which are determined in the pure strategy equilibrium of the delegation stage. This paper adopts a different approach. We assume that each firm asks its manager to maximize either the profit (profit incentive) or the revenue (revenue incentive) of their firms. Incorporating Cournot equilibrium of the competition stage, the interaction between firms A, B is reduced to a 2×2 game of managerial contract choices. We characterize all Nash and correlated equilibria of this game.

We show that provided B is not too inefficient compared to A, then for intermediate sizes of the market there are Nash equilibria (both pure and mixed) under which the inefficient firm B obtains a higher profit than its more efficient rival A. To the best of our knowledge, the result that a cost disadvantage can lead to a higher profit has never appeared before in the literature of industrial economics.

Subsequently we consider correlated equilibrium to check the robustness of our result. Based on the property that our game of managerial contract choices is an *anticoordination game*, we use the results of Calvó-Armengal (2003) to characterize its set of all correlated equilibrium payoffs. We determine the entire set of points where B obtains more than A. A simple geometric presentation (Figure 1) identifies this set in the two-dimensional plane.

Our approach to the delegation problem (where a firm asks its manager to maximize either its profit or its revenue) is distinct from Fershtman-Judd-Sklivas-Vickers (FJSV) where a firm asks its manager to maximize a combination of profit and revenue. Contracts based on such combinations are not easy to implement and rarely observed in practice. In contrast, managerial contracts based on revenue or profit maximization are simple, clearly defined and frequently observed in real life. Thus, compared to the FJSV formulation, the modeling of delegation in this paper is arguably more natural and realistic.

While this paper considers the specific problem of delegation in a duopoly, our result can hold in more general settings. Consider a two-person game between two different players: strong and weak. Prior to the game each player decides whether or not to make a costly investment which acts as a commitment device. The interaction can be reduced to a 2×2 game of investment choices. Assume that the strong player obtains higher payoff than the weak player under symmetric investment choices, but under asymmetric choices the player who makes investment obtains higher payoff than its rival. If this game is an anticoordination game,[2] then both asymmetric outcomes are Nash equilibria and in one of these outcomes, the weak player obtains a higher payoff. Furthermore its set of correlated equilibria coincides with the set of correlated equilibria of general 2×2 anticoordination games and there is a robust subset of such equilibria where the weak player obtains more than the strong player.

The paper is organized as follows. We describe the model in Section 2. We characterize Nash equilibria in Section 2 and correlated equilibria in Section 3.

[1] In recent years the basic delegation model has been enriched by incorporating aspects such as R&D (Zhang & Zhang [13], Kopel & Riegler [4]), collusion (Lambertini & Trombetta [5], Pal [8]),patent licensing (Saracho [9]), wage bargaining (Szymanski [11]), endogenous mode of market competition (Miller & Pazgal [6]), two-period models (Mujumdar & Pal [7]), and Stackelberg competition (Kopel & Loffler [3]).

[2] If a 2×2 game is an anticoordination game, it is also a game of strategic substitutes. See the discussion after Lemma 1 (p.5).

2 The model

Consider a Cournot duopoly with firms A, B. For $i \in \{A, B\}$, let $q_i \geq 0$ be the quantity produced by firm i and $Q = q_A + q_B$ the market quantity. The market price is given by

$$p(Q) = k - Q \text{ if } Q < k \text{ and } p(Q) = 0 \text{ if } Q \geq k \tag{1}$$

where $k > 0$. Firms A, B operate under constant marginal costs τ_A, τ_B. We assume[3]

$$0 < \tau_A < \tau_B < k \tag{2}$$

Firm $i \in \{A, B\}$ seeks to maximize its profit

$$\pi_i(q_A, q_B) = p(q_A + q_B)q_i - \tau_i q_i \tag{3}$$

Each firm employs a manager that chooses the firm's quantity. Denote the managers of A, B by m_A, m_B. A firm can offer its manager either the *profit incentive* or the *revenue incentive*. If manager m_i works under the profit incentive, he chooses q_i to maximize firm i's profit π_i given in (3). If he works under the revenue incentive, he chooses q_i to maximize i's revenue

$$R_i(q_A, q_B) = p(q_A + q_B)q_i \tag{4}$$

For $i \in \{A, B\}$, the nature of the incentive offered to manager m_i is captured by the indicator variable λ_i defined as

$$\lambda_i := \begin{cases} 1 & \text{if firm } i \text{ uses the profit incentive} \\ 0 & \text{if firm } i \text{ uses the revenue incentive} \end{cases} \tag{5}$$

By (3), (4) and (5), when firm i sets $\lambda_i \in \{0, 1\}$, its manager m_i chooses q_i to maximize

$$\pi_i^{\lambda_i}(q_A, q_B) = p(q_A + q_B)q_i - \lambda_i \tau_i q_i \tag{6}$$

2.1 The game Γ

The strategic interaction among firms A, B and managers m_A, m_B is modeled as an extensive-form game Γ that has the following stages.

Stage 1: Firms A, B simultaneously choose $\lambda_A, \lambda_B \in \{0, 1\}$ for their managers. The chosen (λ_A, λ_B) becomes commonly known in Stage 1.

Stage 2: For every (λ_A, λ_B), a simultaneous-move game $\widetilde{\Gamma}(\lambda_A, \lambda_B)$ is played where managers m_A, m_B, simultaneously choose $q_A, q_B \geq 0$. The payoff of m_i in $\widetilde{\Gamma}(\lambda_A, \lambda_B)$ is given by $\pi_i^{\lambda_i}$ from (6). The payoff of firm i is its profit π_i given in (3). Using (3) in (6) it follows that firm i's payoff in Γ is

$$\pi_i(\lambda_A, \lambda_B, q_A, q_B) = \pi_i^{\lambda_i}(q_A, q_B) - (1 - \lambda_i)\tau_i q_i \tag{7}$$

We seek to determine the Subgame Perfect Nash Equilibria (SPNE) of Γ. To this end, we first determine the Nash equilibrium (NE) of $\widetilde{\Gamma}(\lambda_A, \lambda_B)$.

[3]Throughout the paper, we consider generic values of the parameters τ_A, τ_B, k, so we confront only strict inequalities.

2.1.1 Stage 2 of Γ: NE of $\widetilde{\Gamma}(\lambda_A, \lambda_B)$

Observe from (6) that $\widetilde{\Gamma}(\lambda_A, \lambda_B)$ can be viewed as a standard Cournot duopoly game played between two managers m_A, m_B where m_i has marginal cost $\lambda_i \tau_i$. In addition to (2), assume

$$k > 2\tau_B \tag{8}$$

which ensures that for any $\lambda_A, \lambda_B \in \{0, 1\}$, at the (unique) NE of $\widetilde{\Gamma}(\lambda_A, \lambda_B)$, both $q_A, q_B > 0$. Let $i, j \in \{A, B\}$ and $i \neq j$. By equilibrium conditions, at the NE of $\widetilde{\Gamma}(\lambda_A, \lambda_B)$, manager m_i chooses $q_i = q_i(\lambda_A \tau_A, \lambda_B \tau_B)$ and obtains payoff $\phi_i(\lambda_A \tau_A, \lambda_B \tau_B)$ where

$$q_i(\lambda_A \tau_A, \lambda_B \tau_B) = (k - 2\lambda_i \tau_i + \lambda_j \tau_j)/3 \text{ and } \phi_i(\lambda_A \tau_A, \lambda_B \tau_B) = [q_i(\lambda_A \tau_A, \lambda_B \tau_B)]^2 \tag{9}$$

2.1.2 Stage 1 of Γ: The reduced form game Γ^*

Using the unique NE of $\widetilde{\Gamma}(\lambda_A, \lambda_B)$ in stage 2, firms A, B play the 2×2 reduced form game Γ^* in stage 1, where firm $i \in \{A, B\}$ has two pure strategies: $\lambda_i = 0$ and $\lambda_i = 1$. For $\lambda_A, \lambda_B \in \{0, 1\}$, denote the payoffs of firms A, B, in Γ^* by $a_{\lambda_A \lambda_B}, b_{\lambda_A \lambda_B}$. Taking $\pi_i^{\lambda_i}(q_A, q_B) = \phi_i(\lambda_A \tau_A, \lambda_B \tau_B)$ in (7), we have

$$a_{\lambda_A \lambda_B} = \phi_A(\lambda_A \tau_A, \lambda_B \tau_B) - (1 - \lambda_A)\tau_A q_A(\lambda_A \tau_A, \lambda_B \tau_B) \tag{10}$$

$$b_{\lambda_A \lambda_B} = \phi_B(\lambda_A \tau_A, \lambda_B \tau_B) - (1 - \lambda_B)\tau_B q_B(\lambda_A \tau_A, \lambda_B \tau_B) \tag{11}$$

Using (9) in (10) and (11), the game Γ^* is described as follows.

Table 1: The game Γ^*

	$\lambda_B = 0$	$\lambda_B = 1$
$\lambda_A = 0$	$a_{00} = \phi_A(0,0) - \tau_A q_A(0,0)$ $= k^2/9 - \tau_A k/3$ $b_{00} = \phi_B(0,0) - \tau_B q_B(0,0)$ $= k^2/9 - \tau_B k/3$	$a_{01} = \phi_A(0, \tau_B) - \tau_A q_A(0, \tau_B)$ $= (k + \tau_B)^2/9 - \tau_A(k + \tau_B)/3$ $b_{01} = \phi_B(0, \tau_B) = (k - 2\tau_B)^2/9$
$\lambda_A = 1$	$a_{10} = \phi_A(\tau_A, 0) = (k - 2\tau_A)^2/9$ $b_{10} = \phi_B(\tau_A, 0) - \tau_B q_B(\tau_A, 0)$ $= (k + \tau_A)^2/9 - \tau_B(k + \tau_A)/3$	$a_{11} = \phi_A(\tau_A, \tau_B) = (k - 2\tau_A + \tau_B)^2/9$ $b_{11} = \phi_B(\tau_A, \tau_B) = (k - 2\tau_B + \tau_A)^2/9$

We say that (λ_A, λ_B) *is an SPNE of* Γ if (λ_A, λ_B) is an NE of the reduced form game Γ^*. For the rest of the paper, we consider the game Γ^*.

3 Pure and mixed strategy NE of Γ^*

In this section we identify cases where there are NE of Γ^* in which the inefficient firm B obtains higher profit than its efficient rival A. Denote

$$k_0 \equiv 4\tau_A - \tau_B, k_1 \equiv 4\tau_B - \tau_A, k_2 \equiv 4\tau_A \text{ and } k_3 \equiv 4\tau_B \tag{12}$$

As $0 < \tau_A < \tau_B$, we have $k_0 < k_1 < k_3$ and $k_0 < k_2 < k_3$. In addition to (2) and (8), assume that firm B is not too inefficient compared to firm A:

$$\tau_B < 5\tau_A/4 \tag{13}$$

which implies $k_1 < k_2$. Finally assume that the market size (represented by the demand intercept k) is of intermediate size:

$$k_1 < k < k_2 \tag{14}$$

Under (13)-(14), we have $k_0 < k_1 < k < k_2 < k_3$. The assumptions in (2), (8), (13) and (14) are maintained throughout.

Definition Consider a 2×2 game with players A, B where the set of strategies of each player is $\{0, 1\}$. For $i, j \in \{0, 1\}$, let a_{ij}, b_{ij} be the payoffs of A, B at the cell (i, j). This game is an *anticoordination game* if $a_{01} > a_{11}$, $a_{10} > a_{00}$, $b_{01} > b_{11}$ and $b_{10} > b_{00}$.

Lemma 1 shows that Γ^* is an anticoordination game. Furthermore its payoff structure has additional properties.

Lemma 1 *The following hold for the game* Γ^*.

(i) $a_{01} > a_{11} > a_{10} > a_{00} > 0$ *and* $b_{10} > b_{11} > b_{01} > b_{00} > 0$. *Thus, in particular,* Γ^* *is an anticoordination game.*

(ii) $b_{10} > a_{10}$ *and for all other* $i, j \in \{0, 1\}$, $a_{ij} > b_{ij}$.

Proof See the Appendix. ■

The fact that Γ^* is an anticoordination game will be useful to characterize its correlated equilibria in Section 3. Lemma 1 also shows that the payoffs of any firm are well-ordered over the four cells of Γ^*. A firm obtains its maximum payoff at the cell where it chooses the revenue incentive and its rival chooses the profit incentive. It obtains its minimum payoff at the cell where both firms choose the revenue incentive. Finally, firm B obtains higher payoff than firm A only at the cell $(1, 0)$.

From the property of anticoordination of Γ^*, we have $a_{00} - a_{10} < 0 < a_{01} - a_{11}$ and $b_{00} - b_{01} < 0 < b_{10} - b_{11}$. If the revenue incentive ($\lambda_i = 0$) is viewed as a more *aggressive* strategy for a firm compared to the profit incentive ($\lambda_i = 1$), then these inequalities imply that for any firm, the incremental gain from a less aggressive to a more aggressive strategy is lower when its rival chooses a more aggressive strategy. Therefore Γ^* is a game of strategic substitutes.

Consider now the mixed strategy extension of Γ^*. For $i \in \{A, B\}$, a *mixed strategy* of firm i in Γ^* is given by $\sigma(x)$ for $x \in [0, 1]$. Under $\sigma(x)$, firm i chooses the pure strategy $\lambda_i = 0$ with probability x and the pure strategy $\lambda_i = 1$ with probability $1 - x$. The set of all mixed strategies of any firm is $\{\sigma(x) | x \in [0, 1]\}$. A mixed strategy $\sigma(x)$ is *completely mixed* if $x \in (0, 1)$.

Theorem 1 characterizes all NE of Γ^*. It is shown there is a pure strategy NE as well as an NE in completely mixed strategies where the inefficient firm B obtains higher profit than its efficient rival A.

Theorem 1

(i) Γ^* *has three NE in total. The cells* $(1, 0)$ *and* $(0, 1)$ *correspond to two NE in pure strategies. There is another NE, where both firms play completely mixed strategies. In the completely mixed NE, A plays* $\sigma(x^*)$ *and B plays* $\sigma(y^*)$ *where* $x^* \equiv (k - k_1)/\tau_A$ *and* $y^* \equiv (k - k_0)/\tau_B$.

(ii) *Firm B obtains higher profit than firm A at $(1,0)$. Furthermore, firm B also obtains higher expected profit than firm A under the completely mixed strategy NE. At $(0,1)$, firm B obtains lower profit than firm A.*

Proof See the Appendix. ■

Theorem 1 shows that provided firm B is not too inefficient compared to firm A ($c_B < 5c_A/4$), then for intermediate sizes of the market ($k_1 < k < k_2$), there is an NE in pure strategies where (a) A chooses the profit incentive and B chooses the revenue incentive (cell (1,0)) and (b) the inefficient firm B obtains a higher profit than A.

First we see the intuition for (b), given (a). Under outcome (1,0), the objective of manager m_B is to maximize firm B's revenue, so it effectively acts as a firm that has zero cost. In contrast, manager m_A solves the standard problem of maximizing profit with positive cost. Therefore B would obtain a higher profit than A if firms have the same costs. By continuity, the same result holds even if B's costs are higher, as long as they are not too high.

To see the intuition for (a), note that for two demand curves parallel to each other, at any price, the elasticity is higher at the demand curve that lies on the right. In other words, for any price, demand becomes more elastic as the market expands, where the expansion is presented by parallel rightward shift of the demand curve. This drives (a) as follows.

If both firms choose the revenue incentive, then both supply a high quantity that results in a low price. When the market size is not too large, such a low price corresponds to the inelastic portion of the demand curve. One of the firms can then improve its profit by deviating to the profit incentive that results in higher price. On the other hand, if both firms choose the profit incentive then the price is high. When the market size is not too small, such a high price corresponds to the elastic portion of the demand curve. In that case, one of the firms can improve its profit by deviating to the revenue incentive that results in lower price. This explains why A's choice of profit incentive and B's choice of revenue incentive can be sustained as an equilibrium when the market size is intermediate, i.e., it is not too large or too small.

Theorem 1 shows that the inefficient firm B makes higher profit than its efficient rival A also under the completely mixed NE. This is because that NE gives a sufficiently high probability to the cell $(1,0)$ where $b_{10} > a_{10}$. However, a mixed strategy NE has the undesirable feature that although a player is indifferent between its pure strategies, it must mix its strategies in a specific way so as to make its rival player indifferent. We consider correlated equilibrium to check the robustness of our result.

4 Correlated equilibria

The concept of correlated equilibrium is useful in modeling environments where prior to playing the game, players observe private signals of the same random event. Using the results of Calvó-Armengal (2003) on correlated equilibria of 2×2 games, in this section we determine the set of all correlated equilibrium payoffs of Γ^* where the inefficient firm B obtains more than its rival.

4.1 Set of correlated equilibria

Let $\Delta := \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4_+ | x_1 + x_2 + x_3 + x_4 = 1\}$ be the 3-dimensional simplex of $\mathbb{R}^4$. A *random device* is a probability distribution $\mu = (\mu_{00}, \mu_{11}, \mu_{10}, \mu_{01}) \in \Delta$ over the 4 cells of Γ^* as follows.

	$\lambda_B = 0$	$\lambda_B = 1$
$\lambda_A = 0$	μ_{00}	μ_{01}
$\lambda_A = 1$	μ_{10}	μ_{11}

A random device μ is a *correlated equilibrium* (CRE) of Γ^* if for $i, j \in \{0, 1\}, i \neq j$:

$$\mu_{i0}a_{i0} + \mu_{i1}a_{i1} \geq \mu_{i0}a_{j0} + \mu_{i1}a_{j1} \text{ and } \mu_{0i}b_{0i} + \mu_{1i}b_{1i} \geq \mu_{0i}b_{0j} + \mu_{1i}b_{1j} \tag{15}$$

A *polytope* of $X \subseteq \mathbb{R}^n$ is the convex hull of finitely many points of X. Thus, a polytope is convex and compact. The set of all CRE of Γ^*, denoted by $CRE(\Gamma^*)$, is a non-empty polytope of Δ. The convex hull of the three NE of Γ^* is a subset of $CRE(\Gamma^*)$. Denote

$$\alpha \equiv (a_{10} - a_{00})/(a_{01} - a_{11}) \text{ and } \beta \equiv (b_{01} - b_{00})/(b_{10} - b_{11}) \tag{16}$$

Note from Lemma 1 that $\alpha, \beta > 0$ and the inequalities of (15) are equivalent to

$$\mu_{01} \geq \alpha\mu_{00},\ \alpha\mu_{10} \geq \mu_{11},\ \mu_{10} \geq \beta\mu_{00},\ \beta\mu_{01} \geq \mu_{11} \tag{17}$$

Note from (17) that $CRE(\Gamma^*) = CRE(\gamma)$, where γ is the following anticoordination game.

	0	1
0	$-\alpha, -\beta$	$0, 0$
1	$0, 0$	$-1, -1$

(18)

Following Calvó-Armengal (2003), consider these five points in Δ.

μ	μ_{00}	μ_{11}	μ_{10}	μ_{01}
C	0	0	1	0
D	0	0	0	1
E	$\frac{1}{(1+\alpha)(1+\beta)}$	$\frac{\alpha\beta}{(1+\alpha)(1+\beta)}$	$\frac{\beta}{(1+\alpha)(1+\beta)}$	$\frac{\alpha}{(1+\alpha)(1+\beta)}$
F	$\frac{1}{1+\alpha+\beta}$	0	$\frac{\beta}{1+\alpha+\beta}$	$\frac{\alpha}{1+\alpha+\beta}$
G	0	$\frac{\alpha\beta}{\alpha+\beta+\alpha\beta}$	$\frac{\beta}{\alpha+\beta+\alpha\beta}$	$\frac{\alpha}{\alpha+\beta+\alpha\beta}$

(19)

A point of a polytope is called a *vertex* if it is not a convex combination of two other points of the polytope. Proposition 1 is a geometric characterization of $CRE(\Gamma^*)$.

Proposition 1 *$CRE(\Gamma^*)$ is a polytope of Δ with* 5 *vertices defined in* (19).

Proof Follows from Lemma 2 and Proposition 2 of Calvó-Armengal (2003). ■

Observe that vertices C and D correspond to two pure strategy NE of Γ^*, while E corresponds to its completely mixed NE. The vertices F and G lie outside the convex hull of all three NE of Γ^*.

4.2 Set of correlated equilibrium payoffs

For $\mu \in CRE(\Gamma^*)$, let $\pi_A(\mu) = \sum_{i,j\in\{0,1\}} \mu_{ij}a_{ij} > 0$ and $\pi_B(\mu) = \sum_{i,j\in\{0,1\}} \mu_{ij}b_{ij} > 0$ be the expected payoffs[4] of A and B at μ. Denote $\pi(\mu) = (\pi_A(\mu), \pi_B(\mu))$. Let $CREP(\Gamma^*) := \{\pi(\mu) \in \mathbb{R}^2_+ | \mu \in CRE(\Gamma^*)\}$ be the set of correlated equilibrium payoffs of Γ^*. For $i, j \in \{A, B\}$, let $\mathbb{S}_{ij} := \{\pi(\mu) \in CREP(\Gamma^*) | \pi_i(\mu) \geq \pi_j(\mu)\}$.

To simplify notations, denote the payoff pairs generated by the 5 vertices C, D, E, F, G of $CRE(\Gamma^*)$ by lowercase letters, i.e., $c \equiv \pi(C) = (\pi_A(C), \pi_B(C))$, $d \equiv \pi(D) = (\pi_A(D), \pi_B(D))$ etc. Theorem 2 characterizes[5] $CREP(\Gamma^*)$ and determines the entire set of points in $CREP(\Gamma^*)$ where firm B obtains more than firm A.

Theorem 2

(i) *$CREP(\Gamma^*)$ is a polytope of $\mathbb{R}^2_+$ with 4 vertices c, d, f, g.*

(ii) *Firm B obtains more than firm A at all of these vertices except d.*

(iii) *$\mathbb{S}_{BA}$ is a polytope with 5 vertices and $\mathbb{S}_{AB}$ is a polytope with 3 vertices.*

Proof See the Appendix. ■

The conclusions of Theorem 2 can be seen quite clearly from Figure 1. In this figure, the two axes present the payoffs of firms A, B. The set $CREP(\Gamma^*)$ is the quadrilateral $cfdg$. The points c, d, e correspond to the NE payoff pairs. The convex hull of NE payoffs, the triangle cde, is a proper subset of $cfdg$.

Since $\pi_B(C) > \pi_A(C)$ and $\pi_B(E) > \pi_A(E)$, there are other correlated equilibrium payoffs (e.g., any point on the line ce) where B obtains more than A. To identify the entire set of points where this happens, consider the 45° line drawn in Figure 1. For any point above this line, B has higher payoff than A. The 45° line cuts $CREP(\Gamma^*)$ in two parts. At any point above the line mh in the pentagon $cfmhg$, payoff of B is higher. On the other hand, at any point below the line mh in triangle dmh, payoff of A is higher. For all points on the line mh, both firms obtain the same payoff. To conclude, the result that the inefficient firm B obtains higher profit than its efficient rival A is robust under correlated equilibrium.

[4]Since the payoff of any firm is positive in any cell of Γ^* (Lemma 1), these expected payoffs are also positive.

[5]An aspect of the relation between Γ^* and the game γ (given in (18)) is worth emphasizing: $CRE(\Gamma^*) = CRE(\gamma)$, but $CREP(\Gamma^*) \neq CREP(\gamma)$. For instance, points C, D of (19) give the same payoff pair $(0, 0)$ for γ^*, but C, D generate two distinct payoff pairs for Γ^*.

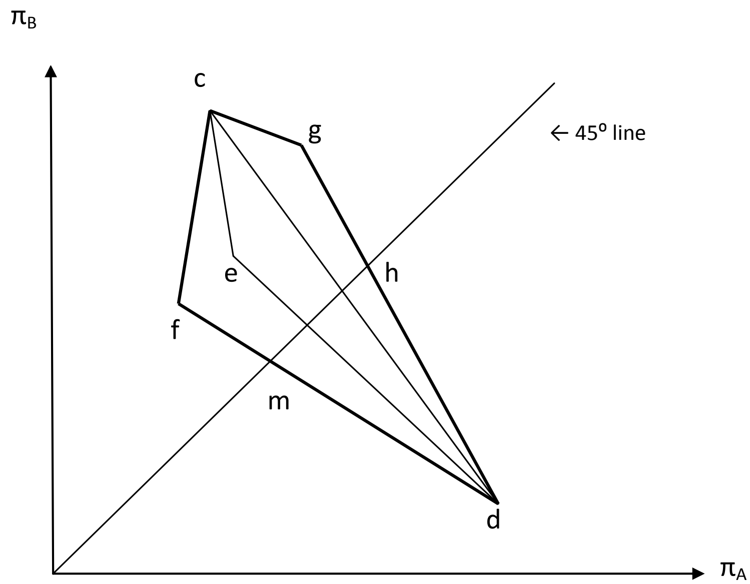

Figure 1

Appendix

Proof of Lemma 1 (i) As $\tau_A, \tau_B > 0$, it is immediate from Table 1 that $a_{10} > a_{11}$ and $b_{01} > b_{11}$. Next observe that $a_{01} - a_{11} = \tau_A(k - k_0)/9$, $a_{10} - a_{00} = \tau_A(k_2 - k)/9$, $b_{10} - b_{11} = \tau_B(k - k_1)/9$ and $b_{01} - b_{00} = \tau_B(k_3 - k)/9$ are all positive, since $k_0 < k_1 < k < k_2 < k_3$. Finally since $\tau_A < \tau_B$, we have $a_{00} > b_{00} = k(k - 3\tau_B)/9 > k(k - k_1) > 0$.

(ii) As $\tau_A < \tau_B$, from Table 1 we have $a_{00} > b_{00}$ and $a_{11} > b_{11}$. By (i), $a_{01} - b_{01} > a_{11} - b_{11} > 0$. Finally, since $k_1 < k < k_2$, we have $b_{10} - a_{10} = [(2k - \tau_A)\tau_A - \tau_B(k + \tau_A)]/3 > [(2k_1 - \tau_A)\tau_A - \tau_B(k_2 + \tau_A)]/3 = \tau_A(\tau_B - \tau_A) > 0$. ■

Proof of Theorem 1 (i) By Lemma 1(i) it is immediate that the only two NE in pure strategies correspond to the cells $(1, 0)$ and $(0, 1)$. Lemma 1(i) also implies that Γ^* does not have an NE where one firm chooses a pure strategy and another firm chooses a completely mixed strategy. Finally let $\sigma(x), \sigma(y)$ be the strategies of A, B in a completely mixed NE. Then we have $ya_{00} + (1 - y)a_{01} = ya_{10} + (1 - y)a_{11}$ and $xb_{00} + (1 - x)b_{10} = xb_{01} + (1 - x)b_{11}$, whose unique solution has $x = x^* \equiv (k - k_1)/\tau_A$ and $y = y^* \equiv (k - k_0)/\tau_B$.

(ii) From Lemma 1(ii), we have $b_{10} > a_{10}$ and $b_{01} < a_{01}$. To complete the proof, let π_A^*, π_B^* be the payoffs of A, B at the completely mixed NE. Using the values of x^*, y^* and simplifying, we have $\pi_B^* - \pi_A^* = (\tau_B - \tau_A)[k - 4(\tau_B + \tau_A)/3] > (\tau_B - \tau_A)(k - k_1) > 0$. ■

Proof of Theorem 2 (i) By Lemma 2 and Propositions 2-3 of Calvó-Armengal (2003), $CREP(\Gamma^*)$ is a polytope of $\mathbb{R}^2_+$ with either 3 or 4 vertices. Points c, d and at least one of f, g are its vertices. In what follows, we show that *both* f and g are vertices of $CREP(\Gamma^*)$.

Consider the $\mathbb{R}^2_+$ plane in Figure 1. We present π_A on the horizontal axis and π_B on the vertical axis. The $45°$ line contains all payoff pairs where $\pi_A = \pi_B$. Hence $\pi_B > \pi_A$ above this line and $\pi_B < \pi_A$ below this line. The proof of (i) proceeds in the following steps.

Step 1: *c lies above and d lies below the $45°$ line and cd is a downward sloping line*: As $c = (a_{10}, b_{10})$ and $d = (a_{01}, b_{01})$, by Theorem 1 c lies above and d lies below the $45°$ line. As $b_{10} > b_{01}$ and $a_{01} > a_{10}$ (Lemma 1(i)), cd is a downward sloping line as drawn in Figure 1.

Step 2: *f lies below and g lies above the line cd*: By (19) and Lemma 1(i) we have $\pi_B(C) = b_{01} > \max\{\pi_B(F), \pi_B(G)\}$, $\pi_A(D) = a_{10} > \max\{\pi_A(F), \pi_A(G)\}$, $\pi_A(G) > a_{10} = \pi_A(C)$ and $\pi_B(G) > b_{01} = \pi_B(D)$. Moreover $\pi_A(F) - \pi_A(C) = \alpha(a_{10} - a_{11})/(1 + \alpha + \beta) > 0$ and $\pi_B(F) - \pi_B(D) = \beta(b_{11} - b_{01})/(1 + \alpha + \beta) > 0$. These imply

$$\pi_A(D) = a_{01} > \max\{\pi_A(F), \pi_A(G)\} \geq \min\{\pi_A(F), \pi_A(G)\} > \pi_A(C) = a_{10}$$

$$\pi_B(C) = b_{10} > \max\{\pi_B(F), \pi_B(G)\} \geq \min\{\pi_B(F), \pi_B(G)\} > \pi_B(D) = b_{01} \quad (20)$$

By the inequalities above, $\exists\ \theta_A^F, \theta_B^F \in (0, 1)$ such that $\pi_A(F) = \theta_A^F a_{10} + (1 - \theta_A^F)a_{01}$ and $\pi_B(F) = \theta_B^F b_{10} + (1 - \theta_B^F)b_{01}$. Let $\widetilde{\beta} \equiv \beta/(1 + \alpha + \beta) \in (0, 1)$. Note from (19) that $\pi_A(F) < \widetilde{\beta}a_{10} + (1 - \widetilde{\beta})a_{01}$ and $\pi_B(F) < \widetilde{\beta}b_{10} + (1 - \widetilde{\beta})b_{01}$ As $a_{10} < a_{01}$ and $b_{01} > b_{10}$, we must have $\theta_A^F > \widetilde{\beta}$ and $\theta_B^F < \widetilde{\beta}$. This proves that $\theta_B^F < \theta_A^F$. Hence f lies below the line cd.

By (20), $\exists\ \theta_A^G, \theta_B^G \in (0, 1)$ such that $\pi_A(G) = \theta_A^G a_{10} + (1 - \theta_A^G)a_{01}$ and $\pi_B(G) = \theta_B^G b_{10} + (1 - \theta_B^G)b_{01}$. Solving these equations using (19), we have $\theta_A^G = \gamma_2(a_{01} - a_{11})/(a_{01} - a_{10}) + \gamma_1$ and $\theta_B^G = \gamma_2(b_{11} - b_{01})/(b_{10} - b_{01}) + \gamma_1$ where $\gamma_1 \equiv \beta/(\alpha + \beta + \alpha\beta)$ and $\gamma_2 \equiv \alpha\gamma_1$. Hence $\text{sign}[\theta_B^G - \theta_A^G] = \text{sign}[(k - \tau_A - \tau_B)] > 0$ (since $k > 2\tau_B > \tau_A + \tau_B$). This proves that $\theta_B^G > \theta_A^G$, so g lies above the line cd.

By Step 2, f is not a convex combination of c, d, g and g is not a convex combination of c, d, f. Then using the results of Calvó-Armengal (2003), it follows that $CREP(\Gamma^*)$ is a polytope of $\mathbb{R}^2_+$ with 4 vertices: c, d, f, g.

(ii) The result for c, d follow from Theorem 1. For f, g, recall from Theorem 1 that at e (the payoff pair of the completely mixed NE), $\pi_B(E) > \pi_A(E)$. Hence by (19), $\psi \equiv (b_{00} - a_{00}) + \alpha\beta(b_{11} - a_{11}) + \beta(b_{10} - a_{10}) + \alpha(b_{01} - a_{01}) > 0$. As $b_{11} < a_{11}$, we have $(b_{00} - a_{00}) + \beta(b_{10} - a_{10}) + \alpha(b_{01} - a_{01}) > \psi > 0$, implying from (19) that $\pi_B(F) > \pi_A(F)$. As $b_{00} < a_{00}$, we have $\alpha\beta(b_{11} - a_{11}) + \beta(b_{10} - a_{10}) + \alpha(b_{01} - a_{01}) > \psi > 0$, so by from (19), $\pi_B(G) > \pi_A(G)$.

(iii) By (ii), both f, g lie above the 45° line. Let m, h be the respective points of intersection of fd, gd with the 45° line. Then the result is immediate from Figure 1. ■

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