Economic Reforms, Frictional Unemployment and Wage Inequality—A General Equilibrium Analysis

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Economic Reforms, Frictional Unemployment and Wage Inequality-----A General Equilibrium Analysis

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Abstract: In this paper we extend the benchmark model of Diamond-Mortensen-Pissarides in a two-sector general equilibrium framework by introducing a frictionless segment of the labour market. The two sectors are the frictionless informal sector and the frictional formal sector where match friction is the root cause of unemployment. Here, both wages are flexible. Informal wage is determined by the marginal productivity rule of the worker and the formal wage is determined by the Nash-bargaining solution. We also examine the effects of trade reforms and labour market reforms on equilibrium rate of unemployment and wage inequality in our stylitized economy. We find that both these reforms reduce equilibrium rate of unemployment. However, trade reforms raise wage inequality but labour market reforms reduce it. These results provide a strong theoretical basis for labour market reform in a small open economy characterized by frictional labour market.

Jel Classification: F 16

Key Words: Economic reforms, Frictional unemployment, Wage inequality, Job-searching, Job-matching, General equilibrium.

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1. **Introduction:**

The striking feature of the labour market is that both jobs and workers are heterogeneous. Workers are specialized with respect to their skills and all jobs are not suitable as well as available to all workers. Workers seek high-paid jobs which ensure good working condition, less exploitation and less harassment. At the same time, firms also seek good workers who are suitable to the existing jobs. Thus, job-searching and job-matching are the two key features in the complicated, versatile and vast labour market.

In the labour market we find flows of jobs, flows of workers, old jobs are destroyed, both firms and workers search each other to match together. All these ideas have been captured in the search and matching models of the labour market. The path-breaking work in the line is the Diamond-Mortensen-Pissarides (called DMP hereafter) model. Others notable works on this front are Diamond (1982a, 1982b, 1984), Pissarides (1979, 1984, 1985a, 1985b, 1986, 2000), Mortensen (1987, 2011), Mortensen and Pissarides (1994, 1998, 1999). The job-matching models generally explain the existence of frictional unemployment where matching plays the central role to dictate unemployment. The use of the matching function has been first observed in Hall (1979), Pissarides (1979), Diamond and Maskin (1979), Bowden (1980).

The traditional literature shows that matching is a function of unemployment rate and vacancy rate and is subject to the constant return to scale. In the matching framework production starts only when workers are firm are matched. However, matching is a costly and time-consuming process. Once match is formed, cost of searching on both sides are reduced and this generates surplus which is distributed between workers and forms. In the existing literature the most commonly used surplus-sharing rule is the Nash-bargaining solution.

The job-matching models have been extended by introducing cyclicality, efficiency wage etc. These have been found in the models of Albrecht et al. (1984), Cole and Rogerson (1989), Abraham et al. (1995), Zenou and Smith (1995), Andolfatto (1996), Shimer (2005), Zenou (2005), Arozamena and Centeno (2007), Sheng and Xu (2007).
It is worth noting that the theoretical literature on search and match-induced unemployment has not been adequately dealt with the dualistic structure of the labour market where informal segment co-exists along with the formal segment of the labour market. It is beyond any doubt that the informal sector plays a very significant role in employment in developing countries where almost 70% of total employment originates in this sector (Agenor, 1996). In case of India this figure rose to 90% if one includes agriculture. It is estimated that over 60% of employment in Nairobi and Kumasi and 50% in Jakarta have been found in this sector. The ILO (2002a) shows that in the South Asian countries more than 90% of workers are in the informal sector. We also find that the percentage of population engaged in the informal sector has significantly increased in the post-reform period (Dev, 2000; Marjit, 2003). Thus, in the analysis of the labour market the inclusion of the informal sector is highly justified and this has not been done in most of the search and match theoretic models.  

The development economists are very much interested regarding the effects of economic reforms on the world economy. However, these reforms have not been explained elaborately in the search and matching models. The liberalized economic policies have increased the size of the informal sector and at the same time have also affected the working conditions and welfare of the labour force. Khan (1998) and Tendulkar et al. (1996) have found that the incidence of poverty has increased in India in the post-reform period. Liete et al. (2006) have shown that a significant decrease in average real wage for informal workers in the South Africa during 200-2004. ILO (2006) has shown that the current pattern of globalization continues to have an uneven social impact with some experiencing rising living standards and others left behind.

Empirical studies e.g. Robbins (1994a, 1994b, 1995a, 1995b, 1996a, 1996b), Wood (1997), Khan (1998) and Tendulkar et al. (1996) have found that in many Latin

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American, African and South Asian countries including India relative wages moved against the poor workers. Wood (1997) argues that diversity in the amount of wage-gap between the East-Asia and the Latin America is probably due to the entry of China into the world economy. Feenstra and Hanson (1996), Marjit et al. (2000, 2004), Chaudhuri and Yabuuchi (2007) and Yabuuchi and Chaudhuri (2007) explain theoretically the deteriorating wage inequality in the developing economies. Empirical studies also show that in the post-reform period the informal sectors have expanded in developing countries. But it could not mitigate the problem of unemployment as vast pool of workers from the formal sector was not absorbed in the expanding informal sector.

Another economic reform is the reform in the labour market. Many developing countries are now thinking to implement such reform so that the rigid labour laws can be relaxed to attract the large foreign as well as domestic investment in the developing countries (Chaudhuri, 2006). It is generally believed that labour market reforms would lead to a rise in wage inequality and unemployment in developing economies. However, Chaudhuri (2006) has shown that labour market reforms may raise social welfare and soften the problem of unemployment. We also hardly find any work on such reforms in the search and match theoretic models.

In this paper, we develop a two-sector job-search and job-match model in general equilibrium framework along the line of DMP. Here, one sector is frictional and the other sector is frictionless. Our analysis embraces that both the trade reform and the labour market reform soften the problem of unemployment. However, wage inequality deteriorates in the case of trade reform and it improves in the case of labour market reform. Thus, there is a trade-off between wage inequality and mitigation of the problem of unemployment as a result of trade reform but labour market reform solves both the problems of unemployment and wage inequality.

Our model differs from the existing theoretical works as follows: firstly, the existing models on job-matching are defined to the partial equilibrium framework which are not handy to analyse trade reforms. Secondly, the existing models that study the effects of
trade reform and the labour market reform in the general equilibrium framework have not considered job-search and job-match in the labour market. We have tried to synchronise the two types of model job-search and job-match model on one hand and general equilibrium model on the other hand to analyze the effects of trade reforms as well as labour market reforms in the developing countries.

2. The Model:

We consider a two-sector small open economy. The two sectors are sector 1 which is the export sector and produces commodity, \( X_1 \) and the other sector is the import-competing sector 2 which produces commodity, \( X_2 \). The prices of the two commodities \( P_1, P_2 \) are given due to the small country assumption. The price of commodity 1 is chosen as numeraire. The two sectors use both labour and capital in production. The production functions are subject to CRS and diminishing marginal productivity. Capital is mobile between the two sectors and this gives a unique rate of return on capital.

Labour is also mobile across the sectors but labour market is segmented. Both workers and the firms search in both sectors. But in one sector (sector 1), job-search and job-match are instantaneous and so this sector is frictionless. Here, workers are paid according to their marginal products. However, the other sector is frictional where matchings are time consuming and costly. In this sector (sector 2) the two-way searching gets fruitful after incurring some costs.

In the frictional labour market job-search is an ongoing process. Jobs are offered to the workers and the workers arrive at the jobs offered. So, there exists job-matching between workers and firm in sector 2. Following DMP we may consider the matching function as \( m = m(u, v) \), where \( m \) stands for matching, \( u \) is the rate of unemployment and \( v \) is the vacancy rate and \( m_1, m_2 > 0, m_{11}, m_{22} < 0, m_{12} = m_{21} = 0 \). Total flow of matches is
\[ m = au \] and total flow of jobs is \( m = vq \). So, \( \frac{m}{u} \equiv a \) is the job arrival rate and \( \frac{m}{v} = q \) is the job offer rate. Matching function is assumed to possess CRS property and so we may write \( q = q(\theta), a = \frac{m}{u} = \frac{m}{v} u = \theta q(\theta) \) where \( \theta = \frac{v}{u} \) is the labour market tightness and \( q'(\theta) < 0, \left| \frac{\theta}{q} \right| < 1. \)

### 2.1 Value Equations:

The Bellman equations for the values of unemployment \( U \), employment \( W \), vacancy \( V \) and jobs filled \( J \) are

1. \[ rU = \theta q(\theta) (W - U) \]  
2. \[ rW = w_2 - \lambda (W - U) \]  
3. \[ rV = -C + q(\theta) (J - V) \]  
4. \[ rJ = \frac{P^*}{2} t - w_2 - rk_2 - \lambda J \]

Equation (1) states that unemployment gives option of a discrete change in the valuation from \( U \) to \( W \). This equation holds at steady state where discount rate, transaction rate and income flows are constant. Equation (2) embraces that the asset value of employment is the wage rate in sector 2 \( (w_2) \) less employment gain when negative shock arises, where \( \lambda \) the job destruction rate which is given exogenously. Equation (3) shows that the asset value of vacancy yields, at the rate \( q(\theta) \), a discrete change in its valuation from \( V \) to \( J \) less a given flow cost \( C \) to maintain vacancy. Finally, Equation (4) states that the

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2 Note that in steady state, \( \frac{1}{q(\theta)} \) is the expected duration of vacancy and \( \frac{1}{\theta q(\theta)} \) is the expected duration of unemployment (Pissarides, 2000).

3 We assume that there is no unemployment benefit.
value of a job filled in is the flow of profit \((P^*_t - w^*_2 - r^*_2)\) to the firm less the jobs destroyed where \(t\) is the match productivity.\(^4\)

### 2.2 Job-Creation Condition:

A firm creates jobs up to the point where \(V = 0\). Putting this condition into Equation (3) one gets

\[
J = \frac{C}{q(\theta)}
\]  

(5)

Substituting (5) into (4) we can write

\[
P^*_t = w^*_2 + r^*_2 + \frac{(r + \lambda)C}{q(\theta)}
\]  

(6)

This is the job-creation condition at steady state. This shows that at steady state, value of the product is equal to the wage cost plus rental cost plus recruitment cost of labour.

### 2.3 Wage function in the frictional Sector:

In the search and matching model, production begins when firm and workers are matched. If the match is broken both of them again search and can produce after new match. But the search is expensive which can be saved by staying together. So, match generates surplus. This surplus can be shared by both the matched workers and firms. The most commonly used surplus sharing rule is the Nash-bargaining solution. The Nash-bargaining solution allocates surplus according to the returns from search on both sides. The Nash-bargaining solution can be obtained from the following exercise:

\(^4\) We may assume full productivity of match i.e. \(t = 1\).
Max \( \Omega = (W - U)^{\beta} (J - V) (1 - \beta) \)

\[ w_2 \quad (7) \]

Where \( \beta \) is the bargaining strength of the workers and \( 1 > \beta > 0 \).

Assuming interior solutions exist, the first order condition is

\[ (W - U) = \beta (W - U + J - V) \quad (8) \]

This is the surplus sharing rule in search equilibrium. This rule states that at steady state the net gain to the workers \((W - U)\) is equal to the fixed proportion, \( \beta \) of the total surplus, \((W - U + J - V)\).

Using Equations (1), (2), (4) and the zero-profit condition for the firm, \( V = 0 \), from (8) we can get

\[ w_2 = \beta \left( P_2^* t + C \theta - rk_2 \right) \quad (9) \]

This is the wage equation for the frictional sector. This shows that wage in frictional sector depends positively on the productivity of the sector and on the market tightness, given the discount rate, \( r \).

Now solving the two basic Equations (6) and (9) we can get the equilibrium values of \( w_2, \theta \).

2.4 Unemployment rate:

The conventional labour force is fixed. Following Pissarides (2000) we may derive the rate of unemployment in the following way:

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5 See Appendix A.
6 At steady state, \( k \) is constant.
Suppose, at time $t$ unemployment is $u_t$ and employment is $(1-u_t)$. In short time interval $\alpha t$, $\theta_t q(\theta_t) u t \alpha t$ workers are matched and $\lambda (1-u_t) \alpha t$ lose their jobs. So, unemployment in this interval is

$$u_t + \alpha t = u_t - \theta_t q(\theta_t) u t \alpha t + \lambda (1-u_t) \alpha t$$

(10)

$$\therefore u_t + \alpha t - u_t = -\theta_t q(\theta_t) u t \alpha t + \lambda (1-u_t) \alpha t$$

$$\therefore \frac{u_t + \alpha t - u_t}{\alpha t} \rightarrow 0$$

$$= -\theta_t q(\theta_t) u t + \lambda (1-u_t)$$

(11)

At steady state, $u = 0$.

$$\therefore u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

(12)

This is the equilibrium rate of unemployment. This is also known as the Beveridge curve which shows an inverse relation between $u, \theta$, given $\lambda$. Putting the equilibrium value of $\theta$ into (12) we can get equilibrium $u$

### 2.5 The General Equilibrium Structure of the Model

The structure of the two-sector general equilibrium model consists of the following equations:

The price equations of the two sectors are

$$w_1 a L_1 + r a K_1 = 1$$

(13)

$$w_2 a L_2 + r a K_2 = P_2^*$$

(14)

Where $P_2^*$ is the tariff-inclusive price of the commodity 2. All $a_{ij}$ are functions of $w_j, r \forall j = 1,2$. 

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The wage equation for the frictional sector is given by
\[ w_2 = rU + \frac{\beta}{(1-\beta)}(r+\lambda)\frac{C}{q(\theta)} \] (15)

An unemployed worker in sector 2 either searches job in this sector or get employed in sector 1. As job-seeker he gets unemployment income \( rU \) and as worker in sector 1 he gets wage, \( w_1 \). The no-arbitrage condition implies that in equilibrium,
\[ rU = w_1 \] (16)

The equilibrium rate of unemployment is
\[ u = \frac{\lambda}{\lambda + \theta q(\theta)} \] (17)

Labour is not fully employed but capital is fully employed. Thus, the two factor endowment equations are
\[ a_{L1}X_1 + a_{L2}X_2 = (1-u)L \] (18)
\[ a_{K1}X_1 + a_{K2}X_2 = K \] (19)

Where \( L, K \) are the fixed supply of labour and capital respectively.

Using (1) and (16) into (13) and (14) one gets\(^7\)
\[ \xi q(\theta)\theta a_{L1} + ra_{K1} = 1 \] (13.1)
\[ \xi \left[ r + \lambda + \theta q(\theta) \right] a_{L2} + ra_{K2} = P^*_2 \] (14.1)

\(^7\) See Appendix A.
Where \( \xi = \frac{\beta}{(1-\beta) \theta} \) is the frictional cost of labour in sector 2. We may write 
\[ \xi = \xi(\beta, \theta) \] 
with \( \xi_{\beta} \xi_{\theta} > 0 \) (\( \xi_{i} \) is the elasticity of frictional cost of labour with respect to \( i \) where \( i = \beta, \theta \).

Now we can determine the equilibrium values of seven endogenous variables: \( w_1, w_2, r, \theta, u, X_1, X_2 \) from seven Equations (13.1), (14.1), (15)-(19). Solving (13.1) and (14.1) we get equilibrium values of \( \theta, r \). Then, from (15), (16) and (17) we get \( w_1, w_2, u \).

Finally, Equations (18) and (19) yield \( X_1, X_2 \).

3. **Comparative Static Exercises:**

Taking total differentials of Equations (13.1), (14.1), (15), (16) and after simplification the following results can be obtained:

\[
\left( \hat{w}_2 - \hat{w}_1 \right) < 0, \quad \left( \frac{\hat{w}_2 - \hat{w}_1}{\hat{P}_2^*} \right) > 0
\]

(20)

These results lead to the following proposition:

**Proposition 1:** In the presence of search friction in the labour market a fall in the tariff-inclusive price of the commodity produced in the import-competing sector raises wage inequality and a fall in the bargaining strength of the labour reduces it in a small open economy.

We may give an intuitive explanation of proposition 1. Trade liberalization reduces \( P_2^* \). It can be verified from Equations (13.1) and (14.1) that a fall in \( P_2^* \) leads to an increase in \( \theta \) and a decrease in \( r \). When \( \theta \) rises the average recruitment cost, \( C\theta \) rises. As a result value of unemployment, \( rU \) rises, given \( \beta \). This, under the no-arbitrage condition,

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8 See Appendix B Appendix C.
implies that \( w_1 \) also rises. From (14.1) it can be observed that \( w_2 \) also rises. Under the capital-intensity condition, \( \left( \theta L_1 K_2 - \theta K_1 L_2 \right) > 0 \) \( w_2 \) rises more than \( w_1 \). Therefore, trade liberalization raises wage inequality in our small open economy where labour market is frictional. On the contrary, a fall in the bargaining power of the labour raises both \( \theta, r \). From (16) it can be verified that \( rU \) falls and so also \( w_1 \). From (14.1) it can be observed that \( w_2 \) also falls. Here also the capital-intensity condition implies that \( w_2 \) falls more than \( w_1 \). So, wage inequality decreases.

Taking total differentials of (17) and using (13.1), (14.1) and after simplification one gets

\[
\left( \frac{\dot{u}}{\dot{P}_2^*} \right) > 0, \left( \frac{\dot{u}}{\dot{\beta}} \right) > 0
\]

(21)

These results give the following proposition:

**Proposition 2:** Both trade reform and labour market reform lower the equilibrium rate of unemployment in a small open economy having frictional labour market.

Proposition 2 can be explained as follows. From (13.1) and (14.1) it can be verified that a fall in \( P_2^* \) and/ \( \beta \) raises \( \theta \). From the Beveridge curve (Equation, 17) it is evident that \( u \) must fall when \( \theta \) rises.

4. **Concluding Remarks:**

In this paper we extend the DMP model in a two-sector general equilibrium framework. Like the DMP model we also assume determination of wage rate in the frictional sector

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9 See Appendix B, Appendix C.
through the Nash-bargaining solution. However, the marginal productivity rule is applied to determine wage rate in the frictionless sector and the unique discount rate.

We introduce no-arbitrage condition in the labour market.\textsuperscript{10} Our theoretical analysis shows that trade reform softens the problem of unemployment but raises wage inequality in a small open economy characterized by search friction in the labour market. However, labour market reform lowers both wage inequality and equilibrium rate of unemployment. Thus, our theoretical results provide a strong ground for labour market reform and weak ground for trade reform in a small open economy where labour market is frictional.

\textbf{Appendix A: Derivation of Some Useful Expressions:}

The Nash –bargaining problem is

$$\text{Max } \Omega = (W - U)^{\beta} (J - V)^{(1 - \beta)}$$

\begin{equation}
\quad w_2
\end{equation}

The first order condition for maximization is

$$\beta (J - V) \frac{\partial}{\partial w_2}(W - U) + (1 - \beta)(W - U) \frac{\partial}{\partial w_2}(J - V) = 0$$

\begin{equation}
\text{(A.1)}
\end{equation}

Using (2), (4) and the zero-profit condition \( V = 0 \) into (A.1) one gets

$$(1 - \beta)(w_2 - rU) = \beta \left( P^*_2 t - w_2 - rk_2 \right)$$

Or

$$w_2 = (1 - \beta) rU + \beta \left( P^*_2 t - rk_2 \right)$$

\begin{equation}
\text{(A.2)}
\end{equation}

\textsuperscript{10} See Sheng and Xu (2007) in this context.
Using (2), (5), (A.1) and V=0 from (1) we can write

\[ rU = \frac{\beta C\theta}{(1-\beta)} \]  \hspace{1cm} (1.1)

Using (1.1) into (A.2) one gets

\[ w_2 = \beta \left( P_2^* t + C\theta - rk_2 \right) \]  \hspace{1cm} (9)

**Appendix B Effects of a Change in \( P_2^*, \beta \) on \( \theta, r, w_1, w_2, u \):**

Using (1.1) from (16) we get

\[ w_1 = \frac{\beta C\theta}{(1-\beta)} = \xi q(\theta)\theta \]  \hspace{1cm} (A.3)

Using (1.1) into (15) we may get

\[ w_2 = \frac{\beta}{(1-\beta)} \frac{C}{q(\theta)} (r+\lambda + \theta q(\theta)) \]
\[ = \xi \left( r + \lambda + \theta q(\theta) \right) \]  \hspace{1cm} (A.4)

Taking total differentials of Equations (13.1) and (14.1) and after simplifications we get

\[
\left( 1 + e^\theta q + e^\theta \xi \right) \theta_{L1} + \theta_{K1} \hat{r} = -\theta_{L1} e^\beta \xi \hat{p} \]
\hspace{1cm} (A.5)

\[
\left[ e^\theta \xi + \frac{\xi q(\theta)\theta}{w_2} \left( 1 + e^\theta \right) \right] \theta_{L2} + \left( \frac{\xi r}{w_2} + \theta_{L2} + \theta_{K2} \right) \hat{r} = \hat{p}_2^* - e^\beta \theta L2 \hat{p} \]
\hspace{1cm} (A.6)

Solving (A.5) and (A.6) we get
\[ \dot{\theta} = \frac{1}{\Delta} \left[ -\theta_{K1} \ddot{\rho}^{*}_2 - \left( \theta_{L1} \theta_{K2} - \theta_{K1} \theta_{L2} \right) e_{\beta} \right] \]  
(A.7)

\[ \dot{\hat{r}} = \frac{1}{\Delta} \left[ \left( 1 + e_{q} + e_{\xi} \right) \theta_{L1} \ddot{\rho}^{*}_2 \left( 1 + e_{q} \right) \right] \theta_{L1} e_{\beta} \]  
(A.8)

where

\[ \Delta = \left( 1 + e_{q} \right) \left[ \left( 1 + e_{q} \right) \left( 1 + e_{q} \right) \right] \theta_{L1} \theta_{L2} \theta_{K1} - \left[ \left( 1 + e_{q} \right) \left( 1 + e_{q} \right) \right] \theta_{L2} \theta_{K1} > 0 \]

\[ \Rightarrow \left( \theta_{L1} \theta_{K2} > \theta_{L2} \theta_{K1} \right) \] and \( 1 + e_{q} + e_{\xi} > 0 \)

From (A.7)-(A.9) we get

\[ \frac{\dot{\theta}}{\ddot{\rho}^{*}_2} = -\frac{1}{\Delta} \theta_{K1} < 0, \quad \frac{\dot{\hat{r}}}{\ddot{\rho}^{*}_2} = -\frac{1}{\Delta} \left( \theta_{L1} \theta_{K2} - \theta_{K1} \theta_{L2} \right) e_{\beta} < 0 \]  
(A.7.1)

\[ \frac{\dot{\hat{r}}}{\ddot{\rho}^{*}_2} = \frac{1}{\Delta} \left( 1 + e_{q} + e_{\xi} \right) \theta_{L1} > 0, \quad \frac{\ddot{\hat{r}}}{\ddot{\rho}^{*}_2} = \frac{1}{\Delta} \left( 1 + e_{q} \right) \left( w_{1} - 1 \right) \theta_{L1} e_{\beta} < 0 \]  
(A.8.1)

Again from (13) and (14) we get

\[ \hat{w}_{1} = -\frac{\theta_{K1}}{\theta_{L1}} \hat{r} \]  
(A.10)
and
\[ \hat{w}_2 = -\frac{\theta K_2}{\theta L_2} \hat{r} \quad (A.11) \]

Using (A.10) and (A.11) we get
\[ \left( \hat{w}_2 - \hat{w}_1 \right) = -\left( \frac{\theta K_2}{\theta L_2} - \frac{\theta K_1}{\theta L_1} \right) \frac{\hat{r}}{P_2^*} < 0 \quad (+) \quad (A.12) \]

Using (A.7.1), (A.8.1) from (A.12 one gets
\[ \left( \hat{w}_2 - \hat{w}_1 \right) = -\left( \frac{\theta K_2}{\theta L_2} - \frac{\theta K_1}{\theta L_1} \right) \frac{\hat{r}}{\beta} > 0 \quad (+) \quad (A.12.2) \]

Taking total differentials of (17) and after simplifying one gets
\[ \hat{u} = -(1-u)\left(1 + e^\theta \right) \hat{\theta} \quad (A.13) \]

Using (A.7.1) from (A.13) we get
\[ \frac{\hat{u}}{\hat{P}_2^*} = -\left(1-u\right)\left(1 + e^\theta \right) \frac{\hat{\theta}}{\hat{P}_2^*} > 0 \quad (+) \quad (A.13.1) \]

\[ \frac{\hat{u}}{\beta} = -(1-u)\left(1 + e^\theta \right) \frac{\hat{\theta}}{\beta} > 0 \quad (+) \quad (A.13.2) \]
Appendix C: Effects of Changes in $p^*_2, \beta$ on Sectoral Output

Taking total differentials of Equations (18) and (19) and using the definitions of the elasticity of factor substitutions and after simplifications we get

$$\lambda_{L1} \hat{X}_1 + \lambda_{L2} \hat{X}_2 = \lambda_{L1} \sigma_1 \theta_{K1} \hat{w}_1 + \lambda_{L2} \sigma_2 \theta_{K2} \hat{w}_2 - \left( \lambda_{L1} \sigma_1 \theta_{K1} + \lambda_{L2} \sigma_2 \theta_{K2} \right) \hat{r} - (1-u) \hat{L} - u \hat{u} \tag{A.14}$$

$$\lambda_{K1} \hat{X}_1 + \lambda_{K2} \hat{X}_2 = -\lambda_{K1} \sigma_1 \theta_{L1} \hat{w}_1 - \lambda_{K2} \sigma_2 \theta_{L2} \hat{w}_2 + \left( \lambda_{K1} \sigma_1 \theta_{L1} + \lambda_{K2} \sigma_2 \theta_{L2} \right) \hat{r} + \hat{K} \tag{A.15}$$

Solving (A.14) and (A.15) by Cramer’s rule one gets

$$\hat{X}_1 = \frac{1}{|\lambda|} \left[ \left( \lambda_{K2} \lambda_{L1} \theta_{K1} + \lambda_{L2} \lambda_{K1} \theta_{L1} \right) \sigma_1 \hat{w}_1 + \lambda_{K2} \lambda_{L2} \sigma_2 \hat{w}_2 - \left( \lambda_{K2} \lambda_{L1} \theta_{K1} + \lambda_{L2} \lambda_{K1} \theta_{L1} \right) \hat{r} + \lambda_{K2} (1-u) \hat{L} - \lambda_{L2} \hat{K} - \lambda_{K2} u \hat{u} \right] \tag{A.16}$$

$$\hat{X}_2 = \frac{1}{|\lambda|} \left[ \lambda_{L1} \hat{K} - \lambda_{L1} \lambda_{K1} \sigma_1 \hat{w}_1 - \left( \lambda_{L1} \lambda_{K2} \theta_{L2} + \lambda_{K1} \lambda_{L2} \theta_{K2} \right) \sigma_2 \hat{w}_2 + \left( \lambda_{L1} \lambda_{K1} \sigma_1 + \lambda_{L1} \lambda_{K2} \theta_{L2} + \lambda_{K1} \lambda_{L2} \theta_{K2} \right) \hat{r} - \lambda_{K1} (1-u) \hat{L} + \lambda_{K1} u \hat{u} \right] \tag{A.17}$$

where

$$|\lambda| = \left( \lambda_{L1} \lambda_{K2} - \lambda_{K1} \lambda_{L2} \right) > 0 \quad \text{(Since sector 1 is assumed to be labour-intensive vis-à-vis sector 2)}.$$
Using (A.7.1), (A.8.1), (A.10) and (A.11) from (A.16) and (A.17) one gets

\[
\begin{pmatrix}
\left(\lambda_{K2} \lambda_{L1} \theta_{K1} + \lambda_{L2} \lambda_{K1} \theta_{L1}\right) \sigma_1 \left(\frac{\hat{w}_1}{\hat{p}_2}\right) + \lambda_{K2} \lambda_{L2} \sigma_2 \left(\frac{\hat{w}_2}{\hat{p}_2}\right) - \\
\left(\left(\lambda_{K2} \lambda_{L1} \theta_{K1} + \lambda_{L2} \lambda_{K1} \theta_{L1}\right) \sigma_1 + \lambda_{K2} \lambda_{L2} \sigma_2\right) \left(\frac{\hat{r}}{\hat{p}_2}\right) - < 0 \tag{A.16.1}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\frac{\hat{x}_1}{\hat{p}_2} = \frac{1}{|\lambda|} \left(\lambda_{K2} \lambda_{L1} \theta_{K1} + \lambda_{L2} \lambda_{K1} \theta_{L1}\right) \sigma_1 \left(\frac{\hat{w}_1}{\hat{p}_2}\right) + \lambda_{K2} \lambda_{L2} \sigma_2 \left(\frac{\hat{w}_2}{\hat{p}_2}\right) - \\
\left(\left(\lambda_{K2} \lambda_{L1} \theta_{K1} + \lambda_{L2} \lambda_{K1} \theta_{L1}\right) \sigma_1 + \lambda_{K2} \lambda_{L2} \sigma_2\right) \left(\frac{\hat{r}}{\hat{p}_2}\right) - < 0 \tag{A.16.2}
\end{pmatrix}
\]
\[
\begin{aligned}
\hat{x}_2 &= \frac{1}{|\lambda|} \left[ \lambda_{L1} \lambda_{K1} \sigma_1 \left( \frac{\dot{w}_r}{\dot{p}_2} \right) - \left( \lambda_{L1} \lambda_{K2} \theta L_2 + \lambda_{K1} \lambda_{L2} \theta K_2 \right) \sigma_2 \frac{\dot{w}_2}{\dot{p}_2^*} \right] + \\
\hat{x}_2 &= \frac{1}{|\lambda|} \left[ \lambda_{L1} \lambda_{K1} \sigma_1 \left( \frac{\dot{w}_r}{\dot{p}_2} \right) - \left( \lambda_{L1} \lambda_{K2} \theta L_2 + \lambda_{K1} \lambda_{L2} \theta K_2 \right) \sigma_2 \frac{\dot{r}}{\dot{p}_2^*} \right] + > 0 \quad (A.17.1)
\end{aligned}
\]

References:


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Economica, 47(185), February, 35-50.


