Real Balance Effects, Determinacy and Optimal Monetary Policy

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Abstract

This paper presents a dynamic New Keynesian macroeconomic model with real balance effects. Both the conditions of equilibrium determinacy under an interest rate rule of the Taylor-type and the implications for optimal monetary policy are considered. We find a number of results that would not appear in the traditional framework. It is shown that the real balance effect makes the so-called “Taylor principle” not necessary for determinacy of rational expectations equilibrium. A relatively “passive” monetary policy is found to be feasible also in the long run, but not necessarily optimal. In particular, within a class of policy rules constrained to be a linear function of state variables, an “active” optimal interest rate rule is more likely to be verified under commitment rather than under discretion.

Keywords: real balance effects, determinacy, optimal monetary policy.

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1 Introduction

Much of the modern literature on interest rate rules uses New Keynesian optimizing models that do not include explicit reference to any monetary aggregate (see, e.g., Rotemberg and Woodford, 1997; Clarida, Galí and Gertler, 1999, and most of the contributions in Taylor, 1999). A common feature of these models is that they specify the demand for output as a function of the real interest rate. The real money stock does not enter the IS relation. These models therefore limit the influence of monetary policy to output and inflation to its effect on the real interest rate. With the nominal interest rate as the policy instrument, it is not necessary specify a money market equilibrium condition (see Romer, 2000).

This common practice of neglecting monetary aggregates in monetary policy analysis has recently been questioned by Meltzer (1999), McCallum (2001), and Nelson (2002, 2003). In particular, McCallum (2001) points out that while no explicit term involving money appears in the standard New Keynesian setup, inflation can still be pinned down in the long run by the economy steady-state nominal money-growth rate. This steady state role for money is also discussed in detail by Nelson (2003).

Moreover, McCallum (2000, 2001) analyzes the case in which the resources used by consumers in conducting transactions are modelled by a non separable function of consumption and real money balances\(^1\). Meltzer (1999) argues that changes in real monetary base generate effects on real aggregate demand not summarized by the real interest rate on short term securities. Nelson (2003) emphasizes the role of money in the transmission mechanism both as a proxy for a whole spectrum of rates of return\(^2\), and as an indicator variable, which contains information about the state of the economy\(^3\).

The main purpose of this paper is to give micro-foundations for an explicit role for money in the transmission mechanism of monetary policy in the current macroeconomic debate. In particular, we derive a real balance effect that explicitly enters the IS relation within an optimizing general equilibrium framework of the New Keynesian-type. According to de Scitovszky (1941), Haberler (1946), Pigou (1943), and Patinkin (1965), the real balance effect describes a channel through which a change in real money balances

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1Monetary models in which the marginal utility of consumption depends on real money balances are also developed by Andres et al. (2001), Ireland (2001a), and Woodford (2003).
2This view has been first developed by Friedman and Schwartz (1982).
3The idea that real money balances capture many channels of monetary transmission is also discussed by Meltzer (2001).
has an impact on the real financial wealth of consumers and therefore affects consumption and output. Notably, this effect is not considered in traditional models. There is, in fact, empirical evidence showing that real monetary base growth is a significant determinant of consumption and output gap in both the United States and in the United Kingdom\textsuperscript{4}.

The present paper extends the New Keynesian setup with money-in-the-utility-function by introducing overlapping generations as modelled by Yaari(1965)-Blanchard (1985). Money in the utility function is the source of money demand\textsuperscript{5}. Besides our desire to be able to evaluate the role of the real balance effect, there are good reasons in basing the demand-side of the economy on a discrete time stochastic version of the Yaari-Blanchard model. First, this kind of extension allow us to maintain the main characteristics of the so-called “New Neoclassical Synthesis”\textsuperscript{6}: forward-looking behavior of optimizing agents and incomplete nominal adjustment of prices featured in New Keynesian theories. Second, the derivation of the real balance effect does not require non-separability in the utility function. Thus, the implications of this effect are analytically more tractable\textsuperscript{7}. Our analysis attempts to examine whether the presence of an explicit real balance effect in the IS relation helps to evaluate the role for money in designing monetary policy rules.

We examine basic issues related to equilibrium determinacy and optimal monetary policy. Specifically, we investigate whether it is necessary for monetary policy to overreact to inflation by raising the nominal interest rate by more than the observed increase in inflation in order to rule out equilibrium multiplicity. It is shown that the real balance effect makes the rational expectations equilibrium determinate also under a relatively “passive” Taylor rule (i.e., that has the long run nominal interest rate to increase by less than point-for-point with inflation). Our result does not imply that the real balance effect makes a relatively “passive” monetary policy optimal. In particular, within a class of policy rules that is constrained to be a linear function of state variables, we demonstrate that the Taylor principle is more likely to be verified under commitment rather than discretion. Our analysis


\textsuperscript{5}Feenstra (1986) establishes an equivalence between money-in-the-utility-function and cash-in-advance models.

\textsuperscript{6}This term is due to Goodfriend and King (1997).

\textsuperscript{7}For the derivation of the real balance effect in the infinite horizon framework à la Weil (1991) with separability in utility, see Ireland (2001b). His analysis shows that the real balance effect eliminates the liquidity trap.
gives support to the view that the observed instability of the U.S. inflation and real economic activity during the 1970’s was due not to indeterminacy of rational expectations equilibrium but to an absence of commitment on the part of the Fed (see Chari, Christiano and Eichenbaum, 1997).

The paper is structured as follows. Section 2 presents a discrete time stochastic overlapping generations framework with staggered nominal price setting. Equilibrium is characterized in Section 3. In Section 4 we study the conditions for a unique bounded solution under a simple Taylor rule. The implications for optimal monetary policy are derived and described in Section 5. Section 6 concludes.

2 An Optimizing IS-LM-AS Model

The objective of this Section is to provide an optimizing general equilibrium model with overlapping generations and staggered price adjustment.

2.1 Consumers optimization

The demand-side of the economy is based on a discrete-time version of the Yaari (1965)-Blanchard (1985) overlapping generations model, with no inter-generational bequest motive. Three kind of extensions are made. First, the model includes endogenous labor-leisure and money holding choices. According to Sidrauski (1965) and Brock (1975), money enters the utility function since it provides transaction services. Second, the economy is characterized by a continuum of consumption goods supplied by monopolistically competitive firms. Third, individuals face uncertainty not only on the duration of their lives, but also on the future time paths of the economic variables. This extension allows us to outline a stochastic macroeconomic model suitable for the evaluation of monetary policy.

All agents have identical preferences, face the same, constant probability of death \( \vartheta \) in each period, and there is no population growth. Each good is produced in a number of varieties or brands indexed by \( j \) and defined over

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8For a discrete-time version of the Yaari-Blanchard model, see Frenkel and Razin (1986).

9A monetary version of the Blanchard-Yaari model was first developed by Marini and van der Ploeg (1988). For a discrete-time version, see Cushing (1999).

10A multi-goods monetary version of the Yaari-Blanchard framework allowing for the existence of nominal rigidities can be found in Leith and Wren-Lewis (2000). For a discrete-time version extended to the open economy, see Smets and Wouters (2002).
the range [0,1]. Each brand is an imperfect substitute in consumption for all other brands, with constant elasticity of substitution \( \theta > 1 \). Following Dixit and Stiglitz (1977), the consumption index of goods at time \( t \) for the representative agent born at time \( s \leq t \) is defined as

\[
c_{s,t} = \left[ \int_0^1 c_{s,t}(j) \theta^{-1} \, dj \right]^{\frac{\theta}{\theta - 1}},
\]

(1)

where \( c_{s,t}(j) \) denotes consumption of brand \( j \). The utility-based price of a consumption bundle of produced goods, denoted by \( P_t \), is derived as

\[
P_t = \left[ \int_0^1 P_t(j)^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}}.
\]

(2)

As usual, the intratemporal individual optimization yields the demand for each brand \( j \) as a function of the relative price of \( j \) and total consumption of goods:

\[
c_{s,t}(j) = \left[ \frac{P_t(j)}{P_t} \right]^{-\theta} c_{s,t}.
\]

(3)

The objective of the representative agent is to maximize the expected utility

\[
E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} (1 - \varphi)^{\tau-t} \left[ \log c_{s,\tau} + \chi \log \frac{m_{s,\tau}}{P_{\tau}} + \kappa \log (1 - l_{s,\tau}) \right],
\]

(4)

subject to the flow budget constraint

\[
m_{s,t} + b_{s,t} \leq [m_{s,t-1} + (1 + i_{t-1}) b_{s,t-1}] (1 - \theta)^{-1}
+ w_{s,t} l_{s,t} + z_{s,t} - t_{s,t} - \int_0^1 P_t(j) c_{s,t}(j) \, dj,
\]

(5)

where all parameters are positive, \( 0 < \beta < 1 \) represents the subjective discount factor, \( l_{s,t}, m_{s,t}, b_{s,t}, w_{s,t}, z_{s,t}, t_{s,t} \) denote labor effort, nominal money balances, holdings of riskless one-period bonds with a nominal interest rate \( i_t \), the nominal wage rate, the share in the profits of firms, and lump-sum net taxes of an individual born at time \( s \), respectively. Our timing convention has \( m_{s,t} \) and \( b_{s,t} \) as agent’s nominal balances and bonds accumulated during period \( t \) and carried over into period \( t + 1 \). The short term nominal rate \( i_t \) is paid at beginning of period \( t + 1 \) and is known at time \( t \). Note that the flow budget constraint incorporates the return on the insurance contract
as modelled by Blanchard (1985)\(^{12}\), \((1 - \vartheta)^{-1}\). The effective discount factor of consumers is given by \(\beta (1 - \vartheta)\). Labor income, the share of profits and lump-sum net taxes are assumed to be equally distributed across agents.

The solution to the individual intertemporal maximizing problem yields the Euler equation and the efficiency conditions on labor supply and money demand choices, respectively:

\[
\frac{1}{P_t c_{s,t}} = \beta (1 + i_t) E_t \left( \frac{1}{P_{t+1} c_{s,t+1}} \right), \quad (6)
\]

\[
\kappa \frac{c_{s,t}}{1 - L_{s,t}} = \frac{w_{s,t}}{P_t}, \quad (7)
\]

\[
m_{s,t} \frac{P_t}{P_{t+1}} = \chi \left(1 + i_t\right) c_{s,t}, \quad (8)
\]

Define now the variable

\[
Q_{t,t+1}(s) = \beta \frac{P_t}{P_{t+1}} \frac{c_{s,t}}{c_{s,t+1}}, \quad (9)
\]

that can be interpreted as the stochastic discount rate of the representative agent of generation \(s\). Comparing (9) with (6) we obtain

\[
E_t Q_{t,t+1}(s) = \frac{1}{1 + i_t} \text{ for each } s \in (-\infty, t]. \quad (10)
\]

In the optimum the flow budget constraint (5) holds with equality in each period. We impose a transversality condition precluding private agents’ Ponzi-game:

\[
\lim_{\tau \to \infty} E_t Q_{t,\tau}(s) (1 - \vartheta)^{\tau \to (t+1)} \left[ m_{s,\tau} + (1 + i_{\tau}) b_{s,\tau} \right] = 0, \quad (11)
\]

where \(Q_{t,\tau}(s) = \prod_{k=t+1}^{\tau} Q_{k-1,k}(s)\). Solving (9) forward, using the budget constraint and imposing the no-Ponzi-game condition, individual consumption can be written as a fraction of total wealth:

\[
P_t c_{s,t} = \Psi \left\{ \left[m_{s,t-1} + (1 + i_{t-1}) b_{s,t-1}\right] (1 - \vartheta)^{-1} + h_{s,t}\right\}, \quad (12)
\]

where \(\Psi = [1 - \beta (1 - \vartheta)] / 1 + \chi\) is the propensity to consume out of wealth, and \(h_{s,t}\) is human capital, defined as

\[
h_{s,t} = E_t \sum_{\tau=t}^{\infty} Q_{t,\tau}(s) (1 - \vartheta)^{\tau - t} \left( w_{s,\tau} I_{s,\tau} + z_{s,\tau} - t_{s,\tau}\right). \quad (13)
\]

\(^{12}\)A perfect insurance market inherits consumers financial wealth contingent on their death and redistributes this in proportion to financial wealth. As a result, zero profits in the insurance industry imply that the gross return on the insurance contract is given by \((1 - \vartheta)^{-1}\).
2.1.1 Aggregation

Aggregate values are the sum across cohorts, weighted by their respective sizes, and thus are defined as

$$X_t = \sum_{s=-\infty}^{t} \vartheta (1 - \vartheta)^{t-s} x_{s,t},$$

(14)

where $x_{s,t}$ indicates the corresponding individual variable.

After aggregation over all the cohorts of consumers, we obtain the aggregate budget constraint, the aggregate consumption function, the aggregate labor supply and the aggregate money demand, respectively:

$$M_t + B_t = M_{t-1} + (1 + i_{t-1}) B_{t-1} + W_t L_t + Z_t - T_t - P_t C_t,$$

(15)

$$P_t C_t = \Psi [M_{t-1} + (1 + i_{t-1}) B_{t-1} + H_t],$$

(16)

$$\kappa \frac{C_t}{1 - L_t} = \frac{W_t}{P_t},$$

(17)

$$\frac{M_t}{P_t} = \chi (1 + i_t) C_t.$$  

(18)

Using (15) into (16) yields the dynamic equation for consumption,

$$E_t \{Q_{t,t+1} P_{t+1} C_{t+1}\} = \beta P_t C_t - \Gamma E_t \{Q_{t,t+1} [M_t + (1 + i_t) B_t]\},$$

(19)

where $\Gamma = [\vartheta/(1 - \vartheta)] \Psi$. Aggregate consumption is a function not only of expected consumption, but also of aggregate non-human wealth.

2.2 The demand for goods

Private demand for differentiated good $j$ is obtained by aggregating (3) across individuals:

$$C_t (j) = \left[ \frac{P_t (j)}{P_t} \right]^{-\theta} C_t.$$  

(20)

We assume that government spending, $G_t$, is allocated amongst differentiated consumption goods in the same manner as individuals’ consumption:

$$G_t = \left[ \int_0^1 g_t (j) \frac{q-1}{q} dj \right]^{\frac{1}{q-1}}.$$  

(21)

It follows that the demand for brand $j$ is

$$Y_t (j) = \left[ \frac{P_t (j)}{P_t} \right]^{-\theta} Y_t,$$  

(22)

where aggregate demand for the composite good is $Y_t = C_t + G_t$. 
2.3 Firms optimization and price setting

The supply-side of the economy has a continuum of monopolistic firms, indexed by $j$, each producing a differentiated product $j$. Each firm $j$ faces a linear production technology,

$$Y_t(j) = A_t L_t(j), \quad (23)$$

where $A_t$ is an exogenous technological parameter. Following Calvo (1983), nominal price rigidity is modelled by allowing random intervals between price changes. Each period a firm adjusts its price with probability $1 - \phi$ and keeps its price fixed with probability $\phi$. The adjustment probability is independent across time and across firms. Firms that do not adjust prices stand ready to adjust output to meet demand (assuming the participation constraint that they operate in a region with a non-negative net markup). In either case, choosing labor to minimize costs conditional on output yields

$$MC^n_t = \frac{W_t}{A_t}, \quad (24)$$

where $MC^n_t$ denotes the nominal marginal cost, that is identical across firms.

The optimal pricing decision of a firm able to revise its price in period $t$ is to choose the price $P_t(j)$ to maximize the following objective:

$$E_t \left\{ \sum_{\tau = t}^{\infty} \phi^{\tau - t} Q_{t,\tau} Z_\tau(j) \right\}, \quad (25)$$

where $Z_\tau(j)$ denotes nominal profits from the sale of good $j$ given by

$$Z_\tau = Y_\tau P_\tau^\theta \left[ P_t(j)^{1-\theta} - MC^n_{\tau} P_t(j)^{-\theta} \right]. \quad (26)$$

The first order condition for the optimal price is

$$E_t \left\{ \sum_{\tau = t}^{\infty} \phi^{\tau - t} Q_{t,\tau} Y_\tau P_\tau^\theta \left[ (1 - \theta) P_t(j)^{-\theta} + \theta MC^n_{\tau} P_t(j)^{-\theta-1} \right] \right\} = 0. \quad (27)$$

Multiplying (27) by $P_t(j)$, dividing by $(1 - \theta)$ and then simplifying we obtain

$$E_t \left\{ \sum_{\tau = t}^{\infty} \phi^{\tau - t} Q_{t,\tau} Y_\tau P_\tau^\theta \left[ P_t(j) - \mu MC^n_{\tau} \right] \right\} = 0, \quad (28)$$

13The factor $\phi^{\tau - t}$ multiplying the stochastic discount factor indicates the probability that price $P_t(j)$ will still be charged in period $\tau$. 

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where $\mu = \frac{\theta}{\theta - 1}$ is the equilibrium gross markup. Condition (28) implies that firms set their price equal to a markup over a weighted average of expected future nominal marginal costs.

In the symmetric equilibrium each producer choosing a new price $P_t(j)$ in period $t$ will choose the same new price $P_t(j)$ and the same level of output $Y_t(j)$. Then, the price index follows a law of motion given by

$$P_t = \left[ \phi (P_{t-1})^{1-\theta} + (1 - \phi) P_t(j)^{1-\theta} \right]^{1/1-\theta}.$$  \hspace{1cm} (29)

### 2.4 Government budget constraint

The government is assumed to run a balanced budget constraint each period:

$$T_t + (M_t - M_{t-1}) = P_t G_t.$$  \hspace{1cm} (30)

For simplicity, let us set $G_\tau = \bar{G} \geq 0$ for all $\tau \geq t$, in what follows.

### 3 Equilibrium

In this Section, we characterize the equilibrium conditions. Specifically, in the aggregate, the nominal money supply must equal nominal money demand, and bonds must be zero in net supply:

$$B_t = \sum_{s=-\infty}^{t} \vartheta (1 - \vartheta)^{t-s} b_{s,t} = 0.$$  \hspace{1cm} (31)

Given these asset market clearing conditions, one can derive the aggregate global goods market clearing condition. Specifically, (15), (30) and (31) together imply the equilibrium requirement that

$$C_t + \bar{G} = Y_t = \frac{W_t}{P_t} L_t + \frac{Z_t}{P_t}.$$  \hspace{1cm} (32)

After combining the aggregate labor supply (17), the costs minimization condition (24), the equilibrium relation (32) and the aggregate production function,

$$Y_t = A_t L_t,$$  \hspace{1cm} (33)

the real marginal cost, $MC_t$, takes the following form:

$$MC_t = \kappa \frac{Y_t - \bar{G}}{A_t - Y_t}.$$  \hspace{1cm} (34)
Condition (34) states that the real marginal cost depends positively on output and negatively on the exogenous technological parameter.

To close the model the behavior of prices and monetary policy must be specified. Section 3.1 examines the global solution under flexible prices. Section 3.2 characterizes the case of nominal rigidities, for which it is necessary a log-linear approximation of the global system around a deterministic steady state.

3.1 Flexible price equilibrium

Under flexible prices ($\phi = 0$) the firm adjusts each period. In particular, it will choose the price for its differentiated good as a constant markup over marginal cost:

$$\frac{P_t(j)}{P_t} = \mu MC_t.$$  

Since the symmetric equilibrium implies that all firms choose the same price ($P_t(j) / P_t = 1$), the flexible price equilibrium is characterized by a constant real marginal cost:

$$MC_t = \frac{1}{\mu}.$$  

Using (36) into (34) yields the natural level of output:

$$Y_t^n = \frac{A_t + \kappa \mu \bar{G}}{1 + \kappa \mu}.$$  

With flexible prices, output is determined independently of monetary factors.

3.2 Equilibrium dynamics under sticky prices

In order to obtain tractable solutions, in this section we develop log-linear versions of all equilibrium conditions under sticky prices around a non-stochastic steady state. The deterministic steady state we consider is defined as follows.

3.2.1 Steady state

We refer to a zero inflation non-stochastic steady state where $P_t = \bar{P}$, $(1 + i_t) = \bar{R}$, $W_t = \bar{W}$, $A_t = \bar{A}$, $L_t = \bar{L}$, $C_t = \bar{C}$, $Y_t = \bar{Y}$, $M_t = \bar{M}$, and $T_t = \bar{T}$. It is straightforward to show that this steady state is also the flexible price non-stochastic steady state.

Specifically, from (17), (18), (19), (28), (30), (31), (32), and (33) it must be that $\kappa \bar{C} / (1 - \bar{L}) = \bar{W} / \bar{P}$, $\bar{M} / \bar{P} = \chi (\bar{R} / \bar{R} - 1) \bar{C}$, $\bar{Q} = 1 / \bar{R}$, $\beta \bar{R} =$
\[1 + \Gamma \tilde{M}/\tilde{P}\tilde{C}, \quad \overline{MC} = 1/\mu, \quad \bar{T} = \bar{G}, \quad \bar{B} = 0, \quad \bar{Y} = \bar{C} + \bar{G}.\] It should be noted that in the general case in which \(\vartheta, \chi > 0\) we have \(\beta\bar{R} > 1.\) Only in the limiting cases of cashless economy \((\chi = 0)\) and/or infinite horizon \((\vartheta = 0)\) the standard property, \(\beta\bar{R} = 1,\) must hold.

### 3.2.2 The linearized model

We now use the steady state defined above as the point around which to log-linearize the model. We use lower case variables with an accent above to denote log-deviations from the deterministic steady state.

On the demand side, log-linear approximations of goods market and money market equilibrium conditions are given, respectively, by

\[\hat{y}_t = s_c\hat{c}_t,\]  
\[\hat{m}_t - \hat{p}_t = \hat{c}_t - \eta\hat{\bar{i}}_t,\]

where

\[s_c = \frac{(\bar{Y} - \bar{G})}{\bar{Y}}, \quad \eta = \left[\frac{1}{\bar{R} - 1} - 1\right], \quad \text{and} \quad \bar{i}_t = \log \left[\frac{(1 + \bar{i}_t)}{(1 + \bar{i})}\right].\]

From (19) (after imposing the bonds market clearing condition (31)), aggregate consumption evolves as

\[\hat{c}_t = -\bar{i}_t + \frac{1}{1 + \Phi}E_t\pi_{t+1} + \frac{1}{1 + \Phi}E_t\hat{c}_{t+1} + \frac{\Phi}{1 + \Phi}(\hat{m}_t - \hat{p}_t),\]

where \(\Phi = \Gamma \tilde{M}/\tilde{P}\tilde{C},\) and \(\pi_{t+1} = \log \left(P_{t+1}/P_t\right)\) is the inflation rate from \(t\) to \(t + 1.\)

On the supply side, approximation to the aggregate production function (33) yields

\[\hat{y}_t = \hat{\alpha}_t + \hat{\bar{i}}_t.\]

Aggregate supply is obtained combining log-linear versions of optimal price setting (28) and of the price index (29):

\[\pi_t = \delta\hat{m}c + \frac{\beta}{1 + \Phi}E_t\pi_{t+1},\]

where

\[\delta = \frac{(1 - \phi)(1 - \phi\bar{y})}{\phi}.\]

Combining the log-linearized version of the relation (34) involving the real marginal cost and the production function (41), one obtains

\[\hat{m}c = v\bar{x}_t,\]

where \(v = \frac{L}{L} + \frac{\bar{V}}{\bar{C}}\) and \(\bar{x}_t = \hat{y}_t - \hat{\bar{y}}_t^\alpha\) denotes the output gap, i.e., the difference between output and the natural level.
The system of equations has essentially the structure of an IS-LM-AS system. Specifically, the IS relation can be obtained substituting (38) into (40) and using the definition of output gap:

$$x_t = -s_c \left( \hat{\eta}_t - \frac{1}{1 + \Phi} E_t \pi_{t+1} \right) + \frac{1}{1 + \Phi} E_t x_{t+1} + s_c \frac{\Phi}{1 + \Phi} (\hat{m}_t - \hat{p}_t) + \hat{\zeta}_t,$$

where $\hat{\zeta}_t = \left( \frac{1}{1 + \Phi} E_t \hat{\eta}_{t+1} - \tilde{\eta}_n \right)$ can be modelled as an exogenous disturbance term. The LM is expressed by (39). The “New Keynesian” Phillips curve can be derived substituting (43) into (42):

$$\pi_t = \lambda x_t + \frac{\beta}{1 + \Phi} E_t \pi_{t+1},$$

where $\lambda = \nu \delta$.

It should be emphasized that, in the general case in which $\Phi > 0$, the LM is not recursive to the model since money appears directly in the structural equation describing aggregate demand determination. In particular, the relevance of money for aggregate demand comes via a micro-founded real balance effect: money is net wealth and tends to stimulate consumption.

How to close the model depends on the path for the nominal interest rate implied by the monetary policy regime.

4 The Taylor Rule and Equilibrium Determinacy

We now consider how the real balance effect may affect the conditions for rational expectations equilibrium determinacy under an interest rate rule of the Taylor-type, given by

$$\hat{\eta}_t = \rho + \phi_{\pi} \pi_t + \phi_x x_t,$$

where $\rho$ is an exogenous intercept, and $\phi_{\pi}, \phi_x$ are constant policy coefficients, indicating the “strength” of monetary policy\textsuperscript{14}. We assume that $\phi_{\pi}$ and $\phi_x$ are non-negative, with at least one strictly positive. A rational expectations equilibrium is a set of processes $\{x_\tau, \pi_\tau, \{\hat{m}_\tau - \hat{p}_\tau\}_{\tau=t}^\infty\}$ satisfying (39), (44)

\textsuperscript{14}It is well-known that Taylor has found this kind of interest rate rule as a good characterization of U.S. monetary policy, as discussed in a number of recent papers (see, e.g., Taylor, 1993, 1999; Judd and Rudebush, 1998).
and (45) each period, for any given specification of the interest rate policy rule (46) and for a given process \( \{ \xi \} \).

In order to analyze the issue of stability of equilibrium, it is convenient to use the LM relation (39) to eliminate real balances from the intertemporal IS equation (44), yielding

\[
x_t = -s_c \left[ (1 + \gamma) (\dot{\pi}_t^n) - E_t \pi_{t+1} \right] + E_t x_{t+1},
\]

where \( \gamma = \Phi (1 + \eta) \) and \( \dot{\pi}_t^n = [s_c (1 + \gamma)]^{-1} \left\{ E_t \ddot{y}^*_{t+1} - \ddot{y}_t^n \right\} \) represents the deviation of the Wicksellian “natural interest rate” from the value consistent with the defined zero-inflation steady state\(^{15}\). Expression (47) can be interpreted as follows. The real balance effect makes even a one-for-one rise in both expected inflation and the nominal interest rate contractionary. This extra-effect on aggregate demand is implied by the reduction in real money demand when an increase in the nominal interest rate occurs.

We now substitute the policy rule (46) into (47), and represent the equilibrium system involving the two endogenous variables \( x_t \) and \( \pi_t \) in the following compact form:

\[
\begin{pmatrix}
  x_t \\
  \pi_t
\end{pmatrix} =
A
\begin{pmatrix}
  E_t \{ x_{t+1} \} \\
  E_t \{ \pi_{t+1} \}
\end{pmatrix} + B \left( \dot{\pi}_t^n - \rho \right), \tag{48}
\]

where

\[
A = \frac{1}{1 + s_c (1 + \gamma) \lambda \phi_x + s_c (1 + \gamma) \phi_x} \left( \begin{array}{cc}
1 & s_c \left[ 1 - \frac{\beta (1 + \gamma) \phi_x}{1 + \Phi} \right] \\
\lambda & s_c \lambda + \frac{\beta (1 + s_c (1 + \gamma) \phi_x)}{1 + \Phi}
\end{array} \right),
\]

and

\[
B = \frac{1}{1 + s_c (1 + \gamma) \lambda \phi_x + s_c (1 + \gamma) \phi_x} \left( s_c (1 + \gamma) \ s_c (1 + \gamma) \lambda \right). \tag{49}
\]

Following Blanchard and Khan (1980), a necessary and sufficient condition for the system (48) to exhibit a unique bounded solution is that the number of non-predetermined endogenous variables equal the number of roots of \( A \) that lie inside the unit circle; otherwise the equilibrium will be indeterminate. Since both \( x_t \) and \( \pi_t \) are free, we can state the following proposition.

**Proposition 1** Let \( \phi_x, \phi_x \geq 0 \), with at least one strictly positive. Under interest rate rules of the form (46) the necessary and sufficient condition for

\(^{15}\)To obtain the natural rate of interest, it should be noted from the Phillips curve that output equals its natural rate \( (x_t = 0) \) at all times if \( \pi_t = 0 \) at all times. Using these paths for inflation and output into the IS relation (after substituting the LM curve), it is straightforward to derive (47) (for this procedure, see Woodford, 2003).
a rational expectations equilibrium to be unique is that

\[ \phi_x + \frac{(1 - \frac{\beta}{1+\Phi})}{\lambda} \phi_x > \frac{1}{1 + \gamma}. \]  

(49)

**Proof.** See Appendix A.

The left-hand side of condition (49) represents the long-run increase in the nominal interest rate prescribed by the policy rule (46) for each unit permanent increase in the inflation rate. It’s clear that condition (49) does not verify what Woodford (2001, 2003) calls the “Taylor principle”: in the event of a permanent one percent rise in inflation, it is not necessary that the cumulative increase in the nominal interest rate be more than one percent. A “passive” monetary policy, that is, a policy that underreacts to inflation by raising the nominal interest rate by less than the observed increase in inflation, is feasible also in the long run, provided that (49) is satisfied. The intuition for this result is as follows. If monetary policy is passive, a deviation of expected inflation from the rational expectations value leads to a decrease in the real interest rate which not necessarily increases the output gap through (47) and then inflation through (45). In fact, the rise in the nominal interest rate implied by the rule (46) increases the opportunity-cost of holding real money balances and therefore tends to reduce aggregate demand through the monetary wealth effect. If condition (49) is satisfied, the real balance effect operates as an “automatic stabilizer”, leading the economy towards the unique rational expectations value.

According to estimates of the rule (46) for the U.S. monetary policy (see, e.g, Clarida, Galí and Gertler, 1999; Taylor, 1999), monetary policy has been significantly “passive” during the pre-Volcker era (1960:1-79:4): Federal Reserve policy tended systematically to accommodate rather than fight increases in inflation. Within the standard “New Keynesian” setup, these estimates suggest that equilibrium was indeterminate in the pre-Volcker regime (see Taylor, 1999; Woodford, 2003). We have shown that the real balance effect implies that the pre-Volcker regime could have been well determinate.

5 Optimal Interest Rate Rules

In this Section, we investigate the implications for optimal monetary policy design. In particular, we discuss optimal interest rate rules, both under

\footnote{See also Bullard and Mitra (2002).}

\footnote{In particular, Taylor (1999) estimates for this period $\phi_\pi = 0.813$ and $\phi_x = 0.252$.}
discretion and under commitment.

In order to introduce a short run trade-off in monetary policy making between output and inflation, we add an exogenous shock, $u_t$, to the aggregate supply curve:

$$\pi_t = \lambda x_t + \frac{\beta}{1+\Phi} E_t \pi_{t+1} + u_t. \quad (50)$$

Following Clarida, Gali and Gertler (1999), the disturbance term can be interpreted as a “cost push shock”, representing anything apart from the output gap that affects marginal costs. We assume that this shock obeys the following stationary first order process:

$$u_t = \nu u_{t-1} + \varepsilon_t, \quad (51)$$

with $0 < \nu < 1$, and where $\varepsilon_t$ is white noise.

The central bank’s objective function is specified as

$$\max -\frac{1}{2} E_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \alpha x_{\tau}^2 + \pi_{\tau}^2 \right] \right\}, \quad (52)$$

where the parameter $\alpha$ measures the relative weight on output deviations.

### 5.1 Optimal discretionary policy

Under discretion, the central bank re-optimizes each period and the optimum is characterized by a “lean against the wind” policy:

$$x_t = -\frac{\lambda}{\alpha} \pi_t. \quad (53)$$

Substituting the above optimality condition into (50) and solving forward yields the following reduced forms for inflation and the output gap:

$$\pi_t = \frac{\alpha}{\lambda^2 + \alpha \left( 1 - \frac{\beta \nu}{1+\Phi} \right)} u_t, \quad (54)$$

$$x_t = -\frac{\lambda}{\lambda^2 + \alpha \left( 1 - \frac{\beta \nu}{1+\Phi} \right)} u_t. \quad (55)$$

Combining the IS relation (47) with the solutions for $\pi_t$ and $x_t$, one obtains the optimal feedback rule for the interest rate:

$$\hat{\pi}_t = \hat{r}_t + \phi^*_\pi E_t \pi_{t+1}, \quad (56)$$
where $\phi_{\pi}^* = \frac{1}{1+\gamma} \left[ 1 + \frac{\lambda(1-\nu)}{\nu s_{\pi} \alpha} \right] \geq 1$. The Taylor principle, according to which a central bank should respond to increases in inflation with a more than one-for-one increase in the nominal interest rate, is not optimal, in general, in our framework. To summarize, we have:

**Proposition 2** Under discretion, it is optimal to implement the Taylor principle if the following condition is satisfied:

$$\gamma < \frac{\lambda(1-\nu)}{\nu s_{\pi} \alpha}.$$  

(57)

This condition is certainly verified in the limiting cases of cashless economies and/or infinite horizon setups, in which $\Phi = 0$ (hence $\gamma = 0$). By contrast, if $\gamma > \frac{\lambda(1-\nu)}{\nu s_{\pi} \alpha}$, the monetary wealth effect described above makes optimal for the central bank to implement a passive monetary policy.

## 5.2 Optimal policy with commitment

Under commitment, the central bank chooses a binding state-contingent rule. In what follows we distinguish between the ”constrained” and the ”unconstrained” commitment.

### 5.2.1 The “constrained” commitment

Consider the case in which the central bank commits itself to conduct monetary policy according to a linear feedback rule on state variables. This approach provides a simple way to clarify the difference of the case of commitment relative to discretion. Specifically, under commitment to a feedback rule of the kind $x_c^t = -\omega u_t$ ($\omega > 0$), it is possible to show that the optimal interest rate is

$$\hat{i}_t = \hat{r}^n_t + \phi_{\pi}^{sc} E_t \pi_{t+1},$$  

(58)

with $\phi_{\pi}^{sc} = \frac{1}{1+\gamma} \left[ 1 + \frac{\lambda(1-\nu)}{\nu s_{\pi} \alpha (1-\beta \nu \frac{1}{1+\Phi})} \right] > \phi_{\pi}^*$. In this case, the central bank optimal policy is to implement a more aggressive response to expected deviations of inflation from target, due to an improved trade-off between output and inflation. Thus we have:

**Proposition 3** Under commitment to a linear policy rule of the kind $x_c^t = -\omega u_t$ ($\omega > 0$), it is optimal to apply the Taylor principle if the following condition holds:

$$\gamma < \frac{\lambda(1-\nu)}{\nu s_{\pi} \alpha \left( 1 - \frac{\beta \nu}{1+\Phi} \right)}.$$  

(59)

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Hence, the main prediction of the model is that the Taylor principle is more likely to be verified under a “constrained” commitment than under discretion. This seems to suggest the idea that the observed instability of the U.S. inflation and real economic activity during the 1970’s could be due not to indeterminacy of rational expectations equilibrium but to an absence of commitment on the part of the Fed, as argued by Chari, Christiano and Eichenbaum (1997).

5.2.2 The “unconstrained” commitment

We now discuss the general solution for the optimal interest rate rule under commitment, which can be derived as follows. In the first stage problem, the central bank chooses a state contingent policy \( \{x_\tau, \pi_\tau\}_{\tau=t}^{\infty} \) in order to maximize (52) subject to the aggregate supply curve (50), which holds in every period \( \tau \geq t \). The solution implies the following optimality conditions:

\[
x_\tau = \frac{1}{1 + \Phi} x_{\tau-1} - \frac{\lambda}{\alpha} \pi_\tau,
\]

(60)

for each \( \tau > t \), and

\[
x_t = -\frac{\lambda}{\alpha} \pi_t.
\]

(61)

Combining (60) with (47), one obtains the optimal interest rate rule:

\[
\hat{i}_t = \hat{r}_t^n + \frac{1}{1 + \gamma} \left[ 1 - \frac{\lambda}{s_c \alpha} \right] E_t \pi_{t+1} + \frac{\lambda \Phi}{s_c \alpha (1 + \gamma) (1 + \Phi)} \pi_t.
\]

(62)

In the limiting cases of a cashless economy and/or infinite horizon (\( \Phi, \gamma = 0 \)), it is well known that this kind of rule involves indeterminacy, since the coefficient associated with expected inflation is less than one\(^{18}\) (see Clarida, Gali and Gertler, 1999; Woodford, 1999). In Appendix B we show that the presence of the real balance effect is not sufficient to make equilibrium determinate.

6 Calibration

In order to evaluate the foregoing results we calibrate the model to quarterly data. The baseline parameter configuration is summarized in Table 1.

\(^{18}\)A further problem is that the optimal plan (60) and (61) is not time consistent, as discussed by Clarida, Gali and Gertler (1999).
We let the steady state share of government spending in GDP be 0.2, which is a conventional estimate of the share of government consumption for the U.S.. The average M2 velocity, $\bar{\bar{P}}\bar{\bar{Y}}/\bar{\bar{M}}$, is set equal to 0.425, consistent with an annual value of 1.7. This is in line with estimates obtained for the U.S. for the period 1960-1995\textsuperscript{19}. We set the steady state real interest rate equal to 0.007 (i.e., 3 per cent per year). Following Leith and Wren-Lewis (2001), the probability of death between two consecutive periods is set equal to 0.015. This parameter can be interpreted as a measure of the individual planning horizon.

The steady state fraction of time in employment is 1/3, according to the standard eight hours working day. We calibrate the probability of keeping the price fixed between two consecutive periods to be 0.85. This is consistent with Gali and Gertler (1999) estimates.

Furthermore, the cost push shock is assumed to have a standard deviation and an autoregressive coefficient equal to 0.07 and 0.7, respectively. The presence of a relatively high degree of persistence is consistent with the estimates of Ireland (2002) obtained for the U.S. during the period 1948-2002.

The remainder of parameters are implied by the steady state relations defined in Section 3.

In terms of condition (49), using the foregoing parameters it is straightforward to find equilibrium determinacy if and only if the nominal interest rate increases more than 0.88 percentage points per percentage point long-run increase in inflation.

In terms of optimal monetary policy, we now investigate how the “sign” of optimal monetary policy (“active” or “passive”) is sensitive with respect to small changes in the parameter values. In particular, we consider the effect of a variation in the parameter reflecting the central bank relative weight on output fluctuations, $\alpha$. Impulse response functions to unit shocks are derived using the toolkit provided by Gerali and Lippi (2003)\textsuperscript{20}.

Figure 1 shows responses to a unit cost push shock in a discretionary monetary policy regime with $\alpha = 0.05$, the value reported by Rotemberg and Woodford (1997). In this case, it is optimal to apply an active monetary policy, since the real interest rate increases on impact. However, setting a value of $\alpha = 1/3$, in line with Broadbent and Barro (1997), one obtains the optimality of a passive monetary policy (Figure 2). Under the previous value


\textsuperscript{20}Thank to the authors, the toolkit is downloadable from their homepages.
of α, Figures 3 and 4 show that in the limit case of infinite horizon (ϑ = 0) and in the case of “constrained” commitment, it is optimal to implement the “Taylor principle”.

7 Conclusions

We have presented a “New Keynesian” setup with the presence of a monetary wealth effect. This kind of extension makes the LM relation non recursive to the equilibrium system. The main finding of the paper is that the micro-founded real balance effect in the IS intertemporal relation has important implications for the design of monetary policy rules. Specifically, we have obtained the following results: (i) the “Taylor principle” relies on the strict assumption of the traditional infinitely-lived representative agent model and disappears as a necessary condition for equilibrium determinacy as soon as we introduce an overlapping generations framework, through a rigorous version of the real balance effect; (ii) this principle is implemented by the central bank more likely under commitment (within a class of linear feedback rules on state variables) rather than under discretion. The view that the pre-Volcker era in the U.S. was not affected by equilibrium indeterminacy but characterized by an absence of commitment on the part of the Fed has therefore sound theoretical micro-foundations.
Appendix A

Proof of Proposition 1

Consider the system (48). The characteristic polynomial of $A$ can be written as

$$P(\xi) = \xi^2 - \text{tr}(A) + \text{det}(A), \quad (A.1)$$

where

$$\text{tr}(A) = \frac{1 + s_c(1 + \gamma)\lambda \phi_\pi + s_c (1 + \gamma) \phi_x}{1 + \frac{s_c(1 + \gamma)\lambda \phi_\pi + s_c (1 + \gamma) \phi_x}{1 + \Phi}}, \quad (A.2)$$

and

$$\text{det}(A) = \frac{s_c\lambda + \frac{\beta(1+s_c(1+\gamma)\phi_x)}{1+\Phi} - s_c\lambda + s_c \frac{\beta(1+\gamma)\phi_\pi}{1+\Phi}}{1 + s_c (1 + \gamma) \lambda \phi_\pi + s_c (1 + \gamma) \phi_x}. \quad (A.3)$$

Conditions for equilibrium determinacy are

$$|\text{det}(A)| < 1, \quad (A.4)$$

$$|\text{tr}(A) - 1| < 1 + \text{det}(A). \quad (A.5)$$

Rearranging (A.3) yields

$$|\text{det}(A)| = \left|\frac{\beta}{1 + \Phi}\right| \frac{1 + s_c (1 + \gamma) \lambda \phi_\pi + s_c (1 + \gamma) \phi_x}{1 + \frac{s_c(1 + \gamma)\lambda \phi_\pi + s_c (1 + \gamma) \phi_x}{1 + \Phi}}. \quad (A.6)$$

Condition (A.4) requires that

$$\frac{\beta}{1 + \Phi} < 1 + s_c (1 + \gamma) \lambda \phi_\pi + s_c (1 + \gamma) \phi_x. \quad (A.7)$$

Given our restrictions about the parameters $\phi_\pi$ and $\phi_x$, this is always verified.

Condition (A.5) requires that

$$\frac{1 + s_c\lambda + \frac{\beta(1+s_c(1+\gamma)\phi_x)}{1+\Phi}}{1 + s_c (1 + \gamma) \lambda \phi_\pi + s_c (1 + \gamma) \phi_x} < 1 + \frac{\beta}{1 + \Phi} \frac{1 + s_c (1 + \gamma) \lambda \phi_\pi + s_c (1 + \gamma) \phi_x}{1 + \frac{s_c(1 + \gamma)\lambda \phi_\pi + s_c (1 + \gamma) \phi_x}{1 + \Phi}}, \quad (A.8)$$

which is satisfied if and only if condition (49) holds.
Appendix B

Equilibrium indeterminacy under the "unconstrained" commitment

Combining (62) with (47) and (50), one can represent the equilibrium dynamics with the system:

\[
\begin{pmatrix}
  x_t \\
p_t
\end{pmatrix} = \mathbf{A}_C \begin{pmatrix}
  E_t \{x_{t+1}\} \\
  E_t \{p_{t+1}\}
\end{pmatrix} + \mathbf{B}_C u_t,
\]

(B.1)

where

\[
\mathbf{A}_C = \frac{1}{1 + \frac{\lambda^2 \Phi}{\alpha(1+\Phi)}} \begin{pmatrix}
  1 + \frac{\lambda}{\alpha} \left[ 1 - \frac{\Phi \beta}{(1+\Phi)^2} \right] \\
  \lambda 
\end{pmatrix}
\]

and

\[
\mathbf{B}_C = \frac{1}{1 + \frac{\lambda^2 \Phi}{\alpha(1+\Phi)}} \begin{pmatrix}
  -\frac{\lambda \Phi}{\alpha(1+\Phi)} & 1
\end{pmatrix}.
\]

We have

\[
\text{tr} (\mathbf{A}_C) = \frac{1 + \frac{\beta}{1+\Phi} + \frac{\lambda^2}{\alpha}}{1 + \frac{\lambda^2 \Phi}{\alpha(1+\Phi)}},
\]

(B.2)

\[
\text{det} (\mathbf{A}_C) = \frac{\beta}{1 + \frac{\lambda^2 \Phi}{\alpha(1+\Phi)}}.
\]

(B.3)

Conditions for equilibrium determinacy are given by (A.4) and (A.5), with \( \mathbf{A} \) replaced by \( \mathbf{A}_C \). Condition (A.4) is always satisfied. Condition (A.5) requires

\[
\frac{1 + \frac{\beta}{1+\Phi} + \frac{\lambda^2}{\alpha}}{1 + \frac{\lambda^2 \Phi}{\alpha(1+\Phi)}} < 1 + \frac{\beta}{1 + \frac{\lambda^2 \Phi}{\alpha(1+\Phi)}},
\]

(B.4)

which is not verified.
References


Table 1 - Baseline Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>Average Ratios</th>
<th>Baseline Parameter Values</th>
<th>Implied Parameter Values</th>
<th>Cost Push Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G/Y = 0.2$</td>
<td>$R = 1.007$</td>
<td>$\chi = 0.02$</td>
<td>$\nu = 0.7$</td>
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<tr>
<td></td>
<td>$C/Y = 0.8$</td>
<td>$\theta = 0.015$</td>
<td>$\beta = 0.994$</td>
<td>$\sigma = 0.07$</td>
</tr>
<tr>
<td></td>
<td>$L = 1/3$</td>
<td>$\phi = 0.85$</td>
<td>$\Phi = 0.001$</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$\gamma = 0.14$</td>
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</tr>
</tbody>
</table>
Figure 1: Inflation, Nominal Interest Rate, Output Gap, and Real Interest Rate Following a Temporary Cost Push Shock: Optimal Monetary Policy under Discretion ($\alpha = 0.05$)
Figure 2: Inflation, Nominal Interest Rate, Output Gap, and Real Interest Rate Following a Temporary Cost Push Shock: Optimal Monetary Policy under Discretion ($\alpha = 1/3$)
Figure 3: Inflation, Nominal Interest Rate, Output Gap, and Real Interest Rate Following a Temporary Cost Push Shock: Optimal Monetary Policy under Discretion ($\alpha = 1/3$) in the Infinite Horizon Limit Case ($\vartheta = 0$)
Figure 4: Inflation, Nominal Interest Rate, Output Gap, and Real Interest Rate Following a Temporary Cost Push Shock: Optimal Monetary Policy under the “Constrained” Commitment ($\alpha = 1/3$)