Global Variance Risk Premium and Forex Return Predictability

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Abstract

In a long-run risk model with stochastic volatility and complete markets, I express expected forex returns as a function of consumption growth variances and equity variance risk premiums (VRPs)—the difference between the risk-neutral and statistical expectations of market return variation. This provides a motivation for using the forward-looking information available in stock market volatility indices to predict forex returns. Empirically, I find that equity VRPs predict forex returns at a one-month horizon, both in-sample and out-of-sample. Moreover, compared to two major currency carry predictors, global VRP has more predictive power for currency carry trade returns, bilateral forex returns, and excess equity return differentials.

Keywords: Global Variance Risk Premium; Excess Foreign Exchange (Forex) Return; Complete Markets; Predictability.

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1 Introduction

Nominal exchange rate movements are difficult to predict out-of-sample (Meese and Rogoff (1983) and Rogoff (2009)), particularly at short horizons (Chinn and Meese (1995) and Rogoff and Stavrakeva (2008)). However, there is no economic reason that nominal exchange rate movements should not have some predictability. Indeed, from a theoretical perspective, equity market information should help to predict forex returns in complete markets (for example see Stulz (1981a and 1981b)). This provides a motivation for studying the predictive power of return predictors across markets; for example, an equity predictor such as the dividend yield has in-sample predictive power for forex returns, and a forex predictor such as the forward premium has in-sample predictive power for equity returns (Bekaert and Hodrick (1992)). This paper studies whether forward-looking information available in stock market volatility indices helps to predict forex returns, in-sample and out-of-sample.

A parsimonious intuition for the predictive power of stock market volatility indices is as follows: a volatility index measures the expectation of stock market volatility over the next period. When a stock market volatility index is significantly higher than the realized stock market volatility, it means that investors expect higher uncertainty over the next period. As a result, domestic agents require higher returns on their domestic and foreign investments. Therefore, domestic stock prices adjust (decrease) to compensate domestic agents to invest in the domestic stock market, and nominal spot exchange-rates adjust (decrease) to compensate them to invest abroad. However, nominal exchange-rates should adjust somehow to compensate both domestic and foreign investors, thus when the expected uncertainty of domestic agents increases (decreases) relative to that of foreign agents, the nominal spot exchange-rates decrease (increase), which predict higher (lower) nominal spot exchange-rates in the future.

This paper finds empirical evidence that the excess foreign exchange (forex) returns are predictable by equity VRPs at the one-month horizon, both in-sample and out-of-sample. This is the main contribution of the paper. In particular, the paper finds that local and global equity VRPs predicts forex returns of major forex returns, namely the pound, the yen, and the euro against the U.S. dollar, with an adjusted R-square of up to 17%. The local equity VRP is measured by the difference between the squared volatility index (e.g., $VIX^2$) and the realized variance of the respective stock index (e.g., $S&P500$) in a country.\footnote{This measure of local VRP is a forward-looking measure of expected stock market volatility risk and may be interpreted as a measure of aggregate economic uncertainty (Drechsler and Yaron (2010)). In theory, the equity VRP is defined as the difference between the risk-neutral and statistical expectations of the market return variation, and Bollerslev, Tauchen, and Zhou (2009) suggest that the equity VRP}
US (in the UK), the expected pound-dollar forex return increases (decreases) by around 5% (3%), and when the global equity VRP increases by one percent, the expected pound-dollar forex return increases by around 3%. The in-sample and out-of-sample R-squares of equity VRP predictions vary up to 17%.

Moreover, I run a “horse race” among three currency predictors—the commodity and the currency volatility factors of Bakshi and Panayotov (2013) and the global VRP—to predict currency carry trade returns, bilateral forex returns, and cross-country equity return differentials. I find first that the global VRP has significantly higher predictive power for the carry trade returns, the bilateral forex returns, and the cross-country equity return differentials. Secondly, I find that the global VRP—which is primarily a equity predictor—has similar predictive power for a bilateral forex returns and the respective cross-country equity return differential, and that the commodity and the currency volatility factors—which are primarily currency predictors—also have similar predictive power for a bilateral forex return and the respective cross-country equity return differential. Documenting the predictive power of these predictors across forex and equity markets is another novel empirical feature of this paper.

To formalize the link between forex returns and equity variance risk premiums, I merge the general currency-pricing model of Bekaert (1996) and Backus, Foresi, and Telmer (2001) and the long-run risk with stochastic volatility model of Bollerslev, Tauchen, and Zhou (2009). As discussed by Bekaert (1996) and Backus et al. (2001), among others, with complete markets the log forex return is equal to the difference between the log of the stochastic discount factor (SDF) variance in the two countries. Therefore, I derive the model-implied log SDF variance in a two-country long-run risk model, which is constructed by merging the aforementioned models for currency pricing and long-run risk. I find that it has two components: consumption growth variance and the variance risk premium. By substituting them in the model of Bekaert (1996) and Backus et al. (2001), I find that the expected forex returns are a function of the consumption growth variances and equity VRPs. The theoretical relation between expected forex returns and the equity VRPs (the global and country-specific VRPs) is a novel result in this paper.

A theoretical explanation for the predictive power of the global and country-specific VRP

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Footnotes:

1 First, this paper is not providing a structural model of real exchange rates. In fact, this paper simply provide a long-run risk intuition behind the main empirical findings of the paper, which is comparable with the theoretical long-run risk models, namely the models in Bansal and Shaliastovich (2013) and Colacito and Croce (2011).

2 Burnside and Graveline (2012) suggest that in the absence of arbitrage opportunities, the log-change in the real exchange rate in a structural model should always equal to the log difference between a stochastic discount factor (SDF) denominated in the real currency units of the two countries. The structural models should separate between the SDFs of distinct agents in two countries and the a SDF denominated in the real currency units of two countries. However, the purpose of this paper is not providing a structural model of real exchange rates. In fact, this paper simply provide a long-run risk intuition behind the main empirical findings of the paper, which is comparable with the theoretical long-run risk models, namely the models in Bansal and Shaliastovich (2013) and Colacito and Croce (2011).

3 Isolate the factor associated with the volatility of consumption growth volatility.
VRPs is that they measure agents’ perceptions of aggregate uncertainty and major shocks to the economic state variables globally and domestically, respectively. In theory, time-varying uncertainty in an economy with agents who have a risk aversion greater than one and Epstein–Zin preferences for early resolution of the uncertainty generate time-varying VRPs (Drechsler and Yaron (2011); Bollerslev et al. (2014)). Moreover, heterogeneity in exposure to the global VRP, specifically the high exposure of the low-interest-rate countries and the low exposure of the high-interest-rate countries, generates a currency risk premium. In a complete market with complete home bias and under lognormality, the domestic agent with a higher exposure to the global VRP is exposed to more uncertainty in periods where global uncertainty is high. As a result, the equilibrium domestic currency appreciates (or the foreign currency depreciates) to compensate the domestic agent for investing in the foreign bonds (as the domestic agent requires higher returns). At the same time, the higher uncertainty pushes down the domestic interest rate. The lower spot exchange rate and the lower domestic interest rate predict a higher excess forex return.

To support the common predictive power of the global equity VRP across markets (and also to motivate the “horse race” among the predictors of different financial assets), I derive the model-implied cross-country equity return differentials and model-implied interest rate in complete markets. I find that they are driven by consumption growth variances and equity VRPs (global and local equity VRPs) as well. This commonality in driving factors supports the global risk interpretation of the global VRP. In addition, it provides an explanation for the high degree of contemporaneous comovements between cross-country equity premium differentials and forex returns documented in Hau and Rey (2006), the predictive power of dividend yield—an equity return predictor—for forex returns documented in Bekaert and Hodrick (1992), the predictive power of forward premium—a forex return predictor—for excess equity returns documented in Bekaert and Hodrick (1992). It also explain the predictive power of global equity VRP for currency returns and the predictive power currency predictors for equity-return differentials documented in this paper.

Several theoretical papers studies real exchange-rate movements in two-country long-run risk models (see for example, Bansal and Shaliastovich (2007)). However, the correlation between the consumption growth variance differential and exchange-rate movements is low in data (Backus and Smith (1993)). As a result, Bansal and Shaliastovich (2013) and Colacito and Croce (2011) augment the consumption process in a two-country long-run risk model to better explain the observed exchange rate movements. Colacito and Croce (2011) develop a two-country long-run risk model with constant volatility to explain the smoothness of exchange rate movements with a persistent long-run component of consumption growth that is highly correlated across countries. Bansal and Shaliastovich
(2013) introduce an inflation-augmented two-country long-run risk model with stochastic volatility where the risk premiums are driven by the volatilities of expected growth and expected inflation. In particular, they use average forecasts of one-year-ahead real GDP growth and inflation from the Survey of Professional Forecasters to estimate the conditional volatilities of expected growth and inflation. This paper augments the consumption process in a two-country long-run risk model with stochastic volatility, where the risk premiums are driven by variance risk.

In addition, Bansal and Shaliastovich (2007 and 2013) and Colacito and Croce (2011) assume complete markets where agents have the same preference specifications across countries. I also assume complete markets, but consider the case where agents have different preference specifications across countries. Moreover, I assume that consumption in each country is driven by a global factor and an orthogonal country-specific factor. As the agents have different exposures to the global factor in the economy, I show that expected forex returns are a function of the global and country-specific components of both consumption growth variance and VRP. On the one hand, consumption growth variance has little or no predictive power for bilateral forex returns, and adding it to the predicting regressions does not change the predictive power of the global and country-specific VRPs. On the other hand, this model simplifies to the model of Bansal and Shaliastovich (2007) when agents have the same preferences and VRPs are equal across countries.

There is a growing empirical literature studying the predictive power of VRPs for different financial assets. Bollerslev et al. (2009), Drechsler and Yaron (2011), and Bekaert and Hoerova (2014) find a strong relation between VRPs and excess equity returns. Bollerslev et al. (2014) find that the global VRP is even a better predictor for local excess equity returns than the local VRPs. Zhou (2009) and Mueller et al. (2011) find a strong relation between the VRP and excess bond returns in the United States. In particular, there is an emerging literature investigating the predictive power of equity and currency VRPs for forex returns. Della Corte, Ramadorai, and Sarno (2014) study the cross-sectional link between currency VRPs and currency returns. Londono and Zhou (2012) investigate the time-series predictive power of currency VRPs for forex returns, and they show that currency VRP is related to currency-specific risks and that adding currency VRPs to the U.S. equity VRP increases the predictability of forex returns. Although others focus on currency VRPs, I examine equity VRPs instead, which has

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3Backus, Gavazzoni, Tehmer, and Zin (2010) also assume asymmetric economies, where central banks react differently (more or less) to the exchange rates. There are several empirical and theoretical papers that support such asymmetries, for example, Benigno (2004), Eichenbaum and Evans (1995), and Engel and West (2006).

4The currency VRP is defined as the risk-neutral and statistical expectations of the future currency return variation.
some advantages from a perspective of complete markets. To the best of my knowledge, the current paper is the first to study and document the predictive power of equity VRPs (and particularly the global equity VRP) for forex returns.

Finally, I perform a number of robustness checks. I find that the global and the country-specific VRPs have predictive power when using Seemingly Unrelated Regressions (SURs), when including global and country-specific consumption growth variances in the regressions, and more importantly when excluding the 2008 crisis months from the sample. Moreover, the global VRP and the U.S.-specific VRP are strong predictors of forex returns for the majority of the other 18 currencies with respect to the U.S. dollar even when their local VRPs are unavailable.

The rest of the paper is organized as follows. Section 2 presents a two-country model in complete markets with stochastic volatilities. I describe the data in section 3 and the empirical findings and the robustness checks in section 4. I compare the predictive power of the global VRP with two currency return predictors across equity and forex markets in section 5. Then, I evaluate the out-of-sample performance of the equity VRPs in section 6. Section 7 concludes.

2 Complete Markets with Stochastic Volatilities: A Two-Country Model

2.1 Preferences

I assume that the representative agents in the economy have the following Epstein and Zin (1989) preferences:

\[ U_t = \left\{ (1 - \delta)C_t^{1+\gamma} + \delta E_t[U_{t+1}^{1-\gamma}] \frac{1}{1+\gamma} \right\} \]

where \( C_t \) is aggregate consumption at time \( t \), \( \delta \) denotes the time discount factor, \( \gamma \geq 0 \) refers to the coefficient of risk aversion, \( \psi \geq 0 \) is the intertemporal elasticity of substitution, and \( \theta \equiv \frac{(1-\gamma)}{1-1/\psi} \). As shown by Epstein and Zin (1991), these preferences imply that the logarithm of the stochastic discount factor (SDF), \( m_{t+1} \), depends on both the log aggregate consumption growth rate, \( g_{t+1} \), and the return from time \( t \) to \( t+1 \) on the asset that delivers aggregate consumption, \( r_{t+1} \):

\[ m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1)r_{t+1} \]

(2)
2.2 The Consumption Process

The consumption asset return is not observable. However, to express the SDF in Equation 2 in terms of state variables and aggregate shocks, I specify consumption exogenously. This is similar to Bansal and Yaron (2004), Bansal and Shaliastovich (2007 and 2013), and Colacito and Croce (2011) and this assumption helps to solve for the equilibrium real value of financial assets. A complete home bias—both representative agents are willing to consume only the goods with which they are endowed—suppresses the motivation to trade endowments across borders in this economy. Moreover, to incorporate stochastic volatility in the equilibrium asset prices, as in Bollerslev et al. (2009), I assume that the geometric growth rate of consumption, \( g_{t+1} = \log(c_{t+1}/c_t) \), and the consumption growth volatility, \( \sigma_{g,t+1} \), have the following joint process

\[
\begin{align*}
    g_{t+1} &= \mu_g + \sigma_{g,t} z_{g,t+1}, \\
    \sigma_{g,t+1}^2 &= a_\sigma + \rho_\sigma \sigma_{g,t}^2 + \sqrt{q_t} \sigma_{\sigma,t+1}, \\
    q_{t+1} &= a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1},
\end{align*}
\]

where \( \mu_g > 0 \) represents the constant mean growth rate, \( \sigma_{g,t}^2 \) denotes the time-varying variance in consumption growth with volatility-of-volatility in the consumption growth process, \( q_t \). The parameters satisfy \( a_\sigma > 0, a_q > 0, |\rho_\sigma| < 1, |\rho_q| < 1, \) and \( \varphi_q > 0; \) and \( \{z_{g,t}\}, \{z_{q,t}\}, \{z_{\sigma,t}\} \) are iid Normal(0,1) processes jointly independent from each other.\(^5\)

2.3 SDF in A Single Country

The return on the exogenous consumption assets, using standard Campbell and Shiller (1988) log-linearization, is given by\(^6\)

\[
    r_{t+1} = -\log\delta + \frac{\mu_g}{\psi} - \frac{(1 - \gamma)^2}{(2\theta)} \sigma_{g,t}^2 + (\kappa_1 \rho_q - 1) A_q q_t + \sigma_{g,t} z_{g,t+1} + \kappa_1 \sqrt{q_t} [A_\sigma z_{\sigma,t+1} + A_q \varphi_q z_{q,t+1}],
\]

where

\(^5\)Please see the details of the global and country-specific consumption processes behind Equation 22 in the Appendix B.

\(^6\)See Bollerslev et al. (2009) for the details.
\[ A_\sigma = \frac{(1-\gamma)^2}{2\theta(1-\kappa_1 \rho_\sigma)}, \quad A_q = \frac{1-\kappa_1 \rho_q - \sqrt{(1-\kappa_1 \rho_q)^2 - \theta^2 \kappa_1^4 \psi^2 A_\sigma^2}}{\theta \kappa_1^2 \psi^2} . \] (7)

As \( \kappa_1 > 0, \gamma > 1, \) and \( \psi > 1, \) both impact coefficients associated with the volatility state variables are negative, i.e., \( A_\sigma < 0 \) and \( A_q < 0. \) Therefore, the return on the consumption asset increases by both higher short-term consumption growth volatility \( \sigma_{g,t}, \) or higher consumption growth vol-of-vol \( q_t \) to compensate representative agents who bear these risks.

To find the equilibrium market prices of consumption and volatility risks, we need to express the conditional mean of the log SDF and its innovations as a function of the state variables:

\[ m_{t+1} = \log \delta - \frac{\mu_g}{\psi} + (\theta - 1 - \frac{\theta}{\psi}) \sigma_{g,t} z_{g,t+1} - \frac{(\theta - 1)(1-\gamma)^2}{2\theta} \sigma_{g,t}^2 + (\theta - 1)(\kappa_1 \rho_q - 1) A_q q_t + (\theta - 1) \kappa_1 \sqrt{q_t} [A_\sigma z_{g,t+1} + A_q \psi q_{q,t+1}] , \] (8)

\[ m_{t+1} - E_t[m_{t+1}] = -\gamma \sigma_{g,t} z_{g,t+1} - (1-\theta) \kappa_1 \sqrt{q_t} [A_\sigma z_{g,t+1} + A_q \psi q_{q,t+1}] . \] (9)

The equilibrium market prices of risks are equal to

\[ \lambda_c = \gamma, \quad \lambda_q = (1-\theta) \kappa_1 [A_\sigma z_{g,t+1} + A_q \psi q_{q,t+1}] . \] (10)

The price of consumption growth risks, \( \lambda_c, \) is equal to the risk-aversion coefficient \( \gamma, \) which is exactly equal to the price of short-run consumption risks in Bansal and Shaliastovich (2013) and Colacito and Croce (2011). When agents have preferences for early resolution of uncertainty (\( \gamma > \frac{1}{\psi} \) or \( 1 > \theta \)), the market price of consumption growth volatility risk, \( \lambda_q, \) is positive.

The priced risk factors that appear in Equation 9 are a result of augmenting the consumption growth risk by the consumption growth volatility risk. However, Bansal and Shaliastovich (2013) and Colacito and Croce (2011) augment the consumption growth risk by different measures of long-run consumption growth risk. Colacito and Croce (2011) use the predictable components of consumption growth rates based on information from domestic financial markets. In particular, these include financial variables such as lagged price-dividend ratio, consumption-output ratios, risk-free rates, lagged consumption growth, and default premiums. Bansal and Shaliastovich (2013) use average forecasts of one-year-ahead real GDP growth from the Survey of Professional Forecasters.

I use equity variance risk premiums (VRPs)—the difference between option-implied and
realized stock market variance—to estimate the consumption growth volatility risk.

Furthermore, I construct a global measure of consumption growth volatility risk factors by averaging local VRPs. This is because in my model, the preference parameters are not equal across countries, which is in contrast to the preference specifications in Bansal and Shaliastovich (2013) and Colacito and Croce (2011). In addition, I assume that consumption in different countries has a common factor. These two assumptions help me to explore a common factor to which the representative agents are heterogeneously exposed. I use a set of local VRPs to construct a global VRP, which mimics the common factor. In other words, I use information available in the set of local equity VRPs to predict their bilateral currency returns. The conditional mean of the log SDF and its innovations, as a function of such global state variables, is presented in the appendix.

2.4 SDFs in Two Countries: Currency Pricing

As noted by Bekaert (1996) and Backus, Foresi, and Telmer (2001), among others, in complete markets with lognormal marginal utilities, the law of one price implies that the relative value of the marginal utility of foreign currency versus domestic currency determines the associated exchange rate. In other words, the logarithm of exchange rate changes is equal to the difference between the logarithms of stochastic discount factors in the two countries;

\[ s_{ij,t+1} - s_{ij,t} = m_{t+1}^j - m_{t+1}^i, \]

where the preference specifications in domestic country i are analogous in the foreign country j. \( m_{t+1}^i \) is the log SDF in the domestic country i, \( m_{t+1}^j \) is the log SDF in the foreign country j, and \( s_{ij,t} \) is the domestic spot price of one unit of foreign currency.

A one-period forex return is given by:

\[ r_{x_{ij,t+1}} = s_{ij,t+1} - s_{ij,t} - r_{f_t}^i + r_{f_t}^j. \]

The variable \( r_{f_t} \) is the risk-free rate at time t. Lustig et al. (2011) measure \( r_x \) as the log excess return on buying a foreign currency in the forward market and then selling it in the spot market in the next period.

Bekaert (1996) and Backus et al. (2001) show that taking expectations of both sides of Equation 12, and substituting \(-\frac{1}{2} \text{var}[m_{t+1}] \) for \( \text{E}[m_{t+1}] + r_{f_t} \), the expected forex returns
will be a function of the difference of the variance of log SDFs in the two countries:

\[
E\left[ r_{x_{ij,t+1}}^f \right] = E\left[ s_{ij,t+1} - s_{ij,t} - r f^i_t + r f^j_t \right]
\]

(13)

\[
E\left[ r_{x_{ij,t+1}}^f \right] = E\left[ m_{i+1}^j - m_{i+1}^i - r f^i_t + r f^j_t \right]
\]

(14)

\[
E\left[ i, j, r_{x_{i,t+1}}^f \right] = \frac{1}{2} \text{var}[m_{i+1}^j] - \frac{1}{2} \text{var}[m_{i+1}^i].
\]

(15)

Therefore, a shock that increases the variance of log SDF in the domestic country relative to that in the foreign country increases the expected excess foreign exchange return.\(^7\)

Under the aforementioned assumptions, I calculate the variance of log SDFs in country i and j as a function of state variables as follows (see the details of derivations in Appendix A),

\[
\text{var}[m_{i+1}^j] = \gamma_{ij}^2 [\sigma_{i,g,t}^2] + (\theta - 1)^2 \kappa_{i,1}^2 (A_{\sigma_i}^2 + A_{i,q}^2 \varphi_{i,q}^2)[q_{i,t}].
\]

(16)

\[
\text{var}[m_{i+1}^j] = \gamma_{j}^2 [\sigma_{j,g,t}^2] + (\theta - 1)^2 \kappa_{j,1}^2 (A_{\sigma_j}^2 + A_{j,q}^2 \varphi_{j,q}^2)[q_{j,t}].
\]

(17)

By substituting Equations 16 and 17 in Equation 15,

\[
E\left[ r_{x_{i,t+1}}^f \right] = \frac{1}{2} \left( \gamma_{i}^2 [\sigma_{i,g,t}^2] - \gamma_{j}^2 [\sigma_{j,g,t}^2] \right)
\]

\[-\frac{1}{2} ((\theta - 1)^2 \kappa_{j,1}^2 (A_{\sigma_j}^2 + A_{j,q}^2 \varphi_{j,q}^2)[q_{j,t}] - (\theta - 1)^2 \kappa_{i,1}^2 (A_{\sigma_i}^2 + A_{i,q}^2 \varphi_{i,q}^2)[q_{i,t}]).
\]

(18)

Equations 16 and 17 show that the variance of log SDF has two components: the consumption growth variance and the consumption growth volatility-of-volatility. A positive shock to either of the volatilities increases the variance of log SDF. In other words, when the domestic consumption growth volatility relatively increases, the equilibrium domestic price of foreign currency drops immediately or the foreign currency depreciates. Thus, relative to the new equilibrium price today, we expect the foreign currency to appreciate tomorrow, as shown in Equation 18. This is similar to the model implications for short-run consumption volatility in Bansal and Shaliastovich (2007 and 2013) and Colacito and

\(^7\)Note that the variance of spot exchange rate changes is \(\text{var}[s_{ij,t+1} - s_{ij,t}] = \text{var}[m_{i+1}^j - m_{i+1}^i].\)

Hence,

\[
\text{var}[s_{ij,t+1} - s_{ij,t}] = \text{var}[m_{i+1}^j] + \text{var}[m_{i+1}^i] - 2 \text{cov}[m_{i+1}^j, m_{i+1}^i].
\]

In other words, the variance of the spot rate changes is a function of "sum" of variance of log SDFs in two countries and their covariance, while the expected forex returns are a function of just the "difference" of variance of log SDFs in two countries. Therefore, we cannot find a direct relation between expected forex returns and the VRP in the exchange market using the variance of spot exchange rates. This is also similar to the argument in Backus et al. (2001) that explains the failure of GARCH-M models.
Equation 18 also suggests that when the domestic consumption growth volatility-of-volatility relatively increases, the equilibrium domestic price of foreign currency drops and the expected forex return increases. This is in line with the expected long-run consumption growth risk in Bansal and Shaliastovich (2013) and Colacito and Croce (2011). However, I use the expected consumption growth volatility-of-volatility instead of the expected long-run consumption growth risk.

2.5 Variance Risk Premiums

Bollerslev et al. (2009) show that the equity VRP, which is the difference between the risk-neutral and realized variances of stock market returns \( (E^Q_t(\sigma^2_{t+1}) - E_t(\sigma^2_{t+1})) \), proxies the factor associated with the consumption growth volatility-of-volatility, \( q_t \);

\[
(\theta - 1)\kappa_1 (A_\sigma + A_q \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2) \varphi_q^2) [q_t] \simeq \text{[VRP]}_t. \tag{19}
\]

By substituting this statement in Equation 16, the variance of log SDF is a function of the consumption growth variance and VRP,

\[
\text{var}[m_{t+1}] = \gamma^2 [\sigma^2_{g,t}] + \lambda [\text{VRP}_t], \tag{20}
\]

where \( \lambda = \frac{(\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2)}{(\theta - 1) \kappa_1 (A_\sigma + A_q \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2) \varphi_q^2)} > 0 \). Finally, if we substitute Equation 20 into Equation 15, we get:

\[
E[rx^f_{ij,t+1}] = \frac{1}{2} \{ \gamma_i^2 [\sigma^2_{g,i,t}] + \lambda_i [\text{VRP}_i,t] \} - \frac{1}{2} \{ \gamma_j^2 [\sigma^2_{g,j,t}] + \lambda_j [\text{VRP}_j,t] \}
\]

\[
E[rx^f_{ij,t+1}] = \frac{1}{2} \{ \gamma_i^2 [\sigma^2_{g,i,t}] - \gamma_j^2 [\sigma^2_{g,j,t}] \} + \frac{1}{2} \{ \lambda_i [\text{VRP}_i,t] - \lambda_j [\text{VRP}_j,t] \} \tag{21}
\]

where \( rx^f_{ij,t+1} \) is the country i vis-à-vis country j forex return (i=domestic and j=foreign), \( \sigma^2_{g,i,t} \) is consumption growth volatility in country i, and \( \text{VRP}_i,t \) is the local VRP in country i. Equation 21 says that the forex return has two components. The first component is a function of the difference in consumption growth variances across countries, and the second component is a function of the difference in VRPs across countries.\(^9\)

\(^8\)Having \( \gamma > 1 \) and \( \psi > 1 \) implies \( \theta < 0 \). In addition, impact coefficients associated with both of the volatility state variables are negative (\( A_\sigma < 0 \) and \( A_q < 0 \)) and \( \kappa_1 > 0 \); these imply that \( \lambda > 0 \).

\(^9\)Several studies find the consumption growth variance differential is not a strong predictor of excess equity or bond returns; see Hasseltoft (2012), Constantinides and Ghosh (2011), Beeler and Campbell (2009), and Lauterbach (1989). Bansal and Shaliastovich (2007) and Backus and Smith (1993) document low correlations between consumption growth variance differentials and the forex returns in the data.
2.6 Decomposing Global and Country-Specific Components

Equation 21 is derived in an economy without a common factor. If we assume an economy in which consumption in each country is driven by a global factor and an orthogonal country-specific factor, we can decompose the global and the country-specific components of both the consumption growth volatility and the VRP. Then, we can express the expected forex return as a function of the global and the country-specific components (see Appendix B for the decompositions and the derivations):

\[
E[r_{x_{ij,t+1}^f}] = \frac{1}{2}(\gamma^g_{i}\tau^g_i - \gamma^g_{j}\tau^g_j)[\sigma^g_{g,t}] + \frac{1}{2}\gamma^{s^i}_{i}[\sigma^s_{i,t}] - \frac{1}{2}\gamma^{s^j}_{j}[\sigma^s_{j,t}]
\]

\[
+ \frac{1}{2}(\lambda^g_{i}\tau^r_i - \lambda^g_{j}\tau^r_j)[VRP^g_{t}] + \frac{1}{2}\lambda^{s^i}_{i}[VRP^{s^i}_{t}] - \frac{1}{2}\lambda^{s^j}_{j}[VRP^{s^j}_{t}].
\] (22)

\(\sigma^g_{g,t}\) is the global consumption growth volatility, \(\sigma^{s^i}_{i}\) is country i-specific consumption growth volatility, \(VRP^g_{t}\) is the global VRP, \(VRP^{s^i}_{t}\) is country i-specific VRP, \(\tau^g_i\) is country i exposure to global consumption growth volatility, and \(\tau^r_i\) is country i exposure to global consumption asset returns.

Equations 21 and 22 provide a motivation for the empirical predictability tests conducted in section 4.

3 Data and Summary Statistics

**Currency and Interest Rates.** The data is from Barclays and Reuters and available via Datastream. I build end-of-month series from December 1999 to December 2011.\(^{10}\) My main focus is on the following major currencies against the U.S. dollar: the euro (EUR), Japanese yen (JPY), and British sterling (GBP). In addition, 18 other countries’ currencies against the U.S. dollar are used for the robustness checks: Australia (AUD), Canada (CAD), the Czech Republic (CZK), Hungary (HUF), Indonesia (IDR), India (INR), Kuwait (KWD), Malaysia (MYR), Mexico (MXN), New Zealand (NZD), Norway (NOK), Poland (PLN), Philippines (PHP), Singapore (SGD), South Korea (KRW), Sweden (SEK), Switzerland (CHF), and Taiwan (TWD). I proxy interest rates by the one-month interbank interest rates (Libor).

**Variance Risk Premiums.** The local variance risk premium in country i, \(VRP^i_{t}\), is measured as the difference between the end-of-month risk-neutral expected variance and the end-of-month realized variance in country i, \(VRP^i_{t} \equiv IV^i_{t} - RV^i_{t}\). The monthly local VRP series are calculated for four major countries: the US, UK, Japan, and Europe.\(^{10}\)

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\(^{10}\)The sample period is limited to the availability of implied equity volatility data.
I construct the end-of-month realized variances $RV_i^t$ as the sum of the daily squared returns for the S&P 500 (US), the FTSE 100 (UK), the Nikkei 225 (Japan), and the DAX (Europe/Germany) from the beginning to the end of month $t$. The corresponding monthly model-free implied volatility indices $(IV_i^t)^{1/2}$ are the end-of-month VIX, VFTSE, VXJ, and VDAX. The volatility indices are available at the one-month horizon and thus I test only predictability at this horizon.\textsuperscript{11} Then the “global” variance risk premium is constructed as the end-of-the-previous-month market capitalization ($w_{t-1}$) weighted average of the local VRPs, as in Bollerslev \textit{et al.} (2014).\textsuperscript{12}

**Consumption Growth Volatility.** I proxy non-durable consumption by the OECD retail sales data, as in Lustig and Nieuwerburgh (2010), Østergaard, Sørensen, and Yosha (2002), Hass and Shin (2006 and 1998), Del Negro (2002 and 1998), and Wilcox (1989). Following Bansal, Khatchatrian, and Yaron (2005), I construct consumption growth volatility measures as a 2-year sum of absolute residuals from AR(3) projections of monthly consumption (retail sale) growth rates.\textsuperscript{13}

Tables 1 and 2 present the summary statistics of the data. The mean of monthly interest rates in the US and Europe are similar (around 3%); this mean is low in Japan (slightly above 0%) and high in the UK (around 4%). According to the carry trade literature, agents should borrow in yen and invest in the dollars, euros, and pounds, or they should borrow in yen, euros, and dollars and invest in the pound.

The US VRP and the UK VRP have means of around 1 percent-squared, standard deviations of above 1 percent, and autocorrelations higher than 30%. The Europe VRP has a small mean of 0.28 percent-squared. The Japan VRP has the highest standard deviation of around monthly 1.3 basis points, with a small autocorrelation of one percent.

Table 3 provides the monthly correlations between forex returns and VRPs in the US, UK, Japan, and Europe, as well as the global VRP. The US VRP is highly correlated with the UK VRP (88%), and less correlated with Japan’s VRP (57%). A high degree of cross-country correlation between local VRPs is consistent with the high cross-country correlation between expected consumption growth factors documented in Bansal and Shaliastovich (2013) and Colacito and Croce (2011). The global VRP is highly correlated with the US VRP and the UK VRP (both more than 90%), and less correlated with Japan’s VRP (60%). From Equation 21, we expect forex returns to be positively corre-

\textsuperscript{11}The model-free volatility indices are limited to VIX (since January 1990), VFTSE (since January 2000), VXJ (since January 1998), and VDAX (since January 1999), as well as France (VCAC) and Switzerland (VSMI). Although the Swiss volatility index is available, I focus on the largest capital markets and high-trading-volume currencies; namely those in the US, UK, Japan, and Europe.

\textsuperscript{12}An alternative measure of the global VRP is an equal-weighted average of local VRPs. The empirical results of the alternative measure of global VRP are similar to those reported in the paper. The results are available on request.

\textsuperscript{13}Therefore the retail sales data is collected from September 1997 to December 2011.
lated with the US VRP and negatively correlated with foreign VRPs. The EUR/USD forex return is positively correlated with the US VRP and is negatively correlated with Europe VRP, which are both in line with our expectation. However, some of correlations between VRPs and expected forex returns are partially inconsistent with Equation 21. For example, the GBP/USD forex return is positively correlated with the UK VRP, and the JPY/USD forex return is also negatively correlated with the US VRP, which are both in contrast to our expectation.

The observed inconsistency in the correlations between VRPs and forex returns might be due to common correlations between VRPs or the global VRP. In the last column of Table 3, the global VRP is positively correlated with the GBP/USD forex return (37%), which is consistent with lower exposure of the GBP (which has higher interest rates compared to the USD). The global VRP is negatively correlated with the JPY/USD forex return (-17%), which is consistent with high exposure of the JPY (which has lower interest rates compared to the USD). The global VRP is less correlated with the EUR/USD forex return (14%). This is consistent with the fact that the US and Europe have similar mean interest rates.

4 Main Empirical Findings

In section 2, the units for excess currency returns and SDFs are real. However, it is customary to explain the observed nominal currency values and their dynamics by a theoretical model of real exchange rates (for example, see Bansal and Shaliastovich (2007 and 2013) and Colacito and Croce (2011)).\footnote{Bansal and Shaliastovich (2007 and 2013) compute the real excess currency return. However, it is still different from the real exchange rate changes, which are not actually investable. Moreover, as there is no monthly survey data for expected inflation in different countries, I simply investigate the nominal excess currency return.} Therefore the real exchange rate equations mentioned above provide a motivation for my empirical tests of nominal forex returns in this section.

4.1 VRPs and its Components in the Data

Equation 21 provides a motivation for using local equity VRPs to predict forex returns, and thus I use the the following specification:\footnote{Consistent with the VRP literature, the consumption growth volatility component is excluded. But I will check for its relevance in the robustness checks to follow.}

\[ r_{x_{t+1}}^{f} = \alpha + \zeta_{VRP}^{US} \times VRP_{US,t} + \zeta_{VRP}^{j} \times VRP_{j,t} + \varepsilon_{t}, \]  

(23)
where $VRP_{US}$ is the local VRP in the US and $VRP_j$ is the local VRP in country $j$ (UK, Japan, and Europe).

The top panel of Table 4 shows the relation between local VRPs at time $t$ and the realized forex return at time $t+1$ during the period from January 2000 to December 2011. The GBP/USD and EUR/USD forex returns are significantly predictable, with adjusted $R^2$s of 17% and 4% respectively. Moreover, their coefficients have the correct signs: positive for domestic local VRP (US VRP) and negative for foreign local VRPs (UK VRP and Europe VRP respectively). The JPY/USD forex is not predictable using the VRPs, and the coefficient of the US VRP is negative, which goes counter to the model predictions. According to Equation 21, the coefficient $\zeta_{VRP}^s$ in specification 23 should be identical for all regressions of the forex returns of a given base currency (U.S. dollar) with respect to all other currencies. Hence, in Table 4 the coefficients for US VRP should be identical in all three regressions.

A possible explanation for the negative coefficient of US VRP in the JPY/USD regression is that there is a common correlation (factor) between local VRPs, and the JPY/USD forex return is negatively correlated with that common factor. In other words, Japanese investors on average are more exposed to the global factor compared to the US investors, and therefore the coefficient of the global VRP in Equation 22 is negative.\footnote{In Appendix D, I show the relation between the model-implied interest rate and the global VRP. There, the exposure to the global VRP is negatively related to the level of interest rates. This may explain why Japanese investors are more exposed to global VRP.}

Equation 22 provides a motivation for using global and country-specific VRPs to predict forex returns, and thus I use the following specification:

$$r_{f,t+1} = \alpha + \zeta_{VRP}^s \times VRP_{US,t} + \zeta_{VRP}^s \times VRP_{j,t} + \delta_{GVRP} \times GVRP_t + \varepsilon_t,$$

(24)

where $VRP_{US,t}$ is the US-specific VRP, $VRP_{j,t}$ is the foreign country $j$-specific VRP, and $GVRP_t$ is a value weighted average of the local VRPs, which is similar to the global VRP in Bollerslev et al. (2011):

$$GVRP_t = \sum_k w_{k,t} VRP_{k,t},$$

(25)

Here, $k$ refers to the US, UK, Japan, and Europe, and $w_{k,t}$ is end-of-(last) month market capitalization of country $k$.

Since I assume that the global VRP and the country-specific VRPs are uncorrelated, the country-specific variance risk premiums are estimated using the following regression:

$$VRP_{k,t} = \alpha_k + \rho_k \times GVRP_t + \xi_{k,t}$$

(26)
where $VRP_{k,t}$ is the local VRP in country k and the country-specific VRP is the orthogonal error term, $(VRP^*_{k,t} = \xi_{k,t})$.

Then, the estimated global and country-specific VRPs are used in the regressions reported in the bottom panel of Table 4. The coefficient of the US-specific VRP is nearly identical for the pound and the euro as predicted by Equation 22, and it is significant for the euro at the 5% level. The coefficients of the global VRP are significant at the 1% level with correct sign for two currency pairs, GBP/USD and JPY/USD. It is not significant for EUR/USD. Table 4 is an important table of the paper and it is a reference for the rest of my empirical investigations.

4.2 SURs

One econometric consequence of excluding the consumption growth variance components is that the error terms of the regressions may be correlated across the equations. According to Zellner (1962), in such cases, seemingly unrelated regression (SUR) is a more efficient procedure. Furthermore, Equations 21 and 22 imply that the coefficient of the US VRP and US-specific VRP should be the same across the regressions, and SUR enables us to include such constraints. Therefore, I repeat the tests with similar explanatory variables in a SUR setup and report them in Table 5. Panels 1 and 3 of Table 5 are respectively without and with constraints on the coefficients of the US VRP to be equal across regressions, and both panels are directly comparable to the top panel of Table 4. In panel 1, the coefficient of US VRP $(\zeta^U_{VRP})$ for JPY/USD is significant at the 10% level but its sign is counter model predictions. In panel 3, the sign of the US VRP coefficients $(\zeta^U_{VRP})$ in all three pairs, as well as the sign of the Japan VRP coefficient, are consistent with model predictions. Compared to the panel 1, the magnitude of the local VRP coefficients changes dramatically and the overall fits of the model dramatically decrease in all three pairs in panel 3.

Panels 2 and 4 of Table 5 are respectively without and with constraints on the coefficients of US-specific VRP, and both panels are directly comparable to the bottom panel of Table 4. The signs of the foreign country-specific VRP coefficients $(\zeta^U_{VRP})$ are consistent with the model predictions in all three pairs in panel 2. The significance of the global VRP coefficient in EUR/USD is increased to the 10% level in panels 2 and 4. Moreover, the magnitude of the coefficients of the global VRP is almost unchanged. The overall fits of the model (R-squares) are similar for all pairs in panel 2 and 4. In sum, compared to the Table 4, the global VRP is the only variable that has similar predictability power in the OLS and SUR regressions.

4.3 Robustness Checks
4.3.1 Missing Variables

In the VRP literature, it is customary to exclude the consumption growth variance (CGV) component. The main reason is that monthly consumption data is not usually available. In this section, I use retail sales data as a proxy for non-durable consumption (as in Lustig and Nieuwerburgh (2010), Østergaard, Sørensen, and Yosha (2002), and the other papers cited in section 3.1) to construct global CGV and country-specific CGVs in a similar way as I constructed the global VRP and the country-specific VRPs. Motivated by Equations 21 and 22, I use them in the predictive regressions of forex returns. Respectively, the specifications are:

\[
rx_{t+1} = \alpha + \beta_{US}^{CGV} \times CGV_{US,t} + \beta_{J}^{CGV} \times CGV_{J,t} + \zeta_{US}^{VRP} \times VRP_{US,t} + \zeta_{J}^{VRP} \times VRP_{J,t} + \epsilon_t \]  

\[
rx_{t+1} = \alpha + \beta_{US}^{CGV} \times CGV_{US,t} + \beta_{J}^{CGV} \times CGV_{J,t} + \theta_{CGV} \times GCGV_t + \zeta_{US}^{VRP} \times VRP_{US,t} + \zeta_{J}^{VRP} \times VRP_{J,t} + \delta_{GVRP} \times GVRP_t + \epsilon_t \]  

Table 6 reports the results of the full-model regressions (specifications 27 and 28). Comparing the results in Table 6 with those in Table 4 shows that: first, the magnitude and significance of the coefficients of local VRPs, country-specific VRPs, and the global VRP are almost unchanged. Second, none of the coefficients of the CGVs are significant in the top panel. Third, the global CGV coefficient and the country-specific CGV coefficient are significant (at the 10% level) only for EUR/USD forex returns in the bottom panel. Fourth, the adjusted R-squares for all three pairs remain almost unchanged, except the little improvement for JPY/USD in the top panel and for EUR/USD in the bottom panel. These results suggest that CGV variables contain little information about forex returns.\(^{17}\) This is consistent with the low correlations between consumption growth rates and exchange rate movements documented in Backus and Smith (1993). Moreover, these results may explain why authors (in this literature) exclude the consumption growth variance component. In the rest of the paper, I exclude the consumption growth variance component as well. Finally, these results suggest that equity VRPs are robust predictors.

\(^{17}\)In Appendix C, I show the theoretical relation between expected forex returns, CGV differentials, and VRP differentials. Then, I use CGV differentials and VRP differentials to predict forex returns. I find that using CGV differentials variables contain little information about forex returns as well. However, if risk-aversions are time-varying, then the result that the CGV does not emerge as statistically significant in the regressions may be because the CGVs are multiplied by the relevant risk-aversion coefficients—which appear in the VRP. In that case, the interpretations of the coefficients from the reduced form estimation are complex.
of forex returns after adding consumption growth variance components in the predictive regressions.

4.3.2 Excluding 2008 Crisis Period

According to the model, when the volatility-of-volatility increases, investors require higher returns to invest in foreign assets. Empirically, the VRP measures the volatility risk and is expected to have more predictive power for excess returns during more volatile periods when volatility uncertainty is high and asset prices change dramatically to adjust to the higher expected returns. Figure 1 shows that both the global VRP and realized forex returns changes dramatically in the recent crisis (particularly during October 2008). However, there is less variation in the global VRP compared to forex returns. Consequently, one concern of the VRPs’ predictive power is that it may arise only from the observations during the 2008 crisis. To address the concern, I drop 5 observations from August 2008 to December 2008, then I re-run the main regressions (specifications 23 and 24) on the sample excluding the crisis months. The results are reported in Table 7.

Table 7 is directly comparable to Table 4. In the simplest specification reported in panel 1, which includes both countries’ local VRPs, without the explicit global and local component decomposition, the sign of the local VRP coefficients are consistent with the model prediction in all three pairs. The US VRP coefficients are nearly identical (as predicted by the model) and statistically significant at the 1% level in GBP/USD and EUR/USD. The UK VRP and Japan VRP are also significant at the 10% and 5% levels. The GBP/USD and EUR/USD forex returns are still significantly predictable, with adjusted R-squares of 5% and 6% respectively. The adjusted R-square is dramatically decreased only for GBP/USD from around 17% (in Table 4) to around 5% (Table 7), and interestingly, it is slightly increased for the EUR/USD from around 4% to around 6%. This means that most of the predictability in GBP/USD comes from the 2008 crisis. In panel 2, the global VRP coefficient is significant in two pairs, GBP/USD and EUR/USD. Also, the coefficient of US-specific VRP is statistically significant with a correct sign for EUR/USD, and Japan-specific VRP is statistically significant with a correct sign for JPY/USD. The adjusted R-squares in panel 2 are similar to those in Panel 1.

Moreover, I use SUR with constraints on the coefficients of local and country-specific VRP in the US to be equal across regressions and report them respectively in panels 3 and 4 of Table 7. Using SUR changes neither the overall fit of the model much (the R-squares are similar) nor the magnitude of the foreign-specific VRPs and the global VRP. To sum up, the equity VRPs still have predictive power in a sample without the

\[ \text{18} \]

\[ \text{I also use SUR without constraints on the coefficients of local and country-specific VRP in the US and report them in the Internet Appendix.} \]
global 2008 crisis. Moreover, in a normal period the local VRPs’ predictive power for forex returns are as good as the global and country-specific VRPs, while in a period of economic downturn or a global crisis decomposing the global and country-specific VRPs improves forex return predictability.

4.3.3 Other Pairs

The global VRP factor constructed in this paper is based on the information available in four major countries. Although it may not fully capture the common factor in the forex market, we expect that this measure mimics the common factor effectively. Therefore, I test whether the global VRP and the US-specific VRP predict the forex returns for the other 18 currencies with respect to the USD. The results of these regressions are reported in Table 8. Table 8 suggests that the global VRP and the US-specific VRP have predictive power for most of the other currency forex returns. The coefficient of global VRP is significant for 15 out of the 18 currencies and has a t-statistic value of more than one in all cases. By the same token, the coefficient of the US-specific VRP is significant for 14 out of 18 currencies and has a t-statistic value of more than one in all cases. Moreover the estimate of the coefficient of the US-specific VRP has the same sign across all pairs, as the model suggests. The model does not completely lack explanatory power (a negative adjusted R-square) for any of the 18 currencies included in the tests. Meanwhile, we should note that some of these currencies are highly correlated with the major currencies (the pound, yen, and euro against the dollar). In particular, the European currencies are highly correlated with the euro (see Table ??) and therefore their robustness checks will not be as informative as those currencies which have lower correlations with the major currencies. Moreover, the major currencies have almost the least correlations with the other currencies, and thus the main regressions for the pound, yen, and euro against the dollar will be sufficiently informative. Overall, these results suggest that the global VRP and the US-specific VRP are robust predictors of most currency forex returns against the USD.

5 A Common Risk Factor

A high correlation between a global currency factor and the return volatility of equity markets (and fixed income markets) supports the global risk interpretation of the currency factor (see for example, Lustig et al. (2011) and Menkhoff et al. (2012)). The high

19The VRP data of most of those 18 countries are not available. Therefore their country-specific VRP factors are not available either.
20The SUR version of these regressions are reported in the Internet Appendix.
correlation also suggests that the time-varying risk premiums co-move across financial markets as a result of the existence of common risk factors.\textsuperscript{21} In this section, I run a "horse race" between the global VRP and two currency carry predictors—the commodity and currency volatility factors of Bakshi and Panayotov (2013)—to predict carry trade, major bilateral forex returns, and excess equity return differentials. I report the results in this section.\textsuperscript{22}

5.1 Excess Currency Returns

We may expect that the global VRP, which is a forward-looking measure of risk (based on option price information used in the volatility indices) to have more predictive power compared to the other two factors, which are based on past information from commodity returns and forex volatilities. To investigate this conjecture, I run a set of regressions where I use alternatively the commodity return, the forex volatility, and the global VRP to predict the returns of the carry portfolio of Lustig \textit{et al.} (2011) as well as the bilateral currency forex returns for the pound, euro and yen against the USD. The results reported in Table 9 show that all three factors have significant predictive power for the carry portfolio returns in the sample from January 2000 to December 2011. Importantly, the global VRP significantly predicts the carry returns with the highest adjusted R-square, 9.88%.\textsuperscript{23} The commodity and the forex volatility factors also have some predictive power for GBP/USD forex returns, however the global VRP is significant in predicting the forex returns of two out of the three bilateral currencies with higher R-squares. In the bottom panel, I use all three predictors together to predict the carry and the bilateral forex returns and I find that the global VRP is the only predictor which has significant coefficient for the carry return as well as two out of three bilateral forex returns. This evidence is suggestive that while all three predictor may provide empirical power to predict currency returns, the global VRP seems to provide more useful information for predicting future currency returns, perhaps because it embeds forward-looking information about aggregate uncertainty reflected in the volatility indices.

\textsuperscript{21}To find whether the level of interest-rate is driven by factors similar to those that drive forex returns, I investigate the model-implied interest rate and report it in Appendix D.

\textsuperscript{22}I compare the predictive power of the three predictors for excess holding interbank-rates as well. In addition, I include the HML carry factor of Lustig \textit{et al.} (2011) and the global forex volatility factor of Menkhoff \textit{et al.} (2012) in the "horse race". I find that neither of these two factors have predictive power for carry and bilateral forex returns and that both of them have some predictive power only for the UK-US excess equity return differentials. However, the global forex volatility factor of Menkhoff \textit{et al.} has significant power for excess holding inter-bank rate returns. The results are reported in the Internet Appendix.

\textsuperscript{23}Brunnermeier, Nagel, and Pedersen (2008) link currency carry returns to the currency crash risk and suggest that the volatility factor (VIX) can predict carry returns. However, I find that the VIX has no predictive power for monthly carry returns or for monthly forex returns from January 2000 to December 2011.
5.2 Excess Equity Return Differentials

Bollerslev et al. (2009) show that their model-implied equity premium, $\pi_{r,t+1} \equiv -\text{cov}_t(m_{t+1}, r_{t+1})$, equals to:

$$\pi_{r,t+1} = \gamma [\sigma_{g,t}^2] + (\theta - 1)\kappa_1^2(Ag^2 + Ag^2 \varphi_q^2)[q_t]$$

or

$$\pi_{r,t+1} = \gamma [\sigma_{g,t}^2] + (\frac{\lambda}{(\theta - 1)})[VRP_t]$$

This relation is very similar to Equation 20. The conditional variance of log SDF and the expected excess equity return, in the model of Bollerslev et al. (2009), are driven by the same two factors: the CGV and the VRP. When consumption growth volatility increases, both the expected forex return and the expected excess equity return increase. Similarly, when the VRP increases, both expected returns increase. In addition, Bollerslev et al. (2014) show that the global VRP predicts local and global equity market returns. This supports that the global VRP (as a risk premiums) commoves across equity and forex markets.

Moreover, Hau and Rey (2006) and Cappiello and De Santis (2005) find significant contemporaneous correlations between the cross-country excess equity return differentials and the forex returns.\(^{24}\) In fact, those contemporaneous comovements are not surprising in light of the factors driving both forex and equity premiums in my model. The cross-country excess equity returns differential is:

$$\pi_{i,t+1} - \pi_{j,t+1} = \gamma^i [\sigma_{g,t}^2] - \gamma^j [\sigma_{g,t}^2] + (\frac{\lambda^i}{(\theta^i - 1)})[VRP^i_t] - (\frac{\lambda^j}{(\theta^j - 1)})[VRP^j_t]$$

or

$$\pi_{i,t+1} - \pi_{j,t+1} = \frac{1}{2} (\gamma^i [\sigma_{g,t}^2] - \gamma^j [\sigma_{g,t}^2]) + \frac{1}{2} \gamma^i [\sigma_{g,t}^2] - \frac{1}{2} \gamma^j [\sigma_{g,t}^2]$$

$$+ \frac{1}{2} \lambda^i [VRP^i_t]/(\theta^i - 1) - \frac{1}{2} \lambda^j [VRP^j_t]/(\theta^j - 1)[VRP^j_t].$$

Firstly, Equation 32 shows that the variations in the cross-country excess equity return differentials are driven by the same factors that drive the variations in forex returns, and that the cross-country excess equity return differentials and forex returns should have contemporaneous comovements. Secondly, Equation 32 suggests that the global and the country-specific VRPs should help to predict equity premium differentials. This supports the empirical findings of Bekaert and Hodrick (1992), which show that an excess equity

\(^{24}\)To test such correlations in my data, I perform similar contemporaneous regressions which support the existence of such contemporaneous correlations and report them in the Internet Appendix.
return predictive variable (e.g. dividend yields) has some predictive power for forex returns, and that a forex return predictive variable (e.g. forward premiums) has some predictive power for excess equity returns as well.

Moreover, we may expect that the currency predictors should also help to predict equity premium differentials. I test this conjecture by regressing cross-country equity premium differentials on the lag commodity factor, the lag forex volatility factor, and the lag global VRP. The results are reported in Table 10. The results show that the commodity and forex volatility factors have some predictive power for US-UK excess equity return differentials, while the global VRP has predictive power for all three excess equity return differentials. Furthermore, the adjusted R-squares of the global VRP predictions are higher for all three excess equity return differentials compared to those of the commodity and the forex volatility predictions. More interestingly, the adjusted R-squares of the global VRP predictions for equity returns differentials in panel 3 of Table 10 are nearly identical to those of the global VRP predictions for forex returns in panel 3 of Table 9.\textsuperscript{25} In the bottom panel, I use all three predictors together to predict the excess equity return differentials, and I find that the global VRP is the only predictor which has significant predictive power for the excess equity return differentials.

To summarize, global VRP is a persistent predictor of excess returns on investments in both forex and equity markets. Among the three predictors—commodity returns, forex volatilities, and global VRP—global VRP has the highest predictive power for carry returns, excess forex returns, and excess equity return differentials.

\section*{6 Out-of-Sample Performance}

While I have shown that global and the country-specific VRPs have predictive power in-sample, an important test of predictive power is how the predictors perform out-of-sample (OOS). Since Meese and Rogoff (1983) first documented the failure of prevalent forex models to beat a simple random walk in out of sample predictions, very little evidence of OOS forex predictability has been offered by the literature (see for example, Rogoff and Stravrakeva (2008) and Rogoff (2009) for surveys of the evidence). In a similar paper, Goyal and Welch (2008) claim that none of equity return predictors can outperform historical means in predicting excess equity returns out-of-sample. However, Cooper and Priestley (2009) and Ferreira and Santa-Clara (2011), among others, find the

\textsuperscript{25}We should note that I calculate excess equity returns in the domestic currencies, and thus there is no bilateral forex return in the excess equity return differentials. In addition, the commodity and forex volatility factors, which are currency predictors have predictive power for both GBP/USD forex returns and US-UK excess equity return differential. They do not have predictive power for either JPY/USD forex returns or US-Japan excess equity return differential, or EUR/USD forex returns and US-Europe equity return differentials.
OOS forecasting power of some equity-return predictors. Moreover, Santa-Clara and Yan (2010), Bollerslev et al. (2009), and many others suggest that implied volatilities (and consequently VRP), which are forward-looking variables and capture the ex-ante crash risk assessed by investors, help to predict the realized returns.

In this section, I test whether predicting forex returns based on the global and the country-specific VRPs outperforms predicting forex returns based on a random walk with drift out-of-sample. I evaluate the performance of the global and country-specific VRPs’ forecasts using an OOS R-square similar to the one proposed by Goyal and Welch (2008). I use a 5-year training sample period from January 2000 to December 2004. The results are reported in Table 11 and displayed in Figure 2. Figure 2 shows that from January 2005 until before the 2008 crisis, the OOS R-squares are positive for GBP and EUR, with a negative slope, which means during this period the historical means smoothly outperform the global and the country-specific VRPs. However, the global and the country-specific VRPs smoothly outperform historical means (positive slope) for JPY in this period.

In the 2008 crisis, the global and country-specific VRPs substantially beat the historical means, and consequently the OOS R-squares are highly positive for all three pairs (they jump up to 8-27%). However, we should note that the high 2008 OOS-R-squares found in this paper are not just due to the low prediction errors of the global and the country-specific VRPs, but they are mainly due to the poor predictive power of the random walk model during the recent crisis.

After the 2008 crisis, the OOS R-squares stay constantly positive for the pound, which means that the global and the country-specific VRPs and historical means have the same performance in predicting GBP/USD forex returns. The OOS R-squares approach zero for Japan and Europe, with negative and positive slopes respectively. In other words, historical means under-perform the global and the country-specific VRPs in Japan and outperform in Europe.

The results of Table 11 show that the final OOS R-square is large and positive for GBP/USD (17%), is small and positive for EUR/USD (2%), and is small and negative for JPY/USD (-2%). More importantly, the final OOS R-square is statistically significant for GBP/USD at the 1% level and for EUR/USD at the 5% level under the McCracken (2007) statistic. The final OOS R-square is not statistically significant. This means equity VRPs significantly outperform the historical mean for GBP/USD and EUR/USD, and they do not significantly under-perform historical means for JPY/USD out-of-sample.\textsuperscript{26} Moreover, we should note that these OOS performances are based on all three-factor predictions (one global VRP and two country-specific VRPs) together, and that using all

\textsuperscript{26}The global and the country-specific VRPs outperform predicting the exchange rate changes (rather than the forex returns) out-of-sample too. The results are available on request.
three factors introduces a lot of noise in the predictions.\textsuperscript{27}

7 Conclusion

This paper provides a way to use forward-looking information available in stock market volatility indices to predict forex returns. By combining the long-run risk with stochastic volatility model of Bollerslev \textit{et al.} (2009) and the general currency-pricing model of Bekaert (1996) and Backus \textit{et al.} (2001), I show that expected forex returns are a function of both consumption growth volatilities (as in Bansal and Shaliastovich (2007 and 2013) and Colacito and Croce (2011)) and equity variance risk premiums (VRPs).

Local equity VRPs can be estimated as the difference between the squared volatility indices and the realized variance of the respective equity indices, and therefore they measure aggregate volatility uncertainties. The global VRP is constructed as the average of local equity VRPs in four major countries (the US, UK, Japan and Europe), and thus it captures investors’ common expectations of aggregate volatility uncertainty.

I find that the equity VRPs predict monthly forex returns for the pound, the euro and the yen against the U.S. dollar at the one-month horizon. When I exclude the 2008 crisis months from the sample, the adjusted R-square of predictions for the GBP/USD forex returns decreases, and the significance and explanatory power of the global VRP for the JPY/USD forex returns decrease as well. However, both the adjusted R-square and the significance of the global VRP for the EUR/USD forex returns increase. The significance and explanatory power of the global VRP is mostly unchanged when using Seemingly Unrelated Regressions (SUR), and when including in the regression country-specific and global consumption volatilities. Moreover, even when the local VRPs are unavailable, as is the case for the other 18 currencies in my sample, the global VRP and the US-specific VRP are strong predictors of forex returns for the majority of these currencies with respect to the U.S. dollar. The in-sample predictive power of global and country-specific VRPs extend out-of-sample, where they outperform the standard random walk benchmark. In addition, in a ”horse race” with two strong predictors of currency carry returns—the commodity and the currency volatility factors of Bakshi and Panayotov (2013)—the global VRP has significantly more predictive power for carry returns as well as individual forex returns.

In complete markets with recursive preferences and under lognormality, the models that are used to derive the drivers of equity returns can also be used to determine the factors driving forex returns. More specifically, the cross-country equity premium differ-

\textsuperscript{27}I find that the global VRP has predictive power of in-sample for JPY/USD before the 2008 crisis and for EUR/USD after 2008 crisis and it may explain the positive slope of the cumulative out-of-sample R-square movements in Figure 2. The in-sample results are reported in the Internet Appendix.
entials are driven by the same global and country-specific VRPs and consumption growth volatility factors as the forex returns. This commonality in driving factors provides an explanation for the high degree of contemporaneous comovements between cross-country equity premium differentials and forex returns documented in the data. Moreover, I find that the global VRP is a strong predictor of equity premium differentials, both in the theory and in the data.

The future development of this literature should address the interpretation of the endowments and the structure of goods markets that generates the observed consumption process and real exchange rates. In addition, the theoretical link among forex returns, equity premiums, and interest rates shown in this paper strongly provides a motivation for investigating further the global and country-specific factors to better understand their dynamics and their predictive power across these markets.
References


Appendix A
The details of derivation for Equation 16 are as follows:

\[
\text{var}[m_{t+1}] = \text{var}[\theta \log(\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{t+1}]
\]

\[
= \left(\frac{\theta}{\psi}\right)^2 \text{var}(g_{t+1}) + (\theta - 1)^2 \text{var}(r_{t+1}) - 2\left(\frac{\theta}{\psi}\right)(\theta - 1) \text{cov}(g_{t+1}, r_{t+1})
\]

\[
= \left(\frac{\theta}{\psi}\right)^2 \sigma^2_{g,t} + (\theta - 1)^2(\sigma^2_{g,t} + \kappa_1^2(A_{\sigma}^2 + A_q^2 \phi_q^2)q_{t}) - 2\left(\frac{\theta}{\psi}\right)(\theta - 1)\sigma^2_{g,t}
\]

\[
= \left(\left(\frac{\theta}{\psi}\right)^2 + (\theta - 1)^2 - 2\left(\frac{\theta}{\psi}\right)(\theta - 1)\right)[\sigma^2_{g,t}] + (\theta - 1)^2\kappa_1^2(A_{\sigma}^2 + A_q^2 \phi_q^2)[q_t]
\]

therefore,

\[
\text{var}[m_{t+1}] = \gamma^2[\sigma^2_{g,t}] + (\theta - 1)^2\kappa_1^2(A_{\sigma}^2 + A_q^2 \phi_q^2)[q_t].
\]
Appendix B
In this appendix, I use a global variance risk premium model similar to Bollerslev et al. (2014) in order to measure the variance of the logarithm of stochastic discount factor.

Assuming that the observed consumption is a function of a constant and the global and the country-specific components:

\[ c_t^i = \theta_c^i + \tau_c^i \times c_t^{gl} + c_t^{si}, \quad (33) \]

and that the geometric growth rate of global consumption, \( g_{t+1}^{gl} = \log(c_{t+1}^{gl}/c_t^{gl}) \), the global consumption growth volatility, \( \sigma_{g,t+1}^{gl} \), and the global variance risk premium, \( q_{t+1}^{gl} \), follow a joint process:

\[ g_{t+1}^{gl} = \mu_g^{gl} + \sigma_g^{gl} z_{g,t+1}, \]
\[ \sigma_{g,t+1}^{gl^2} = \sigma_g^{gl^2} + \nu_g^{gl^2} \sigma_{g,t+1}^{gl^2} + \sqrt{q_{t+1}^{gl^2} z_{g,t+1}}, \]
\[ q_{t+1}^{gl} = a_q^{gl} + \nu_q^{gl} q_{t+1}^{gl} + \varphi_q^{gl} \sqrt{q_{t+1}^{gl^2} z_{g,t+1}}. \]

Also, I assume that the geometric growth rate of country-specific consumption, \( g_{t+1}^{si} = \log(c_{t+1}^{si}/c_t^{si}) \), the country-specific consumption growth volatilities, \( \sigma_{g,t+1}^{si} \), and the country-specific VRPs, \( q_{t+1}^{si} \), follow a similar joint process:

\[ g_{t+1}^{si} = \mu_g^{si} + \sigma_g^{si} z_{g,t+1}, \]
\[ \sigma_{g,t+1}^{si^2} = \sigma_g^{si^2} + \nu_g^{si^2} \sigma_{g,t+1}^{si^2} + \sqrt{q_{t+1}^{si^2} z_{g,t+1}}, \]
\[ q_{t+1}^{si} = a_q^{si} + \nu_q^{si} q_{t+1}^{si} + \varphi_q^{si} \sqrt{q_{t+1}^{si^2} z_{g,t+1}}. \]

where country \( i \in \{\text{US, UK, Japan, Europe, and Switzerland} \} \) in my data. Moreover, I assume that the country-specific components are uncorrelated across countries.

Bollerslev et al. (2009 and 2014) assume that the representative agent in the economy has Epstein–Zin–Weil recursive preferences. Therefore, the logarithm of the stochastic discount factor of the representative agent is:

\[ m_{t+1} = \theta \log(\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{t+1} \]

where \( \theta \equiv \frac{(1-\gamma)}{(1-1/\psi)} \), \( \delta \) denotes the subjective discount factor, \( \psi \) equals the intertemporal elasticity of substitution, \( \gamma \) refers to the coefficient of risk aversion, and \( r_{t+1} \) is the time \( t \) to \( t+1 \) return on the consumption asset.

Moreover, I assume that the global and country-specific logarithm of the price-consumption ratios of the asset that pays the consumption endowment \( p_c^{gl} \) & \( p_c^{si} \) has a conjecturing
solution that is an affine function of the state variables:

\[
p_{c_t}^{gl} = A_0^{gl} + A_\sigma^{gl} \sigma_{t+1} + A_q^{gl} q_t^{gl}
\]

\[
p_{c_t}^{si} = A_0^{si} + A_\sigma^{si} \sigma_{t+1} + A_q^{si} q_t^{si}
\]

Similar to Bollerslev et al. (2014) I use standard Campbell and Shiller (1988) log-linearization of returns:

\[
r_{gt}^{gl} = \kappa_0^{gl} + \kappa_1^{gl} p_{c_t}^{gl} - p_{c_t}^{gl} + g_{t+1}^{gl}
\]

\[
r_{gt}^{si} = \kappa_0^{si} + \kappa_1^{si} p_{c_t}^{si} - p_{c_t}^{si} + g_{t+1}^{si}
\]

(where \(r_{t+1}^{gl}\) and \(r_{t+1}^{si}\) are global and country-specific returns respectively), and a standard no-arbitrage condition \(E_t[exp(r_{t+1} + m_{t+1})] = 0\), to find these equilibrium solution for the coefficients:

\[
A_0^{gl} = \frac{\log \delta + (1 - 1/\psi) \mu_0^{gl} + \kappa_0^{gl} + \kappa_1^{gl} [A_2^{gl} a_0^{gl} + A_q^{gl} a_q^{gl}]}{(1 - \kappa_1^{gl})}
\]

\[
A_\sigma^{gl} = \frac{(1 - \gamma)^2}{2\theta(1 - \kappa_1^{gl} \nu_\sigma^{gl})}
\]

\[
A_q^{gl} = \frac{1 - \kappa_1^{gl} \nu_q^{gl} - \sqrt{(1 - \kappa_1^{gl} \nu_q^{gl})^2 - \theta^2 \kappa_1^{gl} \varphi_q^{gl} A_{\sigma}^{gl^2}}}{\theta \kappa_1^{gl} \varphi_q^{gl^2}}
\]

and

\[
A_0^{si} = \frac{\log \delta + (1 - 1/\psi) \mu_0^{si} + \kappa_0^{si} + \kappa_1^{si} [A_2^{si} a_0^{si} + A_q^{si} a_q^{si}]}{(1 - \kappa_1^{si})}
\]

\[
A_\sigma^{si} = \frac{(1 - \gamma)^2}{2\theta(1 - \kappa_1^{si} \nu_\sigma^{si})}
\]

\[
A_q^{si} = \frac{1 - \kappa_1^{si} \nu_q^{si} - \sqrt{(1 - \kappa_1^{si} \nu_q^{si})^2 - \theta^2 \kappa_1^{si} \varphi_q^{si} A_{\sigma}^{si^2}}}{\theta \kappa_1^{si} \varphi_q^{si^2}}
\]

Therefore, combining the statements for \(r_{t+1}^{gl} \& p_{c_t}^{gl}\), and \(r_{t+1}^{si} \& p_{c_t}^{si}\), and substituting the equilibrium coefficients, I get:
\[ r_{t+1}^{gl} = -\log\delta - \frac{\mu_{t}^{gl}}{\psi^{gl}} \frac{(1 - \gamma)^2}{(2\theta)} \sigma_{\gamma,t}^{gl^2} + (\kappa_1^{gl} \rho_q^{gl} - 1) A_q^{gl} \varphi_q^{gl} + \sigma_{\gamma,t}^{z_{g,t+1}} + \kappa_1^{gl} \sqrt{q_{t+1}^{gl}} [A_{\sigma}^{gl} \varphi_{\sigma,t+1} + A_q^{gl} \varphi_q^{gl} z_{q,t+1}] \]

and

\[ r_{t+1}^{si} = -\log\delta - \frac{\mu_{t}^{si}}{\psi^{si}} \frac{(1 - \gamma)^2}{(2\theta)} \sigma_{\gamma,t}^{si^2} + (\kappa_1^{si} \rho_q^{si} - 1) A_q^{si} \varphi_q^{si} + \sigma_{\gamma,t}^{z_{g,t+1}} + \kappa_1^{si} \sqrt{q_{t+1}^{si}} [A_{\sigma}^{si} \varphi_{\sigma,t+1} + A_q^{si} \varphi_q^{si} z_{q,t+1}] \]

Moreover, calculating VRPs similar to Bollerslev et al. (2009 and 2014), I get:

\[ VRP_t^{gl} \approx (\theta - 1) \kappa_1^{gl} \left[ A_q^{gl} \varphi_q^{gl^2} (A_q^{\varphi^2} + A_q^{gl^2} \varphi_q^{gl^2}) + A_q^{gl} \varphi_q^{gl} \right] q_{t}^{gl} \]

and

\[ VRP_t^{si} \approx (\theta - 1) \kappa_1^{si} \left[ A_q^{si} \varphi_q^{si^2} (A_q^{\varphi^2} + A_q^{si^2} \varphi_q^{si^2}) + A_q^{si} \varphi_q^{si} \right] q_{t}^{si} \]

Finally, I assume that the returns, consumption growth, and variance risk premiums are a function of a constant and the global and the country-specific components:

\[
\begin{align*}
g_t^i &= \vartheta_t^i + \tau_t^i \times g_t^{gl} + g_t^{si}, \\
r_t^i &= \vartheta_t^i + \tau_t^i \times r_t^{gl} + r_t^{si}, \\
VRP_t^i &= \vartheta_{VRP}^i + \tau_{VRP}^i \times VRP_t^{gl} + VRP_t^{si}.
\end{align*}
\]

In this economy, the conditional log of stochastic discount factor and its innovations are equal to

\[ m_{t+1} = \log\delta - \frac{\mu_{t}}{\psi} - \gamma \tau_t^{gl} \sigma_{\gamma,t}^{gl^2} - \frac{(\theta - 1)(1 - \gamma)^2}{(2\theta)} \tau_t^{\gamma,\sigma^{gl^2}} + (\theta - 1) (\kappa_1^{gl} \rho_q^{gl} - 1) \tau_t^{gl} A_q^{gl} \varphi_q^{gl} + (\theta - 1) \kappa_1^{gl} \sqrt{q_{t}^{gl}} [A_{\sigma}^{gl} \varphi_{\sigma,t+1} + A_q^{gl} \varphi_q^{gl} z_{q,t+1}] \]

\[ -\gamma \sigma_{\gamma,t}^{z_{g,t+1}} + \frac{(\theta - 1)(1 - \gamma)^2}{(2\theta)} \sigma_{\gamma,t}^{z_{g,t+1}} - (\theta - 1) \kappa_1^{si} \sqrt{q_{t}^{si}} [A_{\sigma}^{si} \varphi_{\sigma,t+1} + A_q^{si} \varphi_q^{si} z_{q,t+1}] \]

\[ m_{t+1} - E_t[m_{t+1}] = -\gamma \tau_t^{gl} \sigma_{\gamma,t}^{gl^2} - (\theta - 1) \kappa_1^{gl} \sqrt{q_{t}^{gl}} [A_{\sigma}^{gl} \varphi_{\sigma,t+1} + A_q^{gl} \varphi_q^{gl} z_{q,t+1}] \]

\[ -\gamma \sigma_{\gamma,t}^{z_{g,t+1}} - (\theta - 1) \kappa_1^{si} \sqrt{q_{t}^{si}} [A_{\sigma}^{si} \varphi_{\sigma,t+1} + A_q^{si} \varphi_q^{si} z_{q,t+1}] \]

Next, I calculate the variance of the logarithm of stochastic discount factor in country
i: 

\[ \text{var}[m_{t+1}] = \text{var}[	heta \log(\delta) - \frac{\theta}{\psi} g_{t+1}^i + (\theta - 1)r_{t+1}^i] \]

\[ \text{var}[m_{t+1}] = \left(\frac{\theta}{\psi}\right)^2 \text{var}(g_{t+1}^i) + (\theta - 1)^2 \text{var}(r_{t+1}^i) - 2\left(\frac{\theta}{\psi}\right)(\theta - 1)\text{cov}(g_{t+1}^i, r_{t+1}^i) \]

\[ \text{var}[m_{t+1}] = \left(\frac{\theta}{\psi}\right)^2 \text{var}(g_{t+1}^i) + \text{var}(g_{t+1}^s) ] + (\theta - 1)^2 \text{var}(r_{t+1}^i) + \text{var}(r_{t+1}^s) \]

\[ -2\left(\frac{\theta}{\psi}\right)(\theta - 1)[\text{cov}(g_{t+1}^g, r_{t+1}^g) + \text{cov}(g_{t+1}^s, r_{t+1}^s) + \text{cov}(g_{t+1}^s, r_{t+1}^s)] \]

\[ \text{var}[m_{t+1}] = \left(\frac{\theta}{\psi}\right)^2 \text{var}(g_{t+1}^i) + \text{var}(g_{t+1}^s) ] + (\theta - 1)^2 \text{var}(r_{t+1}^i) + \text{var}(r_{t+1}^s) \]

\[ -2\left(\frac{\theta}{\psi}\right)(\theta - 1)[\text{cov}(g_{t+1}^g, r_{t+1}^g) + \text{cov}(g_{t+1}^s, r_{t+1}^s) + \text{cov}(g_{t+1}^s, r_{t+1}^s)] \]

\[ \text{var}[m_{t+1}] = \frac{\theta}{\psi} \text{var}(g_{t+1}^i) + \text{var}(g_{t+1}^s) ] + (\theta - 1)^2 \text{var}(r_{t+1}^i) + \text{var}(r_{t+1}^s) \]

\[ -2\left(\frac{\theta}{\psi}\right)(\theta - 1)[\text{cov}(g_{t+1}^g, r_{t+1}^g) + \text{cov}(g_{t+1}^s, r_{t+1}^s) + \text{cov}(g_{t+1}^s, r_{t+1}^s)] \]

\[ \text{var}[m_{t+1}] = \frac{\theta}{\psi} \text{var}(g_{t+1}^i) + \text{var}(g_{t+1}^s) ] + (\theta - 1)^2 \text{var}(r_{t+1}^i) + \text{var}(r_{t+1}^s) \]

\[ -2\left(\frac{\theta}{\psi}\right)(\theta - 1)[\text{cov}(g_{t+1}^g, r_{t+1}^g) + \text{cov}(g_{t+1}^s, r_{t+1}^s) + \text{cov}(g_{t+1}^s, r_{t+1}^s)] \]

Therefore, variance of the logarithm of stochastic discount factor is a function of the global and the country-specific components. This means that we can write the expected excess foreign exchange returns as a function of the global and the country-specific components as well:

\[ VRF^g_t = (\theta - 1)\kappa_1 [\sigma^g A_{gq}^q (A_{gq}^q + A_{gq}^q) + A_{gq}^q q_{t+1}^{gl}] \]

\[ \wedge \]

\[ VRF^s_t = (\theta - 1)\kappa_1 [\sigma^s A_{qs}^q (A_{qs}^q + A_{qs}^q) + A_{qs}^q q_{t+1}^{si}] \]

then

\[ \text{var}[m_{t+1}] = \gamma^2 [\sigma_{g,t}^2] + \gamma^2 [\sigma_{s,t}^2] + \lambda^g [\tau_{r,g}^2 + VRF^g_t] + \lambda^s [VRF^s_t] \]
where
\[
\lambda^g_t = \frac{(\theta-1)^2 \kappa_1^t (\varphi_1^g r^g_t + \varphi_0^g)}{(\theta-1) \kappa_1^t \kappa_1^t (\varphi_1^g r^g_t + \varphi_0^g \varphi_1^g) + \lambda^g_t \varphi_0^g}
\]
\[
\lambda^s_t = \frac{(\theta-1)^2 \kappa_1^t (\varphi_1^s r^s_t + \varphi_0^s)}{(\theta-1) \kappa_1^t \kappa_1^t (\varphi_1^s r^s_t + \varphi_0^s \varphi_1^s) + \lambda^s_t \varphi_0^s}
\]

moreover, the expected excess foreign exchange return between country i and j is
\[
E[r_{i,j,t+1}^x] = \frac{1}{2} \text{var}[m_{i,t+1}^j] - \frac{1}{2} \text{var}[m_{j,t+1}^i]
\]  

(39)

\[
E[r_{i,j,t+1}^x] = \frac{1}{2} \gamma^2 [\tau_g^2 \times \sigma_{g,t}^2] + \frac{1}{2} \gamma^2 [\sigma_{s,t}^2] + \frac{1}{2} \lambda^g_t [\tau_g^2 \times \text{VRP}_t^g] + \frac{1}{2} \lambda^s_t [\text{VRP}_t^s]
\]
\[
-\frac{1}{2} \gamma^2 [\tau_g^2 \times \sigma_{s,t}^2] - \frac{1}{2} \lambda^g_t [\tau_g^2 \times \text{VRP}_t^g] + \frac{1}{2} \lambda^s_t [\text{VRP}_t^s]
\]  

(40)

by rearranging the Equation 40, I get:
\[
E[r_{i,j,t+1}^x] = \frac{1}{2} A [\sigma_{g,t}^2] + \frac{1}{2} \gamma^2 [\sigma_{s,t}^2] - \frac{1}{2} \gamma^2 [\sigma_{g,t}^2] + \frac{1}{2} B [\text{VRP}_t^g] + \frac{1}{2} \lambda^s_t [\text{VRP}_t^s] - \frac{1}{2} \lambda^s_t [\text{VRP}_t^s].
\]  

(41)

where \( A = \gamma^2 \tau_g^2 - \gamma^2 \tau_g^2 \) and \( B = \lambda^g_t \tau_g^2 - \lambda^g_t \tau_g^2 \).

If we assume that the subjective discount factors and subjective discount factors and the coefficients of VRPs are the same across two countries, i and j:
\[
\lambda^g_t = \lambda^g_j \land \lambda^s_t = \lambda^s_j \land \gamma^g_t = \gamma^s_t,
\]

then
\[
E[r_{i,j,t+1}^x] = \frac{1}{2} A [\sigma_{g,t}^2] + \frac{1}{2} \gamma^2 [\sigma_{s,t}^2 - \sigma_{s,t}^2] + \frac{1}{2} B [\text{VRP}_t^g] + \frac{1}{2} \lambda^s_t [\text{VRP}_t^s - \text{VRP}_t^s].
\]  

(42)

This equation suggests that the excess foreign exchange return between country i and j is a function of the global CGV, \( \sigma_{g,t}^2 \), the cross-country country-specific CGV differential, \( \sigma_{g,t}^2 - \sigma_{g,t}^2 \), the global VRP, \( \text{VRP}_t^g \), and the country-specific VRP differential, \( \text{VRP}_t^s - \text{VRP}_t^s \).
Appendix C

Bansal and Shaliastovich (2007) suggest that the expected forex returns are a function of cross-country CGV differentials. My model suggests that forex returns are a function of both the cross-country CGV differential and the VRP differential. Moreover, my model simplifies to the model of Bansal and Shaliastovich (2007) when VRPs and preferences are equal across countries. Therefore it may be considered to be an extension of their model.

Next, I write the model in a differential setup, as in Bansal and Shaliastovich (2007), where the preference specifications are the same across countries. By rearranging Equation 21, I get:

\[ E\left[ r_{x_{i+1}} \right] = \frac{1}{2} \gamma_i^2 \left\{ \sigma_{g,i,t}^2 - \sigma_{g,j,t}^2 \right\} + \frac{1}{2} \left( \lambda_i \right) \left\{ VRP_{i,t} - VRP_{j,t} \right\} + \frac{1}{2} \left( \gamma_i - \gamma_j \right) \left\{ \sigma_{g,j,t}^2 \right\} + \frac{1}{2} \left( \lambda_i - \lambda_j \right) \left\{ VRP_{j,t} \right\}. \] (43)

Assuming that the coefficients of risk aversion are equal across two countries (\( \gamma_i = \gamma_j = \gamma \)), as in Bansal and Shaliastovich (2007), and assuming also that the coefficients of the local VRP are the same across two countries (\( \lambda_i = \lambda_j = \lambda \)), then the expected forex return still has two components: the cross-country CGV differential and the cross-country VRP differential,

\[ E\left[ r_{x_{i+1}} \right] = \frac{1}{2} \gamma^2 \left\{ \sigma_{g,i,t}^2 - \sigma_{g,j,t}^2 \right\} + \frac{1}{2} \left( \lambda \right) \left\{ VRP_{i,t} - VRP_{j,t} \right\}. \] (44)

Equation 43 reduces to that of Bansal and Shaliastovich (2007) if and only if the risk aversions are equal across the two countries (\( \gamma_i = \gamma_j = \gamma \)) and local VRPs are equal across two countries (\( VRP_{i,t} = VRP_{j,t} \)). In that case, Equation 44 would be:

\[ E\left[ r_{x_{i+1}} \right] = \frac{1}{2} \gamma^2 \left\{ \sigma_{g,i,t}^2 - \sigma_{g,j,t}^2 \right\}. \] (45)

According to Equation 45, the forex return is just a function of the cross-country CGV differential, as in Bansal and Shaliastovich (2007). However, if there is a common factor in the forex market, and local VRPs are not equal across countries, then Equation 44 would be:

\[ E\left[ r_{x_{i+1}} \right] = \frac{1}{2} A \left[ \sigma_{g,i,t}^2 - \sigma_{g,j,t}^2 \right] + \frac{1}{2} B \left[ VRP_{i,t} - VRP_{j,t} \right] + \frac{1}{2} \gamma \left[ \sigma_{g,i,t}^2 - \sigma_{g,j,t}^2 \right] + \frac{1}{2} \left( \lambda_i - \lambda_j \right) \left\{ VRP_{j,t} \right\}. \] (46)

where A is \( \gamma \left[ \sigma_{g,i,t}^2 - \sigma_{g,j,t}^2 \right] \) and B is \( \lambda \left[ VRP_{i,t} - VRP_{j,t} \right] \).
Equation 46 shows that the forex return is a function of CGV differentials, global CGV, country-specific VRP differentials, and global VRP. This relation is similar to the relation between exchange-rate changes, (short-run) CGV differentials, and expected (long-run) CGV differentials introduced in Bansal and Shaliastovich (2013) and Colacito and Corce (2011). The difference is that consumption process in Colacito and Corce (2011) is augmented by the forecast-able consumption growth prospect differential measured by international financial news, and consumption process in Bansal and Shaliastovich (2013) is augmented by the expected CGV differentials measured by professional forecasters for real GDP growth (and inflation) at one-year horizon. While the consumption process is augmented by the monthly VRP differentials in this paper. Equations 44 and 46 provides a motivation for using the CGV differential and the VRP differential to predict forex returns, and thus I use the following specifications:

\[
r_{x_t} = \alpha + \beta_{DCGV} \times DCGV_t + \zeta_{DVRP} \times DVRP_t + \varepsilon_t
\]  
(47)

\[
r_{x_t} = \alpha + \beta_{DCGV} \times DCGV_t + \theta_{GCGV} \times GCGV_t + \varepsilon_t
\]  
(48)

where \( DCGV_t \) is the cross-country CGV differential \( (DCGV_t = CGV_{i,t} - CGV_{j,t}) \), \( DCGV_t^s \) is cross-country country-specific CGV differential \( (DCGV_t^s = CGV_{i,t}^s - CGV_{j,t}^s) \), and \( GCGV_t \) is global CGV.

I test specifications 47 and 48 for the sample period from January 2001 to December 2011. I report the results in the Internet Appendix. In sum, the results suggest that CGV variables contain little information about forex returns, and that using the differentials does not change the significance of the VRP variables and suppresses the little information that is available in the CGV variables.
Appendix D

The cross-country interest-rate differentials are traditionally used as a predictor of forex returns. The previous success of interest rate differentials in predicting forex returns may be due to the fact that they are driven by similar factors as forex returns. In fact, the model-implied interest rate is a function of both CGV and VRP (and hence their global and country-specific components):

\[
rf_t = -C - (\theta - 1)A_\sigma(\kappa_1 \rho_{\sigma c} - 1)[\sigma_{ct}^2] - (\theta - 1)A_q(\kappa_1 \rho_q - 1)[q_t]
\]

\[= -\frac{1}{2} \gamma^2 [\sigma_{g,t}^2] - \frac{1}{2} (\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2)[q_t]
\]

(49)

where \( C = \theta \log(\delta) - \gamma \mu_g(\theta - 1)[\kappa_0 + \kappa_1(A_\sigma a_{\sigma c} + A_q a_q)] + (\theta - 1)(\kappa_1 - 1)A_0 \), and by decomposing the global and country-specific components, we get

\[
rf_t = -C + (D - \frac{1}{2} \lambda^g \gamma_t^2 [r_{g,t}^2]) [\sigma_{g,t}^2] + (D - \frac{1}{2} \lambda^s \gamma^s_t^2) [\sigma_{g,t}^2]
\]

\[+ (E - \frac{1}{2} \lambda^g \gamma_t^2) [VRP^g_t] + (E - \frac{1}{2} \lambda^s) [VRP^s_t]
\]

(50)

where \( D = -(\theta - 1)[A_\sigma(\kappa_1 \rho_{\sigma c} - 1)] \) and \( E = \frac{-(\theta - 1)A_q(\kappa_1 \rho_q - 1)}{(\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2)} \).

Equation 50 shows that when the exposure to the global VRP \( (\lambda^g_t) \) is high the interest rates are low which drives the relation between the heterogeneity in exposures to the global VRP and the level of interest rates.

Moreover, the level of interest rates affects the demand for liquidity and size of investments in interest rates and short-term bonds. As a result, we expect that the expected excess holding interbank rate returns are predictable by the global VRP. Therefore, I test whether the global VRP predicts the excess returns for one-month holding of interbank rates with two-month maturities. I also test the predictive power of the local and country-specific VRPs and report the results in the Internet Appendix. I find that the local, global, and country-specific equity VRPs predict excess one month holding return of interbank rates with two-month maturities in the US, UK and Europe.
Figure 1: The global VRP (GVRP) is plotted at time $t$ and the lead forex returns (FX) are plotted at time $t - 1$. 
Figure 2: The cumulative out-of-sample (OOS) $R^2$ of monthly predictions based on rolling 5-year windows of monthly global and country-specific VRPs for the sample period from January 2005 to December 2011. The 5-year training sample is from January 2000 to December 2004.
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Country</th>
<th>Libor</th>
<th>CG</th>
<th>CGV</th>
<th>VRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>Mean</td>
<td>3.26</td>
<td>37.0</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>S.D.</td>
<td>0.65</td>
<td>36.3</td>
<td>5.23</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.98</td>
<td>-0.39</td>
<td>0.93</td>
</tr>
<tr>
<td>UK</td>
<td>Mean</td>
<td>4.42</td>
<td>4.1</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>S.D.</td>
<td>0.58</td>
<td>34.6</td>
<td>7.14</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.98</td>
<td>-0.25</td>
<td>0.96</td>
</tr>
<tr>
<td>Japan</td>
<td>Mean</td>
<td>0.28</td>
<td>-216.8</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>S.D.</td>
<td>0.07</td>
<td>36.2</td>
<td>3.46</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.95</td>
<td>-0.39</td>
<td>0.98</td>
</tr>
<tr>
<td>Europe</td>
<td>Mean</td>
<td>2.74</td>
<td>-121.8</td>
<td>2.55</td>
</tr>
<tr>
<td></td>
<td>S.D.</td>
<td>0.41</td>
<td>38.3</td>
<td>4.54</td>
</tr>
<tr>
<td></td>
<td>AR(1)</td>
<td>0.98</td>
<td>-0.37</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Monthly interbank rates (Libor) are in annualized percentage form. Consumption is proxied by retail sales. The log of consumption growth (CG) is in annualized basis-point form. The consumption growth variance (CGV) is in annualized squared form. The variance risk premiums are in annualized percentage-squared form. The standard deviation (S.D.) is in percentage form. AR(1) is the estimated one-lag autocorrelations. The sample period extends from January 2000 to December 2011 (except for the consumption growth, which extends from January 1997 to December 2011).
This table reports the average appreciation of the end-of-the-month log spot exchange rates with respect to the U.S. dollar (USD). The monthly means, standard deviations (S.D.), and the one-lag autocorrelations (AR(1)) are reported in percentage points. Data are monthly, from Datastream. The sample period extends from January 2000 to December 2011.
Table 3: Correlations

<table>
<thead>
<tr>
<th>Currency Pair</th>
<th>$VRP_{US}$</th>
<th>$VRP_{UK}$</th>
<th>$GVRP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBP/USD</td>
<td></td>
<td>0.88</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.91</td>
</tr>
<tr>
<td>$FX_{GBP/USD}$</td>
<td>0.39</td>
<td>0.25</td>
<td>0.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Currency Pair</th>
<th>$VRP_{US}$</th>
<th>$VRP_{Japan}$</th>
<th>$GVRP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPY/USD</td>
<td></td>
<td>0.57</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.60</td>
</tr>
<tr>
<td>$FX_{JPY/USD}$</td>
<td>-0.17</td>
<td>-0.12</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Currency Pair</th>
<th>$VRP_{US}$</th>
<th>$VRP_{Europe}$</th>
<th>$GVRP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USD</td>
<td></td>
<td>0.69</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.73</td>
</tr>
<tr>
<td>$FX_{EUR/USD}$</td>
<td>0.16</td>
<td>-0.02</td>
<td>0.14</td>
</tr>
</tbody>
</table>

This table reports monthly correlations among local equity variance risk premiums (VRP), excess foreign exchange returns (FX), and the global VRP (GVRP) for three major currencies in the sample period from January 2000 to December 2011.
This table reports monthly forex returns, $r_{x_{t+1}}$, and the predictive power of local variance risk premiums, $VRP_t$, country-specific VRPs, $VRP_s$, and the global VRP, $GVRP_t$. Monthly forex returns are calculated as returns on buying a forward contract on GBP/USD, JPY/USD, and EUR/USD and selling it at the spot rates after one month from January 2000 to December 2011. I report the slope coefficients ($\zeta$) and ($\delta$), and the adjusted $R^2$ of the following regressions:

\[
r_{x_{t+1}} = \alpha + \zeta^{US}_{VRP_t} \times VRP_{US,t} + \zeta^j_{VRP_t} \times VRP_{j,t} + \varepsilon_t,
\]

\[
r_{x_{t+1}} = \alpha + \zeta^{US}_{VRP_t} \times VRP_{US,t} + \zeta^j_{VRP_t} \times VRP_{j,t} + \delta_{GVRP_t} \times GVRP_t + \varepsilon_t,
\]

Robust t-statistics following Newey and West (1987), with the optimal number of lags according to Andrews (1991), are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.
Table 5: Seemingly Unrelated Regression (SUR)

<table>
<thead>
<tr>
<th></th>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel 1 (w/o)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{US VRP}$</td>
<td>4.53***</td>
<td>-1.39*</td>
<td>2.80***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(4.39)</td>
<td>(-1.74)</td>
<td>(3.02)</td>
</tr>
<tr>
<td>$\zeta_{j VRP}$</td>
<td>-2.04*</td>
<td>0.04</td>
<td>-1.77**</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.89)</td>
<td>(0.06)</td>
<td>(-2.41)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>17.82</td>
<td>2.80</td>
<td>5.45</td>
</tr>
<tr>
<td><strong>Panel 2 (w/o)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{US VRP_s}$</td>
<td>14.6</td>
<td>-1.41</td>
<td>19.8**</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.71)</td>
<td>(-0.17)</td>
<td>(1.98)</td>
</tr>
<tr>
<td>$\zeta_{j VRP_s}$</td>
<td>-1.22</td>
<td>-0.15</td>
<td>-0.91</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-0.25)</td>
<td>(-0.19)</td>
<td>(-0.85)</td>
</tr>
<tr>
<td>$\delta_{GVRP}$</td>
<td>2.88***</td>
<td>-1.42**</td>
<td>1.31*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(4.89)</td>
<td>(-2.04)</td>
<td>(1.73)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>18.50</td>
<td>2.87</td>
<td>7.52</td>
</tr>
<tr>
<td><strong>Panel 3 (w/)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{US VRP}$</td>
<td>1.11*</td>
<td>1.11*</td>
<td>1.11*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.79)</td>
<td>(1.79)</td>
<td>(1.79)</td>
</tr>
<tr>
<td>$\zeta_{j VRP}$</td>
<td>0.87</td>
<td>-1.48**</td>
<td>-0.73</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.21)</td>
<td>(-2.20)</td>
<td>(-1.17)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>10.62</td>
<td>-2.74</td>
<td>3.36</td>
</tr>
<tr>
<td><strong>Panel 4 (w/)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{US VRP_s}$</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.21)</td>
<td>(1.21)</td>
<td>(1.21)</td>
</tr>
<tr>
<td>$\zeta_{j VRP_s}$</td>
<td>-1.67</td>
<td>0.33</td>
<td>-1.58**</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.10)</td>
<td>(0.44)</td>
<td>(-2.05)</td>
</tr>
<tr>
<td>$\delta_{GVRP}$</td>
<td>2.88***</td>
<td>-1.42**</td>
<td>1.31*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(4.88)</td>
<td>(-2.03)</td>
<td>(1.71)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>18.17</td>
<td>1.88</td>
<td>6.46</td>
</tr>
</tbody>
</table>

This table reports the SUR version of Table 4. The top two panels are without (w/o) any constraints. The bottom two panels are with (w/) constraints on the coefficients of the local US VRP ($\zeta_{US VRP}$) and the US-specific VRP ($\zeta_{US VRP_s}$) to be equal across regressions. The t-statistics are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.
Table 6: CGV variables, VRP variables, and Forex Returns

<table>
<thead>
<tr>
<th></th>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^{US}_{CGV}$</td>
<td>0.11</td>
<td>-0.26</td>
<td>-0.15</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.81)</td>
<td>(-1.11)</td>
<td>(-0.71)</td>
</tr>
<tr>
<td>$\beta^{j}_{CGV}$</td>
<td>0.12</td>
<td>-0.06</td>
<td>0.37</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.26)</td>
<td>(-0.29)</td>
<td>(1.46)</td>
</tr>
<tr>
<td>$\zeta^{US}_{VRP}$</td>
<td>5.63***</td>
<td>-1.02</td>
<td>2.79*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(4.39)</td>
<td>(-1.17)</td>
<td>(1.91)</td>
</tr>
<tr>
<td>$\zeta^{j}_{VRP}$</td>
<td>-3.37**</td>
<td>-0.39</td>
<td>-1.82</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-2.61)</td>
<td>(-0.46)</td>
<td>(-1.52)</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>16.79</td>
<td>3.07</td>
<td>4.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^{US}_{CGVs}$</td>
<td>0.08</td>
<td>-0.06</td>
<td>-0.61*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.28)</td>
<td>(-0.18)</td>
<td>(-1.76)</td>
</tr>
<tr>
<td>$\beta^{j}_{CGVs}$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.40</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.96)</td>
<td>(0.45)</td>
<td>(1.51)</td>
</tr>
<tr>
<td>$\theta_{GCGV}$</td>
<td>0.09</td>
<td>-0.27</td>
<td>0.62*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.44)</td>
<td>(-1.30)</td>
<td>(1.90)</td>
</tr>
<tr>
<td>$\zeta^{US}_{VRPs}$</td>
<td>19.3</td>
<td>-3.07</td>
<td>24.2**</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.71)</td>
<td>(-0.40)</td>
<td>(2.27)</td>
</tr>
<tr>
<td>$\zeta^{j}_{VRPs}$</td>
<td>-0.22</td>
<td>-0.44</td>
<td>-0.15</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-0.04)</td>
<td>(-0.56)</td>
<td>(-0.11)</td>
</tr>
<tr>
<td>$\delta_{GVRP}$</td>
<td>2.88***</td>
<td>-1.34**</td>
<td>1.34</td>
</tr>
<tr>
<td>t-stat</td>
<td>(7.00)</td>
<td>(-2.56)</td>
<td>(1.23)</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>15.68</td>
<td>0.22</td>
<td>6.78</td>
</tr>
</tbody>
</table>

The top panel reports the slope coefficients and adjusted $R^2$ of the following monthly predictive regression in the period of January 2000 to December 2011:

$$rx_{t+1} = \alpha + \beta^{US}_{CGV} \times CGV_{US,t} + \beta^{j}_{CGV} \times CGV_{j,t} + \zeta^{US}_{VRP} \times VRP_{US,t} + \zeta^{j}_{VRP} \times VRP_{j,t} + \varepsilon_t$$

The bottom panel reports the slope coefficients and adjusted $R^2$ of the following regression:

$$rx_{t+1} = \alpha + \beta^{US}_{CGVs} \times CGVs_{US,t} + \beta^{j}_{CGVs} \times CGVs_{j,t} + \theta_{GCGV} \times GCGV_t + \zeta^{US}_{VRPs} \times VRP_{US,t} + \zeta^{j}_{VRPs} \times VRP_{j,t} + \delta_{GVRP} \times GVRP_t + \varepsilon_t$$

where the global consumption growth variance is a market capitalization weighted average of the local consumption growth variance,

$$GCGV_t = \sum \omega_i CGV^i_t,$$

and country-specific consumption growth variance are the error terms ($CGV^i_t = \xi^i_t$) in the following monthly predictive regression:

$$CGV_t = \lambda' + \rho' \times GCGV_t + \xi'_t$$

Robust t-statistics following Newey and West (1987), with the optimal number of lags according to Andrews (1991), are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.
Table 7: Without Observations from the 2008 Crisis

<table>
<thead>
<tr>
<th>Panel 1</th>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta_{VRP}^{US} )</td>
<td>4.19***</td>
<td>0.85</td>
<td>4.16***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(3.37)</td>
<td>(0.98)</td>
<td>(3.23)</td>
</tr>
<tr>
<td>( \zeta_{VRP}^j )</td>
<td>-2.69*</td>
<td>-1.62**</td>
<td>-0.66</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.96)</td>
<td>(-2.24)</td>
<td>(-0.86)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>5.34</td>
<td>0.64</td>
<td>6.14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 2</th>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta_{VRP}^{US} )</td>
<td>-1.01</td>
<td>4.61</td>
<td>13.7*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-0.05)</td>
<td>(0.59)</td>
<td>(1.65)</td>
</tr>
<tr>
<td>( \zeta_{VRP}^j )</td>
<td>-3.86</td>
<td>-1.52**</td>
<td>-0.03</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-0.68)</td>
<td>(-2.24)</td>
<td>(-0.03)</td>
</tr>
<tr>
<td>( \delta_{GVRP} )</td>
<td>2.00**</td>
<td>-0.34</td>
<td>3.42***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.54)</td>
<td>(-0.34)</td>
<td>(3.15)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>4.67</td>
<td>0.04</td>
<td>5.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 3</th>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta_{VRP}^{US} )</td>
<td>2.46***</td>
<td>2.46***</td>
<td>2.46***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.77)</td>
<td>(2.77)</td>
<td>(2.77)</td>
</tr>
<tr>
<td>( \zeta_{VRP}^j )</td>
<td>-1.49</td>
<td>-1.80**</td>
<td>-0.20</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.63)</td>
<td>(-2.10)</td>
<td>(-0.33)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>5.59</td>
<td>0.72</td>
<td>6.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel 4</th>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta_{VRP}^{US} )</td>
<td>7.11</td>
<td>7.11</td>
<td>7.11</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.99)</td>
<td>(0.99)</td>
<td>(0.99)</td>
</tr>
<tr>
<td>( \zeta_{VRP}^j )</td>
<td>-1.61</td>
<td>-1.00</td>
<td>-0.68</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.09)</td>
<td>(-1.12)</td>
<td>(-1.02)</td>
</tr>
<tr>
<td>( \delta_{GVRP} )</td>
<td>2.07**</td>
<td>-0.31</td>
<td>3.58***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.02)</td>
<td>(-0.26)</td>
<td>(2.94)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>6.63</td>
<td>1.98</td>
<td>7.72</td>
</tr>
</tbody>
</table>

This table reports monthly forex returns, \( r_{t+1}^{fx} \), and the predictive power of local variance risk premiums, \( VRP \), country-specific VRPs, \( VRP^s \), and the global VRP, \( GVRP \) for a sample period from January 2000 to December 2011 excluding five monthly observations during the 2008 crisis (October 2008 and two months before and after it). The third panel is without \( (w/o) \) any constraints. The bottom panel is with \( (w/) \) constraints on coefficients of \( VRP^s \) (\( \zeta_{VRP}^{US} \)) to be equal across regressions. The t-statistics are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.
This table reports the predictive power of the country-specific VRPs ($VRP^c$) and the global VRP ($GVRP$) for monthly forex returns with respect to the U.S. dollar. The local VRP data for most of these currencies are not available. The global and country-specific VRPs are estimated based on information available for the major currencies included in Table 4. The top panel includes developed non-European currencies, the middle panel includes European currencies, and the bottom panel includes developing currencies. Robust t-statistics following Newey and West (1987), with the optimal number of lags according to Andrews (1991), are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.
Table 9: Predictive Power of Currency Predictors in Forex Markets

<table>
<thead>
<tr>
<th>Panel</th>
<th>( \text{Carry}_{t+1} )</th>
<th>( \text{GBP/USD} )</th>
<th>( \text{JPY/USD} )</th>
<th>( \text{EUR/USD} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel 1</td>
<td>( \text{CRB}_t )</td>
<td>0.29**</td>
<td>0.29***</td>
<td>-0.07</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.49)</td>
<td>(3.51)</td>
<td>(-0.73)</td>
<td>(0.84)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>5.15</td>
<td>8.35</td>
<td>-0.21</td>
<td>0.67</td>
</tr>
<tr>
<td>Panel 2</td>
<td>( \text{FXV}_t )</td>
<td>-0.09***</td>
<td>-0.06*</td>
<td>0.03</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-2.59)</td>
<td>(-1.91)</td>
<td>(1.22)</td>
<td>(-1.27)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>6.21</td>
<td>4.05</td>
<td>0.50</td>
<td>1.40</td>
</tr>
<tr>
<td>Panel 3</td>
<td>( \text{GVRP}_t )</td>
<td>3.19***</td>
<td>2.87***</td>
<td>-1.42***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.99)</td>
<td>(5.19)</td>
<td>(-2.67)</td>
<td>(0.97)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>9.88</td>
<td>12.85</td>
<td>2.13</td>
<td>1.23</td>
</tr>
<tr>
<td>Panel 4</td>
<td>( \text{CRB}_t )</td>
<td>0.09</td>
<td>0.14**</td>
<td>0.03</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.72)</td>
<td>(2.05)</td>
<td>(0.33)</td>
<td>(0.36)</td>
</tr>
<tr>
<td>( \text{FXV}_t )</td>
<td>-0.04</td>
<td>-0.00</td>
<td>0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.33)</td>
<td>(-0.13)</td>
<td>(0.45)</td>
<td>(-0.62)</td>
</tr>
<tr>
<td>( \text{GVRP}_t )</td>
<td>2.23*</td>
<td>2.22***</td>
<td>-1.38**</td>
<td>0.81</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.72)</td>
<td>(2.87)</td>
<td>(-2.02)</td>
<td>(0.60)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>10.38</td>
<td>13.36</td>
<td>0.88</td>
<td>0.33</td>
</tr>
</tbody>
</table>

This table reports the predictive power of three currency predictors in the forex market. Carry is the monthly return of portfolios that short low interest rates and long high interest rates, as in Lustig et al. (2011). Monthly excess foreign exchange returns, \( r_{x,t+1}^{fx} \), are returns on buying a forward contract on GBP/USD, JPY/USD, and EUR/USD and selling it at the spot rate next month during the period from January 2000 to December 2011. CRB (Commodity Research Bureau) is the commodity return factor of Bakshi and Panayotov (2013), FXV is the forex volatility factors of Bakshi and Panayotov (2013), and GVRP is the global VRP. Robust t-statistics following Newey and West (1987), with the optimal number of lags according to Andrews (1991), are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.
Table 10: Predictive Power of Currency Predictors in Equity Markets

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel 1</td>
<td>CRB&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.20***</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>(2.60)</td>
<td>(0.21)</td>
<td>(0.21)</td>
</tr>
<tr>
<td></td>
<td>Adj. R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>4.88</td>
<td>-0.67</td>
<td>-0.70</td>
</tr>
<tr>
<td>Panel 2</td>
<td>FXV&lt;sub&gt;t&lt;/sub&gt;</td>
<td>-0.06*</td>
<td>-0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>(-1.94)</td>
<td>(-0.86)</td>
<td>(-1.01)</td>
</tr>
<tr>
<td></td>
<td>Adj. R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>4.79</td>
<td>0.43</td>
<td>-0.23</td>
</tr>
<tr>
<td>Panel 3</td>
<td>GVRP&lt;sub&gt;t&lt;/sub&gt;</td>
<td>2.53***</td>
<td>2.70**</td>
<td>1.69**</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>(6.05)</td>
<td>(2.47)</td>
<td>(2.12)</td>
</tr>
<tr>
<td></td>
<td>Adj. R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>13.08</td>
<td>3.00</td>
<td>1.21</td>
</tr>
<tr>
<td>Panel 4</td>
<td>CRB&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.04</td>
<td>-0.20</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>(0.54)</td>
<td>(-1.35)</td>
<td>(-1.20)</td>
</tr>
<tr>
<td></td>
<td>FXV&lt;sub&gt;t&lt;/sub&gt;</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>(-0.59)</td>
<td>(-0.47)</td>
<td>(-0.21)</td>
</tr>
<tr>
<td></td>
<td>GVRP&lt;sub&gt;t&lt;/sub&gt;</td>
<td>2.17***</td>
<td>3.27**</td>
<td>2.16**</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>(3.42)</td>
<td>(2.31)</td>
<td>(2.04)</td>
</tr>
<tr>
<td></td>
<td>Adj. R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>12.35</td>
<td>2.62</td>
<td>0.44</td>
</tr>
</tbody>
</table>

This table reports the predictive power of three currency predictors for cross-country excess equity return differentials. The sample period is from January 2000 to December 2011. CRB (Commodity Research Bureau) is the commodity return factor of Bakshi and Panayotov (2013), FXV is the forex volatility factors of Bakshi and Panayotov (2013), and GVRP is the global VRP. Monthly equity premium differentials, \(d\pi_{r,t+1} = \pi_{r,t+1} - \pi_{r,t+1}^\prime\), are cross-country differences between the excess returns of equity indices, including S&P 500 (US), FTSE 100 (UK), Nikkei 225 (Japan), and DAX (Europe). Equity premiums are in domestic currencies. I report the slope coefficients (δ) and adjusted R<sup>2</sup> of the following predictive regression:

\[
d\pi_{r,t+1} = \alpha + \delta_{GF} \times GF_t + \varepsilon_t,
\]

where \(GF \in \{CRB, \, FXV, \, and \, GVRP\}\). Robust t-statistics following Newey and West (1987), with the optimal number of lags according to Andrews (1991), are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.
Table 11: Out-of-Sample $R^2$

<table>
<thead>
<tr>
<th>GBP/USD</th>
<th>JPY/USD</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2_{OOS}$ (%)</td>
<td>17.06***</td>
<td>-2.67</td>
</tr>
</tbody>
</table>

This table reports the cumulative $R^2$ out-of-sample as in Goyal and Welch (2008). In each period, the one month out-of-sample $R^2$ is calculated as

$$R^2_{OOS} = 1 - \frac{MSE_{VRP}}{MSE_{HM}},$$

where

$$MSE_{VRP} = \frac{1}{T - s_0} \sum (r^F_{t+1} - \hat{r}^F_{t+1})^2,$$

$$MSE_{HM} = \frac{1}{T - s_0} \sum (r^F_{t+1} - \hat{r}^HM)^2.$$

$MSE_{VRP}$ is the mean squared error of the out-of-sample prediction using the global and the country-specific VRPs, $MSE_{HM}$ is the mean squared error of the historical sample mean, $T$ is the time period at the end of sample, $s_0$ is the time period at the end of initial or training sample, $r^F_{t+1}$ is the monthly excess exchange return, $\hat{r}^F_{t+1}(= \hat{\zeta}_{VRP}^{US,t} + \hat{\zeta}_{VRP}^{j,t} + \hat{\delta}_{GVRP})$ is out-of-sample estimation of the monthly excess exchange return using the global and country-specific VRPs, and $\hat{r}^HM$ is the historical mean of monthly excess exchange returns.

I evaluate the statistical significance of the out-of-sample R-squares by using the MSE–F statistic proposed by McCracken (2007),

$$MSE_F = (T - s_0) \left( \frac{MSE_{HM} - MSE_{VRP}}{MSE_{VRP}} \right).$$

I use a 5-year initial sample from January 2000 to December 2004. The 5-year rolling $R^2_{OOS}$ is calculated from January 2005 to December 2011. Asterisks denote significance of the out-of-sample MSE–F statistic of McCracken (2007). ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.