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# ENDOGENOUS GROWTH WITH PUBLIC FACTORS AND HETEROGENEOUS HUMAN CAPITAL PRODUCERS[*]

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If government revenues from a flat–rate income tax are spent on public factors and public factors are used for human capital production and human capital is used for the production of technical progress, then a higher rate of taxation will lead to a higher rate of technical progress if steady states are not unstable. If human capital producing households have different abilities they will have different desired (Lindahl) tax rates and a golden rule is no longer an acceptable welfare function. Therefore tax policy determines the rate of technical progress without a generally accepted welfare function. People with lower abilities want lower tax rates at least in the short run. When the rate of time preference is larger than the rate of technical progress people with greater abilities want higher levels of public factors and taxes also in the long run if indirect marginal utility is inelastic with respect to net income. The outcome of this political decision will determine the rate of technical progress.
JEL.–Class.: O41, H11, H5; Growth, technical change, public factors.

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# Endogenous Growth with Public Factors and Heterogeneous Human Capital Producers

## 1. Introduction

Evaluating empirical evidence the World Bank (1990, 1991), Larre and Torres (1991), Stern (1991), Reynolds (1983), Hughes (1982) and Adelman (1980) have put much emphasis on education, health and infrastructure as a justification for a strong role of government in growth and development policy because of their public goods properties. An early non–formal theoretical justification for government interference was given by T.W. Schultz (1964). He argued that technical progress depends on human capital and the production of human capital requires public factors such as basic education and basic scientific research; because the provision of public factors is opposed in the political sphere the levels of human capital and technical progress are very low. In the development literature this opposition to the provision of public factors is often discussed under the topic of "tax resistance" (see, e.g., Mutén, 1985).

This paper provides a formal model to interpret the theory of T.W. Schultz. In the contributions to the literature, at least one of the essential elements, technical progress, heterogeneous (human capital) production functions, public factors and taxation, is absent.

The relation between technical progress and human capital has been modelled in different ways by Lucas (1988), Romer (1990) and Ziesemer (1991). In these contributions public factors are absent. Endogenous technical progress and taxes are related in Mino (1989), Lucas (1990) and Rebelo (1991) but in these papers government spending does not enter (marginal) utility or production functions. Public factors in growth models can be found in Shell (1967), Romer (1986), Barro (1990) and Ziesemer (1990). In the latter two models with publicly provided public factors there is no production of technical progress. In Romer (1986) with privately provided public factors there is no technical progress without externalities. In Shell (1967) and Barro (1990) users of public factors have identical production functions and therefore public factors do not produce the problems of defining welfare functions and designing tax systems. However, the literature marrying endogenous growth theory with the theory of endogenous policy has dealt with such problems (see Persson and Tabellini, 1992, Perotti, 1992, and the text below for brief surveys). To locate the contribution of this paper within the body of literature on endogenous growth with public economics features we briefly discuss this literature and summarize the discussion in Scheme 1.

Scheme 1

Endogenous Growth Models in Public Economics

| Public Investment→<br>↓ Individuals | yes | no |
|---|---|---|
| are identical in | Shell 1967<br>Barro 1990<br>Sorensen 1993 | Mino 1989<br>Rebelo 1991<br>Romer 1986, 1990<br>Lucas 1988, 1990<br>Ziesemer 1991<br>Trostel 1993<br>Justman 1992 |
| differ in endowments in | Saint-Paul&Verdier 1993<br>Alesina&Rodrik 1992<br>Glomm&Ravikumar 1992 | Perotti 1990<br>Creedy&Francois 1990 |
| differ in abilities in | this paper | Persson&Tabellini 1991 |
| differ in endowments and abilities in | Ziesemer 1990 | |

The contributions in this field can best be distinguished with respect to two properties. Firstly, individuals in the various models are either identical or heterogeneous; if they are heterogeneous they either differ with respect to initial endowments or with respect to parametrically given abilities or both. This differentiation goes across rows in Scheme 1. Secondly, models contain either public investment or they don't. This differentiation goes across columns in Scheme 1. The reasons for putting the papers into their respective categories are as follow.

In Romer (1986) and (1990), Lucas (1988) and Ziesemer (1991), individuals are identical and there are no public investments. Lump–sum taxes are raised to correct for private externalities; in Lucas (1988) this is only implicit. In Mino (1989), Lucas (1990), Rebelo (1991) and Trostel (1993) individuals are identical and there is no public investment. Distortionary taxes are raised to finance some government consumption (or transfers in Trostel's paper) that enters neither the utility nor the production functions. Lucas (1990) and Trostel (1993) analyze tax reform proposals. In Justman (1992) indivisible infrastructure is supplied by a regulated monopolistic private firm.

In Shell (1967) users of a public, non–rivalrous stock of technology are identical. A flat–rate income tax is raised to finance the change in the public stock of knowledge. The expenditure effect of the tax increases growth but the distortion through capital income taxation decreases it (see also Grossman and Helpman, 1991, Chap. 2.4). In Barro's (1990) models identical firms use non–cumulated government factors which are also financed by a flat–rate income tax with the same properties as in Shell's paper. Sorensen (1993) extends Lucas' (1988) model to include public investments which are financed by capital and labour income taxation as well as tuition fees.

In Creedy and Francois (1990) households differ in initial incomes which also determine an individual's productivity in forming human capital. Education is paid for partly privately and in part by the government who finances the subsidies through a flat–rate income tax. The larger the portion payed by the government, the greater the number of people who get an education and the higher the growth which benefits all in the second period. Even the median voter who receives no education and therefore no subsidy, may vote for redistributive taxes because he benefits from the growth effect generated by other peoples investment in human capital. The tax revenues are also used for other government expenditures which enter neither the utility nor the production functions. No public factors are considered and the model is constructed only for two periods. Perotti (1990) considers individuals who differ in their initial pre–tax incomes. There are no public investments. In order to pay for indivisible investment in education without access to a capital market, subsidies which are equal across individuals paid out of income tax revenues may be needed to give poor families access to finance and education. A median voter chooses an optimal tax rate which depends both on his income and on the average income of the population because the latter determines the value of the subsidies received in the same period. Depending on the income of the median voter and the average value of income in the economy this redistribution may be conducive or damaging to growth.

Alesina and Rodrik (1992) modify the model of Barro (1990) to allow for households differing in their initial holdings of capital and taxation of capital, a higher rate of which generates lower growth. The more capital poor is an individual, the higher his preferred tax on capital. A poorer median voter therefore generates lower growth and in a democracy income inequality is unfavourable for growth rates. In Glomm and Ravikumar (1992) individuals differ in their initial values of human capital. In the public education regime of their model, revenues from a flat–rate tax on human capital income are spent on the quality of public education which is an argument in the utility function. As the latter are of the additively separable type, all individuals prefer the same tax rate. In Saint–Paul and Verdier (1993) households differ in their initial endowments of human capital. They pay a flat–rate income tax, the revenues of which are spent on public education which is not a public good but an egalitarian supply of education. This supply is a non–cumulative factor that benefits all individuals identically because it enters the production function for human capital in an additively separable way. Therefore poorer people will prefer higher tax rates. When human capital levels converge, preferred tax rates converge as well. The impact of political rights, democracy and inequality on growth are analyzed.

In Persson and Tabellini (1991) individuals differ in their parametrically given abilities. A flat–rate income tax is raised, the revenue of which is rebated at equal amounts to all households. A higher rate of taxation yields lower growth because of the distortion of capital accumulation. The higher the abilities of the median voter the lower the tax he prefers and the higher the growth rate will be.

Different endowments and abilities have been treated in Ziesemer (1990). Having more capital provides a disincentive to higher taxation but higher abilities make public factors more desirable. A median voter wants higher taxes than the average individual. The growth effects of a higher tax rate are transitional. In the long run only the level of the growth path can be changed because, as in Arrow's (1962a) learning by doing model, the rate of growth is proportional to that of population growth. This is in accordance with Koester and Kormendi (1989) who find in a cross–country study, that tax rates have only level effects but no growth rate effects. However, that study is only loosely related to growth theory and in view of the arguments presented in the theory and empirics of the political–equilibrium literature (see Persson and Tabellini 1992, and Alesina and Rodrik, 1992) and their time–series character, the question is whether or not this is the final word on the subject. The literature discussed here tries to improve the basis for such investigations through endogenization of the growth rate.

From the state of the art discussed so far it is quite straightforward to look at the growth rate effects of changes in tax rates when individuals have different levels of appreciation of public factors due to different abilities. Scheme 1 indicates that this has not been done. It therefore will be the subject of this paper. It provides a model which combines all the necessary elements contained in Schultz' theory. Households have different production functions to produce human capital using public factors and therefore have conflicting views on individually optimal taxation. Human capital is used in the production of output and technical progress. Therefore this is the first model of growth with endogenous technical progress that takes the standard justification for government activity – public goods or factors in individually different utility or production functions – explicitly into account. This changes the source of the distributional conflict in the literature. In the literature discussed above the source of distributional conflict is the taxation of people with different endowments, with the exception of Glomm and Ravikumar where the desired tax rates are identical and the distributional conflict lies in the choice of the public or private education system. Moreover, in Creedy and Francois only people getting an education receive the subsidy. In this paper they have different tax burdens and they benefit unequally from public investment in public factors. Tractability of the model depends on the neglect of capital income. However, the contributions of Shell, Barro, Alesina and Rodrik, Saint–Paul and Verdier, Lucas (1990) and Trostel, all cited above, make it rather unlikely anyway that something new on capital could be learned. Whenever the results would have to be modified in view of this knowledge, we indicate this only verbally and the integration of capital is left to future research.

The results are that

i) at sufficiently low rates of taxation there are two steady states one of them saddle point stable and the other unstable; for higher rates of taxation the existence of a unique, no or more than two steady state(s) is also possible;

ii) under a higher flat–rate income tax, which is excess burden free in this simple model, the rate of technical progress at the saddle point stable steady state will be higher;

iii) a median voter will prefer a lower steady state rate of taxation than that of a golden rule if he has lower income and indirect marginal utility from net income is inelastic, implying lower technical progress as well;

iv) under each allocation there are individuals who have higher or lower preferred (Lindahl–) tax rates than the actual one because they have higher or lower abilities, implying that there is no optimal tax rate unless perfect government information would allow perfect compensation which is equivalent to setting Lindahl prices and is generally held to be infeasible because the informational requirements would be equivalent to those of central planning. As a consequence the rate of technical progress and other growth rates are explained as the outcome of a political distributional conflict and not as a result of different parameters or different government choices of taxes and subsidies that merely convert x–best into first best growth paths (see Barro and Sala i Martin, 1992, for a discussion of this type of policy) and not as a result of unfavourable constellations of time preference and marginal productivity of knowledge due to low initial values of knowledge which produce a lock–in (see Becker, Murphy and Tamura, 1990). The redistributional effects may be different once capital income is introduced (see section 5) or if indirect marginal utility is strongly decreasing in net income (see section 6).

The structure of the paper is as follows. In the next section the basic model is set up. In section 3 decisions of individuals are presented. Market equilibrium and a definition of the steady state are given in section 4. The existence, multiplicity and stability of steady states and the impact of taxation on the steady state allocation are the subject of section 5. Section 6 shows that people with higher abilities and low (high) elasticity of indirect marginal utility with respect to net income have higher (lower) shadow prices for public factors and technology if they act as selfish persons in the position of the government (in a dual approach to a government decision where they maximize their indirect utility function) and the discount rate is higher than the rate of technical progress; these higher (lower) shadow prices lead to a desired decision of higher (lower) tax rates. However, the tax rate chosen is a matter of how the distributional conflict implied by the infeasibility of Lindahl prices is solved. One of these possible outcomes is that of a median voter democracy.

## 2. A model with three forms of knowledge

The crucial factor in Schultz' theory is human capital H, which is defined as the knowledge people have personally as a result of schooling. It is assumed here that human capital and labour $L_1$ are the factors which produce output Y under conditions of labour saving technological progress with level $A(\tau)$ at time $\tau$, the firm specific knowledge. The production function used here is of the Cobb–Douglas type:

$$Y = F(H, AL_1) = H^{\beta}(AL_1)^{1-\beta} \tag{1}$$

Increases in firm specific knowledge, $\dot{A}$, are assumed to be produced using human capital and the accumulated knowledge A itself. However, knowledge has to be transferred from those who produced it (human capital) in the research division to those who use it (labour $L_1$) in the production division. The more workers there are in the production division the more human capital will be needed to transfer the knowledge and the lower the productivity of human capital in the production of knowledge. Therefore the number of workers exerts a negative influence on the production of firm–specific knowledge. The production and transfer function for firm–specific knowledge is assumed to be linearly homogenous in $H/L_1$ and A (a dot on a variable indicates a derivative with respect to time):

$$\dot{A} = G(H/L_1, A) = g(h)A \quad \text{with} \quad h \equiv H/AL_1, \lim g = \bar{g} < \infty \tag{2}$$

$$G(0, A) = G(H/L_1, 0) = G_1(H/L_1, 0) = G_2(0, A) = 0,$$

$$G_1, G_2 > 0 \text{ for } H/L_1 > 0 \text{ and } A > 0, G_1(0, A) < \infty, G_{11} < 0, G_{21} > 0.$$

It is assumed that the factors used in innovation are the same as those in production, an

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assumption indicating that close cooperation between production and innovation is advantageous for success in providing the production division with adequate technical progress. Freeman (1988, p. 335) and Mowery and Rosenberg (1989, chap. 6) report research results on the higher efficiency of closely cooperating production and research divisions which in Japanese firms sometimes goes so far that the divisions are almost indistinguishable. Human capital is a crucial factor in innovation because a good education is a precondition for good ideas in research, whereas capital (ignored here) and labour can contribute only indirectly (see also Nelson, 1959 on this point; for a different interpretation see footnote 1.). Making use of $H/L_1$ as an argument is mainly done to ensure zero profits and a constant rate of technical progress independent of exogenous employment growth as in Uzawa (1965), Lucas (1988), Neumann (1989) and Ziesemer (1991); the latter result cannot be obtained in models that use specifications like Arrow (1962a), Phelps (1966) or Shell (1967). This specification can be justified as follows: The relation between F and G could best be interpreted as follows: One could think of H used in technology production, G, and exerting a positive externality on output production F when transferring the knowledge to the production division, which in turn hinders technology production further the more workers $L_1$ have to take over the new knowledge thus exerting a negative externality on G as explained above.

The ultimate reason for dividing H by $L_1$ is nevertheless technical and makes use of some degree of freedom which always remains in the specification of technology production functions. There is nothing in the literature which excludes any of the diverse specifications used in different models. The upper limit on g, $\bar{g}$, ensures that $\dot{A}$ cannot go to infinity and therefore A cannot jump as it would, e.g., under a Cobb–Douglas specification.

Human capital is produced by a constant number of families (i = 1,..., N) using labour services $L_2^i$, basic scientific research results B, which are public factors, e.g., the information contained in libraries, different parametrically given labour augmenting abilities $e^i$ and the knowledge A of the firm where he works. Abilities $e^i$ can stem either from genetic inheritance or socialization or both. As they are exogenous their explanation has no impact on the argument of this paper. Human capital is completely depreciated each period because – unlike basic scientific research results and firm specific knowledge, which can be inherited in written form – it cannot be handed over to the next generation if an individual dies, because $H^i$ is contained in his brain. Moreover, human capital must always be updated by the latest research results to be able to have an idea for inventions. Complete depreciation is the simplest way to capture both arguments.[2] It has the advantage of limiting the number of differential equations to a minimum. Taking it literally it implies that each individual lives one period, i.e. infinitely short lived individuals instead of infinitely long lived individuals often used in this type of model. The production function for a family or clan which contains these assumptions is

$$H^i = H\,(e^i A L_2^i,\ B) = (e^i A L_2^i)^{\alpha} B^{1-\alpha} \qquad i = 1,\ldots,N \tag{3}$$

B is public knowledge and A is knowledge costlessly taken over from the firm simply because family members work there and there is no market for services of A by assumption. Therefore A is an externality here. As long as the firm is a price taker it cannot internalize this externality. "A" in (3) poses therefore a similar public goods problem as B. The difference in the treatment is that the change in the stock of B is provided publicly whereas changes in A are provided privately and, as in Romer (1986) it is assumed that no public action is taken to influence its supply. This role of A also mirrors the fact that there is not always a clear dividing line between private and public knowledge in the real world. Using the Cobb–Douglas function implies an assumption that without public knowledge no production of human capital is possible.

The stock of public knowledge B is enhanced at some costs that are covered by taxes on households income $Y^i$ which sum up to Y:[3]

$$\dot{B} = tY \tag{4}$$

Basic education in practice has an excludability property. However, this is due to the transaction costs involved in its transfer to pupils. The knowledge itself is non–rivalrous and we concentrate on its public factors character here. Equilibrium in the human capital market requires that firm demand H equals households supply, i.e. the sum over all $H^i$:

$$H = \sum_{i=1}^{N} H^i \tag{5}$$

Labour supply, which may differ in levels across families, is assumed to have identical growth rates $\epsilon$

$$L^i(\tau) = L^i(0)e^{\epsilon\tau}$$

A families labour endowment can either be supplied to the firm, $L_1^i$, or used for education, $L_2^i$:

$$L^i = L_1^i + L_2^i$$

The sum over families' labour supply is total labour supply

$$L(\tau) = \sum_{i=1}^{N} L^i(0)e^{\epsilon\tau}$$

which in equilibrium equals labour demand of the firm, $L_1 = \Sigma L_1^i$, plus households own demand, the sum over $L_2^i$:

$$L = L_1 + \sum_{i=1}^{N} L_2^i \tag{6}$$

Due to the simplifying assumption of complete depreciation of $H^i$, it is used in (2) at the same moment when it is produced by $L_2^i$ persons according to (3).

(1) – (6) contain some basic assumptions of the model (those on behaviour follow in the next section) with three types of knowledge supplied by firms, households and the government respectively as represented by equations (2) – (4). In Uzawa (1965) "A" was interpreted to contain all three types of knowledge. Many papers still only deal with one of these forms of knowledge, although in all developed societies it is obvious that all of these are present to quite a large extent. The distinction however is crucial, because household knowledge vanishes with the death of persons, whereas firm and public knowledge can survive in written form but have different properties concerning appropriability (see also Arrow, 1962b and Dosi, Pavitt and Soete, 1990, p.82/83, on this point). Schultz emphasized the role of the household as a human capital producing firm and the role of public factors in producing it and the role firm technology in using it. Thus this distinction is contained in his work although in an unformalized way. What has been added here is the tax resistance argument replacing the role of landowners in Schultz' (1964) argumentation, because the role of agriculture may diminish in the long run whereas the problems of distributional conflict from families' different benefits from public goods or factors and redistributional taxation remain. The public provision of the public factors is only public with respect to their financing but not necessarily so with respect to ownership of knowledge producing organizations.

## 3. The dynamic entrepreneur and the household as a human capital producing firm

The representative firm is assumed to maximize profits over an infinite horizon, discounted at some rate $\rho$ subject to (1) and (2) while acting as a price–taker. The latter assumption is made here because it seems to be still the simplest of all market structures used in models which include production of technical progress. All households are assumed to have the same discount rate $\rho$. These assumptions lead to the current value Hamiltonian

$$\Pi = F(H, AL_1) - qH - wL_1 + \phi\, G(H/L_1, A)$$

where w is the wage rate and q the price for the use of a unit of H and $\phi$ a costate variable. The necessary conditions for a maximum are (2) and

$$\partial\Pi/\partial H = F_1 - q + \phi G_1/L_1 = 0 \tag{7}$$

$$-\partial\Pi/\partial A = -F_2L_1 - \phi G_2 = \dot{\phi} - \rho\phi \tag{8}$$

$$\partial\Pi/\partial L_1 = F_2A - w + \phi G_1H(-1)L_1^{-2} = 0 \tag{9}$$

$$\lim_{\tau\to\infty} e^{-\rho\tau}\,\phi = 0 \tag{10}$$

A lower index 1 or 2 in connection with functions F, G and H indicates the derivative with respect to the first or second argument. (7) means that the marginal product of human capital in output production, $F_1$, plus that in production of technical progress, $\phi G_1/L_1$, must equal the rental q of human capital. (8) says that the marginal product of a stock unit of A on output, $F_2L_1$, plus that on technology production, $\phi G_2$, must equal the negative of the current value of the rate of change of the discounted shadow price, $\phi e^{-\rho\tau}$. (9) tells that the marginal product of labour in output production, $F_2A$, plus its (negative) marginal product in technology production, $\phi G_1 H(-1) L_1^{-2}$, must equal the wage rate. Finally, (10) means that the discounted shadow price of technology must go to zero as time goes to infinity. As (8) implies $\hat{\phi} - \rho < 0$, (10) holds.

Multiplying (7) by H and (9) by $L_1$ and replacing w and q in the Hamiltonian, yields zero current profits. The value of the firm is the value of technical change. Here price taking behaviour inspite of increasing returns in H, A and $L_1$ is possible because the function F is linearly homogeneous in the control variables which leads to a horizontal cost function at each point in time which is driven down over time through the change in technology A.[4,5]

Assuming the absence of a capital market and complete depreciation of human capital both for the mere sake of simplicity, households have no intertemporal consumption allocation problem and therefore can be viewed as pure human capital producers without making any assumption about how long they live. The households' problem then is a purely static one. They are assumed to maximize consumption (equal to after tax income), $(1 - t)\, Y^i = (1 - t)[qH^i + w(L^i - L_2^i)]$, equating the marginal value product of human capital to wages:

$$w = q\, H_1^i e^i A \tag{11}$$

Applying Euler's theorem to (3), insertion of w/q according to (11) and division by B yields

$$H^i/B = [(w/A)/q]AL_2^i/B + H_2^i$$

INSERT FIGURE 1 OVER HERE

This is drawn as a tangent to the production functions in Figure 1 with slope w/q and vertical intercept $H_2^i$ , which indicates the families willingness to pay per unit of public factors which by assumption is unknown to the government and therefore makes Lindahl prices impossible. Families choose the same $H^i/L_2^i$ ratios due to the labour augmenting differences $e^i$ in families abilities, which implies that families with higher abilities choose higher $L_2^i$ and $H^i$ and have higher willingness to pay, $H_2^i$, per unit of public factors. Putting it the other way round, a higher level of public goods and a higher rate of taxation is more in the interest of people with higher abilities. However, if the $e^i$ are perceived to be constant for eternal time preferred tax rates have to consider the long run as well (see section 6).

As the $H_1^i e^i A$ in (11) are identical for all households, because they are all faced with identical market prices w/q, the marginal products of A, $H_1^i e^i L_2^i$, will be higher for families with higher abilities who choose higher $L_2^i$. Again Lindahl–prices would be necessary to reach an optimum. Therefore A can be viewed as a privately supplied public good, whereas B is a publicly provided one (in the sense of financing through taxation).

## 4. Market equilibrium and definition of the steady state

In this section market equilibrium conditions are analysed by eliminating market prices and steady states are defined.

As all individuals choose the same ratio $H^i/L_2^i$ due to the labour saving property of $e^i$ in (3) this must equal the aggregate ratio $H/L_2$:

$$H/L_2 = H^i/L_2^i \tag{12}$$

Insertion of (3) on the right–hand side, division of both sides by A, multiplication by $L_2/L_1$ and (using $h \equiv H/AL_1$, $l_1 \equiv L_1/L$, $b \equiv B/AL$) division by $l_1/(1 - l_1)$ yields the modified human capital production function

$$I_1 \equiv h - H\left(\frac{1-l_1}{l_1} e^i, \frac{b/l_1}{L_2^i/L_2}\right) = 0 \qquad (12')$$

Increasing $l_1$ reduces H per unit $AL_1$ whereas higher b increases it. As h is a macro–variable the value of the H(.) function must be identical for all individuals. $L_2^i/L_2$ must be constant for the following reason. $\hat{L}_2^i$ must be identical for all individuals because with constant elasticity of production $\alpha$ for the first argument of (3) one may write (3) in growth rates using (12):

$$(\hat{H}^i - \hat{L}_2^i =)\ \hat{H} - \hat{L}_2 = \alpha(\hat{A} + \hat{L}_2^i) + (1-\alpha)\hat{B} - \hat{L}_2^i$$

As only the $\hat{L}_2^i$ in the second equation are variables concerning the individual, the $\hat{L}_2^i$ are equal for all i. Together with $\sum_{i=1}^{N} L_2^i/L_2 = 1$ it follows that $\hat{L}_2^i = \hat{L}_2$. Therefore $L_2^i/L_2$ in (12') is constant, although dependent on $e^i$.

Equating w/q from (11) with its value from (7) and (9) yields:

$$H_1^i e^i A = \frac{F_2 A - \phi G_1 H L_1^{-2}}{F_1 + \phi G_1/L_1} \qquad (13)$$

The marginal product of labour in human capital production on the left side of (13) must equal the ratio of the marginal products of labour and human capital in output and technology production on the right side of (13). As $\alpha$ is the constant elasticity of the first argument of (3), the left side of (13) may be written as $\alpha H^i/L_2^i$. As all households choose the same ratio $H^i/L_2^i$ one may again replace it by $H/L_2$. Therefore the left side of (13), after division by A, may be written as $\alpha H/(AL_2) = \alpha h l_1/(1-l_1)$. Defining $\psi \equiv \phi/L$ and replacing

$\phi$ by $\psi L$, the result of all these transformations is

$$I_2 \equiv \frac{F_2 - \psi G_1 h/l_1}{F_1 + \psi G_1/l_1} - \alpha h l_1/(1 - l_1) = 0 \qquad (13')$$

(13') has the same interpretation as (13) and was given there. (12') and (13') are two equations in h and $l_1$ yielding a solution $h = h(b, \psi)$ and $l_1 = l_1(b, \psi)$ which can be used in differential equations for b and $\psi$, which we consider next.

Dividing (4) by B and subtracting the natural rate of growth $g(h) + \epsilon$ and multiplying by b yields

$$\dot{b} = tl_1F(h, 1) - [g(h) + \epsilon]b \ ( = 0 \equiv J_1 ) \qquad (4')$$

Finally, using $\phi \equiv \psi L$ in (8) the system is completed by

$$\dot{\psi} = [\rho - \epsilon - G_2(h)]\psi - F_2 l_1 \ ( = 0 \equiv J_2) \qquad (8')$$

$\dot{b} = 0$ and $\dot{\psi} = 0$ define the steady state. One can consider $J_1$ and $J_2$ as a system in the two variables b and $\psi$ with h and $l_1$ depending on b and $\psi$ according to equations $I_1$ and $I_2$. In the steady state $I_1$, $I_2$, $J_1$ and $J_2$ are four equations for the four variables h, $l_1$, b and $\psi$, all of them depending on t, $\epsilon$ and $\rho$. This will be used in the following analysis of the impact of a change in the rate of taxation on the four variables.

5. Existence, multiplicity and stability of steady states and the economic effects of reducing tax resistance

The reduction of tax resistance is defined here as $dt > 0$. In the following we proceed in three steps. Firstly, we show for given $l_1$ (for merely didactical purposes) that a higher rate of taxation leads to higher values of public knowledge per labour efficiency unit, b, and higher human capital per efficient labour unit used in the firm and therefore to higher technical progress. Secondly, we show that there exists either no steady state or at least two steady states, one of which supports the first result but the other does not. Thirdly, we show in the analysis of the dynamics that the steady state at which the first result holds is stable in the saddle point sense whereas the other is unstable.

In the first (merely didactical) step $J_1$ is solved for b which yields

$$b = \frac{tF(h, 1)l_1}{g(h) + \epsilon} \tag{4''}$$

The interpretation is that c.p. a higher tax rate t leads to higher steady state levels of public factors per efficiency unit of labour. A higher share of employment in output production, $l_1$, also leads to higher b. Higher employment growth leads c.p. to lower steady state levels of public factors per efficiency unit, which is quite analogous to the Solow growth model. The c.p. impact of higher h on b is analyzed as follows. (4'') is drawn as $J_1$ in Figure 2. It's slope is

INSERT FIGURE 2 OVER HERE

$$\partial b/\partial h = \frac{(g+\epsilon)F't - g'tF}{(g+\epsilon)^2} l_1 = \frac{\epsilon F't + h^{-1}gtF(F'h/F - g'h/g)}{(g+\epsilon)^2} l_1 > 0 \text{ if } F'h/F \geq g'h/g$$

This condition is sufficient to ensure a positive slope. The interpretation is that h always has a stronger influence on output than on technical progress and therefore leads to higher steady state values of the public factor stock per labour efficiency unit. Taking the second derivative of (4'') with respect to h it can be shown that

$$\partial^2 b/(\partial h)^2 < 0 \quad \text{if} \qquad (F'h/F)/(g'h/g) > |g''h/g'|/|F''h/F'|$$

Therefore b(h) is concave if (sufficient) [for $\beta = F'h/F$, $\gamma(h) \equiv g'h/g$, $\beta - 1 = F''h/F'$ and $\gamma(h) - 1 = g''h/g'$, all from the Euler theorem applied to f,g and F',g' respectively]

$$(1-\beta)\beta > \gamma(1-\gamma)$$

is fulfilled. This means that elasticities of production in output must be more balanced than those for technology production. For each given h, a higher t (higher $\epsilon$) requires higher (lower) levels of b which is indicated by $J_1'$ drawn as a dashed line in Figure 2. Higher $l_1$ shifts the curve to the right.

The second curve drawn in Figure 2 is $I_1$ [equation (12')], the modified production function for human capital production. Here h is a concave function of b for each given $l_1$. A higher $l_1$ decreases both arguments in the H function of $I_1$ and therefore shifts the curve down, which is drawn as the dashed $I_1'$ curve in Figure 2. A higher $l_1$ therefore leads to a lower (higher) h and b if the $l_1$–shift in $I_1(J_1)$ is stronger and therefore to lower (higher) rates of technical progress. Here the increase of $l_1$ with respect to the $I_1$–curve represents the impact of the labour allocation on higher human capital production which leads to a decrease in the rate of technical progress. The effect of higher $l_1$ on the $J_1$–curve mirrors the impact of the allocation of labour on output which increases b and therefore h. Which of these two effects is stronger is an open question. Formally, $I_1$ and $J_1$ determine h and b depending on $l_1$, t and $\epsilon$:

$$h = h(t, l_1, \epsilon) \text{ and } b = b(t, l_1, \epsilon) \text{ with } \partial h/\partial t,\ \partial b/\partial t > 0 \text{ and } \partial h/\partial\epsilon,\ \partial b/\partial\epsilon < 0$$

Existence of an intersection of $I_1$ and $J_1$ for each $l_1 \epsilon (0,1)$ is guaranteed by the assumption that F and H are both Cobb–Douglas functions. This guarantees that the curvature is sufficiently strong to let the functions intersect.

Figure 2 gives a preliminary formulation of Schultz' theory: Given $l_1$, a higher value of taxation t leads to a higher value of public factors b and human capital h, both in labour efficiency units, and therefore to a higher rate of technical progress. So what remains to be derived are conditions that movements in $l_1$ do not change this result.

The following analysis is less intuitive but more exact because $l_1$ is no longer kept constant. The condition of more balanced elasticities of production in output than in technology production will not be needed here. In the next step we take also $I_2$ and $J_2$ into account to show through elimination of $\psi$ and b that steady state values for h and $l_1$ are not unique.

Solving $J_2$ for $\psi$ and inserting it into $I_2$ yields

$$I_2 \equiv \frac{F_2 - \dfrac{F_2 l_1}{\rho - \epsilon - G_2(h)} G_1 h / l_1}{F_1 + \dfrac{F_2 l_1}{\rho - \epsilon - G_2(h)} G_1 / l_1} - \alpha h l_1/(1 - l_1) = 0 \qquad (13'')$$

The interpretation was already given in connection with equation (13). In the first term the $l_1$ terms can be cancelled. Multiplying successively by the denominator of the large fraction, $1/F_2$ and $(1 - l_1)$ we obtain after some simple rearrangements

$$1 - l_1 - \frac{G_1(h) h}{\rho - \epsilon - G_2(h)} (1 - l_1 + \alpha l_1) = (F_1 h / F_2)\, \alpha l_1 \qquad (13''')$$

Using the Cobb–Douglas function assumption $\beta/(1 - \beta) = F_1 h / F_2$, it can be shown that this equation is a falling line in the h–$l_1$–plane that intersects both axes as drawn in Figure 3 (see appendix B.1 for a derivation). The $l_1$ axis is intersected at value

$l_1 = (1-\beta)/(1-\beta+\alpha\beta) < 1.$

INSERT FIGURE 3 OVER HERE

The second curve of Figure 3 – which approaches the vertical axis as $l_1$ goes to zero – is based on (12') after insertion of (4'') to eliminate b:

$$I_1 = h - H\{\frac{1-l_1}{l_1}e^i, \frac{tF(h)}{[g(h)+\epsilon]L_2^{\frac{1}{2}}/L_2}\} = 0 \tag{12''}$$

The slope of h with respect to $l_1$ is

$$dh/dl_1 = \frac{-e^i H_1/l_1^2}{1-(1-\alpha)[\beta - \frac{\gamma}{1+\epsilon/g(h)}]} < 0$$

with $\gamma(h) \equiv g'h/g < \beta \equiv F'h/F$. For $l_1 \to 0$ we have $h \to \infty$ and $dh/dl_1 \to (-\infty)$ (for a proof see appendix B.2). Moreover, (12'') exists on the whole intervall $l_1 \epsilon [0, 1]$ and as $l_1$ is one, h is zero. Therefore in Figure 3 (12'') cuts (13''') twice or not at all. The first cut at low values of $l_1$ is such that (12'') comes from above and because (13''') ends at $l_1 < 1$ but (12'') goes to $l_1 = 1$ there is a second cut from below. In Figure 3 this case is drawn. An exceptional case could be that the curves are tangential to each other, yielding a unique solution. A higher rate of taxation t shifts (12'') upwards and to the right because it requires a higher value of $l_1$ (or h) for any given value of h (or $l_1$). In the limit for $t \to 0$ it follows from (12'') that h goes towards zero for all $l_1$. This implies that there are always sufficiently low tax rates at which two steady states exist whereas the case of a unique or no steady state at high tax rates cannot be proven but also not excluded. These unproven cases will only be treated in appendices B. If there are two steady states, the impact of t on h is that h decreases with higher t at the steady state with the higher h and lower $l_1$ thus contradicting the idea of this paper and increases with t at the one with higher $l_1$ and lower h thus supporting the idea of this paper.

In the third step we show that the steady state with lower (higher) h and higher (lower) $l_1$ is (un–)stable. Therefore at the stable steady state higher taxes will lead to higher growth rates. In appendix A it is shown that the curves for $\dot{b} = 0$ and $\dot{\psi} = 0$ and the arrows for the dynamics can be drawn as in Figure 4 below. The slopes can be understood

INSERT FIGURE 4 OVER HERE

intuitively as follows (for the full complications of a rigorous analysis see appendix A). For the $\dot{b}$=0–line the partial derivative of $\dot{b} = 0$ with respect to $\psi$ is negative because the higher the shadow price of technology the higher the growth rate of technology and therefore the higher the growth rate of the denominator of b. This effect dominates the output enhancing effect of the increase in human capital which increases the numerator of b via taxation. The partial effect of b on $\dot{b} = 0$ is ambiguous. Higher b also increases technical progress and therefore the denominator of b. One can show that for low values of b the impact is negative and for higher values of b it must become positive. Therefore the $\dot{b}$=0–line must be u–shaped. This bears some similarity to the stationary line for the capital–labour ratio in the standard neoclassical growth model. What differs from the standard model is the line for a constant shadow price. It is not vertical as it would be in the standard model. Society can invest in A and B now. Its simplicity is lost in a model with three production functions. For the locus of a constant shadow price we have the usual unstable effect of the shadow price $\psi$ on its rate of change and a higher b decreases $\dot{\psi}$, because it induces a higher rate of technical progress and output and therefore decreases the shadow price of technology. Therefore this slope must be positive. Arrows follow quite analogously form $\partial\dot{b}/\partial\psi < 0$ and $\partial\dot{\psi}/\partial b < 0$. The steady state at the higher level of b is completely unstable. The steady state at the lower level of b is (un–) stable in the saddle point sense. The unique stable trajectory has a positive slope. A higher shadow price of technology provides an incentive for producing more technology and demanding more human capital (per labour efficiency unit) and a higher stock of public knowledge provides an incentive for households to produce more human capital (per labour efficiency unit supplied to firms). This generates a higher equilibrium value of h which leads to a higher rate of technical progress.

Starting from the saddle point stable steady state one can see from (4') that a higher value of taxation t leads to $\dot{b} > 0$. This means that the economy is below the $\dot{b} = 0$ line after such a shift in taxation. The $\dot{b} = 0$ line is therefore shifted upward. The new steady state will be at higher values of b and $\psi$ and therefore at higher values of human capital, h, and technical progress. Moreover, the analysis in the h–$l_1$–plane showed that higher h is associated with lower $l_1$. Labour will be shifted to education. Other movements than those of saddle point stable trajectory are discussed in connection with the possible case of no existence of steady states in appendix B. In the models by Shell (1967), Barro (1990) and Saint–Paul and Verdier (1993) there is also a distortionary effect from taxation which biases against the formation of (human) capital. Whether or not this effect outweighs the effect presented above is an empirical question. The evidence on this distortion is weak until now. Easterly and Rebelo (1993) find a negative but insignificant impact on growth rates in a cross–section study. Koester and Kormendi (1989) find that there is an negative and significant effect on the level of the path but not on the growth rate. But Otani and Villanueva (1990) find a positive effect of the share of public expenditure on human capital per unit of GDP on growth rates. Given the simplicity of the model it is rather straightforward to interpret the tax rate t as that share because the distortionary effect of taxation is absent. What remains is the expenditure effect. Putting it differently, the result by Koester and Kormendi (1989) suggests absence of effects of taxation on growth rates whereas the result of Otani and Villanueva (1990) suggests that the way of spending matters. The result that a higher share of output should be shifted to public educational purposes to reach more growth bears great similarity with the World Bank's (1990, 1991) suggestion to further education. The World Bank suggests to take the resources from the military complex which may be considered as part of consumption from which it is taken in this model. In view of this model this policy is of the "redistribution with growth" type (see Chenery et.al., 1974) because the increase in the tax rate is a permanent one.

Finally, the impact of higher discount rates and the rate of employment growth is analyzed. A higher rate of time preference used by the firm will lead to $\dot{\psi} > 0$ from (8'). The $\dot{\psi} = 0$ line will be shifted downwards. The new steady state will therefore be at lower $\psi$ and higher b. The firm has a weaker incentive to produce technical progress and demands c.p. less human capital but households have a higher incentive to supply human capital because of more public knowledge available. Higher b and lower $\psi$ lead to higher $l_1$ because it was shown above that $\partial l_1/\partial b > 0$ and $\partial l_1/\partial \psi < 0$. The analysis in the $h$–$l_1$–plane as summarized in Figure 3 showed that at the stable steady state higher $l_1$ is associated with lower h and therefore less technical progress. Higher impatience will thus lead to less technical progress. Higher employment growth will have the opposite effect, shifting $\dot{\psi}=0$ line up leading c.p. to higher technical progress, but the $\dot{b}=0$ line will be shifted downward through higher employment and therefore has the opposite effect on technical progress. The net effect of employment growth on technical progress is thus unclear. Figure 2 contained only the latter effect.

## 6. If i were the government: Household preferences for optimal taxation levels

Up to this point of the analysis the tax rate t has been exogenously varied. The question then is whether or not the different willingness to pay taxes also exists in the long run (steady state) or only in the short run. Growth theory has provided some examples of effects that are present in the short run but may be irrelevant in the long run. This is the reason why we consider individuals' optimal choices of steady states. In particular, differences in individuals' willingness to pay taxes may be small if compared to the income increases from higher technical progress. We want to show the conditions under which even in the long run, the shadow prices of technology, A and public knowledge, b, and therefore the rate of technological progress $\hat{A}$ is higher for individuals with higher abilities if they have identical preferences. A second objective of this section is to relate the willingness to pay of an individual with average income to that of a median voter as it is done in the literature on political equilibrium in endogenous growth models discussed in the introduction. In this section the individual choice of the tax rate is treated under the assumption that the individual takes the working of the whole economy into account without expecting any transactions like compensations between government and household except taxation.

The individual is assumed to maximize his utility from consumption which equals net income:

$$\underset{t,\ l_1, h}{\mathrm{Max}} {}_0\!\int^{\infty} e^{-\rho\tau} U^i[C^i(\tau)\} \, d\tau = {}_0\!\int^{\infty} e^{-\rho\tau} U^i[Y^i(\tau)(1-t)] \, d\tau$$

$$= {}_0\!\int^{\infty} e^{-\rho\tau} U^i\Big[ \{ w(L^i - L_2^i) + qALH^i[e^i(1-l_1)L_2^i/L_2, b] \} (1-t) \Big] d\tau$$

where $H^i$ is the production function (3) after having divided its arguments by AL. The time index has been dropped in the last equation. Here w is an abbreviation for $F_2A + \phi AhG_1/l_1$ according to (9) and q is an abbreviation of $F_1 + \phi G_1/l_1$ according to (7). The maximization is subject to (4'), (8') and $g(h)A = \dot{A}$ with $h = h(\psi, b)$ and $l_1 = l_1(\psi, b)$. A special case of this maximization is that of an individual with average income and abilities. His problem is identical to that of maximizing utility from per capita consumption. If transfers could correct the result of the flat–rate income tax such that each individual would be in the same position as under Lindahl prices this aggregate consumption maximization would be a reasonable objective. If information to find Lindahl prices is not available, aggregate consumption maximization becomes a dubious goal. It seems equally plausible then to assume that no compensation takes place and each individual expects no transfers and cares only for his ideal tax rate in the political process. This case is considered here. Two of the necessary conditions for an optimum (with $\lambda^{i,A}$ and $\lambda^{i,b}$ as co–state variables for A and b respectively and $\Psi$ for the Hamiltonian) are

$$\partial\Psi^i/\partial t = -U^{i\prime}Y^i + \lambda^{i,b}\, l_1 F = 0 \tag{14}$$

which implies

$$\lambda^{i,b} = U^{i\prime}Y^i/l_1F \tag{15}$$

An individual with higher income weighted with marginal utility has a higher shadow price for public factors.

$$-\,\partial\Psi^i/\partial A = -\,U^{i\prime}(Y^i/A)\,(1-t) - \lambda^{i,A}g(h) = \dot{\lambda}^{i,A} - \rho\lambda^{i,A} \tag{16}$$

implies for a steady state with $\hat{\lambda}^{i,A}-\epsilon = \hat{\lambda}^{i,b} = \hat{U^{i\prime}} = -\,\sigma(g+\epsilon)$ with $\sigma \equiv -\,U''Y^i(1-t)/U'$

$$\lambda^{i,A} = U^{i\prime}[(1-t)Y^i]/(\rho-g)A \tag{17}$$

(17) means that an individual with higher net income $(1-t)Y^i$ weighted with marginal utility will give a higher steady–state value to technology A expressed through higher $\lambda^{i,A}$ if $\rho > g$. Given an initial value A(0) this is an incentive to have a higher A in the future which requires higher technical progress g(h). Therefore individuals with higher abilities $e^i$ who have higher shadow prices of technology in their function as hypothetical governments prefer to have higher technical progress which they can achieve through higher taxation as was shown in the previous section. Deriving steady–state values of $\lambda^{i,b}$ and $\lambda^{i,A}$ with respect to $Y^i$ shows that they are higher (lower) for higher $Y^i$ (from higher abilities) if

$$\sigma \equiv -U^{i\prime\prime}(1-t)Y^i/U^{i\prime} < (>)\ 1 \tag{18}$$

In determining desired tax rates not only does the marginal product of public factors in human capital formation matter but the elasticity of marginal utility from consumption as well. The suggestion that more able people want higher tax rates and higher levels of public factors and technical progress only holds if marginal utility has a low elasticity with respect to net income (if $\rho > g$) which means that marginal utility does not decrease strongly as consumption grows. In this case preferences can be viewed as very materialistic. The reason is that the willingness to pay as expressed by the marginal product of public capital in human capital formation indicates the current benefits from an existing stock of public capital. The tax rate, analogous to the saving rate in the standard optimal growth model, determines how much will be invested in the stock of public capital. In this model without capital market this is the only form of savings which feeds public investment $\dot{B}$ directly and private investment $\dot{A}$ indirectly by having a negative impact on the cost of forming human capital. The households preference for high or low tax rates in the dual problem defined above therefore are quite analogous to those of private savings: low (high) $\sigma$ indicates a preference for high (low) savings or taxes. If the elasticity $\sigma$ is zero under an assumption of linear utility or income or consumption maximization as in Creedy and Francois (1990) and Perotti (1990), preferences do not appear explivcitly in the formula for the desired tax rate. If this elasticity equals unity all households want the same steady–state tax rate in this model as in Glomm and Ravikumar (1992) where log–linear preferences have been used and education of the future generation for which the tax revenues are used is an argument of the utility function. In Alesina and Rodrik (1992) we also find a unit elasticity value of the utility function, but the revenues do not go completely back into current utility. They are used for public investment which increases output which in turn is only partly consumed. Therefore, the more capital poor the voter the less he is damaged by a distortion and the higher the tax rate he desires. Saint–Paul and Verdier (1993) also use a log–linear utility function but its argument, 'childrens income' consists of two additively separable parts one of which is government expenditure. The result here is also that poorer people prefer higher tax rates. However, inequality and public support for education vanish in their model. This is not the case in this paper because individuals differ with respect to abilities and not with respect to initial endowments. In Persson and Tabellini (1991) preferences are homothetic with the implication that indirect utility is linear in households abilities. The result is that desired tax rates depend on the abilities of voters such that their impact cannot be outweighed by properties of preferences: less able (poorer) people want higher taxes. In sum, the only result where the expected outcome of poorer people wanting higher tax rates does not hold is that of Glomm and Ravikumar (1992) and in the case of an elasticity larger than or equal to one and $\rho > g$ in this model. As a consequence, if a median voter who is poorer then the average citizen he will desire a higher tax rate in this model (as in all other models except that of Glomm and Ravikumar, 1992), if he has inelastic marginal utility.

The last result may be different once capital income is introduced and taxed at the same rate because the distribution of capital then matters as well. An individual who is "median" with respect to abilities may have a high capital wealth and therefore a low willingness to pay income taxes, also leading to tax resistance. The impact of both, different abilities and capital endowments on the level of the growth path has been analyzed in Ziesemer (1990). The analysis of their joint impact on growth rates would require introduction of capital markets in to this model which would make it rather complicate. Whether a person is on the favoured or the disfavoured side of a distributional conflict will always depend on all the sources of heterogeneity and all the public goods and factors modelled as well as on the tax system used. Up till now all the papers with endogenous long-run growth rates contain only one of the sources of inequality and very simple tax systems. Future work will hopefully be able to construct models with both sources of inequality (or even more as land endowments emphassized by Persson and Tabellini, 1992) and less simple tax systems.

## 7. Summary and conclusion

In section 3 we have shown that individuals prefer different tax rates from the point of view of their temporary private decisions, because families with higher $e^i$ have higher marginal products of public factors B. In the previous section we have shown that the same results can be supported for the steady state from the point of view of a dual approach in which the tax rate is viewed as a governmental decision variable and an indirect utility or consumption function is maximized, provided that marginal utility is inelastic with respect to net income in the case that the rate of discount is higher than the rate of technical progress. In section 5 we have shown that higher tax rates or shares of public expenditure on education in the GDP lead to a higher level of public factors and higher technical progress in a saddle-point stable steady state. As households with higher abilities will prefer higher tax rates and rates of technical progress than households with lower abilities, the crucial question is which of these distributional interests are implemented by politicians. In this view the political resolution of conflict is as important as corrective taxation or historical accidents determining lock–in or lock–out. In this paper the conflict has been modeled in a more satisfactory way than in others because i) unlike the paper by Glomm and Ravikumar (1992) the conflict has not been suppressed by too narrowly modelling special cases, ii) unlike the paper by Alesina and Rodrik (1992) public factors are modeled rigorously as non–rivalrous and iii) unlike Saint–Paul and Verdier (1993) the conflict does not vanish.

## Footnotes

1 An alternative interpretation of the technical progress function could start from a learning–by–doing assumption $A=b\int_0^t Y(s)/L_1(s)ds$, with $b > 0$ so that $\dot{A} = bY/L_1$ as in Conlisk (1967). Insertion of the production function (1) also yields the technical progress function (2). However, learning–by–doing also ends with the life of a worker. Intergenerational knowledge transfer is partly the task of a firm (see also Prescott and Boyd, 1987 on this point) and partly of public libraries and schooling institutions as in this paper. A pure learning–by–doing view ignores both.

2 This is of course quite the opposite of the assumption in Lucas (1988), who assumes that each generation inherits more human capital than the previous one. However, Lucas must do so because he identifies technology ($A$ in our model) with human capital owned by the households. A rigorous formulation of human capital accumulation with finite lifetimes of a continuum of generations can be found in Imhoff (1989). There human capital does not have the same effects as technical progress because there is no growing legacy of knowledge.

3 This is a special case of the more general form $\dot{B} = t^{W}wL_1 + t^{q}qH$, where w is the wage rate and q is the price for the use of a unit of H, both in terms of output. The flat–rate income tax used in (4) is the excess burden free form of this more general tax scheme. Under this more general tax system the left side of (11) would have to be multiplied by $(1 - t^{W})$ and the right side by $(1 - t^{q})$. Unfortunately this leads to complications in the model which make it unsolvable.

4 To see that a corner solution $L_1 = 0$ is not superior, rewrite the Hamiltonian as $\Pi = AL_1[F(h,1) - qh - w/A] + \phi Ag(h)$. First order conditions can then be written as $\partial\Pi/\partial h = AL_1(F_1 - q) + \phi Ag' \gtreqless 0$ (7a); $\partial\Pi/\partial L_1 = AF(h,1) - qh - w/A) \gtreqless 0$ (9a), $-\partial\Pi/\partial A = -L_1(F - qh - w/A) - wL_1/A - \phi g = \dot{\phi} - \rho\phi$ (8a). For an interior solution the Hamiltonian was shown to be $\Pi^{*} = \phi^{*}A^{*}g^{*}$ in the text. For a corner solution it is $\Pi^{C} = \phi^{C}\bar{g}A^{C}$, where $g = \bar{g}$ because $\partial\Pi/\partial h = \phi Ag' > 0$ for $L_1 = 0$ and guarantees that $\Pi^{C}$ is finite. If both solutions have finite values in the beginning, their growth rates determine which one is higher in the long run. From (8a) we get $\hat{\phi}^{C} = \rho - \bar{g}$ for $L_1 = 0$. Therefore $\hat{\Pi}^{C} = \rho$. For steady states it is shown later that $\hat{\phi}^{*} = \epsilon$. Therefore $\hat{\Pi}^{*} = \epsilon + g^{*}$. $\rho < \epsilon + g^{*}$ is therefore a sufficient condition to make sure that the Hamiltonian is larger for an interior solution at least in the long run. As the interior solution yields zero profits, losses of an $L_1 = 0$ phase if $H > 0$ cannot be regained later. Therefore the interior solution given above is the only possible one under the assumption of price taking behaviour.

5 In the Solow model with exogenous technical progress and constant returns to scale one can either fix the number of firms or the size of the firm the other of the two variables being determined by equilibrium demand. In the model of this paper with endogenous technical progress one must make additional assumptions on knowledge. If there is only one firm that can keep the knowledge invented secret, e.g., through patents with infinite

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duration, then one receives a horizontal cost function for each point in time which is driven down over time as A increases. If patents can expire or knowledge has leaked out some way, then the knowledge is public and there may be many firms j using the same knowledge, each having the same production function $F(H^j, AL_1^j)$ but not producing $\dot{A}$. Again perfect competition is no problem. But then one makes a distinction between the innovator with production function $\dot{A} = G(H^j/L_1^j, A)$ and the imitators who may have no such function but for example imitate costlessly because knowledge can't be kept secret. Both are making zero profits if they behave as price takers as it is optimal for and thus required by households. So price "takership" is logically possible in this setting. Only if there is an unequal distribution of shares of a firm owners may be interested in firms using monopoly power. However, distribution of capital ownership is going beyond the scope of this paper.

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## Appendix A

$I_1$ and $I_2$ are two equations for h and $l_1$ depending on $\psi$ and b. Total differentiation with respect to h, $l_1$, b and $\psi$ yields a system of the following structure:

$$\begin{bmatrix} I_{1_h} & I_{1_{l_1}} \\ I_{2_h} & I_{2_{l_1}} \end{bmatrix} \begin{bmatrix} dh \\ dl_1 \end{bmatrix} = - \begin{bmatrix} I_{1_b} & I_{1_\psi} \\ I_{2_b} & I_{2_\psi} \end{bmatrix} \begin{bmatrix} db \\ d\psi \end{bmatrix}$$

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The derivatives of $I_1$ and $I_2$ with respect to h and $l_1$ are as follows (for details of the derivations see appendix B.3):

$$I_{1_h} = 1$$

$$I_{2_h} = [1 - l_1 - \alpha\beta l_1/(1-\beta)]\ (F_2 l_1/h)[\beta - \gamma(h)] > 0 \qquad \text{if } \beta > \gamma(h)$$

$$I_{1_{l_1}} = h l_1^{-1}\,\frac{1 - l_1 + \alpha l_1}{1 - l_1} > 0$$

In the neighbourhood of $\dot{\psi} = 0$ we have

$$I_{2_{l_1}} = \frac{\psi G_1 h}{(1-\beta)(1-l_1) - \alpha\beta l_1}\{-\alpha\} < 0$$

The denominator is positive because the existence of steady states requires $l_1 < (1-\beta)/(1-\beta+\alpha\beta)$ which had been derived above from (13').

The first matrix of the system then has the determinant

$$|D| = \underset{+}{I_{1_h}} \cdot \underset{-}{I_{2_{l_1}}} - \underset{+}{I_{2_h}}\ \underset{+}{I_{1_{l_1}}} < 0$$

The elements of the second matrix are

$$I_{1_\psi} = I_{2_b} = 0$$

$$I_{1_b} = -H_2\,\frac{1/l_1}{L_2^{1}/L_2} < 0$$

$$I_{2_\psi} = -G_1 h(1 - l_1 + \alpha l_1) < 0$$

The relation between h, $l_1$ and b, $\psi$ can then be calculated as

$$\partial h/\partial b = \underset{-}{-}\, \underset{-}{I_{1_b}}\, \underset{-}{I_{2_{l_1}}} / \underset{-}{|D|} > 0$$

$$\partial h/\partial \psi = \underset{-}{-}\, \underset{--}{(-I_{2_\psi})}\, \underset{+}{I_{1_{l_1}}} / \underset{-}{|D|} > 0$$

$$\partial l_1/\partial b = \underset{-}{-}\, \underset{+}{I_{2_h}}\, \underset{--}{(-I_{1_b})} / \underset{-}{|D|} > 0$$

$$\partial l_1/\partial \psi = \underset{+}{I_{1_h}}\, \underset{--}{(-I_{2_\psi})} / \underset{-}{|D|} < 0$$

The explicit form of the differential equations then is

$$\dot{b} = t l_1(b,\psi) F[h(b,\psi), 1] - \{g[h(b,\psi)] + \epsilon\} b \; (= 0 \equiv J_1)$$

$$\dot{\psi} = \{\rho - \epsilon - G_2[h(b,\psi)]\}\psi - F_2[h(b,\psi)] l_1(b,\psi) \; (= 0 \equiv J_2)$$

As we know from $I_1$ that h approaches zero as b approaches zero we can see from $J_1$ that all terms approach zero as b approaches zero. Therefore the $\dot{b} = 0$ line must be in the neighbourhood of the vertical axis as b approaches zero. To determine the slope of the $\dot{b} = 0$ line we compute

$$\partial\dot{b}/\partial b = \frac{I_{1_b} b}{|D|} \cdot$$

$$\{\psi G_1(1 - l_1 + \alpha l_1)(\beta - 1 - G_{11}h/G_1)(g + \epsilon)/l_1 +$$

$$+ [(g + \epsilon)F_1 h/F - g'b]\frac{+\psi G_1 \alpha}{(1-\beta)(1-l_1)-\alpha\beta l_1}\} - (g + \epsilon)$$

As $\frac{I_{1_b} b}{|D|} = \frac{-(1-\alpha)hL_2/L_2^i}{|D|}$ becomes zero as b and h approach zero and there is no reason for the term in brackets to go to infinity and $g(0) = 0$ we have

$$\lim_{b \to 0} \partial\dot{b}/\partial b = -\epsilon \qquad \text{(a)}$$

Moreover we have

$$\partial\dot{b}/\partial\psi = \underset{-}{\frac{-I_{2_\psi}}{|D|}} tF(1 - l_1 l_{1_{l_1}} F_1/F) \underset{-}{-} g'b \underset{+}{\partial h/\partial\psi} < 0 \qquad \text{(b)}$$

if the term in brackets is positive which can easily be shown.

From (a) and (b) it follows that

$$d\psi/db\Big|_{\dot{b}=b=0} = -(\partial\dot{b}/b)/(\partial\dot{b}/\psi) < 0$$

Thus the $\dot{b} = 0$ curve is falling in the neighbourhood of the vertical axis.

The derivatives of $J_2$ are

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$$\partial\dot{\psi}/\partial b = -G_{21}\frac{\partial h}{\partial b}\psi - F_{21}\frac{\partial h}{\partial b}l_1 - F_2\frac{\partial l_1}{\partial b} < 0$$

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$$\partial\dot{\psi}/\partial\psi = (\rho - \epsilon - G_2) - G_{21}\frac{\partial h}{\partial\psi}\psi - F_{21}\frac{\partial h}{\partial\psi}l_1 - F_2\frac{\partial l_1}{\partial\psi}$$

+ − + + − + + − + −

This term unfortunately has an unclear sign. Because $\partial h/\partial\psi$ depends on $F_2$, $I_{2_\psi}$ and $I_{1_{l_1}}$, which all approach zero for $h = 0$ and this is the case for $b = 0$, we have

$$\lim_{b,\, h\to 0} \partial\dot{\psi}/\partial\psi = (\rho - \epsilon) > 0$$

Therefore in the neighbourhood of $b = 0$ we have

$$d\psi/\partial b\big|_{b=0} = -(\partial\dot{\psi}/\partial b)/(\partial\dot{\psi}/\partial\psi) > 0$$

Thus the $\dot{\psi} = 0$ line has a positive slope in the neighbourhood of $b = 0$. Moreover, for $\psi \to 0$ we have

$$\lim_{\psi\to 0} \dot{\psi} = -F_2 l_1 < 0$$

Thus in the neighbourhood of the horizontal axis $\psi$ is falling as indicated by arrows in Figure 4 and the $\dot{\psi} = 0$ line can't converge to the horizontal axis. Furthermore, for $h \to 0$ as $b \to 0$

$$\lim_{b\to 0} \dot{\psi} = (\rho - \epsilon)\psi > 0$$

Thus $\psi$ is increasing in the neighbourhood of the vertical axis and the $\dot{\psi} = 0$ line can't converge to the vertical axis. As the $\dot{\psi} = 0$ line can't converge to the axes for $b \to 0$ or $\psi \to 0$ separately and has positive slope for $b \to 0$ it must come from the origin.

Because $\partial\dot{\psi}/\partial b < 0$ implies that $\psi$ is falling to the right of $\dot{\psi} = 0$ and to the left of $\dot{\psi} = 0$ it is increasing, the $\dot{\psi} = 0$ line can't become falling for higher b because then $\psi$ would have to increase to the left of the falling part but also would have to decrease below $\dot{\psi} = 0$ which would imply opposite directions of the arrows on the same side of the curve which is a contradiction. Therefore the $\dot{\psi} = 0$ curve must have a positive slope throughout.

As we know from the steady state analysis in the h–$l_1$–plane summarized in Figure 3 that there are either (at least) two steady states or no steady state, we can conclude that the $\dot{b} = 0$ line must be u–shaped as indicated in Figure 4.

# APPENDIX B (NOT FOR PUBLICATION)

## Appendix B.1 (Derivation of Figure 3 from (13'''))

If $l_1$ is zero in (13''') its right hand side is zero. Therefore the left hand side must also be zero, imlying

$$1 - \frac{G_1(h)}{\rho - \epsilon - G_2(h)} = 0 \qquad \text{or} \qquad G_1(h)h = [\rho - \epsilon - G_2(h)]$$

The functions of the last equation are drawn in Figure 5. If h is positive (zero), $G_2$ is

INSERT FIGURE 5 OVER HERE

positive (zero) too. The right hand side is a falling function of h with vertical intercept $\rho - \epsilon$. The left hand side starts from the origin and is increasing in h if $G_{11}h/G_1 + 1 > 0$ and must therefore intersect the curve of the right hand side. Thus, for $l_1 = 0$ there is a unique solution $h^0$.

The slope of the (13''') is

$dl_1/dh =$

$$\frac{(\rho - \epsilon - G_2)^{-2}[(\rho - \epsilon - G_2)(G_{11}h + G_1) - G_1h(-G_{21})]\;(1 - l_1 + \alpha l_1)}{-1 - \dfrac{G_1 h}{\rho - \epsilon - G_2}(-1 + \alpha) + \dfrac{-\alpha\beta}{1 - \beta}}$$

For $h = 0$ it is negative because

$$dl_1/dh\Big|_{h=0} = \frac{(\rho - \epsilon)^{-2}[(\rho - \epsilon)G_1](1 - l_1 + \alpha l_1)}{-1 - \alpha\beta/(1-\beta)} < 0$$

and for $h = h^0$ (at $l_1 = 0$) it becomes also negative

$$dl_1/dh\Big|_{h=h^0} =$$

$$\frac{(\rho - \epsilon - G_2)^{-2}[(\rho - \epsilon - G_2)(G_{11}h + G_1) - G_1h(-G_{21})](1 - l_1 + \alpha l_1)}{-\dfrac{G_1h}{\rho - \epsilon - G_2}\alpha + \dfrac{-\alpha\beta}{1-\beta}} < 0$$

because $\rho - \epsilon - G_2 > 0$ for $\psi > 0$, $G_{11}h + G_1 > 0$. As the denominator of the slope is a monotonically increasing function of h and independent of $l_1$ and the slope is negative for the highest possible value of $h = h^0$ the denominator cannot change sign between $h = 0$ and $h^0$. $l_1 = 1$ (13''') leads to a contradiction. This shows that the function intersects the axes at $h = 0$ where $l_1 < 1$. At $h = 0$ we have $l_1 = [1 + \alpha\beta/(1 - \beta)]^{-1}$. This is the maximum value for $l_1$. It implies that

$$l_1^{max}(1 + \frac{\alpha\beta}{1-\beta}) = 1, \quad l_1^{max} + \frac{\alpha\beta}{1-\beta}l_1^{max} = 1, \quad 0 = 1 - l_1^{max} - \frac{\alpha\beta}{1-\beta}l_1^{max}$$

So in the neighbourhood of steady states we have that $l_1$ must be such that

$$1 - l_1 - \frac{\alpha\beta}{1-\beta}l_1 \geq 0.$$

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Appendix B.2 [Slope of (12'') at $l_1 = 0$]

The slope can be rewritten as

$$dh/dl_1 = \frac{-\, H_1 \frac{1-l_1}{l_1} e^i \, \frac{h}{(1-l_1)l_1} (e^i)^{-1}/h}{1 - (1-\alpha)\,[\beta - \frac{\gamma}{1+\epsilon/g(h)}]} = \frac{-\,\alpha \, \frac{h}{(1-l_1)l_1} (e^i)^{-1}}{1 - (1-\alpha)\,[\beta - \frac{\gamma}{1+\epsilon/g(h)}]} < 0$$

As $l_1 \to 0$ we have $h \to \infty$, and therefore $\lim_{l_1 \to 0} dh/dl_1 = -\infty$.

Appendix B.3

To derive the arrows drawn in Figure 4 we rewrite $I_1$, and $I_2$:

$$I_1 \equiv h - H\left(\frac{1-l_1}{l_1} e^i, \frac{b/l_1}{L_2^1/L_2}\right) = 0 \qquad (12')$$

$$I_2 \equiv \frac{F_2 - \psi G_1 h/l_1}{F_1 + \psi G_1/l_1} - \alpha h l_1/(1-l_1) = 0 \qquad (13')$$

Multiplication of (13') by the denominator of the fraction yields

$$F_2 - \psi G_1 h/l_1 - (F_1 + \psi G_1/l_1)\alpha h l_1/(1-l_1) = 0$$

Using $F_1 h/F_2 = \beta/(1-\beta)$ this can be rewritten as

$$I_2 = F_2(h)l_1[1 - l_1 - \alpha\beta l_1/(1-\beta)] - \psi G_1(h)h(1 - l_1 + \alpha l_1) = 0 \qquad (13^{iv})$$

From (12') it follows that $h = 0$ if either $b = 0$ or $l_1 = 1$. From ($13^{iv}$) $\psi = 0$ would imply $l_1 = (1 - \beta)/(1 - \beta + \alpha\beta) < 1$. This value had already been derived as a limit for $l_1$ in appendix B.1. A positive $\psi$ therefore requires that $l_1$ be smaller than that value.

$I_1$ and $I_2$ are two equations for h and $l_1$ depending on $\psi$ and b. Total differentiation with respect to h, $l_1$, b and $\psi$ yields a system of the following structure:

$$\begin{bmatrix} I_{1_h} & I_{1_{l_1}} \\ I_{2_h} & I_{2_{l_1}} \end{bmatrix} \begin{bmatrix} dh \\ dl_1 \end{bmatrix} = - \begin{bmatrix} I_{1_b} & I_{1_\psi} \\ I_{2_b} & I_{2_\psi} \end{bmatrix} \begin{bmatrix} db \\ d\psi \end{bmatrix}$$

The derivatives of $I_1$ and $I_2$ with respect to h and $l_1$ are as follows.

$$I_{1_h} = 1$$

$$I_{2_h} = F_{21} l_1 [1 - l_1 - \alpha\beta l_1/(1 - \beta)] - \psi(G_{11}h + G_1)(1 - l_1 + \alpha l_1)$$

Using $I_2 = 0$ to eliminate $\psi$ yields

$$I_{2_h} = F_{21} l_1 [1 - l_1 - \alpha\beta l_1/(1 - \beta)] - F_2 l_1 [1 - l_1 - \alpha\beta l_1/(1 - \beta)](G_{11}h/G_1 + 1) =$$

$$[1 - l_1 - \alpha\beta l_1/(1 - \beta)]\ [F_{21} l_1 - (F_2 l_1/h)(G_{11}h/G_1 + 1)] =$$

$$[1 - l_1 - \alpha\beta l_1/(1 - \beta)]\ (F_2 l_1/h)[F_{21}h/F_2 - (G_{11}h/G_1 + 1)] =$$

$$[1 - l_1 - \alpha\beta l_1/(1 - \beta)]\ (F_2 l_1/h)[\beta - \gamma(h)] > 0 \qquad \text{if } \beta > \gamma(h)$$

$$I_{1_{l_1}} = - H_1 e^{i} l_1^{-2}[l_1(-1) - 1(1 - l_1)] - H_2 \frac{b}{L_2^{i}/L_2}(-1) l_1^{-2}$$

$$= H_1 e^{i} l_1^{-2} + H_2 \frac{b/l_1}{L_2^{i}/L_2} l_1^{-1} = [\alpha h/(e^{i} \frac{1-l_1}{l_1})] e^{i} l_1^{-2} + (1-\alpha) h l_1^{-1}$$

$$= h l_1^{-1}[\alpha(1-l_1)^{-1} + 1 - \alpha] = h l_1^{-1} \frac{\alpha + (1-\alpha)(1-l_1)}{1-l_1} = h l_1^{-1} \frac{1 - l_1 + \alpha l_1}{1 - l_1} > 0$$

To find the sign of $I_{2_{l_1}}$ we consider $I_2$ only in the neighbourhood of $\dot{\psi} = 0$. Eliminating the $F_2$–term in $I_2 = 0$ using $\dot{\psi} = 0$ yields

$$I_2 = (\rho - \epsilon - G_2)\psi[1 - l_1 - \alpha\beta l_1/(1-\beta)] - \psi G_1 h(1 - l_1 + \alpha l_1) = 0.$$

Derivation with respect to $l_1$ yields

$$I_{2_{l_1}} = (\rho - \epsilon - G_2)\psi[-1 - \alpha\beta/(1-\beta)] - \psi G_1 h(-1 + \alpha)$$

Using $I_2 = 0$ again to eliminate the first term in brackets yields

$$I_{2_{l_1}} = \frac{\psi G_1 h(1 - l_1 + \alpha l_1)}{[1 - l_1 - l_1\alpha\beta/(1-\beta)]}[-1 - \alpha\beta/(1-\beta)] - \psi G_1 h(-1+\alpha) =$$

$$\frac{\psi G_1 h}{1 - l_1 - l_1\alpha\beta/(1-\beta)} \cdot$$

$$\{(1 - l_1 + \alpha l_1)[-1 - \alpha\beta/(1-\beta)] + [1 - l_1 - l_1\alpha\beta/(1-\beta)](1-\alpha)\}$$

Multiplying the denominator and the second row of the product by $(1-\beta)$, doing the multiplication in the second line and cancelling terms yields

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$$I_{2_{l_1}} = \frac{\psi G_1 h}{(1-\beta)(1-l_1) - \alpha\beta l_1}\{-\alpha\} < 0$$

From ($13^{iv}$) one can see that the denominator is positive for $\psi > 0$. This was used above to derive the upper limit for $l_1$.

The first matrix of the system then has the determinant

$$|D| = \underset{+}{I_{1_h}} \cdot \underset{-}{I_{2_{l_1}}} - \underset{+}{I_{2_h}}\, \underset{+}{I_{1_{l_1}}} < 0$$

The elements of the second matrix are

$$I_{1_\psi} = I_{2_b} = 0$$

$$I_{1_b} = -H_2 \frac{1/l_1}{L_2^1/L_2} < 0$$

$$I_{2_\psi} = -G_1 h(1 - l_1 + \alpha l_1) < 0$$

The relation between h, $l_1$ and b, $\psi$ can then be calculated as

$$\partial h/\partial b = \underset{-}{-}\, \underset{-}{I_{1_b}}\, \underset{-}{I_{2_{l_1}}}/\underset{-}{|D|} > 0$$

$$\partial h/\partial \psi = \underset{-}{-}\, \underset{-\;-}{(-I_{2_\psi})}\, \underset{+}{I_{1_{l_1}}}/\underset{-}{|D|} > 0$$

$$\partial l_1/\partial b = \underset{-}{-}\, \underset{+}{I_{2_h}}\, \underset{-\;-}{(-I_{1_b})}/\underset{-}{|D|} > 0$$

$$\partial l_1/\partial\psi = \underset{+}{I_{1_h}}\underset{--}{(-I_{2_\psi})}/\underset{-}{|D|} < 0$$

The explicit form of the differential equations then is

$$\dot{b} = tl_1(b,\psi)F[h(b,\psi),\, 1] - \{g[h(b,\psi)] + \epsilon\}b \;(= 0 \equiv J_1)$$

$$\dot{\psi} = \{\rho - \epsilon - G_2[h(b,\psi)]\}\psi - F_2[h(b,\psi)]l_1(b,\psi) \;(= 0 \equiv J_2)$$

The following information will be collected in Figure 6.

As we know from $I_1$ that h approaches zero as b approaches zero we can see from $J_1$ that all terms approach zero as b approaches zero. Therefore the $\dot{b} = 0$ line must be in the neighbourhood of the vertical axis as b approaches zero. To determine the slope of the $\dot{b} =$ 0 line we compute

$$\partial\dot{b}/\partial b = t\{F\underset{+}{\partial l_1/\partial b} + l_1F_1\underset{+}{\partial h/\partial b}\} \underset{-}{-} \underset{+}{(g+\epsilon)} \underset{-}{-} g'b\underset{+}{\partial h/\partial b}$$

$$= t\,\frac{I_{2_h} I_{1_b}}{|D|}\,F + (tl_1F_1 - g'b)\,\frac{-I_{1_b} I_{2_{l_1}}}{|D|} - (g+\epsilon)$$

$$= \frac{I_{1_b}}{|D|}\,\{tI_{2_h}F + (tl_1F_1 - g'b)(-I_{2_{l_1}})\} - (g+\epsilon)$$

Inserting the results for $I_{2_h}$ and $I_{2_{l_1}}$ and using $\dot{b}$=0 one gets

$$\partial\dot{b}/\partial b = \frac{I_{1_b} b}{|D|}\,.$$

$$\{\psi G_1(1 - l_1 + \alpha l_1)(\beta - 1 - G_{11}h/G_1)(g + \epsilon)/l_1 +$$

$$+ [(g + \epsilon)F_1 h/F - g'h]\frac{+\psi G_1 \alpha}{(1-\beta)(1-l_1)-\alpha\beta l_1}\} - (g + \epsilon)$$

As $\frac{I_{1_b}\ b}{|D|} = \frac{-(1-\alpha)h}{|D|}$ becomes zero as b and h approach zero and there is no reason for the term in brackets to go to infinity and $g(0) = 0$ we have

$$\lim_{b \to 0} \partial \dot{b}/\partial b = -\epsilon \qquad \text{(a)}$$

Moreover we have

$$\partial \dot{b}/\partial \psi = t(F\underset{-}{\partial l_1/\partial \psi} + l_1 F_1 \underset{+}{\partial h/\partial \psi}) - \underset{-}{g'}b\underset{+}{\partial h/\partial \psi}$$

$$= tF\frac{-I_{2_\psi}}{|D|} + t l_1 F_1 \frac{I_{2_\psi} I_{1_{l_1}}}{|D|} - g'b\partial h/\partial \psi = \frac{-I_{2_\psi}}{|D|}(tF - t l_1 F_1 I_{1_{l_1}}) - g'b\partial h/\partial \psi$$

$$= \frac{-I_{2_\psi}}{\underset{-}{|D|}} tF\,(1 - l_1 I_{1_{l_1}} F_1/F) - \underset{-}{g'}b\underset{+}{\partial h/\partial \psi} < 0 \qquad \text{(b)}$$

if the term in brackets is positive. This will be shown next through insertion of $I_{1_{l_1}}$.

$$(1 - l_1 I_{1_{l_1}} F_1/F) = 1 - h\frac{1 - l_1 + \alpha l_1}{1 - l_1} F_1/F = 1 - \beta\frac{1 - l_1 + \alpha l_1}{1 - l_1}$$

$$= \frac{1}{1 - l_1}[(1 - l_1)(1 - \beta) - \alpha\beta l_1] = \frac{1 - \beta}{1 - l_1}[1 - l_1 - \alpha\beta l_1/(1 - \beta)] > 0$$

The last result of appendix B.1 was that in the neighbourhood of a steady state with $\psi > 0$,

$l_1$ must be small enough to make the last term positive. This can also be seen from ($13^{iv}$).

From (a) and (b) it follows that

$$d\psi/db\Big|_{\dot b = b = 0} = -(\partial\dot b/\partial b)/(\partial\dot b/\partial\psi) < 0$$

Thus the $\dot b = 0$ curve is falling in the neighbourhood of the vertical axis.

INSERT FIGURE 6 OVER HERE

The derivatives of $J_2$ are

$$\partial\dot\psi/\partial b = -G_{31}\frac{\partial h}{\partial b}\psi - F_{21}\frac{\partial h}{\partial b}l_1 - F_2\frac{\partial l_1}{\partial b} < 0$$

$$-\ +\ +\quad -\ +\ +\quad -\ +\ +$$

$$\partial\dot\psi/\partial\psi = (\rho - \epsilon - G_2) - G_{21}\frac{\partial h}{\partial\psi}\psi - F_{21}\frac{\partial h}{\partial\psi} - F_2\frac{\partial l_1}{\partial\psi}$$

$$+\qquad -\ +\ +\quad -\ +\ +\ -\quad +\ -$$

This term unfortunately has an unclear sign. Because $\partial h/\partial\psi$ depends on $F_3$, $I_{2_\psi}$ and $I_{1_{l_1}}$, which all approach zero for $h = 0$ and this is the case for $b = 0$, we have

$$\lim_{b,\,h\to 0} \partial\dot\psi/\partial\psi = (\rho - \epsilon) > 0$$

Therefore in the neighbourhood of $b = 0$ we have

$$d\psi/\partial b\big|_{b=0} = -(\partial\dot{\psi}/\partial b)/(\partial\dot{b}/\partial b) > 0$$

Thus the $\dot{\psi} = 0$ line has a positive slope in the neighbourhood of $b = 0$. Moreover, for $\psi \to 0$ we have

$$\lim_{\psi\to 0} \dot{\psi} = -F_2 l_1 < 0$$

Thus in the neighbourhood of the horizontal axis $\psi$ is falling as indicated by arrows in Figure 6 and the $\dot{\psi} = 0$ line can't converge to the horizontal axis. Furthermore, for $h \to 0$ as $b \to 0$

$$\lim_{b\to 0} \dot{\psi} = (\rho - \epsilon)\psi > 0$$

Thus $\psi$ is increasing in the neighbourhood of the vertical axis and the $\dot{\psi} = 0$ line can't converge to the vertical axis. As the $\dot{\psi} = 0$ line can't converge to the axes for $b \to 0$ or $\psi \to 0$ separately and has positive slope for $b \to 0$ it must come from the origin.

Because $\partial\dot{\psi}/\partial b < 0$ implies that to the right of $\dot{\psi} = 0$ is $\psi$ is falling and to the left of $\dot{\psi} = 0$ it is increasing, the $\dot{\psi} = 0$ line can't become falling for higher b because then $\psi$ would have to increase to the left of the falling part but also would have to decrease below $\dot{\psi} = 0$ which would imply opposite directions of the arrows on the same side of the curve which is a contradiction. Therefore the $\dot{\psi} = 0$ curve must have a positive slope throughout. This is indicated by the dashed line in Figure 6.

As we know from the steady state analysis in the h–$l_1$–plane summarized in Figure 3 that there are either (at least) two steady states or no steady state, we can conclude that the $\dot{b} = 0$ line must be u–shaped as indicated in Figure 6. If there were three steady states which we can't exclude in view of Figure 3, the u–curve would have to go down again, if there were four it would go up again and so on.

In sum, the analysis of the existence of a steady state in the h–$l_1$–line together with some results for the conventional analysis of the dynamics allowed to find the dynamic properties of the model.

Figure 7 contains the dynamics of the case where there is no steady state. These arrows can be concluded from

$\partial\dot{\psi}/\partial b < 0$ and $\partial\dot{b}/\partial\psi < 0$

INSERT FIGURE 7 OVER HERE

To the right of $\dot{\psi} = 0$ the values for $\psi$ must decrease and to the left they must increase. Above $\dot{b} = 0$ the values for b must decrease and below $\dot{b} = 0$ it must increase. The dashed lines indicate possible paths for $\psi$ and b. On the path where b and $\psi$ are increasing we see from $\partial h/\partial\psi > 0$ and $\partial h/\partial b > 0$ that h will be increasing as long as we are in the neighbourhood of the $\dot{\psi} = 0$ line, because the sign for $I_{2_{l_1}}$ could only be derived for the assumption of being in that neighbourhood. If b is falling and $\psi$ is increasing this induces $l_1$ to fall and if b is increasing and $\psi$ is falling this induces $l_1$ to increase. This movement of $l_1$ is limited by its upper bound derived in appendix B.1. The case for two steady states is

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summarized in Figure 4 in the text.

If such a case of no steady state exists then between that one and the proven case of two steady states there must be a case where the curves are tangential to each other. This case would be stable in the saddle point sense from the left and unstable from the right. Here the rate of technical progress would be higher than in all other stable steady states. If there are more than two steady states those with a falling $\dot{b}=0$ line will be stable in the saddle point sense and those with increasing $\dot{b}=0$ line will be unstable.

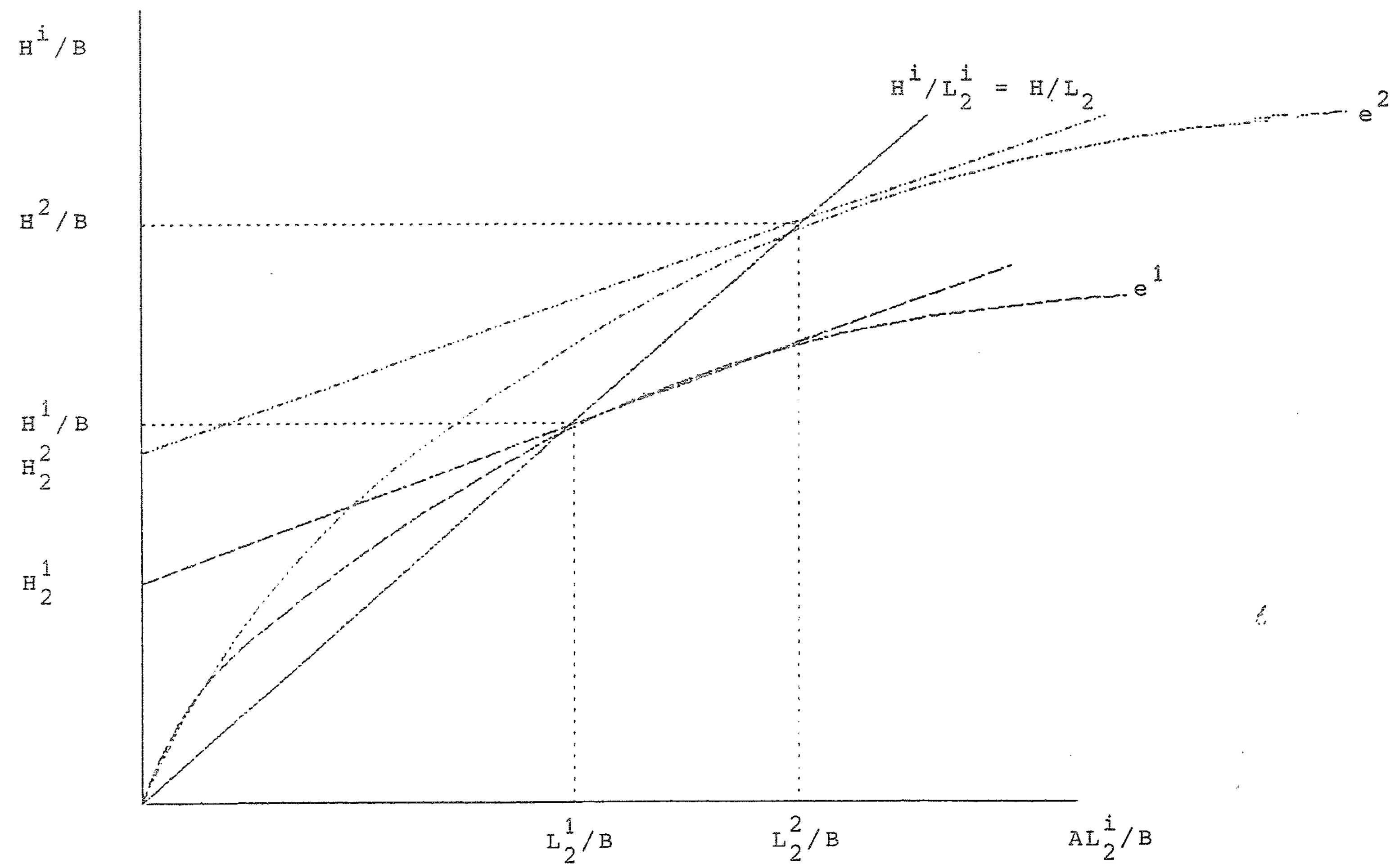
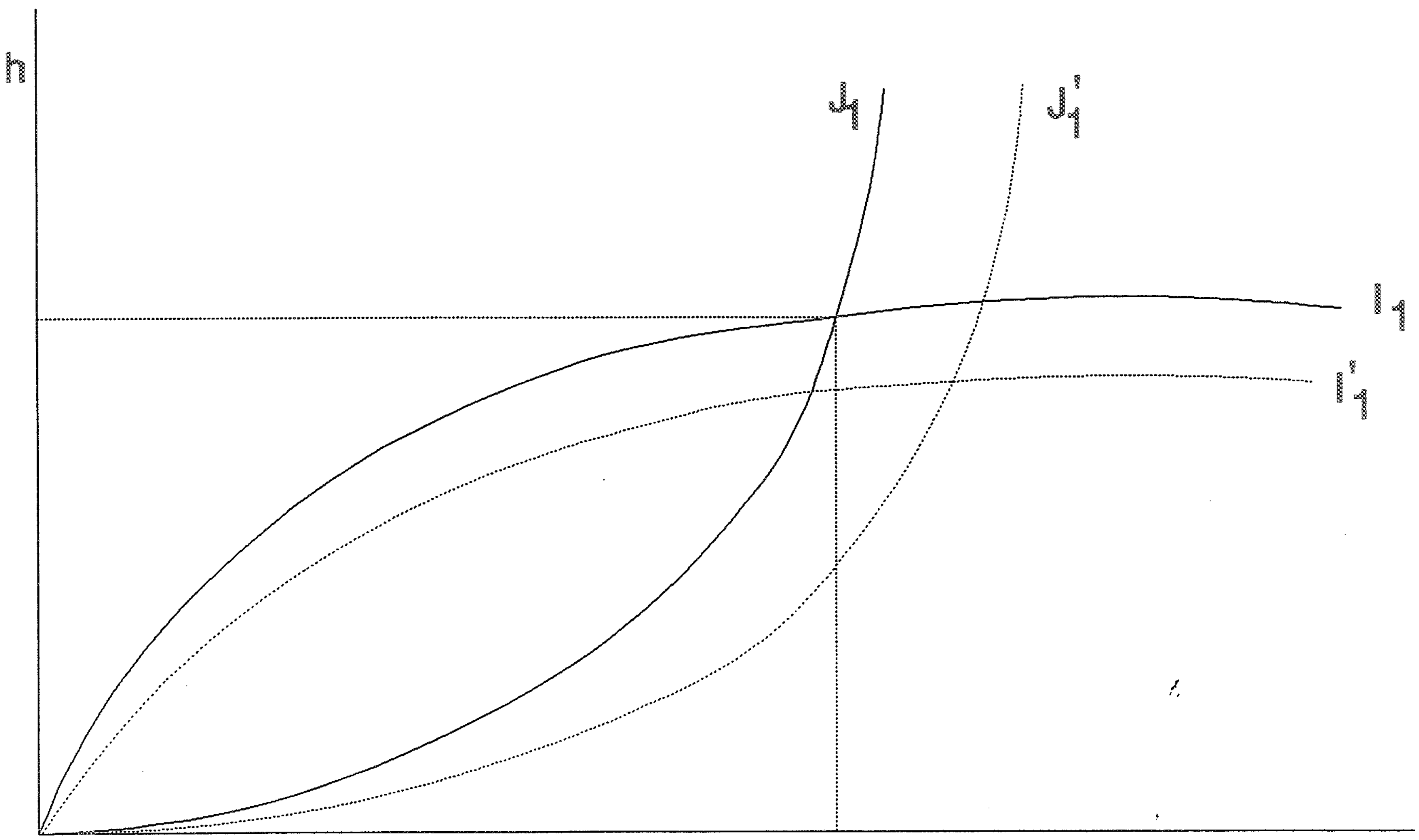



**Figure 1**

Choice of $L_2^i$, $H^i$ for the two families with different abilities $e^1$, $e^2$.




Higher tax rates lead to a higher steady state stock of public knowledge per labour hour efficiency unit, b, and a higher technical progress g(h), given $I_1$, through shifting $J_1$ to $J'_1$. Higher $I_1$ shifts $J_1$ to $J'_1$ and $I_1$ to $I'_1$ leading to unclear net outcomes. Higher population growth shifts $J_1$ to the left reducing h and b, given $I_1$.

Figure 2

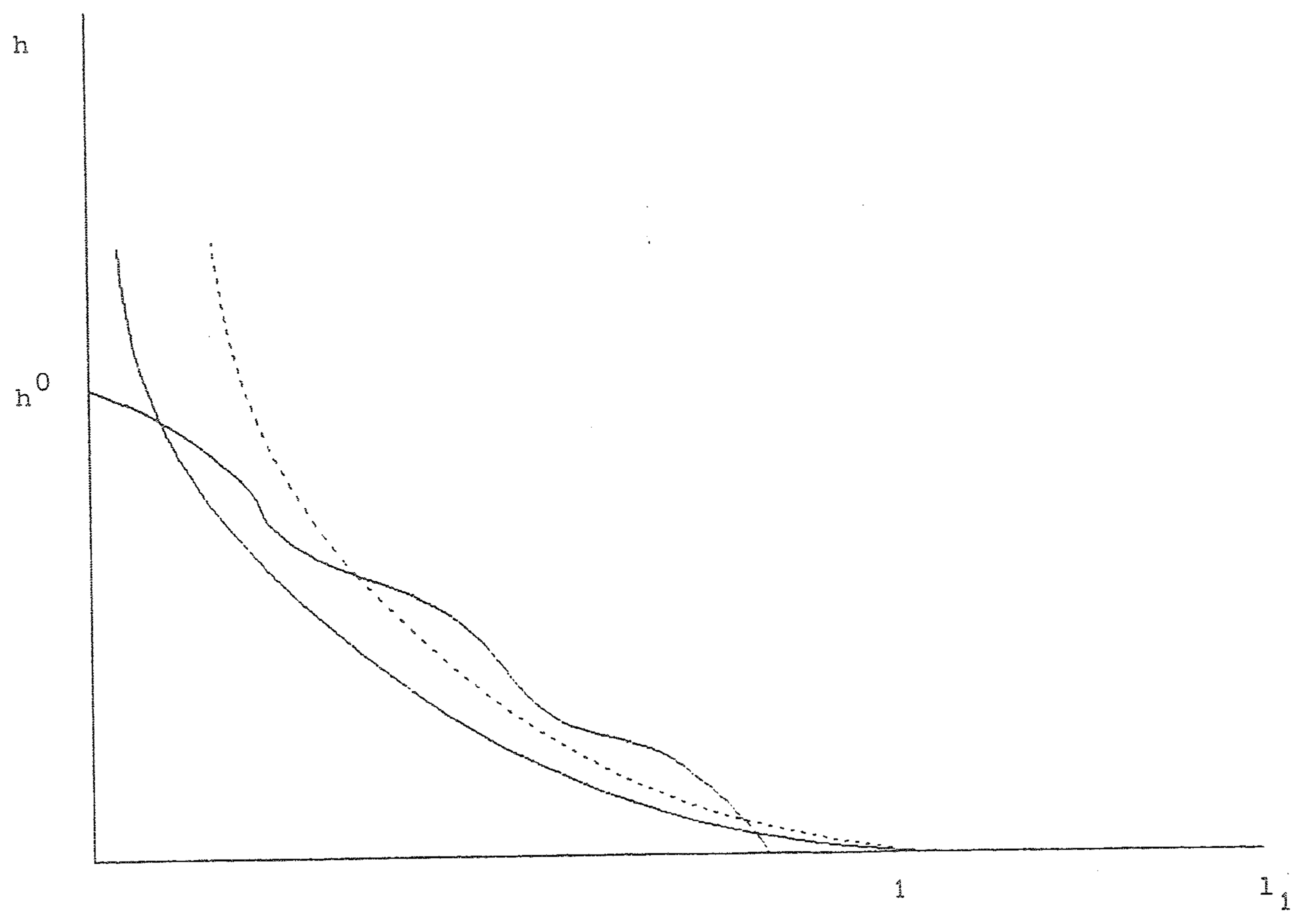



Figure 3

The case of the two steady states. A higher tax rate shifts the asymptotic curve to the right and upwards.

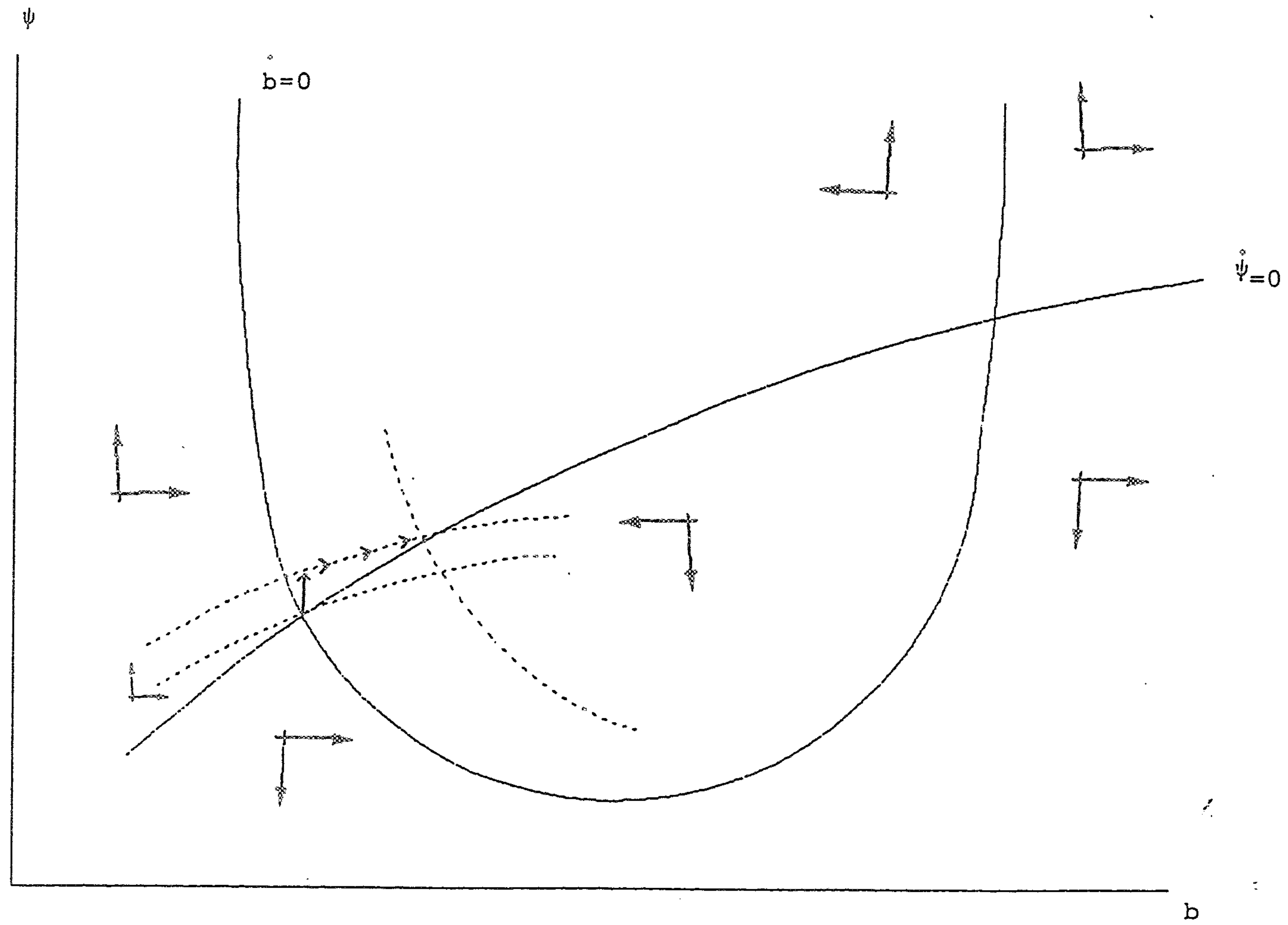



Figure 4
The dynamics of multiple steady states. The left one is saddle point stable, the right one unstable. A higher tax rate shifts the $\mathring{b}=0$ line and the stable trajectory upwards with subsequent dynamics indicated by small arrows.

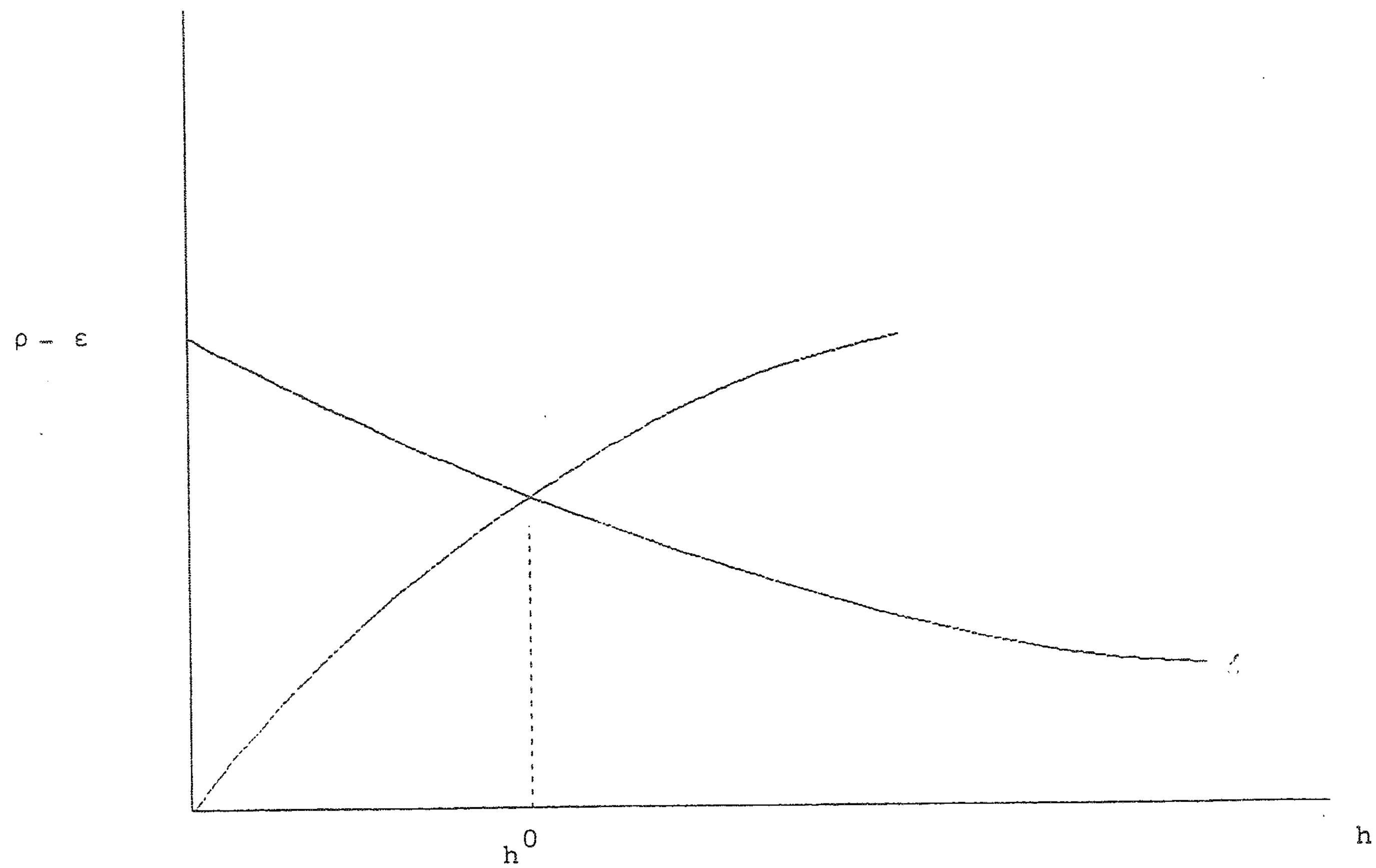



Figure 5

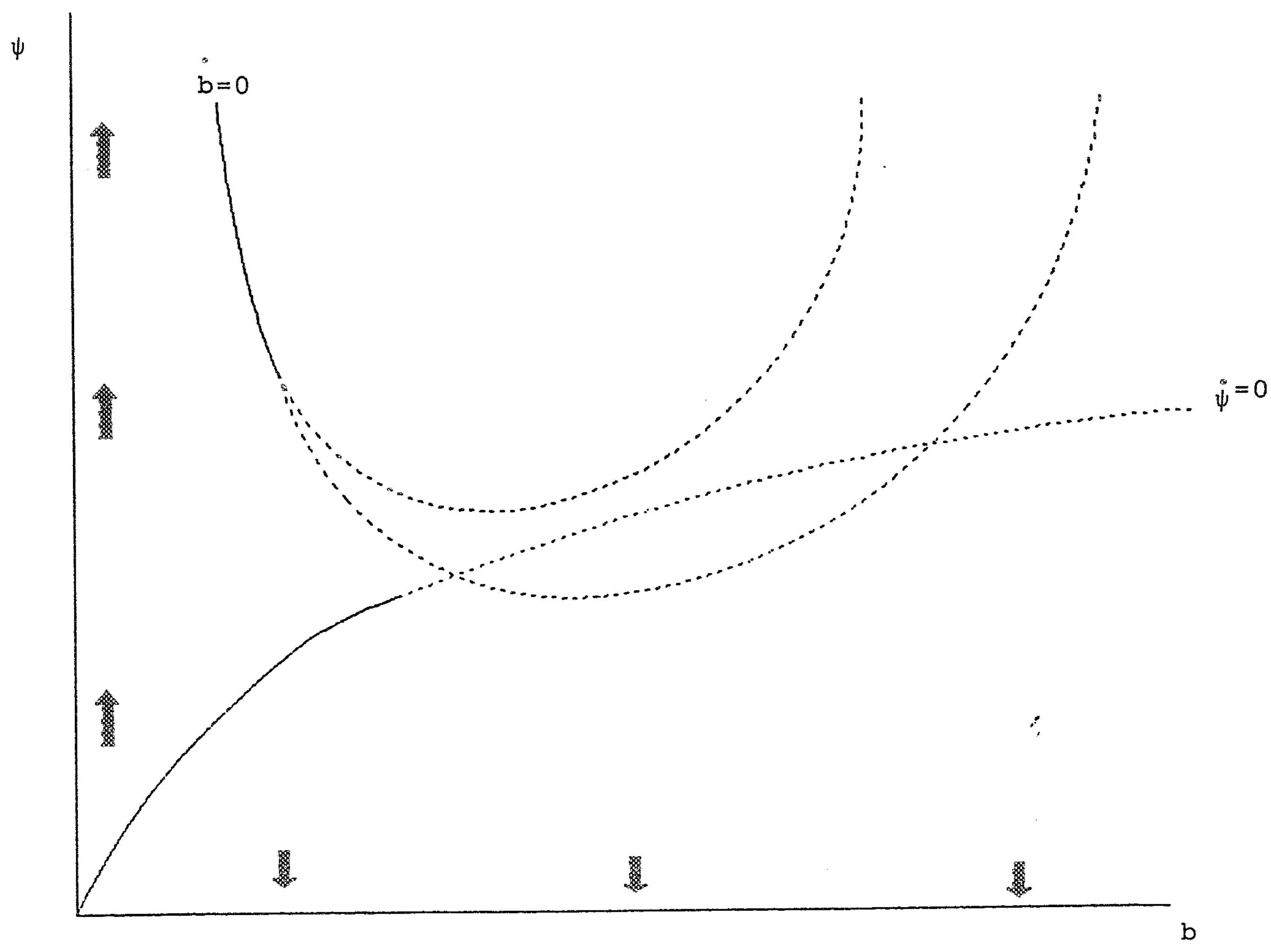



Figure 6

There is either no steady state or there are at least two.

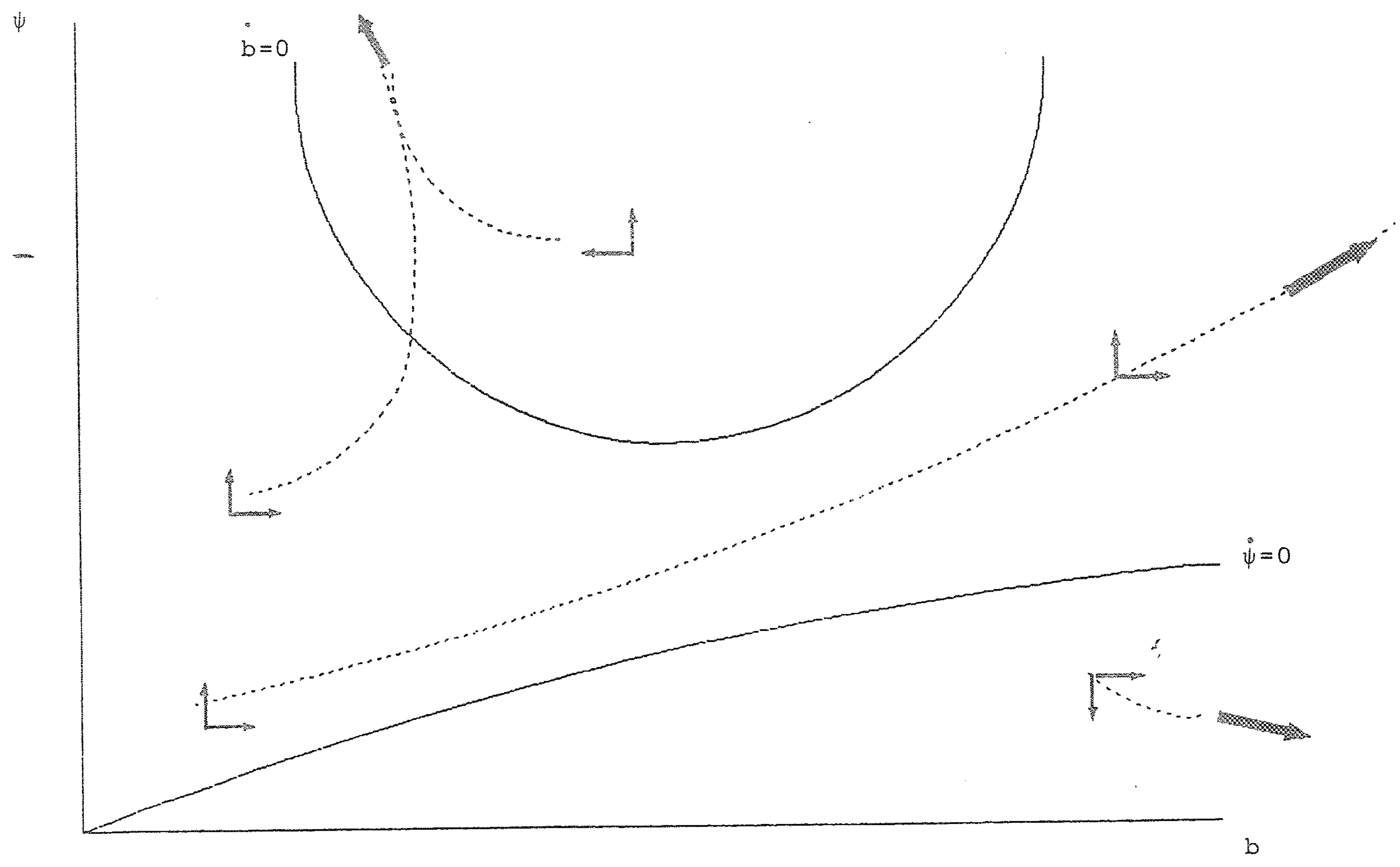



Figure 7

The dynamics of the no-steady-state case. If b increases (decreases) and $\psi$ decreases (increases), $l_1$ will increase (decrease). Increasing $\psi$ and b lead to increasing technical progress through higher h.