

MPRA

Munich Personal RePEc Archive

A computational model for inventory management and planning

Mehar Ayub and Muhammad Ahsanuddin

2000

Online at <http://mpra.ub.uni-muenchen.de/600/>
MPRA Paper No. 600, posted 30. October 2006

A COMPUTATIONAL MODEL FOR INVENTORY MANAGEMENT AND PLANNING

BY

DR. AYUB MEHAR

**DEAN, MANAGEMENT SCIENCES
INSTITUTE OF BUSINESS AND TECHNOLOGY – BIZTEK,
KARACHI (PAKISTAN)
2002**

**Address: R-73 Block 7, Decent Town, Gulistan-e-Jauhar Karachi 75290
(PAKISTAN) Ph.92-021-8130550.
Email: ayubmehar@yahoo.com**

A COMPUTATIONAL MODEL FOR INVENTORY MANAGEMENT AND PLANNING

JEL Classification Number: C61; C63; M41; M11;

Key Words: Inventory Control; Economic Order Quantity; Buffer Stock;
Baumoul's Model; Optimization; Material Cost Accounting.

ABSTRACT

The objective of the study is to determine the factors of the optimal level of merchandizing inventory. The study is based on a mathematical model. The results revealed some interesting findings. The most important conclusion is that the 'Usage of Material' or the Sales Volume is not the real determinate of the inventory volume. It is concluded in the model that the volume of inventories depends on the difference between the return on investment in the inventories and the rate of interest on short-term deposits. The traditional methods in cost accounting - Buffer Stock and Economic Order Quantity - have been reconciled with the profit maximization hypothesis.

A COMPUTATIONAL MODEL FOR INVENTORY MANAGEMENT AND PLANNING

I. INTRODUCTION

The optimal level of inventories has always been remained an important area in the financial management of a company. It is estimated that inventories cover one-third of the value of total assets in a firm's balance sheet (Mats, Curry, Frank and Khan: 1982). The magnitude and appraisals of the inventories have much importance due to the following reasons:

- (1) They are appeared in the balance sheet at their historical value, but their realization depends on the economic conditions and business environment at present and in future.
- (2) They have a significant share in total assets. So, a change in the magnitude of inventories can affect the profitability and financial viability of a business.
- (3) It is generally thought that an optimal size of inventories is required for profit enhancement. The Economic Order Quantity and Buffer Stocks may lead to the cost minimization. The policy regarding the inventory volume is directly concerned with the sales volume and profitability.

The primary objective of the study is to determine the factors of the optimal level of merchandizing inventory. However, the study is important also from the business cycle's point of view, because, the excess inventories have always been being mentioned as a primary cause of the recession in the world economies. It is a common observation that firms have excess inventories at the beginning of a contractionary period. The excess inventories may lead the other problems during the recession, such as decrease in the market prices of the products and decline in the magnitude of the production and employment level etc.

II. THE OBJECTIVE OF THE INVENTORY-HOLDING

The specification of the closing inventories (CLSTOCK) is not a simple task. Identification of the determinants of inventory holdings constitutes one of the more challenging tasks of computational economics. The task consists of defining rational objectives and then identifying the decision variables, and rules required for the achievement of their objectives. The traditional analyses of inventory holding decisions are based on comparing inventory to sales ratio. The various analyses recommended the different tools and options for inventory-level optimization. Among the objectives of the inventory decision discussed in the previous studies, the following stands out: -

A) The buffer stock for unanticipated expansion in demand:

The textbooks on production management and operation research refer this objective of inventory holding in the techniques of production planning. Inventory may be

maintained for the objective of the extra-ordinary increase in the demand for goods. This case highlights the cost-benefit elements in terms of inventory holding costs versus the cost of foregone sales and goodwill. For this objective a part of the goods is remained in holding, which is known as a buffer stock.

B) Inventories for smooth production:

This objective is largely discussed in the literature on cost accounting. An inventory may also be held for smoothness in the production process. The optimal stock for this objective is known as Economic Order Quantity (EOQ). The economic order quantity (EOQ) depends on the required-time to receive the material after issuance of a purchase order. However, opportunity cost of interest on investment in inventories is also an important factor of inventories holding.

C) Inventories as some desired ratio to sales:

This concept is led by the accelerator principle in macroeconomics. A higher inventory leads the higher growth. According to the principle, the inventory holding for a year will be determined by the expected sale in the next year.

D) The speculative factors in inventories:

Another stand of analysis has emphasized the speculative element in the inventory decisions. One of the major objectives of the inventory-holding policy is to capitalize the anticipated price changes. This factor is always applied in the industries where heavy fluctuations are expected in the products' prices. It may also be applied in case of the expected volatility in the inputs' prices. The inventories of the precious metals, coffee, cotton, and tea are best examples of such an objective. If an industry consumes the imported raw material, the inventory may depend on the speculative factor of exchange rate. Similarly, the speculative factors are also applied in the export-oriented industries. A leading hypothesis in this connection concerned the importance of spot and forward exchange rates, transport costs and the preponderance of domestic over foreign sales by affiliated companies. The importance of exchange rates suggests that the speculative role often attributed to inventories may have a significant international dimension (Stern: 1993)

Stern (1993), concluded that the effect of 'just-in-time' inventory policies, which have driven US inventory-sales ratio to historically low levels, is not in evidence in the year-to-year inventory-sales ratio of the foreign affiliates of US parent firms during the 1982-87 span.

III. THE DETERMINANTS OF INVENTORIES

Traditional studies in cost accounting recommend Economic Order Quantity (EOQ), and Buffer Stock techniques. Baumol's (1952) formula for optimal cash transactions is applied in the typical textbooks of the cost accounting. The formula is based on the mathematics of optimization. But, it has been observed in a number of cases that the Economic Order Quantity was not related with the closing inventories in the balance sheet of a firm. It is important that the inventories in the balance sheet do not necessarily mean the average inventory. The magnitude of the inventory in the balance sheet of a firm depends on the length and variation of the inventory cycle. According to the standard accounting practice, the maximum inventory-level can be

defined as the level of the Economic Order Quantity (EOQ) plus the Buffer Stock. The optimal level of the inventory will be the maximum level determined through the buffer stock and the Economic Order Quantity depending upon the pattern and volume of the required sales and production.

The accountants divide the factors of inventories into two classes:

A) The factors influence the Economic Order Quantity (EOQ):

The usage of the raw material, the acquisition cost of the material, the cost of inventory storage and management and the opportunity cost of the investment in the inventories are included in those factors.

B) The Factors Influences the Buffer Stock:

The opportunity cost of investment in the inventories and the opportunity cost of unearned profit in case of the demand could not be met due to the non availability of the material are two major determinant of the buffer stock.

We have reconciled the short-term accounting with the long-term economic planning for inventories. It is concluded in the forthcoming model that the volume of inventories depends on the difference between the profit margin and the rate of interest. It is interesting that the financial variables determine the physical inventories in the long-term. In other words, the volume of inventories depends on the relative returns on the short-term deposits and the profit margin on the investment in the inventories.

IV. THE MODEL

The derivations are based on the realistic assumptions that have been taken for the formulation of a basic model. In case of the violation of one or more assumptions, the model will remain valid after some modifications. The inventories have been factorized into three parts, for the analysis:

- 1) The inventories for the smooth production process, i.e. Economic Order Quantity (EOQ)
- 2) The Buffer Stock, to meet the requirement of the unexpected surge in the demand
- 3) The incremental inventories due to the lower sales on credit

Furthermore, the Buffer Stocks, Economic Order Quantity and Closing Stocks have been taken in monetary term (not in physical term).

A) Basic assumptions.

- 1) The firm has a linear cost function.
- 2) The linear demand function is applied for the firm's products.
- 3) No change in the inventory due to price effects.
- 4) No, extra cost will be involved in the recovery of the receivables. The profit margin on cash sales will be equal to the profit margin on credit sales.

- 5) The elasticity of profit with respect to the quantity stock (b) will be remained constant.
- 6) The Profit margin (p) is not equal to the short-term interest rate (i).

B) The Derivations.

Proposition (I): "The inventory-holding decision depends on three objectives: The Economic Order Quantity for smooth production, The Buffer Stock to meet the unexpected surge in the demand and the credit sales policy of a firm. In case of a soft credit policy the volume of inventory will be declined because of the shifting of inventories into the induced sales".

Proposition (II): "The firm will minimize its cost of inventory holding. The inventory holding cost is divided into two components, carrying cost (i) and ordering cost. The carrying cost includes the storage, insurance, and opportunity costs of investment in the inventories. Such costs are directly related with the volume of inventories. A higher volume of the inventory indicates the higher cost. A normal distribution in the pattern of the utilization of the inventories has been assumed. It implies that the carrying cost will be equal to the arithmetic mean of the maximum and minimum levels of the inventories. The Ordering Cost (OC) is directly concerned with the number of purchase' orders in a year. The fixed costs of shipping, handling, and documentation are included in the Ordering Cost (OC)".

Proposition (III): "The Ordering Cost (OC) will be subject to the diminishing return with respect to the Economic Order Quantity (EOQ). The annual Ordering Cost (OC) will decrease in case of the higher Economic Order Quantity. It is because of the certain discounts in the price, handling and transportation, in case of the higher quantity".

1. Economic Order Quantity (EOQ):

$$TC = i (EOQ/2) + OC (Q/EOQ^c) \text{ ----- (1)}$$

Where,

'TC' is total cost of inventory holding

'EOQ' is Economic Order Quantity

'i' is inventory-carrying cost including storage, insurance and opportunity cost of investment, as a percentage of the Economic Order Quantity.

'OC' is inventory acquisition cost for per order. This includes shipping, handling and documentation expenses.

'Q' is annual sales or demand for the company's products.

'c' is constant elasticity of Economic Order quantity for the inventory holding cost.

Minimization condition:

$$\partial TC / \partial EOQ = 0.5i - c.OC.Q.EOQ^{c-1} = 0$$

$$0.5i = c.Q.OC / EOQ^{1-c}$$

$$EOQ^{1-c} = 2.c.OC.Q / i$$

$$EOQ = (2.c.OC.Q / i)^{1/1-c}$$

Where,

$$'\alpha' = 1 / (1 - c)$$

$$EOQ = (2.c.OC/i)^\alpha Q^\alpha \text{ ----- (2)}$$

2. Buffer Stock:

Proposition (IV): "The firm will minimize the holding cost of the buffer stock. The holding cost of the buffer stock includes the carrying cost (i) and opportunity costs of additional profits in case of non availability of the inventory to meet the supply orders for unexpected surge in the demand. The carrying cost (i) covers the storage, insurance and the opportunity cost of investment in the inventories. While, the cost of arrangement of the additional production in urgency (HL) and the profit (p) that could not be earned due to the shortage of the material are included in the opportunity cost of profits".

The basic condition in the 'Buffer Stock' decision making is to equalize the stock carrying cost and opportunity cost of profit. (The opportunity cost must be equal to the opportunity profit).

So,

$$pBFRSTK \cong HL + iBFRSTK \text{ ----- (3)}$$

Where,

'BFRSTK' is the buffer inventory to meet the requirement of extra ordinary surge in demand.

'HL' is the fixed cost to arrange the extra of production.

'p' is the profit margin on the inventory.

$$pBFRSTK - iBFRSTK \cong HL$$

$$BFRSTK = HL / (p - i)$$

(C) The Induced Inventories due to the lower Sales on Credit:

Proposition (V): "There is a negative relation between the inventories and the receivables from the customers. The magnitude of the inventory will be smaller for those firms, where the soft credit sales policy will be available to the customers".

$$\text{STKOUT} = -\beta \text{DBTRS} \text{ ----- (4)}$$

Where,

'STKOUT' is the induced inventory that is created by the soft credit policy of a firm.

'DBTRS' is the accounts receivables (including notes receivables) from the customers.

' β ' is the partial effect of the magnitude of the receivables on the inventories. (The expected sign of ' β ' is negative).

$$\text{AVGSTK} = [(2\text{OC}/i)^\alpha Q^\alpha] / 2 + [\text{HL}/(p-i)] - \beta \text{DBTRS}$$

Where,

'AVGSTK' is the average inventories.

$$\begin{aligned} &= [(2^\alpha \text{OC}^\alpha Q^\alpha) / i^\alpha 2] + [\text{HL}/(p-i)] - \beta \text{DBTRS} \\ &= [2^{\alpha-1} \text{OC}^\alpha i^{-\alpha} Q^\alpha] + [\text{HL}(1/p-i)] - \beta \text{DBTRS} \\ &= \gamma Q^\alpha - \beta \text{DBTRS} + [\text{HL}(1/p-i)] \end{aligned}$$

Where,

$$\gamma = 2^{\alpha-1} \text{OC}^\alpha i^{-\alpha}$$

$$\lambda = \text{HL}$$

$$= \gamma + \alpha \ln Q - \beta \text{DBTRS} + \lambda(1/p-i)$$

$$\text{AVGSTK} = \gamma + \lambda(1/p-i) + \alpha \ln Q - \beta \text{DBTRS} \text{ -----(5)}$$

From the assumption (1) and (2) we can infer the following results:

$$\text{Cost Function: } C = a_0 + a_1 Q \text{ ----- (6)}$$

$$\text{Demand Function: } Q = b_0 - b_1 P \text{ ----- (7)}$$

Where,

C = Total cost of goods sold

Q = The sales (or demand) of the firm's products

P = is the per unit price.

$$P = (b_0/b_1) - (1/b_1)Q$$

$$P = c_0 - c_1 Q$$

Where,

$$c_0 = b_0/b_1$$

$$c_1 = 1/b_0$$

$$\text{TR} = c_0 Q - c_1 Q^2 \text{ ----- (8)}$$

$$\Pi = c_0 Q - c_1 Q^2 - a_0 - a_1 Q \text{ ----- (9)}$$

$$\Pi = (c_0 - a_1)Q - c_1 Q^2 - a_0$$

$$\partial \Pi / \partial Q = (c_0 - a_1) - 2c_1 Q = 0$$

$$\begin{aligned} \text{Second Order Condition: } \partial^2 \Pi / \partial Q^2 &= -2c_1 \\ \partial^2 \Pi / \partial Q^2 &= -2(1/b_1) \text{ ----- (10)} \end{aligned}$$

Corollary I:

" The profit will decrease with respect to the quantity sold, after an optimal level of sales. The magnitude of the rate of decrease in the profit will be twice of the reciprocal of the change in the demand with respect to the change in price".

$$\begin{aligned} \text{Optimal } Q &= (c_0 - a_1) / 2c_1 \\ &= [(b_0/b_1) - a_1] / (2/b_1) \\ &= [b_0 - a_1 b_1] / 2 \text{ ----- (11)} \end{aligned}$$

Where, 'b₀' is the maximum level of the demand in the market.

$$\begin{aligned} &= [\text{Max demand} - \{(\Delta C / \Delta Q)(\Delta Q / \Delta P)\}] / 2 \\ &= [\text{Max demand} - (\Delta C / \Delta P)] / 2 \\ &= 0.5 [\text{Max demand} - (\Delta C / \Delta P)] \text{ ----- (12)} \end{aligned}$$

Corollary II:

" The Optimal Sales volume for the profit maximization of a company will be 50 percent of the difference between the maximum demand in the market and the ratio of the incremental cost to the incremental price".

$$\begin{aligned} \text{AVGSTK} &= \gamma + \lambda \{1 / (p - i)\} + \alpha \ln Q - \beta \text{DBTRS} \\ &= \gamma + \lambda \{1 / (p - i)\} + \alpha \ln \{(b_0 - a_1 b_1) / 2\} - \\ &\quad \beta \text{DBTRS} \\ &= \gamma + \alpha \ln \{(b_0 - a_1 b_1) / 2\} + \lambda \{1 / (p - i)\} - \\ &\quad \beta \text{DBTRS} \\ \text{AVGSTK} &= \varpi_0 + \varpi_1 \{1 / (p - i)\} + - \varpi_2 \text{DBTRS} \text{ ----- (13)} \end{aligned}$$

Where,

$$\varpi_0 = \gamma + \alpha \ln \{(b_0 - a_1 b_1) / 2\}$$

$$\varpi_1 = \lambda$$

$$\varpi_2 = \beta$$

So,

$$\text{AVGSTK} = f(p, i, \text{DBTRS}) \text{ -----(14)}$$

Corollary III:

" The average volume of the inventory of a firm depends on the volume of receivables from customers and the reciprocal of the difference between the profit margin and the short-term interest rate".

Corollary IV:

If, profit margin (p) is greater than the short-term rate of interest (i), the volume of inventory will has been increasing with the increase in the difference between the profit margin (p) and interest rate (i). If profit margin (p) is less than the interest rate (i), the level of the inventory will be increase with the decrease in the difference of the profit margin and the interest rate.

V. THE EMPIRICAL TESTING AND RECOMMENDATIONS:

The model provides a simple formula for the estimation of the optimal level of inventories. However, to test the model is not a simple task. The data in the uniform accounting bases is the main hurdle in the empirical testing. We tested the model in the context of Pakistan. For this purpose, the statistical parameters have been estimated through the pooled data of the annual audited accounts of 225 companies listed on the Karachi Stock Exchange. The accounts cover the period of 1980 to 1994, giving us 3,375 observations (225 companies and 15 years). The data for the model have been obtained from a variety of sources, depending on the definition and the nature of the variables. However, most of the data on different variables have been extracted from the annual reports of the listed companies. However, data for some variables have also been extracted from the various issues of the annual reports of the State Bank of Pakistan (State Bank of Pakistan: 1995-96, 19990-91, 1986-87, 1982-83).

All the data are in million of the rupees, except the interest rate and the profit margins which are in the percentage.

STATISTICAL RESULTS

Variables	Coefficients	T-Statistics
Intercept	53.887	11.93
,(1/i-p)	0.329	1.21
DBTRS	-0.923	125.78
Adjusted R-Square		0.9152

The results revealed some interesting observations: First, the traditional formula for the inventory decision-making is not longer valid in the present business environment. The recession in the recent past gave a lesson to the large corporation that the problem of excess inventories must be addressed. Our analysis provides a solution in this scenario. It is found that the receivable is a substitute of the inventories. The firm should adopt the soft credit policies during the recession. By such a way, the problem of the excess inventories may be solved. The statistical results also confirm the acceptability of the hypothesis.

Second, the magnitude of the inventory depends on the reciprocal of the difference of the rate of the interest and the marginal returns.

The most important conclusion is that the 'Usage of material' or sale volume are not the real determinates of the volume of inventory. The 'Usage of material' depends on the sales, while the sale is a function of the profitability. It may be inferred that the optimal profit level determines the sales volume. So, the traditional methods of computations in the cost accounting and the production management should be modified accordingly. Baumol's formula may be valid for the optimal cash transaction, where marginal return are remain constant and the concept of discount can not be applied. But, it is not valid for the merchandising inventory management, in a competitive business environment.

It should be remembered that the model is based on the certain assumptions. In case of the violation of such assumptions the model will remain valid, but some modification will be required.

REFERENCES

1. AICPA, 1986 “**Accounting Trends and Techniques**”, New York, The American Institute of Certified Public Accountants.
2. Altman, Edward, 1968 “**Financial Ratios Discrimination Analysis and the Prediction of Corporate Bankruptcy**”, Journal of Finance, September 1968, 589-609.
3. Karachi Stock Exchange (1980-1994), “**Annual Reports of the Companies**”. Karachi.
4. Baumol, William, 1952 “**The Transaction Demand for Cash: An Inventory Theoretic Approach**”, Quarterly Journal of Economics, November 1952, 545-556.
5. Brackus D., Brainard W.C., Smith G., and Tobin J., 1980 “**A Model of US Financial and Non-financial Economic Behavior**”, Journal of Money, Credit and Banking, 1980 (12), 259-293.
6. Government of Pakistan, Various Issues, “**Statistical Year Book of Pakistan**”, Islamabad, Federal Bureau of Statistics.
7. Matz, Curry, Franks and Khan, A.R. (1982); “**Cost Accounting: An Emphasis to Management**”, Karachi : ICMAP.
8. Mayer C., 1988 “**New Issues in Corporate Finance**”, European Economic Review, 1988, 1167-1189.
9. Mehar, M. Ayub K., 1994, “**A Simulation Model of Corporate Finances: A Study of Companies Listed on the Karachi Stock Exchange**”, Karachi. Ph.D. Dissertation.
10. Mehar, M. Ayub K., 1992 “**A Forecasting Model of Public Finance: An Experience from the Pakistan Economy**”, M. Phil Dissertation, Karachi: AERC.
11. Mehar, M. Ayub K., Sohail Javed, and Samina Aijaz, 1993, “**Short Term Financing, Accounts Receivables, and Liquidity Position of the Firm**”, Pakistan Economic and Social Review, 1993 (2).
12. Mian L. Shehzad, and Clifford W. Smith, 1992, “**Account Receivable Management Policy: Theory and Evidence**”, The Journal of Finance, 1992 (1), 169-200.
13. Myers, S. 1977; “**Determinants of Corporate Borrowing**”; Journal of Financial Economics, 1977 (November) 145-177

14. Myers S.C; 1984; "Capital Structure Puzzle", Journal of Finance, July 1984, 575-592.
15. Robert B. Anthonyson and J. Philip Cooper, 1982, "**Simulation Model for Corporate Planning**", Cambridge, Mass. Dynamic Associates Incorporated, 1982.
16. Smith, Clird W., Jr., and Ross L. Watts, (1992), " **The Investment Opportunity Set and Corporate Financing, Dividend and Compensation Policies**", Journal of financial Economics, 32, 263-292
17. Smith, Janet Kiholm, 1987, "**Trade Credit and Informational Asymmetry**", Journal of Finance, 1987 (62), 863-871.
18. Smith, Keith,1980, "**Readings on the Management of Working Capital**", West Publishing Company.
19. State Bank of Pakistan, (Various Issues), "**Annual Report** ", Karachi .
20. State Bank of Pakistan, Various Issues, "**Balance Sheet Analysis**" Karachi: The State Bank of Pakistan.
21. Stern, Andrew (1993),"**Inventories in US Parent Companies and their Foreign Affiliates: An Analysis of Recent Data**"; Applied Economics. 1993 (25), 335-342.
22. Stulz, Rene M. and Herb Johnson (1988), " **An Analysis of Secured Debt**", Journal of Financial Economics, 14; 501-522.
23. Tobin, J., 1958, "**Liquidity Preference as a Behavior Towards Risk**", Review of Economic Studies, February 1958, 65-86.