From monopsonistic insurgent groups to oligopolistic cocaine traffickers: the market of cocaine in Colombia

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From monopsonistic insurgent groups to oligopolistic cocaine traffickers: the market of cocaine in Colombia

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Abstract
The main purpose of this note is to model an imperfect competitive and vertically integrated market structure of production and trafficking of cocaine. We consider the particular case of colombian cocaine market, but the results could be generalized to different scenarios. We model three main participants: farmers, producing the coca-leaf and being price-takers in its market; insurgent groups, producing paste of cocaine and being a local monopsony in the coca-leaf market; and cocaine traffickers, being an oligopoly competing a la Cournot. We find out an explicit relationship between the price of coca-leaf and paste of cocaine, with the coca-leaf elasticity of supply. An inelastic coca-leaf supply allows the insurgent groups to increase the gap between the price of coca-leaf and the price of the paste of cocaine. Additionally, the insurgent groups obtain important profits from the oligopolistic market structure of cocaine market, because the increase in the price of cocaine also increases the price of paste of cocaine, through the increase in its demand. These profits feed every step in the pyramid of cocaine production exacerbating the problem and making more difficult its solution. These remarks offers important information to explain the reasons behind the ineffectiveness of some national and international policies in the war against illegal drugs.

Key words: Colombia, coca-leaf, paste-of-cocaine, cocaine, insurgent-groups, monopsony, oligopoly.

JEL classification: D43, J42, K42.
1 Introduction

It is well known that Colombia is a very important participant in the market of cocaine. The critical consequences from this market in the social and political development of the country has been subject of intense debate, into the academic work and outside. Different national and international policies have tried to solve the problem. In particular, the Plan Colombia was designed with two main purposes, on one hand, reducing the production and trafficking of cocaine and, on the other hand, reducing the violent power of insurgent groups.

According to DNP (2006), the National Department of Planning of Colombia, its results are ambiguous, especially in the war against cocaine. Becker, Murphy and Grossman (2006) suggest an answer for this sort of puzzle by considering the price in-elasticity of demand. Decreases in the supply of cocaine increase its price and, given an in-elastic demand function, it increases the profits, so we end up with a very profitable business. This paradox motivates the research on the policies against the supply of illegal drugs, its functionality and results.

For the particular case of Colombia, the existing literature models the cocaine market as a perfect competitive and vertically integrated one. According to Mejía and Posada (2008) there are producers of coca-leaf, base or paste of cocaine, and finally cocaine. The farmers and insurgent groups participate in the production of coca-leaf and paste of cocaine, and the drug traffickers operate in the final market. The coca-leaf is a necessary input for producing base or paste of cocaine and, in turn, this is a necessary input for producing cocaine.

Toward a policy analysis, Grossman and Mejía (2008) models the war against drugs by considering some supply policies such as eradication and interdiction. With eradication the government control the crop of coca-leaf; and, with interdiction, it decreases the amount of traded illegal drug. Within a competitive partial equilibrium model, they discuss the work of insurgent groups in the conflict on productive factors, in particular, arable land for producing coca-leaf.

Mejía and Restrepo (2013) extends the later result by considering a competitive vertical integration of the market. There is also conflict on the control of arable land for producing coca-leaf. They found that interdiction is a more efficient policy than eradication. In the present note, we also model the market as vertically integrated, but we explicitly incorporate some imperfect competition factors in the market. However, we do not model the conflictive problem on the arable land. We assume the presence of insurgent groups in a region and study their market power.

The general idea of our model is as follows. There are some farmers producing the crop of coca-leaf in the mountains. They are price-takers facing a trade-off between producing coca-leaf, with important profits but risky, and producing other commodities, with low profits without risk. There are some insurgent groups with control on the national territory. They are a local monopsony in its territory by fixing the price of coca-leaf to induce the farmers to produce the crop\(^1\). Its function in the market is of producing paste of cocaine in laboratories with standardized techniques. They obtain important profits in trading it to cocaine traffickers.

Cocaine traffickers are the final group of the pyramid. The cocaine traffickers buy the paste of cocaine to insurgent groups, and produce the cocaine for consumption.

\(^{1}\)This market structure is remarked in the UNODC and Gobierno de Colombia (2013) report, and it is analyzed in Mejia and Rico (2010) with empirical data about the production of illegal drugs in Colombia.
They are an oligopoly competing a la Cournot with homogeneous marginal costs. The important profits from trading cocaine hold important profits from producing paste of cocaine which, in turn, hold important profits from producing coca-leaf, and it works by feeding the circle.

Considering imperfect competitive factors in the colombian cocaine market offers important elements in the discussion. First, the price of coca-leaf and paste of cocaine is related with the coca-leaf elasticity of supply. The in-elasticity of this supply enables insurgent groups to acquire important profits by putting relatively low prices to coca-leaf, and obtaining relatively high prices to the paste of cocaine. This elasticity is related with the scale returns of the coca-leaf production function and the eradication or interdiction programs from the national government.

Second, the price of the cocaine is not only determined by marginal costs and risks, but also the number of the traffickers. In an oligopoly we have some traffickers producing and trading a higher quantity than a monopolistic market, but a lower quantity than a perfect competitive market. This limitation in the production enables the traffickers to obtain important profits from the in-elasticity of the demand of cocaine, and it determines the prices from the market demand function.

This note is organized as follows. After this introduction, we study the production of coca-leaf. Then, we model the participation of insurgent groups as a local monopsony by fixing the price of the coca-leaf. Then, we study the market of cocaine as an oligopoly competing a la Cournot. Finally, we present a brief discussion on the elasticity of substitution in the coca-leaf production function, and the references.

2 Production of coca-leaf

Let us start with the farmers. They are the base of the pyramid of the cocaine production. They produce coca-leaf ($cl$) by using land ($l$) and other factors ($f_{cl}$), which may be capital or labour. Let $a, b \in \mathbb{R}_{++}$ be the technological factors of $l$ and $f_{cl}$ respectively, $0 \neq \rho < 1$ the elasticity of substitution between $l$ and $f_{cl}$, and $0 < \beta < 1$ the returns to scale of the production function.

Let us write the production function of $cl$ as a CES-production function:

$$cl = [al^\rho + b f_{cl}^\rho]^{\frac{\beta}{\rho}}$$

The governmental policies may have important effects on this production function. In particular, eradication and interdiction may decrease $a$ and $b$ by inducing the $cl$-producers to produce the crop using both non-conventional labour techniques and non-productive portions of land. In general, the production may be inefficient. These policies may also have a long-run negative effect on $\beta$.

The government aims to eliminate the production of $cl$ by going to the jungle. In one case, it could find and destroy the crop with probability $\sigma_{cl}^p$ and puts a penalty of $\chi_{cl}$ to the producers. In the other case, it could interdict a proportion $\tau_{cl}$ of traded $cl$ with probability $\sigma_{cl}^c$. Let $p_{cl}$ be the price of $cl$, and $r, w$ the prices of $l$ and $f_{cl}$ respectively. Let us define the expected profits from $cl$:

**Definition 1** A $cl$-producer maximizes the following expected profit function:

$$\mathbb{E} [\pi_{cl}] = \left(p_{cl} [al^\rho + b f_{cl}^\rho]^\frac{\beta}{\rho} \left(1 - \tau_{cl}\sigma_{cl}^c\right) - rl - wf_{cl}\right) \left(1 - \sigma_{cl}^p\right) - \chi_{cl}\sigma_{cl}^p$$

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Notice that \((1 - \tau_d \sigma_d^c)\) is the portion of \(cl\) that could effectively survive the interdiction of the government. Also, if \(\sigma_d^p = 1\) then the farmer loses its profits, and gets a penalty of \(\chi_d\). The farmer decides to produce \(cl\) if \(\mathbb{E}[\pi_d] > 0\), so \(cl\) increases with the increases of \(p_{cl}\) and the decreases of \(\sigma_d^p\), \(\sigma_d^c\), \(\chi_d\) and the costs.

**Proposition 1** The supply function of \(cl(p_{cl})\) is given by:

\[
cl(p_{cl}) = \left[ \frac{\beta p_{cl}(1 - \tau_d \sigma_d^c)ab}{[a(br)^{\rho - 1} + b(aw)^{\rho - 1}]^{1/\rho}} \right]^{\frac{1}{1 - \beta}}
\]

**Proof**: Maximize the expected profit function of Definition 1. □

The decrement in the factor productivity, decrease the optimal supply of \(cl\) but, again, the effect is neutralized by \(p_{cl}\). If \(p_{cl}\) is highly enough to give \(\mathbb{E}[\pi_d] > 0\) then the farmer becomes a \(cl\)-producer even with an inefficient system of production. The \(p_{cl}\) ends up being a fundamental variable in the production of \(cl\) and it depends crucially on the price-elasticity of the supply as we show in the next section.

The price elasticity of the supply of \(cl\) is given by \(\beta / (1 - \beta)\). The more decreasing returns to scale in the production function, the more inelastic supply of \(cl\). In the short-run, the elasticity depends on the technology of producing \(cl\); in the long-run, it may also depend on the result of governmental policies such as crop substitution, eradication and interdiction.

**Proposition 2** The optimal expected profit function of a \(cl\)-producer is given by:

\[
\mathbb{E}[\pi_d] = \left[ \frac{p_{cl}(1 - \tau_d \sigma_d^c)(\beta ab)^{1/\beta}}{[a(br)^{\rho - 1} + b(aw)^{\rho - 1}]^{1/\rho}} \right]^{1 - \beta} (1 - \beta)(1 - \sigma_d^p) - \chi_d \sigma_d^p
\]

**Proof**: Use Proposition 1 in the expected profit function of Definition 1. □

As we will see in the next section, the insurgent groups are able to control \(p_{cl}\). They use it as an instrument for inducing the farmers to produce the coca-leaf crop. The farmers face a trade-off between, from one hand, a risky activity penalized by the government, and on the other hand, a very profitable activity assisted by the insurgent groups. The market power and the efficiency of the governmental policies define the dynamic of this structure.

### 3 Production of paste of cocaine

Let us continue with the insurgent groups. They are the second stage of the pyramid, with the main purpose of connecting the \(cl\)-producers with the cocaine traffickers. They produce past of cocaine \((pc)\) by using coca-leaf \((cl)\) and other factors \((f_{pc})\), which may also be capital or labour. Let \(d \in \mathbb{R}_{++}\) be the Hicks-neutral technological factor of producing \(pc\), and \(\alpha \in (0, 1)\) the elasticity of \(pc\) to \(cl\).
Let us write the production function of $pc$ as a Cobb-Douglas-production function:

$$pc = df_{pc}cl^\alpha$$  \hspace{1cm} (2)$$

The government is also in conflict with the insurgent groups. In particular, it looks for the producers of paste of cocaine, which is a necessary input for producing cocaine. Let us suppose it could interdict a proportion $\tau_{pc}$ of $pc$ with probability $\sigma^c_{pc}$. The $pc$ is usually produced in the jungle, near to the $cl$-crop so $\sigma^c_{pc} \approx \sigma^c_{cl}$.

**Definition 2** The profits of producing $pc$ are given by:

$$\pi_{pc} = p_{pc}df_{pc}cl^\alpha (1 - \tau_{pc}\sigma^c_{pc}) - w_{pc} - p_{cl}(cl)cl$$

Where $p_{pc}$ is the price of paste of cocaine, and $p_{cl}(cl), w$ are the prices of coca-leaf and other factors, respectively. The $p_{cl}$ depends on $cl$ because the insurgent groups have local monopsonistic power. They are the unique $pc$-producer in the region, so they are the only one $cl$-buyer. They use this market power in fixing $p_{cl}$.

**Theorem 1** The insurgent groups fix $p_{cl}$ according to the following rule:

$$p_{cl} = \left[ \frac{\alpha p_{pc}df_{pc}(1 - \tau_{pc}\sigma^c_{pc})}{1 + \frac{\varepsilon_{cl,pcl}}{\varepsilon_{cl,pcl}} + \beta(1 - \tau_{cl}\sigma^c_{cl})ab} \right]^{\frac{1}{1-\alpha\beta}}$$

**Proof**: Taking $\frac{d\pi_{pc}}{dcl} = 0$ we have:

$$\alpha p_{pc}df_{pc}cl^\alpha(1 - \tau_{pc}\sigma^c_{pc}) = p_{cl}(cl) + \frac{dp_{cl}(cl)}{dcl}$$

Let $\varepsilon_{cl,pcl} = \frac{dcl}{dp_{cl}}$ be the price elasticity of the supply of $cl$. Then we have,

$$\alpha p_{pc}df_{pc}cl^\alpha(1 - \tau_{pc}\sigma^c_{pc}) = p_{cl} \left( 1 + \frac{1}{\varepsilon_{cl,pcl}} \right)$$

Replace Proposition 2 and, after some calculations, we have the required result. $\square$

There are some insights behind this equation. First, increments in the marginal income of $pc$-producers induce increments in the marginal income of $cl$-producers. The profits in the business benefit every step of the pyramid. Second, the $p_{cl}$ must cover both the marginal costs of production and the associated risk.

Third, an elastic supply of $cl$ induces insurgent groups to increase $p_{cl}$ for sustaining the production of $cl$. Fourth, increments in the technology of $pc$ increases its production, and its demand of $cl$, so $p_{cl}$ also increase. Finally, the effect of $\sigma^c_{pc}$ on $p_{cl}$ is ambivalent because it also affect $p_{pc}$ so we will analyse it in the next section.

We could calculate the supply of $cl$ with the previous information.

**Proposition 3** The supply of $cl$ is given by

$$cl = \left[ \frac{\alpha \beta abdp_{pc}f_{pc}(1 - \tau_{pc}\sigma^c_{pc})(1 - \tau_{cl}\sigma^c_{cl})}{a^{\frac{\rho}{\rho - 1}} + b^{\frac{\rho}{\rho - 1}} + \frac{\rho - 1}{\rho} \left( 1 + \frac{1}{\varepsilon_{cl,pcl}} \right)} \right]^{\frac{\rho - 1}{\rho}}$$
Proof: Replace Theorem 1 in Proposition 1. □

There is a positive relationship between \( cl \) and \( p_{pc} \). A possible reason from the model is because the increment in \( p_{pc} \) increase the production of \( pc \), and in turn it increases the demand of \( cl \), increasing \( p_{cl} \). It is a possible way to transfer profits from an step of the pyramid to another. Now, we are able to estimate the supply function of \( pc \):

**Proposition 4** The supply function of \( pc \) is given by:

\[
p_{pc} = \left[ \frac{\alpha \beta ab p_{pc} (d_{p_{pc}})^\frac{1}{\sigma} (1 - \tau_{pc} \sigma_{p_{pc}})^{(1 - \tau_{cl} \sigma_{p_{cl}})}}{\left[ a (br)^{\frac{2}{\rho}} b (aw)^{\frac{2}{\rho}} \left( 1 + \frac{1}{\tau_{cl} \sigma_{p_{cl}}} \right) \right]^{\frac{2 - 1}{\rho}}} \right]^{\frac{\alpha \beta}{1 - \alpha \beta}}
\]

Proof: Replace Proposition 3 in the equation 2. □

The insurgent groups have local monopsonistic power in the \( cl \) market. However, there are some insurgent groups in the country, so we assume they do not have any market power in the \( pc \) market. In this case, the \( p_{pc} \) is determined through the interaction between insurgent groups and cocaine traffickers. We study this interaction in the following section.

## 4 Cocaine traffickers

Let us finish the vertical structure of the market with the cocaine traffickers. They are on the top of the pyramid of cocaine business, obtaining important profits and also assuming important risks. They produce cocaine, and trade it in a country different where it was produced. The reason of that is the important difference in the willingness-to-pay for cocaine of the consumers in both countries.

There are \( n \) cocaine traffickers, and let \( i \in I = \{1, 2, \ldots, n\} \) be their counter. Let \( c = \sum_{i=1}^{n} c_i \) be the total quantity of cocaine, where \( c_i \) is the portion of the market corresponding to trafficker \( i \in I \). The production function is \( c_i = (1/n)pc \) where \( (1/n) \) is the amount of \( pc \) used by trafficker \( i \) in producing \( c_i \).

As buyers, cocaine traffickers have no important power in the market of paste of cocaine. However, as sellers, they operate in an oligopolistic market competing each other a la Cournot. They try to increase the quantity of \( c \) for obtaining an important participation in the market, nevertheless, it is constrained to the marginal cost of production. Suppose the marginal cost is homogeneous among them.

Let us assume that each cocaine trafficker has a marginal cost of \( \mu \). The government wants to eliminate the production of \( c \) by seeking cocaine traffickers. It could eliminate the production of \( c_i \) with probability \( \sigma_{p_{ci}}^p \), and puts a penalty of \( \chi_c \) to the trafficker. It could also interdict a proportion \( \tau_c \) of \( c_i \) with probability \( \sigma_{p_{ci}}^c \).

The expected profits are given by:

**Definition 3** The profits from producing \( c_i \) are given by:

\[
E[\pi_{c_i}] = (p_c c_i (1 - \tau \sigma_{p_{ci}}^e) - \mu c_i) (1 - \sigma_{p_{ci}}^p) - \chi_c \sigma_{p_{ci}}^p
\]
Where $p_c$ is the price of cocaine in the consumer country. It is clear $p_{pc}$ is part of $\mu$ but, given the huge difference $p_c - p_{pc}$, we consider $p_{pc}$ is not relevant in the maximization process of the cocaine trafficker. We prefer to consider $p_{pc}$ as a non-significantly part of $\mu$, and add it to the transportation cost, and the efforts of passing the interdiction, or trading the cocaine in the market.

As it is usual, the term $(1 - \tau_c \sigma_c^e)$ is the portion of $c_i$ passing the interdiction of the government in the producer country. Also, If $\sigma_c^p = 1$ then the drug trafficker loses its profits and gets a penalty of $\chi_c$. The government of the consumer country puts a penalty of $\kappa$ for being caught consuming $c$ with probability of $\sigma_c^d$. Let $\eta$ be a parameter of persistence in the consumption of $c$.

**Definition 4** The cocaine market demand function is given by:

$$p_c = \eta c^{-\theta} - \kappa \sigma_c^d$$

Where $\theta \in \mathbb{R}_{++}$ is a parameter affecting the elasticity of the demand of $c$. With this demand function we use the following expected profit function:

**Definition 5** The profits of the cocaine trafficker $i \in I$ are given by:

$$\mathbb{E} [\pi_{ci}] = \left( (\eta c^{-\theta} - \kappa \sigma_c^d) c_i (1 - \tau_c \sigma_c^e) - \mu c_i \right) (1 - \sigma_c^d) - \chi c^p$$

The cocaine trafficker maximizes its profit function in terms of $c_i$. The solution of this problem gives us the optimal individual quantity produced for each trafficker, the total quantity traded in the market and its price. With this information we are able to solve the problem for the market of $pc$.

**Theorem 2** The total quantity of cocaine traded in the market is given by:

$$c = \left( \frac{\eta (1 - \tau_c \sigma_c^e)}{[\mu + \kappa \sigma_c^d(1 - \tau_c \sigma_c^e)]} \right)^{\frac{1}{\theta}}$$

The price of the cocaine traded in the market is given by:

$$p_c = \frac{n \mu + \theta (1 - \tau_c \sigma_c^e) \kappa \sigma_c^d}{(n - \theta)(1 - \tau_c \sigma_c^e)}$$

**Proof**: We proceed in two steps. First, we prove that each cocaine trafficker has the same participation in the market. Let $i \neq j \in I$ be two cocaine traffickers with the following optimal conditions:

$$\left[ \eta (c^{-\theta} - \theta c_i c^{-\theta - 1}) - \kappa \sigma_c^d \right] (1 - \tau_c \sigma_c^e) = \mu$$

$$\left[ \eta (c^{-\theta} - \theta c_j c^{-\theta - 1}) - \kappa \sigma_c^d \right] (1 - \tau_c \sigma_c^e) = \mu$$

From these conditions, we have

$$c^{-\theta} - \theta c_i c^{-\theta - 1} = \frac{\mu + \kappa \sigma_c^d (1 - \tau_c \sigma_c^e)}{\eta (1 - \tau_c \sigma_c^e)}$$

$$c^{-\theta} - \theta c_j c^{-\theta - 1} = \frac{\mu + \kappa \sigma_c^d (1 - \tau_c \sigma_c^e)}{\eta (1 - \tau_c \sigma_c^e)}$$
Then, we have each cocaine trafficker has the same participation in the market because \( c_i = c_j \). Write \( c_1 = c_2 = \ldots = c_n \). We are able to write \( c \) as \( nc_i \) in the \( i \)'s optimality condition. After some calculations we have:

\[
c_i = \frac{1}{n} \left( \frac{\eta(1 - \tau_c \sigma^c_c)}{\mu + \kappa \sigma^d_c(1 - \tau_c \sigma^c_c)} \right) \frac{n - \theta}{n} \right)^{\frac{1}{2}}
\]

From this, we derive directly the claimed result. □

The risk associated to the traffic and consumption of \( c \) increase its price. Because the risk is so important, the marginal income must be so important. If the demand is inelastic, the more price the more total income of cocaine traffickers, so it increases the disposition to produce \( c \). However, the more number of cocaine traffickers, the less price of cocaine because it also increases the total amount of cocaine.

Let us estimate the demand of paste of cocaine:

**Proposition 5** The demand of \( pc \) is given by:

\[
pc = \left( \frac{\eta(1 - \tau_c \sigma^c_c)}{\mu + \kappa \sigma^d_c(1 - \tau_c \sigma^c_c)} \right) \frac{n - \theta}{n} \right)^{\frac{1}{2}}
\]

**Proof**: Because \( c = pc \). □

One idea behind this equation is that the more persistence \( \eta \) in the consumption of \( c \), the more production of \( c \), and \( pc \) and \( cl \). We then have a rainfall effect, through the prices and quantities, from the top to the base of the pyramid. The more profitable cocaine market, the more profitable paste of cocaine and coca-leaf markets.

Finally, we are able to estimate the \( p_{pc} \) previously considered.

**Proposition 6** The \( p_{pc} \) is given by:

\[
p_{pc} = \frac{a \left( \frac{1}{b} \right)^{\frac{1}{\tau_c}} + b \left( \frac{1}{a} \right)^{\frac{1}{\tau_c}}}{\alpha \beta \alpha \beta} \left( \frac{\eta(1 - \tau_c \sigma^c_c)}{\mu + \kappa \sigma^d_c(1 - \tau_c \sigma^c_c)} \right) \frac{n - \theta}{n} \right)^{\frac{1}{2}}
\]

**Proof**: Equal Propositions 4 and 5. □

This equation relates insurgent groups, simultaneously, with \( cl \)-producers and cocaine traffickers. First, the more in-elastic supply of \( cl \), the higher \( p_{pc} \). With an in-elastic supply curve of \( cl \) the insurgent groups are able not only to decrease \( p_{cl} \) but increase \( p_{pc} \). It increases the gap \( p_{pc} - p_{cl} \) ending up with a very profitable business for them.

Second, the increment of \( c \)-production increases \( p_{pc} \) through its demand. There is a perfect positive relation in the production of \( c \) and \( pc \) so they move in the same way. Third, the risk may also modify \( p_{pc} \). The riskier \( cl \) production, the higher \( p_{cl} \) and the higher \( p_{pc} \) to compensate the difference. Also, the riskier \( c \) production, the higher \( p_c \) and, given an in-elastic demand of \( c \), that increases \( c \), so it also increases \( pc \) and \( p_{pc} \).
5 Appendix

In this appendix we are going to estimate $p_{cl}$, $cl$, $pc$ and $p_{pc}$ when $\rho$ changes. In particular, we consider two scenarios. First, we could have $\rho = 0$, with no substitution between $l$ and $f_{cl}$. The eradication policy may have important effects in reducing $cl$ by reducing the proportions of $l$ and $f_{cl}$. In this case, the factors are perfect complements and we are able to write the production function as $cl = \min\beta\{al, bf_{cl}\}$.

Second, we could have $\rho \to -\infty$, with perfect substitution between $l$ and $f_{cl}$. The policies of eradication or land substitution may have no important effects, because the producer is able to alter the usage of one factor to another. In this case, the factors are perfect substitutes and the production function may be written as a linear function $cl = (al + bf_{cl})^\beta$.

**Corollary 1** The insurgent groups fix $p_{cl}$ according to the following rule:

a) If $\rho \to 0$:

$$p_{cl} = \left(\frac{\alpha p_{pc}df_{pc}(1 - \tau_{pc}\sigma_{pc})}{1 + \frac{1}{\varepsilon_{cl,p_{cl}}}}\right)^{1-\beta} \left(\frac{br + aw}{\beta(1 - \tau_{cl}\sigma_{cl})ab}\right)^{\beta(1-\alpha)}$$

b) If $\rho \to -\infty$:

$$p_{cl} = \left(\frac{\alpha p_{pc}df_{pc}(1 - \tau_{pc}\sigma_{pc})}{1 + \frac{1}{\varepsilon_{cl,p_{cl}}}}\right)^{1-\beta} \left(\min\{r_{a}^{\sigma_{pc}}, w_{b}^{\sigma_{pc}}\}\right)^{\beta(1-\alpha)}$$

**Corollary 2** The supply of $cl$ is given by:

a) If $\rho \to 0$:

$$cl = \left[\frac{\alpha\beta abdp_{pc}f_{pc}(1 - \tau_{pc}\sigma_{pc})(1 - \tau_{cl}\sigma_{cl})}{(br + aw)^{\alpha\beta}(1 + \frac{1}{\varepsilon_{cl,p_{cl}}})}\right]^{\frac{1}{1-\alpha\beta}}$$

b) If $\rho \to -\infty$:

$$cl = \left[\frac{\alpha\beta dp_{pc}f_{pc}(1 - \tau_{pc}\sigma_{pc})(1 - \tau_{cl}\sigma_{cl})}{\min\{r_{a}^{\sigma_{pc}}, w_{b}^{\sigma_{pc}}\}(1 + \frac{1}{\varepsilon_{cl,p_{cl}}})}\right]^{\frac{1}{1-\alpha\beta}}$$

**Corollary 3** The supply of $pc$ is given by:

a) If $\rho \to 0$:

$$pc = \left[\frac{\alpha\beta dp_{pc}(df_{pc})^{\frac{1}{\alpha\beta}}(1 - \tau_{pc}\sigma_{pc})(1 - \tau_{cl}\sigma_{cl})}{(br + aw)^{\alpha\beta}(1 + \frac{1}{\varepsilon_{cl,p_{cl}}})}\right]^{\frac{1}{1-\alpha\beta}}$$

b) If $\rho \to -\infty$:

$$pc = \left[\frac{\alpha\beta p_{pc}(df_{pc})^{\frac{1}{\alpha\beta}}(1 - \tau_{pc}\sigma_{pc})(1 - \tau_{cl}\sigma_{cl})}{\min\{r_{a}^{\sigma_{pc}}, w_{b}^{\sigma_{pc}}\}(1 + \frac{1}{\varepsilon_{cl,p_{cl}}})}\right]^{\frac{1}{1-\alpha\beta}}$$
Corollary 4 The $p_{pc}$ is given by:

a) If $\rho \rightarrow 0$

$$p_{pc} = \frac{(br + aw) \left( 1 + \frac{1}{\tau_{cl,pc}} \right)}{\alpha \beta ab (df_{pc})^{\frac{1}{\alpha \beta}} (1 - \tau_{pc} \sigma_{pc}^c)(1 - \tau_{cl} \sigma_{cl}^c)} \left( \frac{\eta(1 - \tau_c \sigma_c^c)}{[\mu + \kappa \sigma_c^d(1 - \tau_c \sigma_c^c)] n} \right)^{1 - \alpha \beta \frac{n}{\alpha \beta}}$$

b) If $\rho \rightarrow -\infty$

$$p_{pc} = \frac{\min\{\frac{w}{a}, \frac{w}{b}\} \left( 1 + \frac{1}{\tau_{cl,pc}} \right)}{\alpha \beta (df_{pc})^{\frac{1}{\alpha \beta}} (1 - \tau_{pc} \sigma_{pc}^c)(1 - \tau_{cl} \sigma_{cl}^c)} \left( \frac{\eta(1 - \tau_c \sigma_c^c)}{[\mu + \kappa \sigma_c^d(1 - \tau_c \sigma_c^c)] n} \right)^{1 - \alpha \beta \frac{n}{\alpha \beta}}$$

In each case, there are differences in the impact of the technological factors and factorial costs on each estimated variable. With perfect complements, both technological factors and factorial costs have a significant impact on the estimated variables, however, with perfect substitutes, there is only one technological factor or only one factorial cost with a significant impact. With them, we have the same sort of previously considered relations but with different proportions.

References


