An Explanation of Economic Change and Development

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2014

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1. Intoduction

The contribution to the explanation of economic change that this paper sets out is centered on a core of interconnected endogenous variables, mainly innovation, radical uncertainty and entrepreneurship, which current economic analyses consider only in part and separately, sometimes as endogenous but for the most as exogenous. The article (and the formalized model) suppose that the functioning of the economy is not disturbed by the operation of pathological factors mainly concerning public sector, as largely happens in current time, for instance: excessive public debt and public deficit; great inefficiencies and wastes in public sector and administration, and hence high taxation; inefficiencies, slowness and arbitrariness of judicial power; diffused organized criminality; financial capital operating, mainly at the international level, as master instead of servant of production, that is, largely devoted to speculation. A proper and efficient operation of the economy needs that those anomalies are absent. We attempt to explain economic change and development with regard to modern dynamic economies where the above pathologies have been removed. This supposition would be strengthened by the reduction of the model to only 'necessary' variables, as devised in sub-section 3.1.2

The theoretical frame of the proposed explanation is a dynamic competitive process: that is, a competition based not merely on prices but also put into action by entrepreneurs' search for opportunities of profit attached to successful innovations, which generate profits through temporary monopolies and also engender disequilibria and radical uncertainty that will provide additional opportunities of profit.

This dynamic competitive process is a great agent of economic change and evolutionary motion. As a first stage approximation, it can be thought of as a combination of Schumpeterian innovative entrepreneurship and action with the neo-Austrian market process and entrepreneurship: a combination describing the advent of innovations and the subsequent adaptive push enacted by the imitative diffusion of innovations and the search for other opportunities of profit allowed by rising disequilibria and uncertainty; a push that leads towards the reduction of the inconsistencies and radical uncertainty caused by innovation and (hence) towards a reorganization and re-equilibration of the economy on new structural bases.

The understanding of the process of change and development is greatly obscured by the current separation of the two theoretical perspectives above. But it must be added that the explanation of such processes requires more than the simple combination of the two perspectives. In particular, it is essential that the notion of radical uncertainty – of which the Schumpeterian theory of economic development gives no explicit importance – is deepened. For its part, the neo-Austrian analysis of the market process, while attributing a great importance to radical uncertainty, thinks of it simply as a fog, an exogenous variable. We shall see that the explanation and measurement of radical uncertainty is a crucial – albeit very controversial and delicate – element of the understanding of the process of economic change and development. Moreover, we shall underline that the two theoretical perspectives (Schumpeterian and neo-Austrian) lack an adequate explanatory analysis of both the main agent of the whole process, that is, entrepreneurship (mainly its availability) and innovations.

It must be underlined that the notion of profit relevant with regard to the envisaged dynamic competition process does not include interest on the employed capital; it concerns only true profits, the so-called extra-profits resulting from entrepreneurial gains from successful innovations and the profit opportunities attached to the consequent disequilibria and uncertain perspectives. The ratio between those profits and the capital employed, expressed as the profit rate, is relevant mainly in that it is the only reliable indicator of the degree of success of an entrepreneur's decision making, primarily in introducing innovations and meeting disequilibria and uncertainty. However, here we are not interested in the distribution of profits, that is, whether profit takes on a capitalist nature or is yielded by public or self-managed firms, etc. Such distributive characteristics express simply a choice of civilization, which is incidental to the mere question of economic change and development.

Our model is not limited to the explanation of the core variables (that is, various kinds of innovation, such as radical and incremental process innovations and innovations of product, the demand and supply of entrepreneurship, and radical uncertainty) crucial in the representation of the whole process of change and the inherent disequilibrating and re-equilibrating evolutionary motion. The specified model also includes (and explains) other important variables such as output, employment, investment, prices, and wages. It refers to the maximum level of sectoral disaggregation, a sector for each specific good, and describes long waves. A specification with a restricted number of sectors is used for simulations.

The structure of the paper is as follows: Section 1 concerns the introduction, while a second section is dedicated to a literary presentation of the theoretical construction, concerned mainly with the main variables enacting dynamic competition (entrepreneurship, radical uncertainty, innovation, profit) and long waves. A third section presents the formal specification of the model. This section is divided into five blocks. Block 1 concerns the explanation of radical process innovations and the advent of new products (that occur as soon as their explanatory functions reach some specified trigger values) and incremental innovations, while some Gamma distributions describe the diffusion of the radical process innovations across the economy, that is, the adaptive process following the innovative breakthroughs. Block 2 includes the equations explaining uncertainty, the availability of entrepreneurship, its demand and hence the excess of entrepreneurial skills. Block 3, which includes the equations of prices, wages and profits, has a conventional content, with the exception of some explanations of mark up and the definition of the rate of true profit, which excludes interests on capital. Block 4 concerns consumption and, in particular, the diffusion of new goods. Block 5 concerns capital and investment. A fourth section presents three simulations of the model that suppose different degrees of intensity of dynamic competition. A final section exposes some reference to a previous micro-specification of the model at the level of the firm.

2. The main factors of economic change and development

Premise

Explanation of economic change is one of the most deficient components of modern economics. It is also one of the most embarrassing, for the ever more important competition based on innovations fuels that intense and growing dynamism that has become one of the primary characteristics of modern economies. The poverty of the treatment of the primary sources of economic change is striking: in economic modeling dynamic motion is often expressed merely by the inclusion of time in equations.

Any serious analysis of economic change and development is obliged to pay attention to entrepreneurship and its counterpart, profit, as well as to radical uncertainty and innovation along with the associated process of dynamic competition and the implied long waves. But, even if not ignored by economists, these variables and processes are usually represented in a fragmented and incomplete way. This implies, among other things, a scarcity of available data on those variables: a lack that the emphasis of economics on their importance should hopefully remedy.

a) Entrepreneurship

The entrepreneur is a primary dynamic force within the economy and the main agent of economic change. In the absence of the entrepreneur, technological inventions would remain useless from an economic point of view, as happened in ancient (and also recent) bureaucratic-centralized empires.[1] The transformation of inventions into innovations and its acceleration through R&D are the work of entrepreneurship. A dynamic economy, that is, one with technological change and innovation, cannot do without the entrepreneur for promoting innovation, meeting the resulting radical uncertainty, and, in sum, governing the whole process of disequilibration and re-equilibration expressing change.

However, it may be useful to underline that the capitalist character of both entrepreneurship and economic dynamics must be considered, from a scientific point of view, a contingency resulting from the operation across history of specific kinds of spontaneous forces. More generally, the social sciences should consider capitalism as but one particular choice of civilization. But the figure of the entrepreneur as innovator and manager of uncertain perspectives cannot be erased by modern societies (be they capitalist or not) without destroying their dynamic content. The entrepreneur represents an organizational 'necessity', who both legitimates and strengthen his necessity through innovative actions generating true or radical uncertainty and, therefore, also the need of an agent able to face it.

Economics has largely disregarded the entrepreneur, and has too readily opted to conceive of entrepreneurship simply as an expression of animal spirits, thereby avoiding the question of the demand and supply of this important agent. In particular, growth economics substantially neglects entrepreneurship. As a consequence, quantitative data on entrepreneurship are lacking.[2]

Our model will explain the supply of entrepreneurship as a function of the degree of radical uncertainty multiplied by the level of production and the dynamics of innovations, these expressing proxies of entrepreneurial learning by doing; eventually some sociological explanatory factor should be added. The demand for entrepreneurship will be explained by the same variables but with different adjustment speeds, that is, as a function of the level of production weighted with the degree of radical uncertainty and the intensity of innovative action. For zero uncertainty and innovation, there will be no learning by doing of entrepreneurship and no need for entrepreneurship; indeed there will not be entrepreneurship at all.

So, entrepreneurship generated and absorbed by business activity is directly correlated to the degree of uncertainty. Entrepreneurial skills increase when deep uncertainty compels entrepreneurs to put into motion all their capabilities, and deteriorate when the economic situation does not require such an effort as happens, for instance, in equilibrium. But the demand for entrepreneurial skills varies with a shorter lag than the supply, with respect to the same independent variables of both equations. The result is the variation of the 'excess' of entrepreneurial skills, the difference between the supply and demand of entrepreneurship, a variable that, as we shall see, is crucial for the explanation of innovation and the representation of the dynamic competition process.

[1] Think, for instance, of some important inventions of ancient China and of the Alexandrian academicians (an institution financed by Ptolemaic absolute state of Egypt), such as the piston, the connecting rod and the aeolipile. These important mechanical and steam power inventions, which two millennia later would spur industrial revolutions, remained confined to the condition of toys. For their part, the modern systems of 'real socialism' excelled only in innovations pushed by the military system. For a detailed treatment of this subject, see A. Fusari (2000), *Human adventure, an inquiry on the ways of people and civilizations*, SEAM, Roma.

[2] It is significant that in Italy the best statistical data on entrepreneurship are not provided by ISTAT but by CERVED Elsewhere the situation is not better.

b) Radical uncertainty

This is the uncertainty in regard to which "there is no scientific basis on which to form any calculable probability whatever".[3] Radical (or true) uncertainty just expresses a lack of knowledge. In this respect, such uncertainty is completely different from expectations that allow reaction to the uncertainty of perspectives and constitute an attempt to penetrate the future; expectations constitute an expression of hope quite different to the expression of the limitation of knowledge of radical uncertainty. As such, radical uncertainty has to do with the violation of expectations, their variability and dispersion; in a sense, it expresses the degree of unreliability of expectations.

F. H. Knight, who may be considered the father of the notion of uncertainty, has insisted on its non-measurability; and economics, while proposing various ways of estimating expectations (sometimes implausible – think, for instance, of the notion of rational expectations), follows Knight in denying the measurability of radical uncertainty, effectively designating this a fog. Such a denial can be accepted with reference to many specific events that the entrepreneur encounters; after all, the measurement of the degree of uncertainty of these specific events would imply the erasing of uncertainty, its transformation into insurable risk, and hence would expunge entrepreneurship. But the sectoral degree of uncertainty is quite another thing: *the denial of the measurability of sectoral uncertainty constitutes a substantial and unwarranted limitation.*

Various measures of radical uncertainty by sector can be proposed. For example: the *volatility* of opinions as expressed by the EU surveys on business tendency and concerning firms' expectations on delivery orders, production, prices, cost of financing and liquidity assets. Another indicator of uncertainty may be represented by the sectoral standard deviation of profit rates across firms; for, in the absence of institutional monopolies, these standard deviations are the consequence of different abilities to meet uncertainty and hence increase with uncertainty, becoming zero in its absence. Again, sectoral uncertainty may be measured by a minimum-maximum range of expectations, with the distance between the minimum and the maximum expressing the degree of uncertainty. Yet another potential measure is provided by the standard deviation of foresights.[4] But statistical data for these measures are unfortunately lacking.

Radical uncertainty is strictly linked to the figure of the entrepreneur. As previously seen, it implies the 'necessity' of the entrepreneur and, at the same time, it is the result of entrepreneurial innovative action. In the absence of this action, only exogenous uncertainty would survive, that is, a very limited portion of uncertainty.

Our model explains the variations of radical uncertainty as a function of innovation: radical process and product innovations, as well as the diffusion of radical and incremental process innovations; an exogenous term is added in the explanation, meaning that in the absence of innovations uncertainty would tend to decrease toward the operation of exogenous factors.

c) Innovation

The explanation of innovation is a central aspect of the model that we are going to specify. Innovations can be radical, that is, concerning completely new processes and products, or incremental, if they simply improve existing processes. As is well known, Schumpeter distinguished five kinds of innovations: the production of new goods; the introduction of a new method of production; the opening of a market; the conquest of a new source of supply of raw materials and of semi-finished products; industrial reorganization. With some approximation and useful simplification, our model will put together Schumpeterian innovations of types one, three and four, under the category *new products*, while types two and five may be considered under the category *new processes*. Of course, the introduction of innovations is delayed with respect to the

[3] See J. M. Keynes (1937), p. 214.

[4] On the standard deviation of foresights, see H. Ekstedt and A. Fusari (2010), chapter 5 by Fusari; see also A. Fusari (2014).

corresponding inventions and discoveries. The delay is shortened by the combination of the two aspects in the firms' strategy of R&D.

The explanation of innovation is one of the most unsatisfactory aspects of economics. Often the need for such explanation is simply ignored, with innovations treated as exogenous. Attempts to explain them operate, for the most part, in the context of the function of production – adding into this function some additional factors, such as: human capital (Lucas); the operation of an innovative sector producing knowledge (Shell and Romer); the advent of new intermediate products and quality based innovation (Grossman and Helpman and Aghion and Howitt). But these explanations ignore the following: the role of entrepreneurship and its availability; radical uncertainty; the distinction between radical and incremental innovations and between new products and new processes (in the context of the dynamic competition process that we are going to discuss). A different and distinctly better landscape is offered by some micro models, for instance the model of Saviotti and Pyka that, unfortunately, only concerns the advent of new products and "does not contemplate process innovations, notwithstanding the insistence of both the authors on their indispensability to cause, through the productivity rise, the deficiency in the demand for the existing goods, a deficiency that pushes product innovation".[5]

Innovation constitutes a main component of entrepreneurial action and is stimulated by the excess (difference between supply and demand) of entrepreneurship. A crucial aspect, from an explanatory point of view, is the relation between innovation and radical uncertainty: while the first tends to stimulate uncertainty, as just seen, the second discourages innovation since, if uncertainty is high, a large part of entrepreneurship is absorbed by ordinary activity and hence deflected from innovation. Moreover, innovations are negatively influenced by the profit rate: when this is negative, the firm is forced to innovate (Mensh's 'innovate or perish'). Innovations of process are also stimulated by the apparition of new capital goods, which imply changes of processes. For its part, the explanation of innovations of product, while excluding (of course) the role of new capital goods, adds (to the factors considered above) both the stimulus due to the saturation of the demand of existing goods (emphasized by Saviotti and Pyka) and the degree of inequality in income distribution, since such inequality generates market niches that are inclined to buy new products.

Radical innovations constitute important discontinuities. In our model, they materialize as soon as their explanatory equations reach a value equal to some established threshold. Moreover, we specify the diffusion of radical product and process innovations through, respectively, a Gamma distribution and a Logistic. For their part, incremental innovations are negatively influenced by uncertainty and positively influenced by the excess of entrepreneurship. They are also stimulated by the diffusion of radical process innovations weighted by the excess of entrepreneurship; in fact, many incremental innovations are a result of entrepreneurs' attempt to achieve the best performance of radical innovations; as a consequence, incremental innovations become a dominant feature of the dynamic motion after the complete diffusion of radical innovations.

d) Profit and profit rate. 'Necessity' and 'choice-possibility' in the organization of the economy

Profit is a counterpoint of entrepreneurship; it does not make sense to conceive of entrepreneurship independently of profit. In fact, the profit rate is economically essential in that it is the only reliable and overall measure of the degree of success of the entrepreneur's decision making; it represents an organizational 'necessity'. However, it is important to underline that what is relevant in measuring such a degree of success is the rate of the so called extra (or true) profit on used capital – that is, not including the interest rate on the capital employed.

Our very general reference to the profit rate should not be thought of as establishing a category of income distribution. The kind of attribution of profit intended here simply expresses a choice of civilization; for instance, the capitalist character of social organization. This feature of our analysis must be strongly underlined in order to avoid the frequent prejudices that arise concerning the

[5] See A. Fusari, p. 144, in H. Ekstedt and A. Fusari (2010).

organization of the economic systems – an emphasis that constitutes, not a subversive assessment, but a scientific assertion.[6] The problem that arises here is part of a more general question: the distinction between 'necessity' and 'choice-possibility-creativeness' in the organization of social systems, a distinction that has repeatedly appeared in the previous subsections. In order to better clarify this point we shall delineate, at the beginning of section 3 (dedicated to the formal model), some transformations of the model directed so as to express only 'necessities', that is, directed to reduce the model to *necessary institutions,* which vary only in the long run, across historical ages as scanned by the variation of the general conditions of development.[7]

e) Dynamic competition

Now we unify the above treatments in the notion of dynamic competition. This should provide a much more articulated and comprehensive explanatory model of economic change and development than those based on the notion of the production function. Our approach joins together all the factors considered from a) to d), that is, entrepreneurship, the various kinds of innovations and their diffusion, radical uncertainty, the role of the profit rate. Economic change is practically impossible, at least as an enduring feature, in the absence of competition of such a kind.

The dynamic competitive process is a result of the entrepreneurial search for profit. A description of some of the main contents of such a process has been provided both by the Schumpeterian theory of entrepreneurship, innovation and development, and the neo-Austrian market process centered on the entrepreneur's alertness to the existence of price differences between inputs and outputs, to new goals, new available resources and other unnoticed opportunities. One of the most surprising aspects of current economics is that the two approaches remain separated, as products of two different and opposing schools of thought, notwithstanding their strong and stimulating complementariness, mainly in explaining economic change.

It is indubitable that the entrepreneurial search for profit through the various kinds of innovations previously considered constitutes a main engine of economic change and development. But this is not enough. Innovation is a powerful source of radical uncertainty and disequilibria, which represent further crucial challenges for the entrepreneur and provide additional opportunities to put into effect his business ability to obtain profits. This gives rise to a phase of re-equilibration and organizational structuring that follow the initial innovative dash. In the envisaged process, the neo-Austrian entrepreneur acts as an equilibrating force, while the Schumpeterian entrepreneur acts as a disequilibrating one. In this respect, the combination of Schumpeterian and neo-Austrian theoretical perspectives is precious from an analytical point of view. It is instructive to analyze the dynamic competitive process of change and development in the light of such a combination. Let us so proceed.

Approaching the equilibrium of the economy as a consequence of the action of the neo-Austrian adaptive entrepreneur tends to erase radical uncertainty (with the exception of that caused by exogenous factors, such as natural disasters), and also to erase economic change and profit. But low uncertainty makes it easy to invest and innovate and increases the excess of entrepreneurship to be dedicated to innovation; moreover, innovation is powerfully stimulated by the fact that the neo-Austrian push towards equilibrium makes innovation the main effective way to make profits. The rise, with the consequent innovation dash, of disequilibria and radical uncertainty recreates new adaptive opportunities for profit, as implied by the neo-Austrian arbitrage and market process, and hence a renewed push towards equilibrium; and so on, and so forth, in a cyclical process of innovation and subsequent reorganization that gives rise to a more advanced structural base of the economy.

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[6] Israel Kirzner, who is not a subversive but certainly is a serious student, has insisted on the fact that ownership and entrepreneurship are completely separate functions.

[7] In this regard and, in particular, on the definition of the notion of historical phase, see my book on *Methodological Misconceptions in the Social Sciences*, chapter 4, entitled 'Social Development and Historical Processes'.

It may be useful to insist in the description of the process, notwithstanding the risk of some repetition. As we can see, variations in the degree of radical uncertainty and of the excess of entrepreneurship are crucial for representing the working of the dynamic competition process: adaptive equilibrating competition implied by the market process causes the reduction of disequilibria and uncertainty and, therefore, reduces both the need for entrepreneurship and the existing opportunities of profit; but, in this way, adaptive equilibrating competition increases the excess of entrepreneurship and, hence, the entrepreneurship available to introduce innovations aimed at creating new opportunities of profit; so that, the reduction in radical uncertainty generates a favorable climate for the introduction of innovations, and vice-versa. In sum, the variations of radical uncertainty determine the way entrepreneurs' alertness is oriented towards the two complementary kinds of entrepreneurial competition (the Schumpeterian and the neo-Austrian one) and, more generally, the way the disequilibrating-re-equilibrating process of dynamic competition operates.

Innovation by itself would result in a paralyzing confusion, while adaptive structural organization leads toward equilibrium, thus erasing innovation, change and entrepreneurship. To avoid those inconveniences, the combination of the two theoretical perspectives, the Schumpeterian and neo-Austrian, is needed. But this combination is insufficient for the adequate representation of the whole process of change and development: some further variables implied by the process relating to radical uncertainty and entrepreneurship must be considered and explained. In fact, and as just seen, it is the rise of radical uncertainty caused by innovation that generates the push toward adaptation and structural organization, while the consequent decrease in radical uncertainty opens the door to the subsequent new rise of innovation. The whole process is the work of entrepreneurship and hence depends on its availability.

Unfortunately, and as we saw, radical uncertainty is not specified but only implied by the Schumpeterian theory of development; and although emphasized by neo-Austrians it is nevertheless treated simply as a fog and an exogenous variable. Moreover, the explanation of the availability (demand and supply, and hence the excess) of entrepreneurship is disregarded by both theories. Probably, the lack of such a theoretical deepening is the main reason behind the surprising persistence of the separation of the two theoretical perspectives. Our model will attempt to give a rigorous formal specification of all the aspects of dynamic competition specified from (a) to (d). This will enable us to provide a representation and explanation of economic change and business cycles more complete than that offered by the current models.

f) Long waves[8]

It may be interesting to say something about the long waves attached to innovation and economic change in the context of the dynamic competition process described above.

The cyclical interaction between innovation and adaptation, disequilibrating and re-equilibrating motion implies long waves. Van Duijn (1983) has set out the following table on the intensity of the various kinds of innovation during the phases of the long wave.

Table 1: Propensity to innovate during the phases of the long-wave

(Van Duijn, 1983, p. 137)

	Stagnation	Expansion

[8] This sub-section offers a synthesis of some development by A. Reati in the article by A. Fusari and A. Reati, SCED 2013.

	Depression	Recovery	Prosperity	Recession
1. Product innovations (new industries)	*	****	**	*
2. Product innovations (existing industries)	***	***	*	*
3. Process innovations (existing industries)	***	*	**	**
4. Process innovations (basic sectors)	*	**	***	**

The more stars, the greater the propensity to innovate.

As we can see (and as A. Reati writes), "during the depression phase of the long-wave major innovations tend to appear in existing industries and concern processes as well as products. In the latter case, a radical product innovation concerns the satisfaction of an already existing need by a completely new product (the PC replaces the mechanical typewriter; the photocopying machine is a substitute for carbon paper, etc.). During the recovery, the number of major process innovations in existing industries falls sharply while the flow of product innovations continues. However, the dominant feature of this phase is the appearance of radical product innovations leading to the creation of new industries, which means that there is the creation of a new need. This is summarised in table 1"[9]

Now come to the process of diffusion of innovations, which has much to do with waves. A. Reati has written: "In order to understand the dynamics of *product* innovations, let us recall that new (final) *products* pass through a four stages life cycle: *(i)* market development (introduction), when the product is first brought to the market; *(ii)* growth, when demand begins to accelerate and the size of the total market expands rapidly; *(iii)* maturity, when demand levels off and grows, for the most part, only at the replacement and new family-formation rate; *(iv)* decline, when the product begins to lose consumer appeal and sales drift downwards.

The reasons underlying consumer behaviour in the first and second stages are primarily due to the gradual spread among consumers of information on the existence of the new commodity, its characteristics and its appropriateness in satisfying a particular need: it is the "epidemic" model. Next we turn to prices. Very often, the introduction of a new product requires heavy investment in research and development as well as considerable marketing expenditure. In such circumstances, the price at the initial stage in the product life-cycle will be set at a high level to allow the innovator to recoup his costs before too many imitators enter the market. The ensuing high profits will attract imitators while the decrease in inequality occurring when the economy enters into the recovery and the prosperity phases will gradually enlarge the potential market for the new product. Diffusion is also facilitated by the fact that the initial price level, which "skims the cream of the demand", will be progressively abandoned during the later stages of the product life cycle so as to stimulate demand from other segments of the market. Further price reductions of this kind will be engendered by the growing competition from newcomers as well as by process innovations in the sector concerned and in the corresponding capital goods sector.

The pattern of diffusion of *process* innovations is explained by the fact that, for instance, the enterprises in the sector do not have the necessary information to perceive immediately the advantage of imitating the first innovator or, if they are fully aware of the new opportunities, they prefer to wait so as to avoid the cost of accelerated scrapping or they are unable to adopt the new

[9] See A. Fusari and A. Reati (2013) SCED, p. 78

technology for organizational or institutional reasons (they do not know how to master the new technology or do not have the necessary skills; managers are reluctant to change radically the organization of the company).

Product and process innovations can also be favoured by organizational factors. "As the S-curve matures...., the business becomes bigger and more bureaucratic. That's where 'creative destruction' comes in because the more something becomes bureaucratized, the more room it leaves at the bottom for individuals and small teams of heretics to redefine the game in new ways" (Stefik and Stefik 2006, p. 4). The number of years it takes to reach a complete diffusion of radical innovations differs substantially according to the type of innovation.

For *process* innovations, we assume that diffusion is almost complete by the end of the phase of the long-wave in which the technological revolution started off.

For *product* innovations, the length of the diffusion period varies a great deal from one product to another, but in general, the maturity stage is reached much later than in the case of process innovations. Empirical research by Gort and Klepper (1982) based on a sample of "basic" product innovations first commercially introduced between 1887 and 1960 shows that, on average, the maturity stage was reached within 37 years. However, the interval required for successful imitation has systematically declined over time. While the overall average length of the first stage (introduction) is 14.4 years, for products introduced before 1930 this interval was 23.1 years; it was 9.6 years for those introduced in the period 1930-39 and only 4.9 years for products introduced in 1940 or later (Gort and Klepper, 1982, p. 640; see also Stefik and Stefik 2006, p. 203-204 for further evidence concerning the USA)."[10]

A. Reati has also set out this second table concerning the relation between some variables and the various phases of the waves

Table 2: The main features of the long-wave

	STAGNATION		EXPANSION	
	Depression	**Recovery**	**Prosperity**	**Recession**
Rate of profit	Decreasing & very low	Increasing	High and stable	Decreasing
Degree of inequality	High	High	Decreasing to low	Low but increasing
Unused entrepreneurial skills ("excess")	High	High but decreasing	Low	Low but (slightly) increasing
Degree of radical uncertainty	Low then increasing	High	Low	Low
Expectations	Negative	Slightly improving	Favourable	Favourable but deteriorating

3. The model[11]

[10] See A. Fusari and A. Reati (2013) SCED, p. 79.

[11] This model is taken from the book by H. Ekstedt and A. Fusari, 'Economic theory and social change', Routledge 2010, precisely from chapter 5 by Fusari, entitled 'Innovation. Uncertainty, entrepreneurship'. Moore precisely, the

Our model has a double purpose: First, to deepen the mechanism of economic change and evolution by complementing the theory of section 2 with analytical details; secondly, to prepare the ground for ascertaining the overall consistency of the model through numerical simulations.

3.1. General features

3.1.1. The conventional duration of the long wave

In our model we conventionally assume that a long wave lasts 50 years, long expansion and stagnation 25 years each, while individual phases last 20 years (prosperity), 5 years (recession), 15 years (depression) and 10 years (recovery).[12]

3.1.2. The structure. The reduction of the model to only 'necessary' variables

Initially, our model is composed of two sectors – the consumer (final) goods sector, grouping *n* industries, and the capital goods sector, grouping *m* industries. Radical product innovations are taken into account by adding new industries to the sector in which they appear, which means that the structure of the economy is made up of two "big" capital goods and consumer goods sectors operating with the traditional technology, to which are now added the new sectors.

We generalize this by supposing that we have *h* new capital goods industries and *g* new consumer goods industries. Therefore, our model formalizes, for each endogenous variable, four groups of *n*, *m*, *g* and *h* equations expressing a complete disaggregation of the economy – one industry for each commodity.

The discontinuities represented by radical innovations are introduced into the model through a "switch" binary variable similar to a Dirac function δ. Value 1 of the "switch" opens the door to the innovation while value 0 precludes it. To see how this operates in practice, consider a function z (e.g. the level or the rate of change of productivity of an enterprise) and posit that a radical innovation materializes when z exceeds or is equal to a given threshold k ($z \geq k$). Then, defining a variable y as the difference between z and its threshold ($y = z - k$), our "switch" function δ is such that

$\delta(y) = 1$ if $y \geq 0$, otherwise $\delta(y) = 0$.

Alternatively, when a radical innovation appears when z is below the threshold, we change the direction of δ by a new "switch" ω, which is

$\omega = 1 - \delta$.

In this case $\omega(y) = 1$ if $y < 0$, otherwise $\omega(y) = 0$.

By way of an example, let us consider depression and put z = rate of profit − the variable that, as we have seen, represents one of the main triggering factors for radical innovations. If, for a given time-span (e.g. 3 years), the rate of profit is on average less than a certain threshold level (e.g. 1.5%) and we fix the threshold $k = 1.5$, the diffusion of the innovation will start the 4th year.

We indicate now, as promised above in subsection (d), some modifications intended to reduce the formal model to only necessary variables. The modifications are as follows: a) the real interest rate should be null, so that the nominal interest rate equates with inflation; b) money wages should be expressed as a function of only the demand and supply of labor, so as to represent just an element of cost charged on prices and not a component of income distribution; c) the degree of

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model conforms to the version of such a model reproduced, with some change, in the article by A. Fusari and A. Reati 'Endogenizing technical change: Uncertainty, profits, entrepreneurship. A long-term view of sectoral dynamics'. Structural Change and Economic Dynamics (SCED), (2013).

[12] In our numerical simulations reported below, changes in the parameters (particularly those concerning the degree of dynamic competition) produce slight changes in this periodization

inequality of income distribution should be erased in the equation for new products in its role as a variable stimulating product innovation, such a stimulus being relevant only in a particular social system, capitalism; d) in the equation of consumption, total money wages should be replaced by some other component of income distribution, wages being excluded from income distribution and only intended as an element of cost (see b).[13]

3.1.3. The process of adaptation

For the sake of realism, we posit that the theoretical ("normal") level of some variables does not fully materialize immediately but instead determines the actual level within a temporal lag. The rationale for this adjustment process – which applies to entrepreneurship, wages, prices, mark-up and consumption – will be set out below when illustrating the relevant equations.

3.1.4. The diffusion process

The diffusion process of radical *product* innovations is represented by a logistic function, whose rates of change are decreasing and the derivative is "bell shaped". For *process* innovations, we instead use a gamma distribution in order to formalize more efficiently the successive appearance of new radical innovations[14] and also to take into consideration the fact that some improvements of a previous process innovation can be considered as an additional radical innovation. By choosing the appropriate parameters for the gamma distribution, it is possible to approximate the sigmoid pattern of diffusion, with its "bell shaped" derivative.

3.1.5. Market structures

We assume the ubiquitous presence of economic power and "domination effect" (Perroux 1964; 1965). This means, among other things, that in every industry there is either monopolistic competition or an oligopoly in its various forms.

Competitive sectors are defined on the basis of "workable competition", i.e. the rivalry that is possible within the prevailing market structures. Thus, monopolistic competition implies that each firm has a certain market power resulting from product differentiation; concerning oligopolistic market structures, we assume that there are no agreements restricting competition and also that rivalry entails that selling prices roughly follow the evolution of productivity.

Market power evolves over time in relation to the changing structures of the economy which characterizes the long wave. The stylized facts are:

- during the depression phase of long stagnation, competition strongly intensifies, and this reduces the market power of existing firms; this phenomenon is exacerbated by the low level of demand
- pressures on the market power of firms are slightly reduced during the recovery phase as demand begins to recover
- during the long expansion phase, enterprises can fully exploit the potentialities offered by buoyant demand, charging prices that fully reflect their potential market power. In the final phase of the long expansion the market power of firms tend to remain stable.

3.1.6. Rate of profit

The rate of profit is determined with respect to the net fixed capital stock at current replacement prices. To quantify the "true" appropriation for entrepreneurship (i.e. the part of surplus exceeding the imputed wage), we deduct from the rate of profit the rate of interest. This

[13] For a broad exposition of this matter, see H. Eksted and A. Fusari 'Economic theory and social change', (2010), chapter 8 by Fusari, entitled 'Towards a non-capitalist market system: spontaneous order and organization'.

[14] This is because the gamma distribution has a "memory", in such a way that the new radical innovations are grafted on the events of the past.

adjustment of the profit rate is based on the fact that the owner of capital has to decide either to buy securities (obtaining an interest) or to invest his capital in a productive activity, thereby becoming an entrepreneur.

3.1.7. Inequality

The best way to take into consideration the degree of inequality would be to introduce exogenously into the model a Gini index. However, considering that we want to limit the number of exogenous variables, as a proxy for inequality we consider the share of wages with respect to net output at current prices. In this way, we assume a negative correlation between the two variables: when the wage share is low inequality is high and vice versa. At present, this global indicator is reinforced by the fact that the current technological revolution in information technologies has strongly increased the wages dispersion. Thus, we have a rather low wage share and strong inequality among wage earners.

3.2. Notations

Endogenous variables

CON = Consumption
CT = Average consumption level over a specified number of recent years
Ec = Unused entrepreneurial skills ("excess" of entrepreneurship)
Ed = Demand for entrepreneurship
Es = Supply of entrepreneurship
I = Gross investment (at constant prices)
K = Net fixed capital stock (at constant prices)
KD = Demand of existing capital goods
KDT = Demand of existing capital goods: average level over a specified number of recent years
L = Employment (demand for labour), economy as a whole. The variable with a subscript refers to specific industries
M = Variable that singles out the advent of a new sector
MKP = Mark-up
ND = Demand for new goods
NDT = Demand for new goods: average level over a specified number of recent years
Pr = Labour productivity (VA/L) ; subscript (av) means that productivity refers to the average of the economy ($Pr_{(av)}$)
PRE = Prices
PrI = Labour productivity of incremental innovations
$PrR_{(f)}$ = Labour productivity of radical process innovations for the individual innovator
r = Rate of profit
R = Rate of profit: average level over a specified number of recent years
SW = Variable that singles out the appearance of radical process innovations
u = Radical uncertainty
VA = Value added at constant prices
$VA_{(nom)}$= Value added at current prices
w = Nominal wage rate
X = Production at constant prices
XT = Average level of output over a specified number of recent years
Γ = Gamma distribution for the diffusion of radical process innovations
γ = Variable to reduce the second order derivative concerning Γ to a first order one

Exogenous variables

i_r = Interest rate
Ls = Supply of labour
t = Time

Other notations

a = constant term
D = Derivative with respect to time
e = Unit column vector
I = Column vector (*i+j+(n)i+(n)j* components) of investments in existing capital goods
$k_{(M)}$ = Threshold level for the advent of radical product innovation
$k_{(PrR)}$ = Threshold level for the advent of radical process innovation
$\mathbf{K}^{(d)}$ = Diagonal matrix (of order *i+j*) of net fixed capital stock (constant prices)
KD = Column vector (*j* components) of demand of existing capital goods
ln = Natural logarithm
PRE = Row vector (*j* components) of the prices of capital goods
Q = Percentage increase in the productivity level of the individual innovator resulting from a radical process innovation
TI = Transition matrix [*j*×*(i+j+(n)i+(n)j)*] of investment from sectors of utilization to sectors of origin
TK = Transition matrix [*j* × *(i+j+(n)i+(n)j)*] of capital from sectors of utilization to sectors of origin

α = Adjustment parameter
β = Other parameters
δ = Switch variable (Dirac function)
η = Obsolescence as a percentage of new capital goods
λ = Scaling parameter
μ_I = Depreciation rate of capital (worn-out capacity)
ν = Technical coefficient linking VA to X
ω = Complement to one of the switch variable ($\omega = 1 - \delta$)

Indexes and subscripts

′ = Symbol indicating partial equilibrium variables in adjustment equations
i = Subscript indicating consumer goods
j = Subscript indicating capital goods
(n)i = Subscript indicating new consumer goods
(n)j = Subscript indicating new capital goods

3.3. The equations

Our model is structured around five blocks of equations. Block I concerns the production side of the economy, including employment; block II introduces uncertainty and the behavioural hypotheses relating to entrepreneurship; block III sets out the distributive variables while blocks IV and V refer to the demand side of the system.

We usually take the natural logarithms of the variables for the sake of convenience, as the derivative of the logarithm indicates the rate of change and the coefficient of the logarithm the elasticity.

Where the equations for sectors *i*, *j*, *(n)i* and *(n)j* are similar, they will be specified only for sector *i* and, to simplify matters, the parameters of each explanatory variable are almost always identical for every *i*.

3.3.1. *Production, productivity and employment*

♦ *Gross output*

(I.1) $DlnX_i = \beta_1 r_i + \beta_2 ln(Es_i/Ed_i) + \beta_3 lnCON_i$ $\quad i = 1, 2, 3….n$

(I.2) $DlnX_j = \beta_1 r_j + \beta_2 ln(Es_j/Ed_j) + \beta_4\ lnKD_j$ $\quad j = 1, 2, 3….m$

(I.3) $DlnX_{(n)i} = \omega(-M_{(n)i})\ \beta_5 D(ND_{(n)i})$ $\quad (n)i = 1, 2, 3…g$

(I.4) $DlnX_{(n)j} = \omega(-M_{(n)j})\ \beta_6 D(ND_{(n)j})$ $\quad (n)j =1, 2, 3….h$

Behavioural equations (I.1) and (I.2) explain the changes in output of consumer and capital goods in existing industries in terms of three factors: the current rate of profit, the entrepreneurial skills and demand.[15] Profitability and demand indicators should be seen as *proxies* for the *expected* values of these variables (firms extrapolate the present situation).

For new consumer and capital goods (equations I.3 and I.4) production is fundamentally driven by demand.

♦ *Output level in recent years*

(I.5) $DlnXT_i = \alpha_1\ ln(X_i / XT_i)$

This equation is derived from the expression for distributed lags, with a weight of 0.63 for X_i for the last $1/\alpha_i$ years

The equation concerning sectors *j* is similar

♦ *Value added*

(a) Constant prices

(I.6) $VA_i = \nu_1\ X_i$

(b) Current prices

(I.7) $VA_{(nom)i} = \nu_2\ X_i\ PRE_i$

For the sake of convenience, we posit that value added is linked to gross output through the technical coefficients ν, that are supposed to be constant

[15] In eq. (I.1) and (I.2) we take the level of demand instead of its rate of change just to reduce the number of derivatives entering the model that complicates the simulations. The resulting inconvenient is minor because the importance of such factors within the equations is low. In equations (I.3) and (I.4), the rate of change in demand is expressed by the rate of change of the logistics formalizing the diffusion of these products, a derivative that in any case exists in the model

♦ *Trigger functions expressing the advent of new products*

$$(I.8)\quad D(M_{(n)i}) = \delta\{[-\beta_7 u_i + \beta_8 \ln(Es_i/Ed_i) - \beta_9 R_i - \beta_{10}\ln(CT_i/XT_i) + \beta_{11}\ln Es_i + - \beta_{12}\ln(w\,L\,/VA_{nom})\,] - k_{(M)i}\}$$

$$(I.9)\quad D(M_{(n)j}) = \delta\{[-\beta_7 u_j + \beta_8 \ln(Es_j/Ed_j) - \beta_9 R_j - \beta_{13}\ln(KDT_j\,/XT_j) + \beta_{14}\ln Es_j\,] + - k_{(M)j}\}$$

Equations (I.8) and (I.9) summarize our previous analysis of the endogenous factors that generate a radical product innovation in consumer goods (eq. I.8) as well as in capital goods (eq. I.9). These innovations materialize when the expression in square brackets is higher than or equal to the trigger value $k_{(M).}$

Some terms within the square brackets (such as uncertainty or "excess" entrepreneurship (Es_i/Ed_i)) do not require specific comments. We just note the following:

- R represents the profit motive for innovation: if profitability is persistently negative, this will stimulate the radical change represented by the innovation[16]
- Terms CT_i/XT_i and $KDT_j\,/XT_j$ express the saturation effect of the demand for existing products, which is inversely related to the propensity to innovate.
  CT_i/XT_i or $KDT_j\,/XT_j$ less than one means stagnating demand, which is a positive incentive to innovate)
- Terms Es mean that the higher the supply of entrepreneurship the more likely innovation will occur
- The last term in square brackets in formula (I.8) – i.e. the inverse of the wage share – is a *proxy* for the degree of inequality in income distribution, inequality being directly related to the propensity to innovate[17]
- In eq. (I.9) the term representing the degree of inequality does not appear because it is not relevant for launching new plant and equipment onto the market

♦ *Labour productivity of the individual innovator resulting from radical process innovations*

As already noted, radical process innovation entails a leap in the productivity level of the innovator. We explain the process of adoption of such innovations and the subsequent change in productivity in two steps. First we define the factors that determine the occurrence of the innovation in question. Then we derive the productivity function for the individual innovator.

The switch function that marks the presence of the radical innovation is:

$$(I.10)\quad D(SW_i) = \delta\{[-\beta_7 u_i + \beta_8\ln(Es_i/Ed_i) - \beta_9 R_i + \beta_{15}\,\omega(-M_{(n)j})] - k_{(PrR)i}\}$$

SW = 1 if $k_{(PrR)i}$ is lower than or equal to the expression in square brackets, otherwise SW is zero. The equation expresses the constraint $D\ln PrR_{(f)i} \geq k_{(PrR)i}$.

$k_{(PrR)i}$ is the minimum productivity increase that a radical process innovation must yield, i.e., a trigger for the advent of radical process innovation.

[16] Considering that during the depression phase R is supposed to be negative, $-\beta_6 R_i$ becomes positive. We shall see below that we define the rate of profit in a particular way, by deducting the rate of interest from the profits. Thus, when actual profits are below the threshold level represented by the rate of interest, R becomes negative

[17] The sign minus could give the wrong impression of an inverse relation between inequality and innovation instead of a direct one. This is because we take the logarithm of the wage share, i.e. $\ln(w\,L) - \ln(VA_{nom})$, which is negative, instead of its inverse.

Term $\omega(-M_{(n)j})$ refers to the fact that the appearance of a new capital goods (product innovation in the capital goods sector) could produce a process innovation in another industry.
$\beta_{15} = 0$ if the new capital goods in question do not materialize

From this we derive the individual innovator's productivity function

(I.11) $\quad DlnPrR_{(f)i} = \omega(-SW_i)\, \beta_{16}\, (Q - PrR_{(f)i})$

Q may take different values for radical process innovations in consumer and capital goods (respectively Q_1 and Q_2)

The equations for sectors *j* are similar, as are those for *(n)i* and *(n)j* provided that they are multiplied by $\omega(-M_{(n)i})$ and $\omega(-M_{(n)j})$, that mark the advent of new sectors (see eqq. II.8 and II.9). This proviso must be extended to all other groups below.

- *Gamma distribution for the diffusion of radical process innovations*

(I.12) $\quad D^2\Gamma_i = \beta_{17}\, \beta_{18}(DlnPrR_{(f)i} - \Gamma_i) - (\beta_{17} + \beta_{18})D\Gamma_i$

The Gamma function aggregates the productivity of the individual innovators of industry *i*, thus showing how the productivity of this industry evolves under the effect of the diffusion of innovation.[18]

Equation (I.12) can be reduced to a first order derivative (and hence the model to a first order one) by adding a first order identity $D\Gamma_i = \gamma_i$, thus obtaining
(I.12bis) $\quad D\gamma_i = f(x)$
where f(x) is the right-hand side of equation (I.12).
Substituting $D\Gamma_i$ in γ, we come back to the second order derivative of equation (I.12).

The endogenous time does not appear explicitly because it is embodied in the "memory" of the Gamma function.

The Gamma distribution is the device we adopt to obtain the evolution of productivity of a given sector – something that occurs at the end of the process of the progressive adoption of radical technical change by individual innovators. As explained in section 2, there is a first innovator who is then followed by others. This process of imitation/diffusion follows a sigmoid path that in turn shapes the curve of the sector's productivity.

- *Labour productivity yielded by incremental innovations*

(I.13) $\quad DlnPrI_i = -\beta_{19}u_i + \beta_{20}\ln(Es_i/Ed_i) + \beta_{21}\Gamma_i \ln(Es_i/Ed_i)$
with the positivity constraint $DlnPrI_i > 0$

This equation formalizes the two sources of incremental innovations, the endogenous ones, i.e. uncertainty ($-\beta_{19}u_i$) and the "excess" of entrepreneurship (the term $\beta_{20}\ln(Es_i/Ed_i)$), as well as the "imported" innovations, i.e. the incremental productivity promoted by the diffusion of radical process innovation (the third term on the right) weighted by the "excess" of entrepreneurship that favours the diffusion of the incremental innovations in question.

[18] What we do here on the basis of the Gamma function is not to describe the micro-economic process of diffusion but just to consider its final effect on the productivity of the sector.

♦ *Labour productivity in each industry*

(I.14) $DlnPr_i = \Gamma_i + DlnPrI_i$

The productivity of the industry is the sum total of radical and incremental innovations.

♦ *Employment*

(a) Demand for labour

(I.15) $L_i \equiv VA_i/Pr_i$

(b) Supply of labour

(I.16) $Ls = (Ls_0 \quad e^{\beta_{22} t})$

The supply of labour is exogenously determined on the basis of an exponential trend with respect to an initial level Ls_0

3.3.2. Uncertainty, entrepreneurship, time

♦ *Radical uncertainty*

(II.1) $Du_i = \beta_{23}\Gamma + \beta_{24}DlnPrI_i + \beta_{25}DlnPrR_{(f)i} + \beta_{26}\Sigma D(ND_i) - a_1$

The third and fourth terms on the right-hand side are the result of the fact already referred to that radical uncertainty is increased when radical process and product innovations jeopardise the industry's equilibrium. By the same token, the diffusion of radical and incremental process innovations produces supplementary uncertainty, which is taken into account by the first and second terms on the right of the equation. The constant term a_1 means that, in the absence of innovations, uncertainty tends to decrease to the level caused by the exogenous factors.

♦ *Entrepreneurship*

(a) Availability of entrepreneurship

As already noted, we start by defining the theoretical ("normal") specification of the variable in order to show the adjustment process leading to the actual value.
Considering first the existing sectors we have:

(II.2) $lnEs'_i = \beta_{27}ln(u_i\ X_i) + \beta_{28}\ DlnPr_i + \beta_{29}\ u_i\ t$

Equation (II.2) shows that entrepreneurial skills vary in relation to three factors:

(i) learning by doing, that is proportional to the quantity produced, to be weighted by the degree of uncertainty associated to the entrepreneurial activity. As already noted, strong uncertainty obliges entrepreneurs to mobilize all their potentialities, thus improving their skills. Conversely, where uncertainty vanishes, no entrepreneurial capabilities are required;

(ii) the learning by doing that is also influenced by innovation, expressed here by the percentage change in productivity

(iii) the third term on the right-hand side is a trend element capturing the changes in entrepreneurship not resulting from learning

Considering that the process of learning takes time, the actual supply of entrepreneurial skills appears after some delay with respect to the factors described in equation (II.2). Thus we have:

(II.3) $DlnEs_i = \alpha_2 ln(Es'_i/Es_i)$

where $1/\alpha_2$ refers to the length of the adjustment process

As usual, for the existing capital goods sector the equation is similar.

Entrepreneurship in the new sectors (both consumer and capital goods) evolves in a more complex way. For the new consumer goods we have:

(II.4) $lnEs'_{(n)i} = \beta_{30} Dln X_{(n)i} + \beta_{31} ln(u X_{(n)i}) + \beta_{32} DlnPr_{n(i)} + \beta_{33} r_{(n)i}$

The learning component appears here in three ways. Looking at the right-hand side of equation (II.4) we see that it depends on:

(i) the percentage rate of change in output (the first term); considering that output follows a logistic path, this implies a substantial entry into the sector
(ii) the quantity produced, weighted by uncertainty (the second term)
(iii) the innovation, as before (the third term)

In addition, the supply of entrepreneurship is also positively influenced by the rate of profit of the sector, an element which can stimulate new initiatives.

As in the previous case, we have a process of adaptation:

$DlnEs_{(n)i} = \alpha_3 ln(Es'_{(n)i}/Es_{(n)i})$

For the new capital goods sector the equation is analogous

(b) Demand of entrepreneurship

(II.5) $ln Ed'_i = \beta_{34} ln(u_i X_i) + \beta_{35} DlnPr_i$

Equation (II.5) shows that the demand of entrepreneurship is:
(i) positively related to the quantity of output, weighted by the degree of uncertainty, since without uncertainty no entrepreneurial skills are requested (see section 2)
(ii) positively related to the degree of innovation, represented by the increase in productivity

In this case too, the actual demand for entrepreneurship follows its theoretical level with some time lag – which is nevertheless smaller than in the case of the creation of skills.

(II.6) $DlnEd_i = \alpha_4 ln(Ed'_i/Ed_i)$

(c) Unused entrepreneurial skills ("excess" of entrepreneurship)

"Excess" of entrepreneurship is the difference between the supply of and the demand for entrepreneurial skills

(II.7) $ln E_c = ln(E_s/E_d)$

The adjustment process for the "excess" of entrepreneurship is the result of the different speed of adjustment of supply and demand, as reflected by parameters α_2 and α_4 of (II.3) and (II.6).

♦ *Time*

(II.8) $Dt_i = \omega(-M_{(n)i})$ $(n)i = 1,2,3,\ldots g$
(II.9) $Dt_j = \omega(-M_{(n)j})$ $(n)j = 1,2,3,\ldots h$

These equations mark the advent of radically new products in final sector *i* as well as in capital sector *j* (when M is positive). For radical process innovations, as indicated above, time is embodied into the "memory" of the Gamma function.

3.3.3. *Wages, prices and profits*

♦ *Nominal wage rate*

As before, we first define the "normal" wage rate, as resulting from the wage bargaining which occurs periodically; then we consider the adjustment process of the actual wage rate with respect to its "normal" level".

(III.1) $\ln w' = \beta_{36} \ln Pr_{(av)} + \beta_{37} \ln(\text{index } PRE_i) + \beta_{38} \ln L - \beta_{38} \ln Ls$

Formula (III.1) shows that the "normal" wage rate depends on three factors, i.e.

(*i*) the productivity of the economy as a whole,
(ii) the rate of inflation
(iii) the relative strength of the Unions. As a *proxy* of this element, we take the difference between the demand for (L) and the supply of labour (Ls)[19]

The linkage between wages and the productivity of the system (instead of the sectoral productivity) means that workers in all industries (including the least progressive) benefit from technical progress. This criterion is suggested as a measure of social fairness. In addition, if competition works effectively, the proposed link between wages and productivity results in the stability of the general price level. In fact, the industries with a percentage increase in their productivity higher than the average of the system will decrease their prices and vice versa for the industries whose productivity increases less than the average.[20]

A uniform wage rate can be interpreted either as a normative criterion which is at odds with the prevailing trends in our societies, or, as it is done in Sraffa (1960, p. 10), by assuming that any differences in the quality of labour "have been previously reduced to equivalent differences in quantity so that each unit of labour receives the same wage".

The fact that wage bargaining occurs only at discrete intervals of time (every few years) implies that current wages follow prices and productivity evolution with considerable delay. Thus, actual wage rate (*w*) does not coincide all the time with the "normal" one (w'), to which is nevertheless supposed to converge. Equation (III.2) describes this adjustment process, where parameter α_5 quantifies the speed of the adjustment ($1/\alpha_5$ being the time lag)

(III.2) $D\ln w = \alpha_5 \ln(w'/w)$

♦ *Prices*

Contrary to what is generally done here – where the formulae for capital goods as well as new consumer and capital goods correspond *mutatis mutandis* to the formulae concerning consumer

[19] One could observe that, in order to obtain percentage rates of change, formula (III.1) should be *D*ln w' and the symbol of derivative should also be inserted in the right-hand side of (III.1). In actual fact, the percentage rate of change in wages results from the structure of the adjustment of eq. (III.2) below.
[20] For a detailed analysis of this last point see Pasinetti (1981).

goods (the only differences being the subscripts) – for prices it is preferable to write *in extenso* the formulae for the individual sectors.

Similarly to what has been done for the wage rate, the price equations express the adjustment process of market prices in relation to the "normal prices" defined below. We thus suppose that market prices can differ from the "normal prices" because of temporary disequilibria between supply and demand.

(a) Prices of consumer goods

(III.3) $\ln PRE_i' = \beta_{39} \ln(w/Pr_i) + \beta_{39} \ln(1 + MKP_i) + \beta_{40} \ln(CON_i/X_i)$

and

(III.4) $DlnPRE_i = \alpha_6 \ln(PRE'_i/PRE_i)$

The first term on the right-hand side of formula (III.3) means that prices are directly related to the unit wage costs (wage rate divided by productivity), that are taken here as a *proxy* for total unit costs. Parameter β_{39} refers to the relative importance of the set of enterprises of the sector adopting administered prices (the oligopolistic industries), while β_{40} gives the weight of the other enterprises. For the first group of enterprises prices are relatively sticky because they reflect the evolution of the mark-up; on the contrary, for the second group of enterprises prices are more subject to pressures from competition and, consequently, reflect more closely temporary fluctuations of demand as formalized by the last term of equation (III.3). Market power is reflected in the magnitude of the mark-up over costs. Thus, if the enterprises detain an important monopolistic power β_{39} will be high. The evolution over time of the market power is defined below (eq. III.8 to III.11).

(b) Prices of capital goods

$\ln PRE_j' = \beta_{39} \ln(w/Pr_j) + \beta_{39} \ln(1 + MKP_j) + \beta_{41} \ln(KD_j/X_j)$

and

(III.5) $DlnPRE_j = \alpha_6 \ln(PRE'_j/PRE_j)$

(c) Prices of new consumer goods

$\ln PRE'_{(n)i} = \beta_{42} \ln(w/Pr_{(n)i}) + \beta_{42} \ln(1 + MKP_{(n)i}) - \beta_{43} Dln(ND)$

and

(III.6) $DlnPRE_{(n)i} = \omega(-M_{(n)i}) [\alpha_6 \ln(PRE'_{(n)i}/PRE_{(n)i})]$

The minus sign of Dln(ND) in the first equation is the result of the fact that where the demand for radically new commodities gains momentum, enterprises reduce the price in order to capture new segments of demand

(d) Prices of new capital goods

$\ln PRE'_{(n)j} = \beta_{42} \ln(w/Pr_{(n)j}) + \beta_{42} \ln(1 + MKP_{(n)j})$

$\beta_{42} > \beta_{39}$, since the degree of monopoly and hence the mark-up are higher on new goods than the existing ones.

Thus we posit that, in the sectors of the new consumer and capital goods, prices are administered instead of being fully determined by competition

As in the other cases, the adjustment process is:

(III.7) $DlnPRE_{(n)j} = \omega(-M_{(n)j}) [\alpha_6 ln(PRE'_{(n)j} / PRE_{(n)j})]$

♦ *Mark-up*

Bearing in mind that mark-up is relevant for concentrated (oligopolistic) industries, we have here two patterns of evolution, one referring to the existing consumer and capital goods, the other to the new commodities.

(a) Existing industries

For these industries, empirical evidence shows that mark-up fluctuates pro-cyclically, in relation to changes in demand (Goldstein 1986). During the long stagnation phase, whereby the demand for existing products is sluggish, the mark-up is low. During the long expansion phase, the buoyant demand gives the possibility to charge an increasing mark-up which in turn declines when the cycle approaches maturity.[21]

In our model the stepwise evolution of mark-up is approached by taking the moving average of demand in equations (III.8) and (III.10) and, subsequently, by introducing a time-lag between the actual and the theoretical evolution of the mark-up (equations III.9 and III.11)

(III.8) $MKP'_i = \beta_{44} DlnCT_i + \lambda_i$

(III.9) $DMKP_i = \alpha_7(MKP_i' - MKP_i)$

(III.10) $MKP'_j = \beta_{44} DlnKDT_j + \lambda_j$

(III.11) $DMKP_j = \alpha_7(MKP_j' - MKP_j)]$

(b) New commodities

The evolution of mark-up for (radically) new consumer goods is explained on the basis of the pattern of demand and on the triggering factor for innovation (see paragraph 2.2.1 (b) above). On the one hand, demand follows a logistic path (equation IV.4 below). On the other hand, the unequal income distribution characterizing the long stagnation offers the possibility to exploit a niche in the market and to charge high prices (i.e. high mark-up). As diffusion proceeds, prices are gradually reduced to stimulate demand, thereby entailing a corresponding decline in the mark-up that reaches its minimum at the end of the long-wave. In equation (III.12) this constant decline of the mark-up results from the percentage rate of change of demand.

(III.12) $MKP'_{(n)i} = \beta_{45} Dln(ND_{(n)i}) + \lambda_{(n)i}$

Equation (III.13) refers to the adaptation process between theoretical and actual mark-ups

(III.13) $DMKP_{(n)i} = (\omega)(-M_{(n)i})\{\alpha_7(MKP'_{(n)i)} - MKP_{(n)i})\}$

For the new capital goods, positing the same pattern of demand (logistically shaped), we obtain a similar evolution of the mark-up with a corresponding adaptation process

(III.14) $MKP'_{(n)j} = \beta_{46} Dln(ND_{(n)j}) + \lambda_{(n)j}$

[21] The evidence presented by Goldstein (1986) refers to the business cycle, although his analysis can be extended to the long wave. See however, Oliveira Martins and Scarpetta (2002) for contrasting findings.

(III.15) $DMKP_{(n)j} = (\omega)(-M_{(n)j})\{\alpha_7(MKP'_{(n)j} - MKP_{(n)j})\}$

♦ *Profit rate*

(III.16) $$r_i = \frac{VA_{(nom)i} - L_i\, w}{\mathbf{PRE\ TK\ K^{(d)}\, e}_i} - i_r$$

In our simulations with only 5 sectors, vector **PRE** and the matrices become scalars.

♦ *Profit rates in recent years*

(III.17) $DR_i = \alpha_8(r_i - R_i)$

This equation – as well as equations (IV.3) and (V.4) below – is derived from the expression for distributed lags, attributing a weight of 0.63 to r_i of the last $1/\alpha_8$ years.

3.3.4. Consumption and demand

♦ *Consumption*

(IV.1) $\ln CON'_i = \beta_{47}\ln(w\, L) - \beta_{48}\ln PRE_i - \beta_{49}\ln(CT_i/XT_i) + \lambda_i$

and

(IV.2) $D\ln CON_i = \alpha_9 \ln(CON'_i / CON_i)$

In equation (IV.1), the first two terms on the right-hand side are self-explanatory. The third term refers to the saturation of demand occurring when for a number of years the share of consumption with respect to output rises beyond its "physiological" level. Equation (IV.2) expresses the adjustment of "normal" consumption CON'_i with respect to the current consumption.

♦ *Consumption in recent years*

(IV.3) $D\ln CT_i = \alpha_{10}\ln(CON_i / CT_i)$

See comments on equation (III.13); CON of the last $1/\alpha_{10}$ years has a weight of 0.63

Demand for new goods

This variable evolves according to the derivative of a logistic which, obviously, starts at the moment of the first innovation

(IV.4) $$D(ND_{(n)i}) = (\omega)(-M_{(n)i})\, \frac{ND_{as}\,\beta_{50}\,\beta_{51}\, e^{-\beta_{51}(t_i - t_c)}}{(1+\beta_{50}\, e^{-\beta_{51}(t_i - t_c)})^2}$$

The first term on the right-hand side marks the advent of a new commodity, something that happens when M is positive and consequently the switch is 1. When the switch is zero (i.e. M negative), there is no demand for the new goods. The second term is the derivative of a logistic, where ND_{as} is the asymptote of ND (the saturation level), and t_c is the mid-point of the period considered. In the present simulation t_c=14 and ND_{as}= 800 for new consumer goods.

The demand for new capital goods corresponds to formula (IV.4) *mutatis mutandis*, i.e. attributing to ND the values 50 and 40 for the two sectors of new capital goods.

3.3.5. Capital and investment

♦ *Stock of fixed capital*

For the existing sectors (*i* and *j*) changes in the capital stock are determined by the desired output/capital ratio corrected by the importance of radical uncertainty. Considering sector *i*, we have:

(V.1) $DlnK_i = [\beta_{52} (X_i/K_i) - a_2] - \beta_{53} u_i$

Constant a_2 results from the adjustment (in continuous time) of capital stock.

The formula for sector *j* is similar.

However, for the new sectors (consumer goods *n(i)* and capital goods *n(j)*) the capital stock is driven by the change in output of the sector concerned; radical uncertainty appears on the scene in this case too. For the new consumer goods sector we have:

(V.2) $DlnK_{n(i)} = \omega(-M_{n(i)}) [D \ln X_{n(i)}] - \beta_{54} D u_{n(i)}$

For the new capital goods sectors the equations are similar

♦ *Gross investment*

(V.3) $I_i = (D \ln K_i) K_i + \mu_1 K_i + \eta_i \Sigma X_{(n)j}$

Gross investment is the sum total of three components:

- the enlargement of existing productive capacity, or net investment (the first term on the right-hand side)
- the replacement of worn-up capacity (the second term)
- the obsolescence of existing plants and equipment resulting from the adoption of radical and incremental innovations (the third term).

In our simulation with only 5 sectors, obsolescence is expressed as a percentage (η) of the amount of new capital goods, and total gross investment (I) is the sum of investment in the five sectors, i.e.

$I = \Sigma_{i=1}^{5} (D \ln K) K_i + \mu_1 \Sigma_{i=1}^{5} K_i + \eta_i \Sigma X_{(n)j}$

♦ *Demand for existing capital goods*

(V.4) $\mathbf{KD} = \mathbf{TI} \ \mathbf{I}$

Let us recall that the dimensions of matrix **TI** and vector **I** are respectively:

$[j \times (i+j+(n)i+(n)j)]$ and $[(i+j+(n)i+(n)j \times 1]$

The transition matrix **TI** from sector of utilization to sectors of origin is held constant over time, something that represents an important drawback for a model centered on innovation. This inconvenience could be almost eliminated by introducing in the elements of **TI** some changes proportional to the obsolescence of capital as expressed by equation (V.3) above. However, we deemed that this would have complicated the model unnecessarily and in the absence of anything better we preferred to stick to the simplification in question.

In any case, in our numerical simulations this matrix does not appear since only one sector of existing capital goods is considered. Therefore we have the following scalars:

$$KD = I - \Sigma ND_j \qquad (j = 1\ ,\ 2)$$

The same considerations hold for matrix **TK** above.

♦ *Demand for existing capital goods in recent years*

(V.5) $$D\ \ln KDT_j = \alpha_{11} \ln(KD_j/KDT_j)$$

KD_j is weighted by 0.63 for the last $1/\alpha_{11}$ years

4. Numerical simulations

4.1. General features

The best way to check the plausibility of our model would be an econometric test on the basis of long-term statistical series for a sufficient number of countries. Unfortunately, in our case such an enquiry is not possible owing to a lack of data. Some data – which in theory could exist –are not available in practice at the necessary level of detail (sectoral data at 4 or 5 statistical digits). Some other data, concerning for instance the productivity level for individual innovators, could perhaps be obtained from industrial surveys; however, this could create a problem of coherence with respect to the source of data for the other variables of the model.

For this reason we were obliged to adopt a weaker notion of plausibility, in that we test the consistency of results through numerical simulations. Thus, relying on realistic hypotheses on the structure of the economy and on "reasonable" values for the parameters, it is possible to check whether the dynamic path of the variables resulting from our system of equations conforms to or contradicts our theory.

The numerical simulations were performed using the software WYSEA (2006): System estimation analysis, developed by Clifford R. Wymer. We considered a period of 50 years, starting from the depression phase of the long wave.

To have something manageable, we relied on a simplified model of just five sectors − one for the existing consumer goods (i = 1), another for the existing capital goods (j = 1) and three additional sectors for new commodities: one for consumer goods ($n(i)$ = 1) and two sectors for capital goods ($n(j)$ = 2). For radical product innovations in consumer goods, the additional sector covers both the case of innovations occurring in existing industries and the case in which they give birth to a new industry. Radical process innovations materialize first of all in existing consumer and capital goods sectors but in order to see the combined effects of process and product innovations, we assumed that in the additional sector of consumer goods, radical process innovation also occurs.

The initial values of the variables are arbitrary although, in order to have plausible levels, we referred to the Italian national accounts whenever possible (ISTAT 2007).

We performed three simulations (A, B and C) that differ according to:

(i) the importance of the leap in the productivity level of the individual firm adopting radical process innovation,
(ii) the intensity of dynamic competition, i.e. the relative importance of innovations and the speed of their diffusion within the sector,

(iii) the period in which the innovation first materializes in the various sectors.

This last aspect is quantified by attributing alternative values to some β parameters, to parameter Q referring to the productivity leap and to the threshold levels for the appearance of radical product and process innovations (parameters $k_{(M)}$ and $k_{(PrR)}$). Simulation B refers to a high level of dynamic competition, simulation C to a low level of such competition while simulation A represents an intermediate case.

Tables A1 and A2 in appendix respectively provide the initial values of the endogenous and exogenous variables, the values of the parameters common to the three simulations while Table A3 shows the elements that differ in the three simulations and Table A4 shows the periods of appearance of radical innovations.

The present model can be extended to include successive long waves, by simply taking the outcome of the previous simulation as initial values; what were previously new sectors are now incorporated into the existing ones and the model can be specified as in the first run.

4.2 Main Results

Figures 1 to 14 provide a selection of the results of our simulations. We have limited the number of figures to a bare minimum in order to save space although we shall be pleased to communicate the entire set on request. For the sake of brevity, we usually focus on the consumer goods sector; this means that where we add no comments on the peculiarities of the other sectors, their evolution is quite similar to the consumer goods sector.

Figure 1

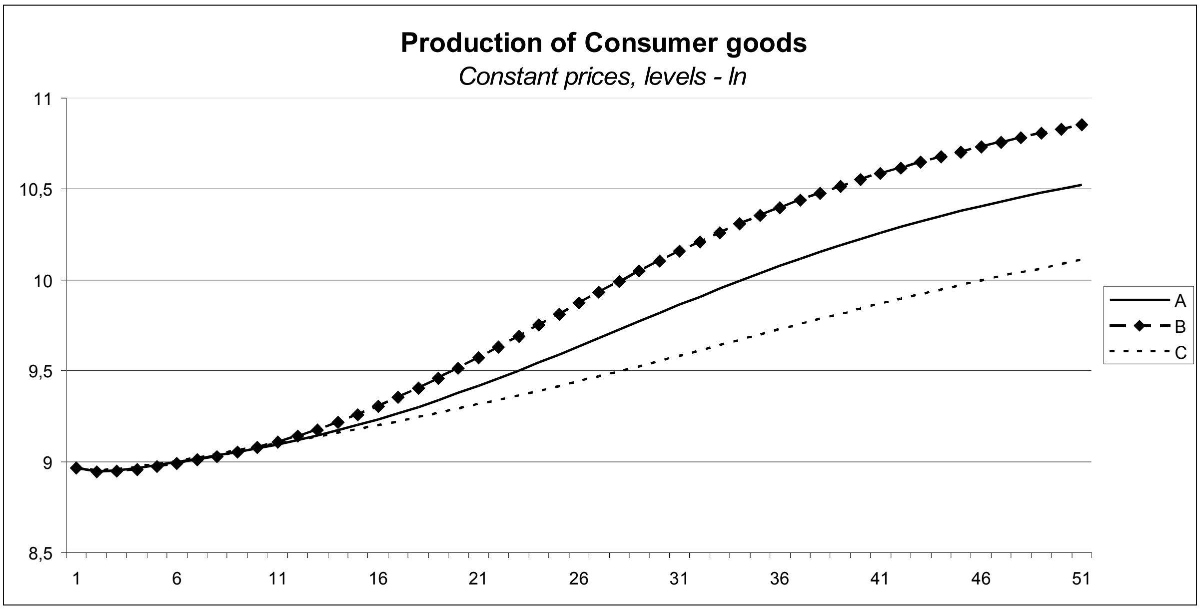


Figure 2

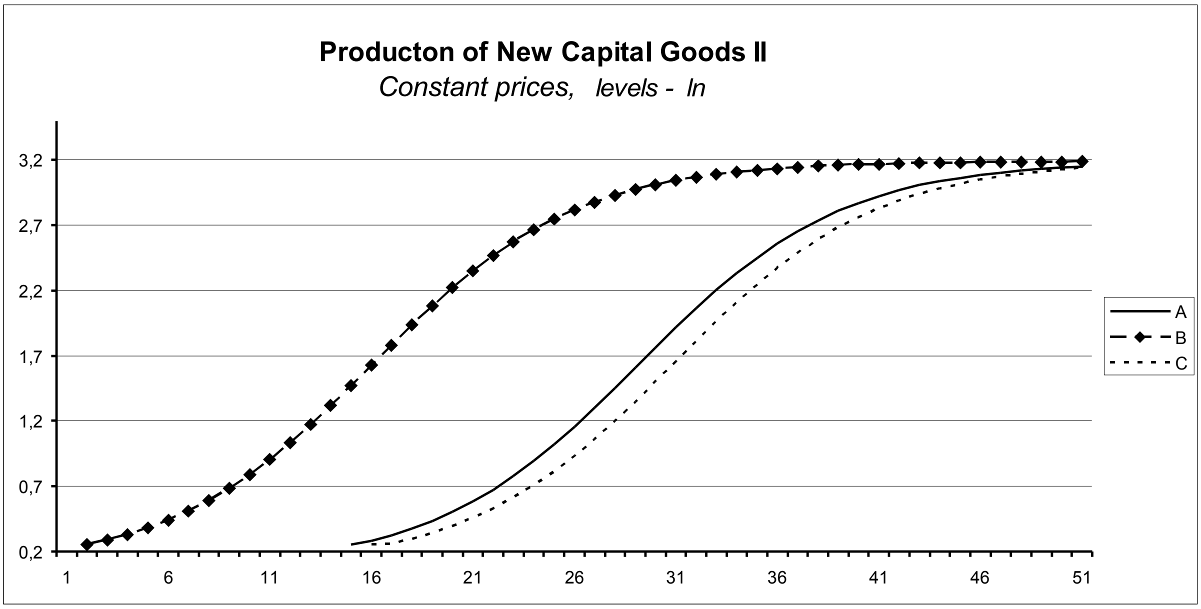
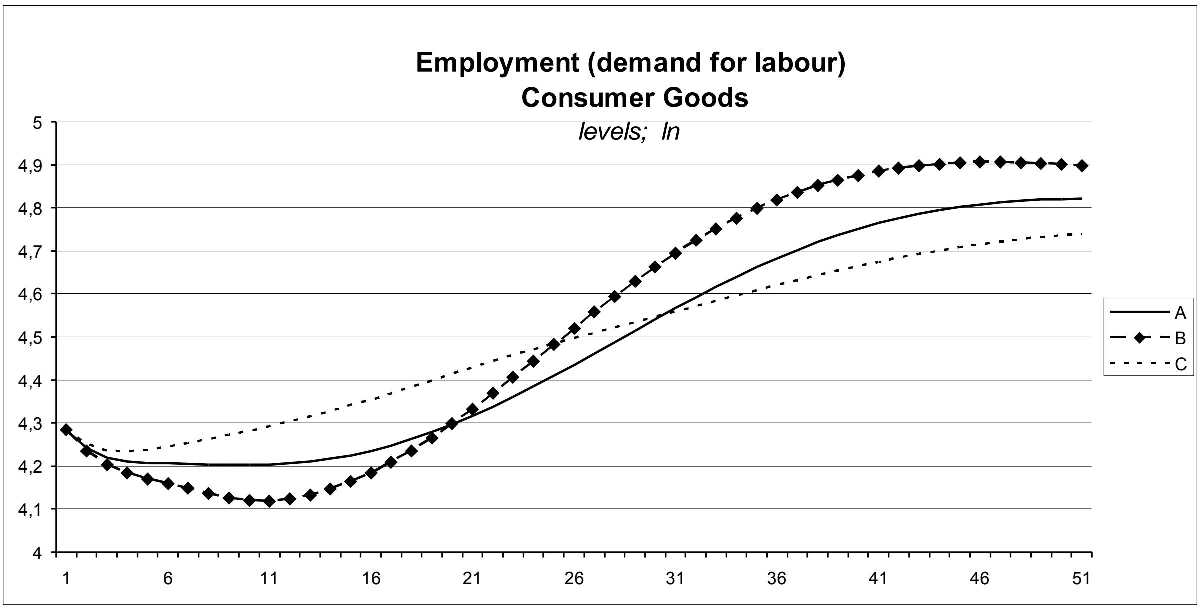


Figure 3



The main aggregates such as total output, value added, consumption, employment and investment display the typical long-wave (S-shaped) pattern. Figures 1, 2 and 3 provide examples. Figure 2, on the output of new capital goods, has been chosen to illustrate our hypotheses regarding this sector. As a general feature, simulation B – based on the assumption of stronger dynamic competition – shows higher values than the other two simulations.

Figure 4

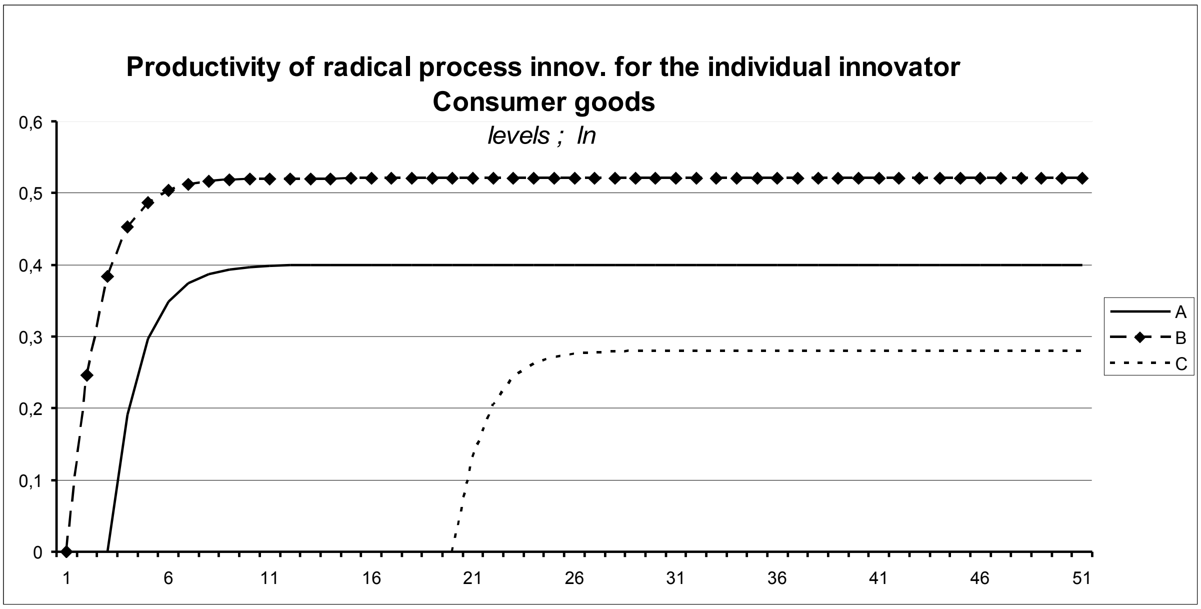
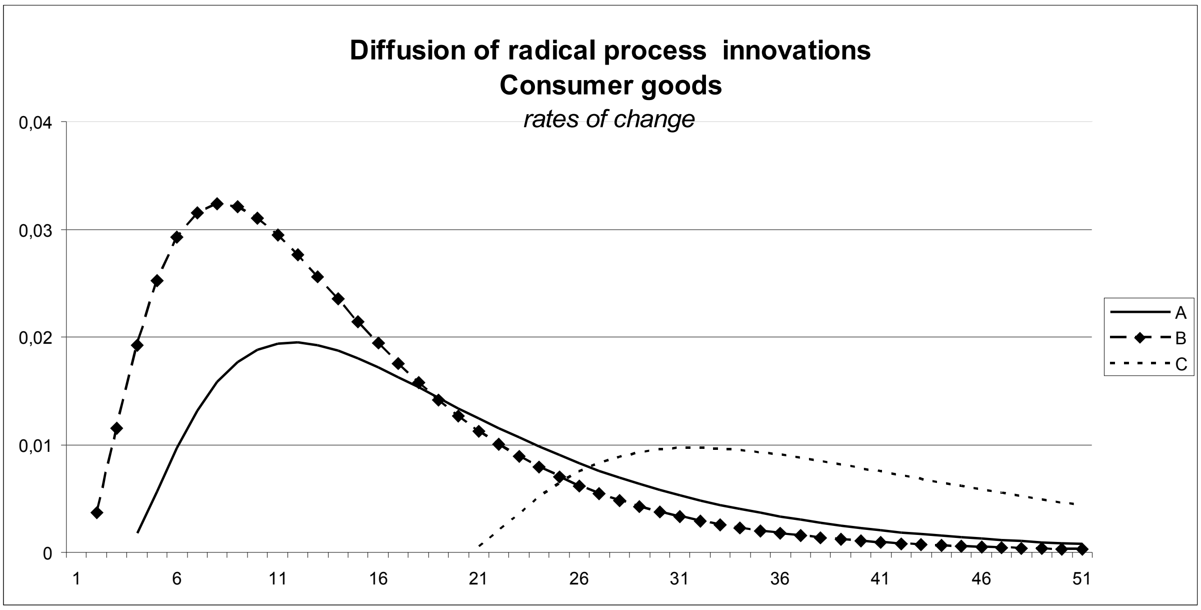


Figure 5



Referring to consumer goods, fig. 4 depicts the productivity function for the individual innovator resulting from radical process innovations, while fig. 5 gives the percentage rate of productivity change of the whole sector. Fig.5 conforms to the results of historical analysis summarized in table 1 above, as the pace of innovation is stronger at the beginning of the period, when the system is in the phase of long stagnation.

Figure 6

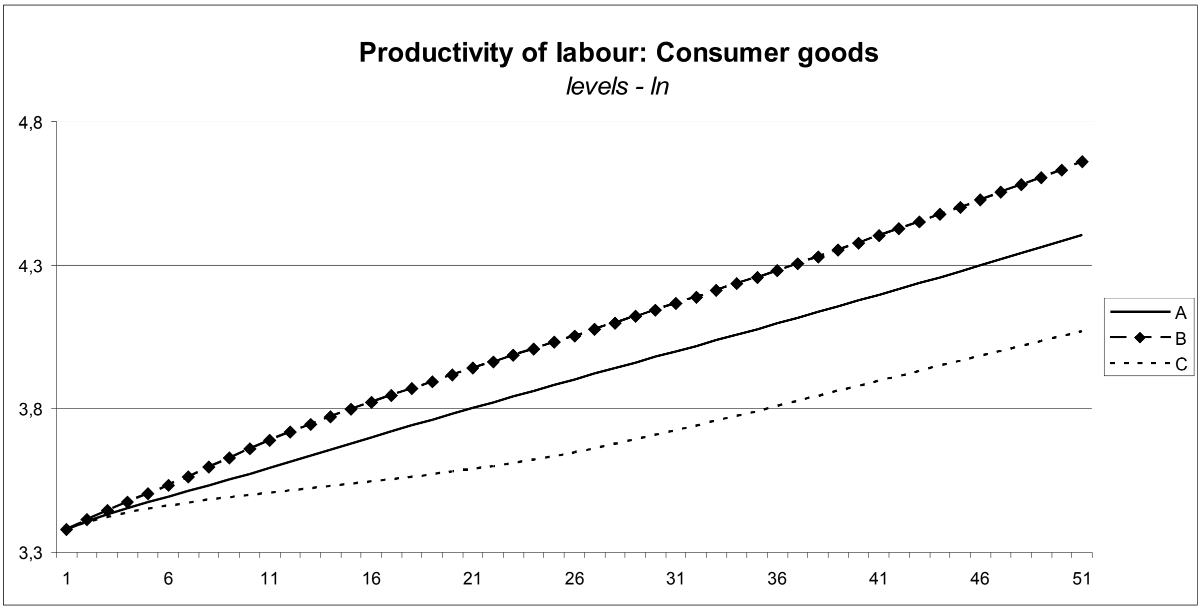
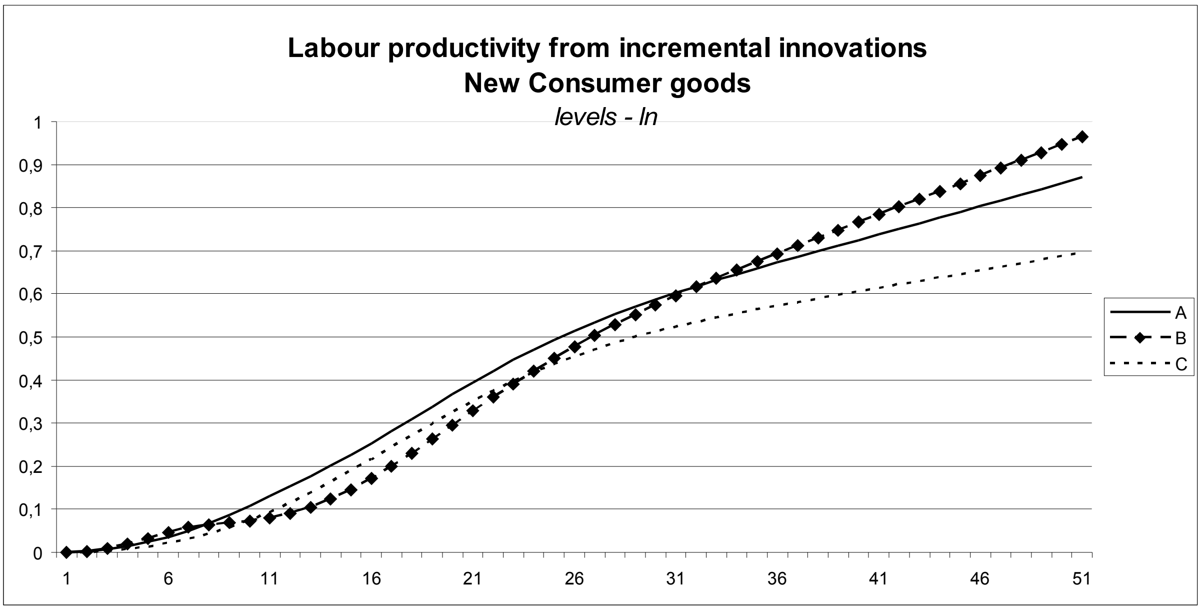


Figure 7

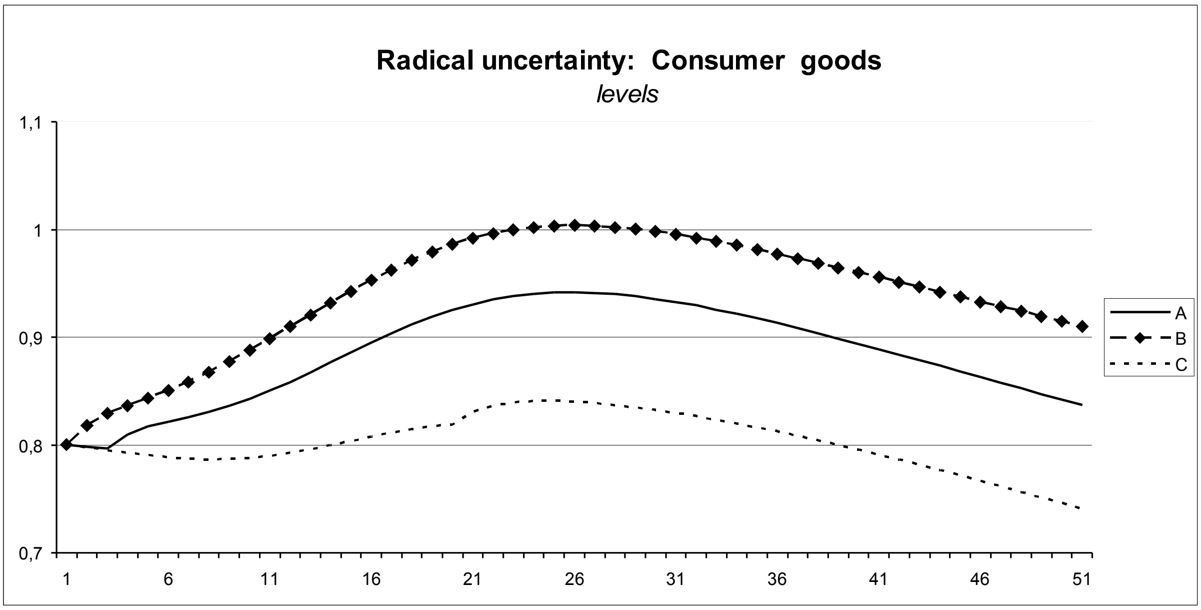


The productivity level of the consumer goods industry –summing up the effects of radical and incremental innovations – appears in figure 6. For the new consumer goods productivity grows faster because we posit that in this case the interaction between radical and incremental innovations exerts a strong stimulus for the adoption of incremental technical change (fig. 7 which shows the level of productivity determined by this kind of technical change).

In the two sectors of new capital goods (not reported) – where there are no radical process innovations (see subsection 4.1) – the pace of productivity is driven by incremental innovations.

However, we assume here that incremental innovations entail a productivity growth higher than in the other sectors because of the wider scope for improvements inherent to new commodities

Figure 8



Radical uncertainty (figure 8) behaves as expected under the theory: it increases substantially during long stagnation – when the "creative destruction" of the innovation activity is stronger and destabilizes the system – and slows down when the pace of radical technical change normalizes. In the sector of new consumer goods (not reported on) the increase in uncertainty during the first two decades is even more prominent as a result of the combined effect of process and product innovations.

Figure 9

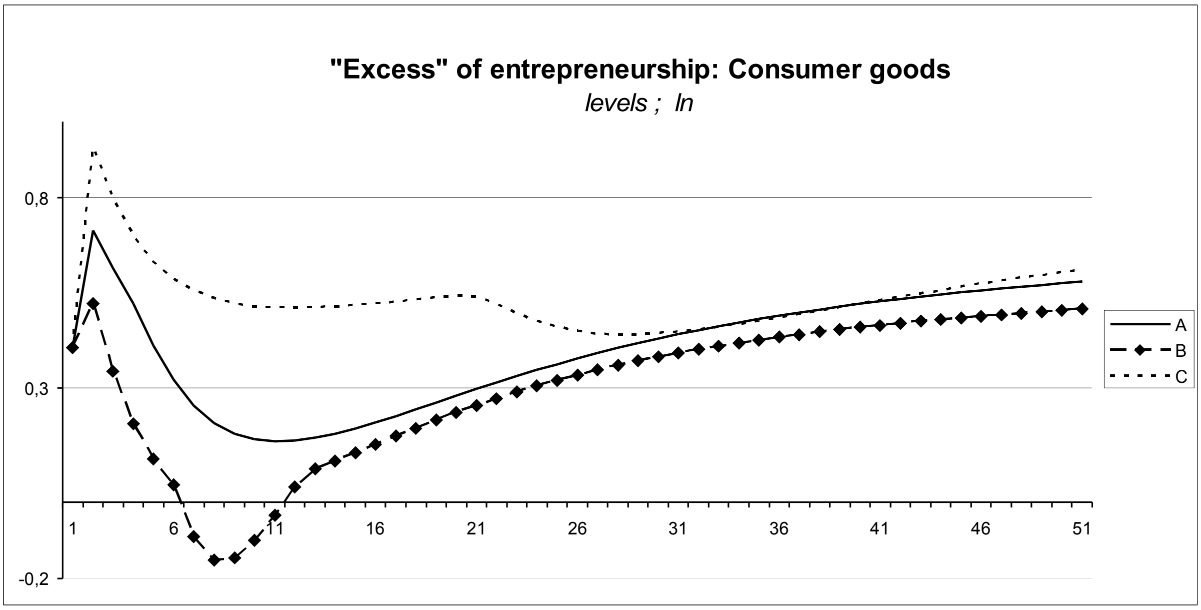
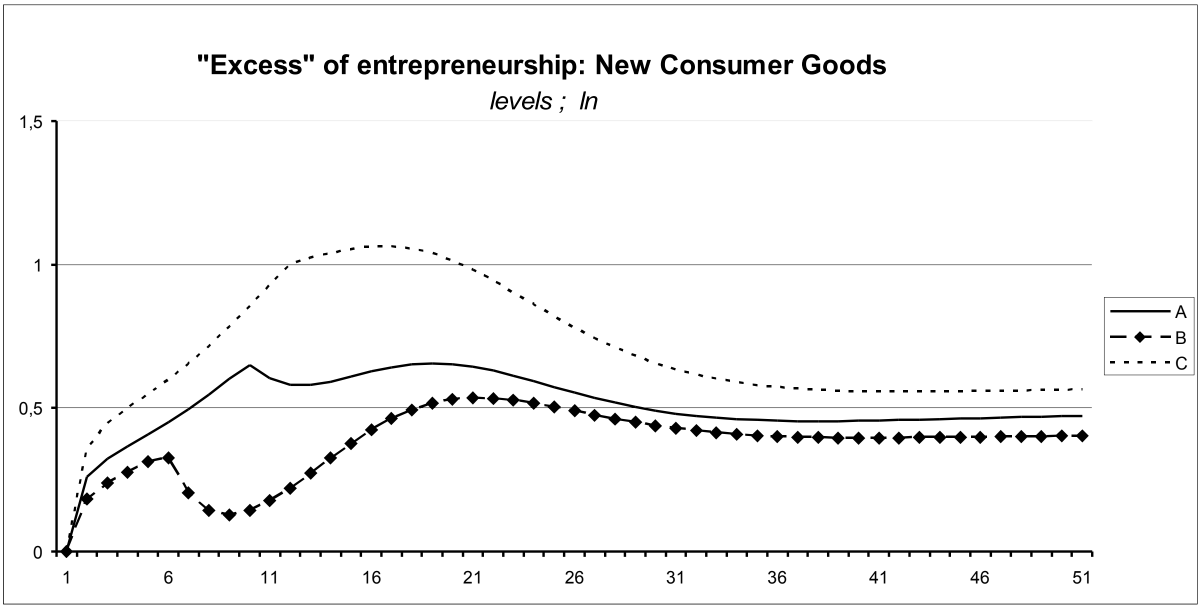


Figure 10



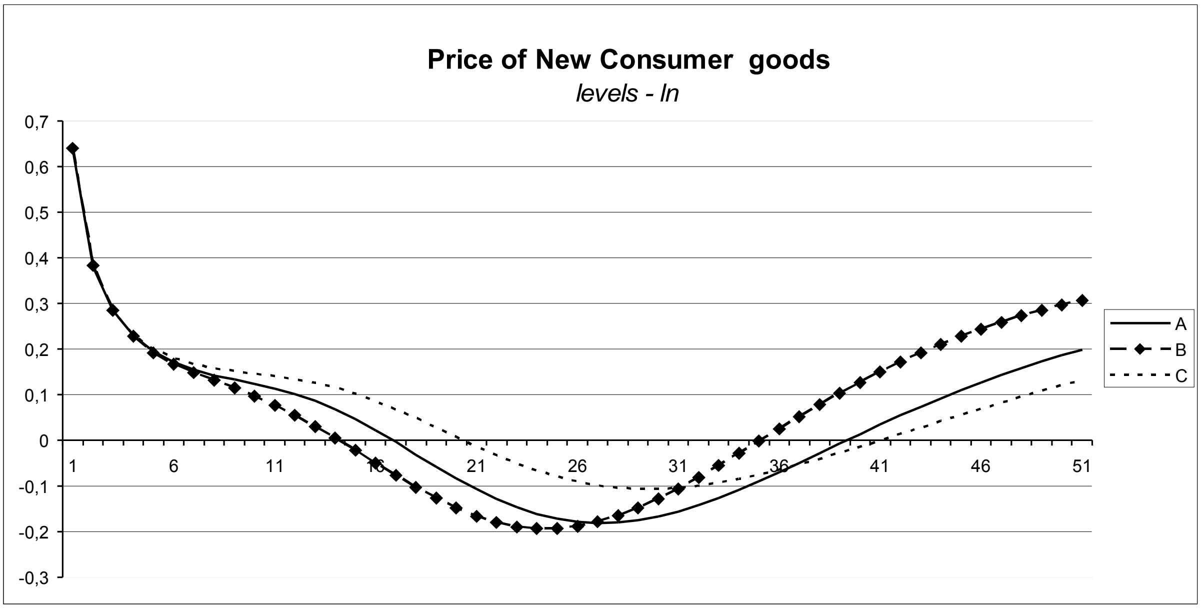
Unused entrepreneurial skills ("excess" of entrepreneurship) in the existing sectors (figure 9) mirror the process of diffusion of radical technical change (see fig. 5 above). When the pace of innovation is strong, entrepreneurs must make full use of their capabilities to cope with the turbulent environment. During the second half of the long wave, when the system is on a steady state growth path, the general environment is less demanding and this can justify an increase in unused entrepreneurial skills.

The characteristics of the new sectors (both consumer and capital goods) entail completely different dynamics (fig. 10). In fact, when a new sector appears, unused entrepreneurial skills are very low but as the new product progresses and new competitors enter the market, this increases the supply of entrepreneurship and correspondingly increases "excess" entrepreneurship. When the sector reaches maturity, the available skills are fully mobilized while the "excess" tends to vanish.

Fig. 11

Consumer goods prices (figure 11) are strongly influenced by the evolution of productivity. In simulations A and B, during the depression phase of the wave – when the long-run trajectory of radical innovations and productivity is incipient – they remain stationary or decline until about the beginning of the long expansion, following the dynamics of productivity; the following ascending trend is produced by the buoyant demand which characterizes the long expansion phase.
The different level and dynamics of prices in simulation C constitute the obvious effect of less dynamic productivity.
The price evolution of the capital goods sector (not reported) is quite similar.

Fig. 12

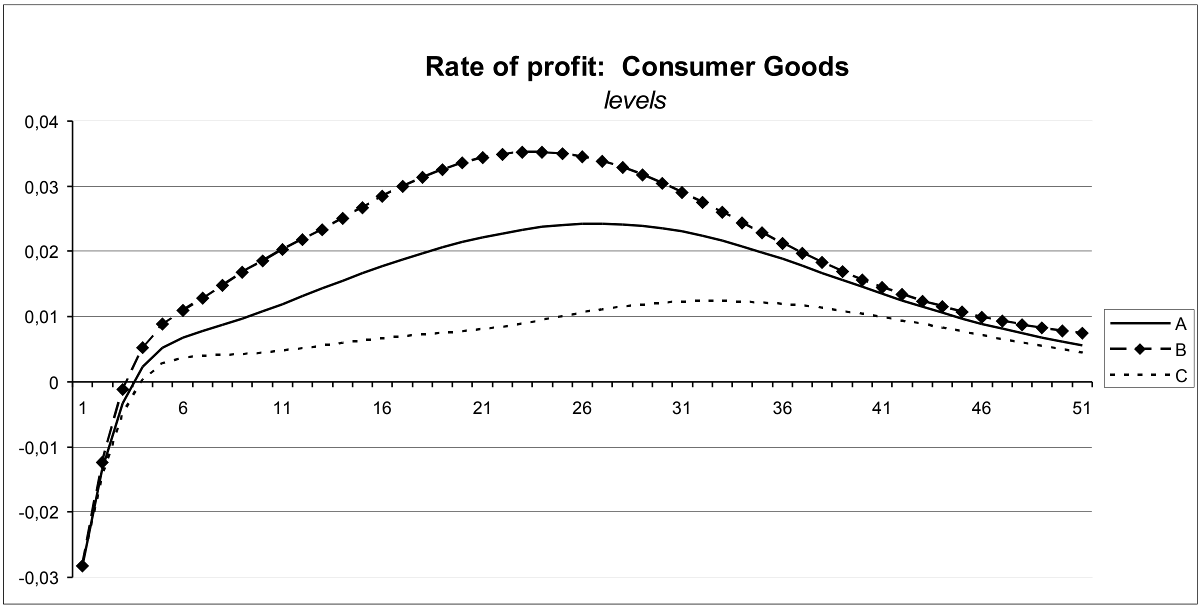


N.B. Considering that prices are expressed in natural logarithms, the negative values mean that the natural levels are positive and less than 1

The new consumer goods sector displays quite different price dynamics (figure 12). As noted above, the main reason for the first successful launching on the market of radically new products is the inequality in income distribution that marks the depression phase of the wave and the possibility for the innovating firms to exploit a niche in the market. This justifies the relative high level of prices at the beginning of the period and, also, explains why, as far as the long-wave displays its potentialities, such prices decline. In fact, when the income distribution becomes less unequal, firms can attract new layers of demand by reducing their prices. This strategy is also reinforced by the fact that the period of high prices gives enterprises the possibility to recover part of the money invested in the new product during the launching period. The ascending trend in the second half of the long wave is caused by cost increases because, during that period, wages tend to increase more than productivity.

The prices of new capital goods (omitted here) show a shorter decline at the beginning of the wave and a longer growing trend rooted in cost increases.

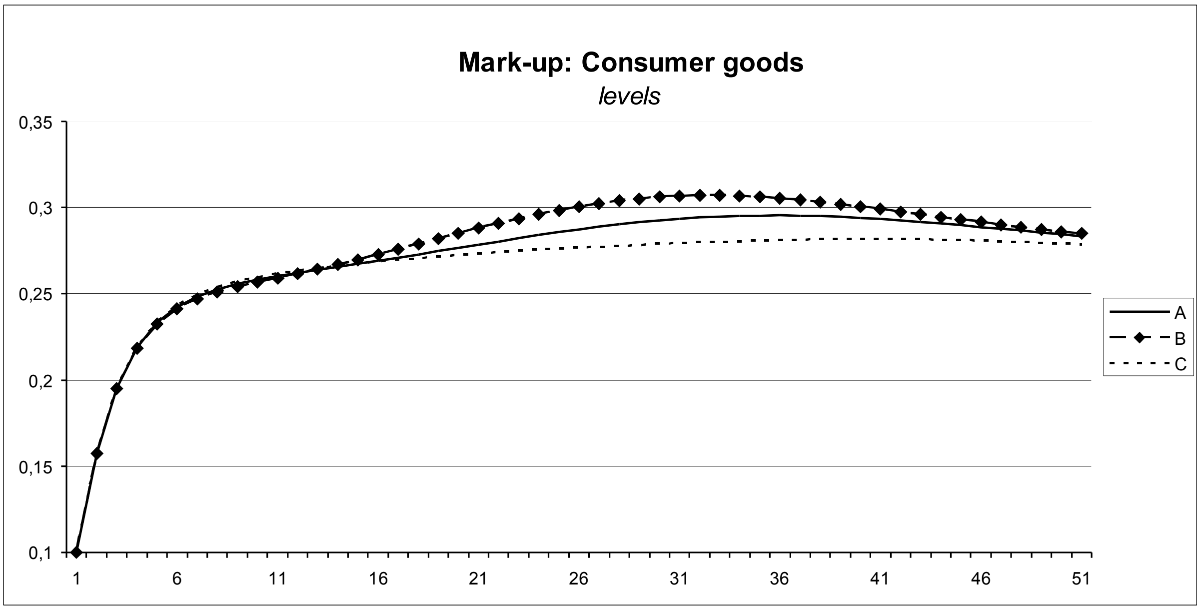
Fig. 13



N.B. – See the above footnote concerning the negative logarithms. Here profit rates of the first three periods, being expressed in level, are true negative number

As regards the profit rate, figure 13 shows, for the consumer goods sector a strong increase until the end of the long stagnation, then stability followed by decline. The mark-up displays a similar path (fig. 14)

Fig. 14



Conversely, in the new sectors (both consumer and capital goods) the mark-up (not represented here) follows a decreasing trend that is justified by the price policy for these commodities: starting from a high level, the mark-up progressively reaches its minimum level towards the end of the wave.

We notice an almost identical evolution for the rate of profit of the new consumer goods while in the case of the new capital goods profitability shows a growing trend since the beginning of the recovery phase of the wave.

5. A micro version of the model

A previous article by the author[22] provides a micro (firm level) version of the model here discussed, but referred only to one sector and to process innovations. The model considers n firms, m of which display both innovation and adaptation, while n-m only practice adaptation. Moreover $h \leq m$ firms display both radical and incremental innovations.

The model with only adaptive entrepreneurship (that is, without innovation) converges toward a stationary equilibrium. In fact, in the absence of innovation, the search for profits implies the convergence to uniform technique as well as the annulment of (endogenous) radical uncertainty and, hence, of the need for entrepreneurship. In sum, in this merely adaptive case the entrepreneurial convergence, through imitation, toward the best available technology would imply a 'suicide' of entrepreneurship, a steady state solution of the model and its stability. More precisely, the simulation of the model for a long period of time after the above convergence shows a limit cycle about the steady state solution.

A micro model with innovation, imitation and adaptation is then considered, and hence simulated. In this model, the stationary state is impossible in that it fades away before the system converges. In fact (and as we know), the reduction in radical uncertainty and the progressive increase in the excess of entrepreneurial skills will stimulate both radical and incremental innovation. But the rise in innovation will cause, in turn, a parallel rise in radical uncertainty and a reduction in the excess of entrepreneurship, i.e. the rise in the need of entrepreneurship, in opposition to the 'suicide' of this (both directly and through the increase of uncertainty) that will provoke the reduction in innovation and a recovery of adaptation.

Innovation of products, and hence a variety of products (that is, more than one sector), can be included in the micro model. In this regard, our one good micro model, which only considers process innovation, may obtain some inspiration from Pyka and Saviotti's micro model, which only considers product innovation.[23]

6. Concluding comments

1. In this research we pursue an ambitious objective that, if successful, should advance the frontier of our knowledge regarding the long-term dynamics of the economy. In fact, in section 2 of the paper we try to provide a fully endogenous explanation of the factors that trigger the technological revolutions underlying a new long upswing. More particularly, we emphasize the role of the profit rate in explaining the appearance of radical process and product innovations and also the impact of inequality and the stagnation of demand on the adoption of radical product innovations.

The framework – that implies an analysis at macro and meso level – marries Schumpeter with the neo-Austrian market process; moreover, it is enriched by some microeconomic components such as entrepreneurship and radical uncertainty, and is open to a full micro specification. The introduction of radical uncertainty also implies some reference to Keynes. But we make a step forward with respect to the current Keynesian (and Hayekian) approach which

[22] See A. Fusari (2005), 'A model of the innovation-adaptation mechanism driving economic dynamics: a micro representation'. *Journal of Evolutionary Economics*.

[23] See P.P. Saviotti and A. Pyka (2004), *Economic development by the creation of new sectors.* Journal of Evolutionary Economics.

conceives radical uncertainty as something that by its very nature escapes measurement. Indeed, we show that such a measure is possible at sectoral level and establish a link between radical uncertainty and the behaviour of innovation. The incipient diffusion of radical process and product innovations in existing industries destabilizes the incumbents' position and creates fundamental (or radical) uncertainty, which we identify with the volatility of expectations. The depression phase is thus characterized by the coexistence of diffuse negative expectations and hence low (volatility of these) but increasing radical uncertainty. During the recovery phase, radical uncertainty persists because the diffusion of product innovations in existing industries gains momentum while the same occurs for product innovations that create new needs. The dominant traits of long expansion are favorable expectations and relatively low radical uncertainty because the process of diffusion of radical process and product innovations is very advanced and the innovative activity concerns mainly incremental innovations – the kind of innovations that are less destabilising for the system.

Continuing our effort to quantify what is usually deemed to be unquantifiable, we put into evidence how the innovation process is also shaped by unused entrepreneurial skills (the "excess of entrepreneurship") – a notion that compares the available skills with the skills required to run the enterprise at a specific moment in time. If the current level of activity and the low uncertainty do not imply full utilization of the available skills, there is an "excess" of entrepreneurial skills that could be mobilized for innovations.

2. Our theoretical analysis is enhanced by further analytical details in the multi-sectoral dynamic model presented in section 3. One aim of this model is to test the overall consistency of our theoretical construction. For this purpose, we have numerically simulated a reduced version of the model with only five big sectors: one for the existing consumer goods, another for the existing capital goods and three additional sectors for new commodities: one for consumer goods and two sectors for capital goods. Radical process innovations materialize first of all in the existing consumer and capital goods sectors, but in order to see the combined effects of process and product innovations, we assumed that in the additional sector of new consumer goods radical process innovation also occurs. We performed three simulations that differ essentially according to the characteristics and the importance of radical technical change. The selection of results presented at the end of the paper confirms the robustness of our approach.

Appendix

Table A1 Initial values of the endogenous and of exogenous variables

(natural logarithms, except for SW, Γ, γ, M, MKP, ND, u, R, r. i_r)

Endogenous variables

	Sectors				
	i	***j***	***(n)i***	***(n)j1***	***(n)j2***
X	8.9628	6.47383	0.42	0.3	0.25
PrR	0.0	0.0	0.0		
SW	0.0	0.0	0.0		
K	8.7008	6.5028	0.5	0.4	0.33
PrI	0.0	0.0	0. 0	0.0	0.0
γ	0.0	0.0	0.0		
Γ	0.0	0.0	0.0		
ND			1.0	1.0	1.0
M			0.0	0.0	0.0
Pr	3.38	3.38	3.6	3.6	3.68
W *	3.435				
PRE	0.0115	0.065	0.64	0.51	0.51
u	0.8	0.8	1.0	1.0	1.0
CON *	8.9748				
t			0.0	00	0.0
CT	8.94				
XT	8.95				
XT2		6.36			
KDT		6.35			
R	–0.03	–0.018	0.09	0.028	0.028
Es	5.858	3.367	0.0	0.0	0.0
Ed	5.452	3.063	0.0	0.0	0.0
MKP	0.1	0.1	0.34	0.34	0.34

Exogenous variables

L_{s0}	4.367
i_r	0.05

* whole economy

Table A2 Parameters

$\alpha_1 = 0.166$	$\alpha_2 = 0.52$	$\alpha_3 = 0.7$	$\alpha_4 = 2.8$
$\alpha_5 = 0.57$	$\alpha_6 = 0.53$	$\alpha_7 = 0.43$	$\alpha_8 = 0.25$
$\alpha_9 = 0.65$	$\alpha_{10} = 0.166$	$\alpha_{11} = 0.166$	
$\beta_1 = 1.5$	$\beta_2 = 0.02$	$\beta_3 = 0.00017$	$\beta_4 = 0.0044$
$\beta_5 = 0.00828$	$\beta_6 = 0.078$	$\beta_7 = 0.095$	$\beta_8 = 0.35$
$\beta_9 = 2.3$	$\beta_{10} = 0.87$	$\beta_{11} = 0.0025$	$\beta_{12} = 0.008$
$\beta_{13} = 1.17$	$\beta_{14} = 0.0035$	$\beta_{15} = 0.161$	$\beta_{16} = 0.7^a$ $\beta_{16} = 4.0^b$
$\beta_{17} = 0.1$	$\beta_{18} = 0.2$	$\beta_{19} = 0.093$	$\beta_{20} = 0.049$
$\beta_{21} = 0.5$	$\beta_{22} = 0.001$	$\beta_{23} = 0.34$	$\beta_{24} = 0.17^c$
$\beta_{25} = 0.075$	$\beta_{26} = 0.000293^d$	$\beta_{27} = 0.4$	$\beta_{28} = 30.5$
$\beta_{29} = 0.01$	$\beta_{30} = 6.0$	$\beta_{31} = 0.48$	$\beta_{32} = 20.5$
$\beta_{33} = 2.5$	$\beta_{34} = 0.33$	$\beta_{35} = 55.7$	$\beta_{36} = 0.99$
$\beta_{37} = 0.58$	$\beta_{38} = 0.43$	$\beta_{39} = 0.79$	$\beta_{40} = 0.27$
$\beta_{41} = 0.1$	$\beta_{42} = 1.2$	$\beta_{43} = 0.1$	$\beta_{44} = 0.9$
$\beta_{45} = 0.17$	$\beta_{46} = 0.18$	$\beta_{47} = 0.983$	$\beta_{48} = 1.02$
$\beta_{49} = 0.56$	$\beta_{50} = 1.0$	$\beta_{51} = 0.2$	$\beta_{52} = 0.03^e$
$\beta_{53} = 0.01$	$\beta_{54} = 2.6$		
$a_1 = 0.0091$	$a_2 = 0.026$		
$\mu_1 = 0.08$	$\eta_1 = 0.19$	$\nu_1 = -1.298$	$\nu_2 = -1.298$

Scaling parameters λ: 1.01 in the consumption equation; 0.11 in all Mark-up equations

[a] existing consumer and capital goods

[b] new goods

[c] 0.41 for new goods

[d] 0.0022 for new goods

[e] 0.0215 for capital goods

Table A3 Hypotheses underlying the simulations

Parameters and constants

	Intermediate case *Simulation **A***	High dynamic competition *Simulation **B***	Low dynamic competition *Simulation **C***
β_{19}	0.0093	0.0065	0.0121
β_{20}	0.049	0.0637	0.0343
β_{21}	0.5	0.65	0.35
β_{17}	0.1	0.13	0.07
β_{18}	0.2	0.26	0.14
Q_1	0.40	0.52	0.28
Q_2	0.50	0.65	0.35
$k_{(PrR)1}$	0.32	0.192	0.42
$k_{(PrR)2}$	0.21	0.13	0.31
$k_{(PrR)3}$	0.23	0.2	0.38
$k_{(M)1}$	0.11	0.13	0.11
$k_{(M)2}$	0.175	0.127	0.218
$k_{(M)3}$	0.195	0.13	0.285

Table A4   First appearance of radical process and product innovations

Sector	Simulation	Period
Process innovations		
Consumer goods (*i*)	A	4
	B	2
	C	21
Capital goods *(j)*	A	4
	B	2
	C	4
New consumer goods *(n)i*	A	10
	B	6
	C	12
Product innovations		
New consumer goods *(n)i*	A	2
	B	2
	C	2
New capital goods I $(n)j_1$	A	4
	B	2
	C	4
New capital goods II $(n)j_2$	A	16
	B	3
	C	17

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