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Abstract

This study is based on positivism research philosophy and utilizes deductive approach. The study uses a dataset of 117 firms listed on KSE-100 Index from 2005 to 2012 to analyze the predictability of Capital Asset Pricing Model (CAPM) under different data frequencies and time frames. Six months daily data, in contradiction to the recommended five years monthly data, provides the best estimates. However, the performance of model can be regarded poor as it only explains 7.39% difference in returns.

Key words: CAPM; Beta; Markowitz Mean-Variance Framework

JEL Classification: G11; G12; G31

1. Introduction

Decision making in finance particularly in portfolio management, capital budgeting, and equity valuation requires return estimation on individual firm basis. Capital Asset Pricing Model (CAPM) and Fama and French (1992) are single and three factor models respectively commonly used for such decision making. These models are of great importance when estimating returns. Academic literature favors Fama and French when portfolio perspective is considered. However, CAPM is preferred by practitioners for estimation on individual stock basis [e.g., Bruner et al.]

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Fama and French model performs no better than CAPM when applied for individual stock’s return estimation. It requires more calculation and is not cost effective. Considering the practitioners’ preferences, the objective of this paper is to analyze the CAPM estimates, for individual stocks, under different data frequencies, time frames and indices.

The CAPM developed by Sharpe (1964) and Lintner (1965) based on its simplistic approach is widely used by practitioners and academicians to calculate the required returns for the portfolio management and the cost of equity for the capital budgeting decisions and assets valuation. CAPM was developed for a hypothetical environment with certain assumptions about the market players. The assumption encompass risk-averse attitude of investors, risk free lending and borrowing, assets divisibility and market perfection etc. The model implies mean-variance efficiency of the market portfolio in a manner that (a) the expected return on security is a positive linear function of its market ‘β’ (the slope in regression of a security’s return on the market’s return), and (b) the Market ‘β’ describes the cross-section of expected return. The empirical evidence for the validity of CAPM to predict future returns is mixed. The basic principle of the model is that the investor should be compensated for taking risk in addition to risk free return (time value). CAPM assumes the opportunity of risk free investment and then justifies the excess return as risk premium for the risky portfolio (Railey et al 2007). Further it divides the total risk of an asset in two parts; unsystematic and systematic. There is no risk premium justified for unsystematic risk as it can be eliminated through efficient diversification. Unsystematic risk is the risk of security itself which can be calculated by application of Markowitz mean-variance model and diversified by selection of efficient portfolio with less than zero correlation among the securities. CAPM talks about the systematic risk which covers the risk of an economy as a whole. The measure of systematic risk is beta i.e. relationship of security value with the market index.

Application and validity of CAPM posed intellectual challenge ever since foundations for its development were laid by [e.g., Markowitz (1952)] by proposing mean-variance framework. Since inception, CAPM version developed by Sharp (1964) and Lintner (1965) remained under criticism. Model has been modified in many ways to capture the true nature of risk/return relationship and to increase the predictability of returns so that the accuracy in financial decisions can be achieved. After decades, CAPM is still widely used in financial applications and provides basis for financial decisions. The model has been securitized time and again by researchers however; the research efforts were unable to invalidate the underlying principle of the model.
CAPM is a powerful and intuitive tool to predictions to measure relationship between expected return and risk. Unfortunately, the empirical record of the model has been unsatisfactory ever since its inception. The CAPM’s empirical insufficiency may reflect theoretical limitations and the result of many unrealistic assumptions. However, they may also be caused by difficulties in implementing valid tests of the model.

All valuation decisions require discounting of future cash flows by appropriate rate, which is not observed but estimated. To compare beta estimates given by CAPM under different frequencies, these estimates must qualify as best for a plausible comparison. The word best in this study implies the estimation of a beta, which provides a perfect fit between the econometric model and the data. A measure of best fit, $R^2$, is therefore used to identify the best beta estimates in this study. Despite the existence of a rich academic literature which discusses implementation of CAPM, particularly in relation to the estimation of key parameter beta, there is no consensus with regards to how a best estimate should be obtained. There is also no consensus with regard to the index, timeframe, and data frequency that should be used for estimation. Previous research has focused on reasons for differences in estimated betas between periods and the ability of historical betas to predict future betas [e.g., Blume (1975); Carleton and Lakonishok (1985); Klemkosky and Martin (1975); Reilly and Wright (1988)].

As illustrated by the following quote, the situation is no better among professional beta providers².

“A commercial provider of betas once told the authors that his firm, and others, did not know what the right period was, but they all decided to use five years in order to reduce the apparent differences between various services’ betas, because large differences reduced everyone’s credibility!”

[Brigham and Gapenski (1997), p354, footnote 9]

This lack of consensus manifests itself in different beta estimates for the same company by different beta providers. The objective of this paper is, therefore, to find the best index, time frame, and data frequency for estimating beta, and thereafter expected return, based on CAPM. Two other issues include: (1) Calculation of returns with or without dividend, and (2) Raw return or excess return are not addressed in this particular setting as [e.g., Bartholdy and Peare (2005)] findings state these issues have no significant impact on estimation procedure.
The analysis of selected sample suggests that six months daily data is the appropriate time period and data frequency for estimation of beta based on the R² criteria. High correlation (0.80) between equal weighted index and market capitalization value weighted index (KSE-100) indicates that there is no difference between beta estimates of these indices. General empirical evidence is not in support of beta, in explaining portfolio returns [e.g., Roll (1988)]. For practical usage, it is the performance on individual basis which requires further evidence. Best estimates obtained using six months daily data and value weighted index are only able to explain seven percent of differences in returns. Thus, use of CAPM by practitioners seems strange.

2. Recent Debate

Dempsey (2013) claims that CAPM which made finance an appropriate subject for econometric studies may need to be replaced with a paradigm of market, vulnerable to unpredictable behavior. In his words, “The empirical studies of CAPM are nothing more than fitting of data rather than theoretical principles”, he criticized on Black’s³ CAPM, the replacement of risk free rate \( R_f \) with returns on zero beta portfolio \( R_z \). The higher value of \( R_z \) was fitted as an econometric technique in an effort to improve CAPM results found during the early study of [e.g., Black, Jensen and Scholes (1972)]. The love affair with data mining emanates from the desire to support one’s prior convictions [e.g., Moosa (2013)]. In conclusion, Dempsey (2013) says “An inexact science becomes even more inexact for academics” claiming finance now to be an econometric exercise in mining data either for confirmation of a particular factor model or for the confirmation of deviations from a model’s predictions as anomalies.

Moosa (2013) adding to the Dempsey (2013) discussion concludes CAPM as an inadequate model. The belief “economics and finance are allegedly similar to physics” has led to excessive and unnecessary mathematization, producing fancy but unrealistic models (including the CAPM). “Finance, after all, is not physics and CAPM is by no means Boyle’s law” he concludes. Claims of CAPM’s invalidity routing deep in history of finance [e.g., Fama and French (2004); Horn (2009); Lai (2011)] and criticism on Mean-Variance (MV) analysis [e.g., Borch (1969); Borch (1974); Borch (1978)] were discussed under “Logic and philosophy of finance” by Johnstone (2013). He encourages the CAPM debate [e.g., Dempsey (2013)], considering it the philosophical aspect of finance and criticizes the cultural shift within finance from critical philosophical analysis of logic and methods, caused due to more empirical historical focus of researchers in finance.
Bornholt (2013) studied an extensive data set (from July 1963 to December 2009) using value-weighted monthly returns and revealed that CAPM will eventually become a reasonable approximation to market reality. He supported the claim by showing weakening of beta anomaly over recent years and argued that it resulted due to continued exploration by practitioners. Other anomalies (i.e. momentum and value effect) will also diminish overtime and Dempsey (2013) study is relevant for other variants of pricing models.

Brown and Walter (2013), in response to Dempsey (2013), quoted Roll (1977) that so-called tests of CAPM were invalid due to the use of inefficient benchmark portfolio proxy for market portfolio contrary to what the theory suggests that the benchmark should be efficient. The critics have not sufficiently internalized this requirement [e.g., Diacogiannis and Feldman (2007)]. They supported their argument based on the results presented by Graham and Harvey (2005) where the average 10-year bond yield (4.6%) is higher than average market risk premium (3.7%), hence market portfolio used was inefficient. In their view, valid test of CAPM requires efficient benchmark, which in the case of CAPM is world market portfolio which has been proved elusive so far.

Partington (2013) under the title “Death where is thy string?” based his conclusion, “CAPM is the king of asset pricing models”, on two arguments. First, CAPM has passed a very important test which is the test of time [e.g., Graham and Harvey (2001); Truong et al. (2008)]. Second, it is not entirely clear as to what has been tested in empirical tests of the CAPM. He also emphasized the importance of efficient benchmark as proxy for market portfolio. The Mean-Variance (MV) framework was first introduced in 1952 by Professor Henry Markowitz where mean and standard deviation were used as proxies for investment return and risk respectively [e.g., Markowitz (1952)]. In 1960s, Sharpe (1964) and Lintner (1965) extended his work to modern portfolio theory (MPT). MPT resulted in equilibrium expected returns that are linear function of security’s and market’s risk. The single factor Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965) provided basis for return predictability and equilibrium pricing with several unrealistic theoretical assumptions. He and Ng (1994) and Davis (1994) criticized the model’s predictability for providing week empirical evidence on risk-return relationship. Bowers and Heaton (2013) emphasized the importance of CAPM used to calculate cost of equity but failed to provide empirical evidence in support. Efforts were made to overcome the problems of CAPM through alternative approaches [e.g., Merton (1973); Ross (1976); Fama and French (1996);

3. Econometric Specification

This section briefly discusses the estimation process of individual stock returns by CAPM and evaluation methodology of its estimates. Single factor CAPM calculates the expected return on an asset by:

\[ R_i = R_f + \beta_i(R_m - R_f) + \epsilon_i \]  \hspace{1cm} (1)

In Equation (1), \( R_i \), \( R_f \), \( R_m \) and \( \beta_i \) represent return on stock \( i \), risk free rate, return on market portfolio and beta of stock \( i \) respectively. Beta is the systematic risk of asset \( i \) relative to market (world market portfolio) and can be calculated as \( \beta_i = \frac{\text{Cov}(r_i, r_m)}{\sigma_m^2} \). Returns are calculated using natural log of periods’ closing prices as \( R = \text{Ln}(P_t/P_{t-1}) \). The estimate for \( \beta_i \) is typically calculated by running time series regression of market and stock excess returns as follow:

\[ R_{it} - R_{ft} = \alpha_i + \beta_i \gamma_{it} + \epsilon_{it} \]  \hspace{1cm} (2)

\( \gamma_{it} = R_{mt} - R_{ft} \) is known as market risk premium in Equation (2). As the market portfolio, which consists of all assets in the market is not observable so it is necessary to use a proxy (KSE 100 Index) denoted by \( k \) in Equation (3). More precise equation in case of proxy would be:

\[ R_{it} - R_{ft} = \alpha_i + \beta_i(R_{kt} - R_{ft}) + \epsilon_{it} \hspace{1cm} t = 1, \ldots, n. \]  \hspace{1cm} (3)

Equation (3) can also be calculated by using raw returns instead of excess returns, there is no significant difference in estimates using both returns Bartholdy and Peare (2005).

\[ R_{it} = \alpha_i + \beta_i R_{kt} + \epsilon_{it} \hspace{1cm} t = 1, \ldots, n. \]  \hspace{1cm} (4)

Ordinary Least Square (OLS)\(^5\) methodology is applied using equation (4) to estimate beta (a proposed estimation procedure by Fama and MacBeth in 1974). Then, for all stocks in the sample, an estimate for the risk premium, \( \gamma_{1t} \) in the subsequent year is obtained using, \( \beta_{it}^l \) i.e. betas obtained from Eq.(4), as explanatory variable through following cross section regression:

\[ r_{it+1} - r_{ft+1} = \gamma_{0t+1} + \gamma_{1t+1}\beta_{it}^l + \epsilon_{it+1} \hspace{1cm} i = 1, \ldots, N \]  \hspace{1cm} (5)
Equation (5) is a tool to measure the statistical fit of beta. The data on left hand side of equation represent actual annual return of stocks, irrespective of the data frequency used for estimation of beta in equation (4), the general application of model by practitioners. This procedure has also been applied in previous academic literature. However, the objective of studies was to test factor models for detection of anomalies in stock returns. To serve the purpose, the studies used portfolio for estimation of equation (5). Since the objective of this study is to obtain estimates for individual stock, use of portfolio is not appropriate.

The above mentioned estimation procedure can be evaluated based on two criteria [e.g., Bartholdy and Peare (2005)]. First criterion is the significance of $\gamma_{1t+1}$, estimated risk premium for specific period, as it is a necessary condition for the model to be of any use. Second criteria will be $R^2$ of equation (5), a direct measure of beta’s ability to explain differences in returns on stocks in the period following estimation, used as yardstick following Roll (1988).

The theory of CAPM is very specific to use a value-weighted index comprising all the assets in the world which is unrealistic in practice. An Equal Weighted Index (EWI) of selected stocks is constructed to evaluate the importance of proxy in addition to KSE-100 Index (market capitalization value weighted index). Two important estimation issues of thin trading and mean reversion in beta are not part of this study as more basic situation (frequently traded stocks) can find the best way to estimates beta before dealing with complexities.

4. Data and Results

This study utilizes data of firms listed in Karachi Stock Exchange (KSE) of Pakistan. Sample based on data availability and survival of firms from 2005 to 2012 resulted in 117 stocks. In order to avoid thinly traded stocks (Interval effect as highlighted by [e.g., Hawawini (1983)]), only those stocks are included those have been traded for at least 80% of trading days during the sample period. Selection of observation frequency and length of time is an interesting question. Generally greater number of observations is recommended. Using longer time period, however, may cause mean reversion or time variation bias in beta estimates. Thus requires shortening in selected time period. Other way to increase observations is to use high frequency data, which induces noise in data. Thus there is a tradeoff between the length of the time period and sampling frequency. In this paper, therefore, the performance of monthly data for four, five and six years, weekly data for two, three and four years, and daily data for one, one half and two years, are examined.
Daily prices are extracted from the official website of Karachi Stock Exchange from 2005 to 2012. Daily returns are calculated as simple holding period rates of return between days. Weekly returns are calculated from Wednesday to Wednesday to avoid any contaminating effects from weekends and Mondays and end of month prices are used for calculating monthly returns. 12 month T-bill rate is used as proxy for the yearly risk-free rate in cross sectional regression (Eq.5). The estimation procedure based on CAPM involves first obtaining an estimate for beta using either (3) or (4) and then using the resulting estimate to obtain an estimate for expected return using (5). Thus if five years of monthly data is used then, for each stock, first an estimate for beta is obtained using the years 2005 to 2009. Then the resulting estimate is used in (5) to obtain an estimate for expected return in 2010. If estimation is based on two years of weekly data then 2007 and 2008 are used to obtain an estimate for expected return in 2009 and if one year of daily data is used then 2008 is used for beta. Thus the sample of stocks used for monthly, weekly, and daily data is the same. The process is then repeated with monthly role forward using different time frames and data frequencies to obtain estimates for expected return of next periods.

Beta estimation results using time series regression through Eq. (4) show highest number of significant cases of beta using six years monthly data. The average $R^2$ for all sample of stocks ranges from 9.99% (one year daily data) to 32.32% (four year monthly data). Diagnostic test are also applied to further examine the situation. Serial correlation, conditional heteroskedasticity and non normality of error terms are found in almost all cases. Serial correlation cases are highest with daily frequencies. Conditional heteroskedasticity (CH) is highest in three years weekly data. More than 30 out of 117 cases of CH are found in daily return data. Similar finding are evident for non normality of error terms. Correlation between EWI and KSE is above 0.82 for all samples. Beta estimates (Eq.4), of all time frames and data frequencies, are cross regressed against next year actual returns. As defined in methodology section, the slop “$γ_{1t+1}$” must be significant for the model to be of any use. Results of the procedure are reported in table 1. Estimates of risk premium for the subsequent period are significant for all time periods and data frequencies. However the number of significant cases varies with the use of different time periods and data frequencies. Average risk premium is 9%. $R^2$ also varies with used frequencies and time frames. It is highest for 0.5 year daily data i.e. 7.39% and lowest for 6 years monthly data. Higher $R^2$ values are found for daily frequency data. Average $R^2$ of all time periods and data frequencies is 5.3% which
Table 1: Results of Cross Sectional Regressions

<table>
<thead>
<tr>
<th>Code</th>
<th>Data Frequency</th>
<th>Time Period (years)</th>
<th>Monthly Rolls</th>
<th>No. of Positive Significant cases</th>
<th>$\gamma_{1t+1}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>Daily</td>
<td>0.5</td>
<td>34</td>
<td>11</td>
<td>8.16%</td>
<td>7.39%</td>
</tr>
<tr>
<td>D2</td>
<td>Daily</td>
<td>1</td>
<td>34</td>
<td>12</td>
<td>10.47%</td>
<td>6.95%</td>
</tr>
<tr>
<td>D3</td>
<td>Daily</td>
<td>1.5</td>
<td>34</td>
<td>12</td>
<td>11.05%</td>
<td>6.66%</td>
</tr>
<tr>
<td>W1</td>
<td>Weekly</td>
<td>1</td>
<td>34</td>
<td>10</td>
<td>5.84%</td>
<td>4.83%</td>
</tr>
<tr>
<td>W2</td>
<td>Weekly</td>
<td>2</td>
<td>34</td>
<td>11</td>
<td>7.21%</td>
<td>3.91%</td>
</tr>
<tr>
<td>W3</td>
<td>Weekly</td>
<td>3</td>
<td>34</td>
<td>10</td>
<td>10.08%</td>
<td>4.82%</td>
</tr>
<tr>
<td>M1</td>
<td>Monthly</td>
<td>4</td>
<td>34</td>
<td>10</td>
<td>8.03%</td>
<td>4.60%</td>
</tr>
<tr>
<td>M2</td>
<td>Monthly</td>
<td>5</td>
<td>24</td>
<td>9</td>
<td>8.68%</td>
<td>4.75%</td>
</tr>
<tr>
<td>M3</td>
<td>Monthly</td>
<td>6</td>
<td>12</td>
<td>5</td>
<td>11.46%</td>
<td>3.81%</td>
</tr>
</tbody>
</table>

Average 9.00% 5.30%

5. Conclusion

Besides the importance of cost of equity and estimation of expected return for financial decision makers and academia, few studies have covered the importance of data frequencies and time frames used for estimation. The recommended framework for CAPM estimates is five years monthly data using value weighted index. Result of this study indicates non-normality of return distribution for almost all data frequencies and time frames. Similar results surfaced in serial correlation, conditional heteroskedasticity and non-normality of error term in time series regression of beta estimation. Results are in line with previous findings of [e.g., Lintner (1965)]. Higher correlation between Equal Weighted Index and Value weighted Index helps to conclude that both indices produce similar estimates of beta [e.g., Bartholdy and Peare (2005)]. Beta’s ability to explain return difference in subsequent period ranges from lowest 3% to highest 8.7% in significant cases using different time periods with varying frequencies. The highest explaining power is found with betas obtained using 0.5 year daily frequency. Study highlights that data frequency and time period impacts the CAPM predictability while estimating the cost of equity and expected returns for individual stocks.

Notes

1. Estimation through CAPM requires construction of a world market index, representative of all tradable assets in the world, which is unobservable and thus makes estimation impossible.
However, the term “estimation based on CAPM” used in this study is consistent with common use.

2. See Reilly and Wright (1988) for a discussion of Merill Lynch’s betas.

3. See for example [Mehrling (2011)].

4. More detailed perspective on use of inefficient portfolio can be found in Diacogiannis and Feldman (2007).

5. Newey West (1987) estimation is also applied; however, no improvement in results is found.


References


