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Horizontal mergers in the presence of vertical relationships

By Arghya Ghosh, Hodaka Morita, AND Chengsi Wang*

Abstract

We study welfare effects of horizontal mergers under a successive oligopoly model and find that downstream mergers can increase welfare if they reduce input prices. The lower input price shifts some input production from cost-inefficient upstream firms to cost-efficient ones. Also, the lower input price makes upstream entry less attractive, reduces the number of upstream entrants, and decreases their average costs in the presence of fixed entry costs. We identify necessary and sufficient conditions for a reduction in input prices and welfare-improving horizontal mergers under a general demand function. Qualitative nature of our findings remains unchanged for upstream mergers.

Keywords: merger, successive oligopoly, welfare, reallocation, rationalization

JEL Classification Numbers: L13, L41, L42

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1 Introduction

Can horizontal mergers improve social welfare? We address this important question through analyzing a model that explicitly incorporates vertical relationships between industries. Vertical relationships are common and important. Final-goods producers often procure intermediate products from upstream firms, and sell their products through downstream retailers. Horizontal mergers yield richer implications under vertical relationships because they affect not only merging firms' market power but also the nature of interactions between vertically related industries.

Standard modeling choices to analyze vertical relationships include bilateral oligopoly and successive oligopoly models. The former types of models view vertical relations through bilateral contracting, whereas the latter types view vertical interactions through market interface with uniform contractual terms. We study horizontal mergers under a successive oligopoly model and demonstrate that a horizontal merger of downstream firms can increase welfare even if merging firms are symmetric and the merger has no synergy or learning effects. We also show that the qualitative nature of the results remain mostly unchanged for a horizontal merger of upstream firms.

Consider a model in which M symmetric downstream firms can produce a homogeneous final product and face a downward-sloping inverse demand. Each downstream firm can transform one unit of an intermediate product into one unit of the final product at a constant unit cost. Let N denote the number of upstream firms that can produce the homogeneous intermediate product with constant marginal costs (which may differ across firms). Upstream firms compete with each other by choosing quantity and downstream firms also engage in quantity competition, in which the input price r is determined at the market-clearing level and is taken as given by all downstream firms.

Using the model outlined above, we study welfare effects of downstream mergers. We first show that the merger reduces the equilibrium input price under a range of parameterizations. We then demonstrate, under two different scenarios, that the lower input price may result in higher welfare.

In the first scenario, we assume that the number of upstream firms N is fixed, and show that the lower input price may increase welfare when upstream firms have asymmetric costs. To understand the logic, suppose that the upstream sector has only two firms, 1 and 2, with constant marginal costs c_1 and c_2 , respectively, satisfying $c_1 < c_2$. We interpret that $(r - c_1)/(r - c_2)$ captures firm 1's competitive

advantage over firm 2 in terms of the price-cost margin. The lower input price increases firm 1's competitive advantage. To see this, suppose that the downstream merger reduces the equilibrium input price from r^* to r^{**} , $r^* > r^{**}$. Then, firm 1's competitive advantage increases from $(r^* - c_1)/(r^* - c_2)$ to $(r^{**} - c_1)/(r^{**} - c_2)$. The higher competitive advantage increases firm 1's equilibrium market share, implying that a larger fraction of the industry output is produced in the cost-efficient firm when the input price is lower.

This effect (referred to as the *production reallocation effect*) works in the direction of increasing welfare under the downstream merger. Even though the merger increases concentration in the downstream sector, it can still increase welfare if the concentration effect is dominated by the production reallocation effect. We find that the downstream merger reduces the equilibrium aggregate output due to the concentration effect. Then, a necessary condition for the merger to increase welfare is that it increases not only firm 1's market share but also its output. We find that this in fact happens under a range of parameterizations.

It is important to notice that the production reallocation effect just mentioned is different from the so-called production reshuffling effect of horizontal mergers. To see the difference, consider a standard Cournot oligopoly model (without a vertical structure) consisting of firms A, B, and C, where firm A is more cost efficient than firm B. Suppose that firms A and B merge. The merged firm would then produce more output in A and less in firm B in equilibrium to minimize its overall production cost. This production reshuffling works in the direction of increasing welfare. Production reshuffling of this kind does not occur in our model because downstream firms are assumed to be symmetric. Downstream mergers change the equilibrium input prices, which in turn change the nature of competition in the upstream firms, leading to the production reallocation effect in our model.

In the second scenario, we rule out the production reallocation effect by assuming that upstream firms are symmetric. Instead, we endogenize the number of upstream firms. Assume that a large number of potential entrants exist for the upstream sector, where each potential entrant can enter by incurring a fixed entry cost. Once the entry process is over, upstream firms engage in quantity competition. The downstream merger again reduces the equilibrium aggregate output in this setup. We find, however, that the merger can increase welfare when it reduces the input price. The lower input price makes upstream entry less attractive, and hence reduces the number of upstream entrants in equilibrium. In the presence of the fixed entry cost, a smaller number of upstream firms implies that their average costs are lower.

This effect, referred to as the *rationalization effect* of the upstream sector, works in the direction of increasing welfare. We show that the positive welfare effect of the downstream merger dominates its negative effect due to concentration under a range of parameterizations.¹

A key element of our results is the effect of horizontal mergers on input price. A necessary condition for downstream mergers to increase welfare is that they reduce the equilibrium input price. Bhattacharyya and Nain (2011) use the data of United States company acquisitions between 1984 and 2003 to study the impact of downstream mergers on upstream suppliers. They find that, in those more concentrated industries or industries with high entry barriers, upstream suppliers indeed experienced large input price declines after consolidation in the downstream sector.

We study a general demand function with standard assumptions for the existence and uniqueness of the Cournot-Nash equilibrium, and identify the necessary and sufficient condition for downstream mergers to reduce the input price. We then show that, although downstream mergers reduce aggregate output regardless of their effects on input prices, they can improve welfare. For each of the two scenarios— asymmetric upstream firms and free entry in the upstream sector—we identify the necessary and sufficient condition for downstream mergers to increase welfare. We also analyze upstream mergers in the successive oligopoly framework. While both the final good price and the input price go up with upstream mergers, welfare can still improve if the difference between the final good’s price and the input price goes down. The squeeze in the downstream price-cost margin can improve welfare through reallocation of output towards more efficient downstream firms (Section 5.1) or through rationalization of downstream sector (Section 5.2). We identify the necessary and sufficient condition for upstream mergers to increase welfare.

2 Relationship to the literature

Welfare effects of horizontal mergers have been investigated previously in the literature (Salant, Switzer, and Reynolds, 1983; Deneckere and Davidson, 1985; Perry and Porter, 1985; Farrell and Shapiro, 1990; McAfee and Williams, 1992).

¹Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) show that, under a Cournot oligopoly model with fixed set-up costs, the level of entry in the free-entry equilibrium is socially excessive. Ghosh and Morita (2007) find that free entry can lead to a socially insufficient number of firms in a successive oligopoly model, but it can still be socially excessive under a range of parameterizations. In the context of socially excessive entry, our findings tell us that the downstream merger can increase welfare by mitigating the negative welfare effect of excessive entry.

Farrell and Shapiro (1990) analyzed a Cournot oligopoly model with quite general cost and demand functions to study the output and welfare effects of horizontal mergers. Production reshuffling and synergy or learning associated with mergers play important roles in their analyses. They found, among other things, that a merger causes the price to rise if a merger generates no synergies or learning. More recent analyses of horizontal mergers emphasize how dynamic interaction between distinct mergers and the private information about synergies can affect post-merger welfare (see, for example, Nocke and Whinston, 2010, 2013). Our contribution to the horizontal merger literature is to evaluate the welfare consequences of horizontal mergers under a model that incorporates vertical relationships between industries.

Several papers have previously studied horizontal mergers in bilateral oligopoly models (see Horn and Wolinsky, 1988; Ziss, 1995; Lommerud, Straume, and Sorgard, 2005; Milliou and Pertrakis, 2007; Symeonidis, 2010). Although modeling details are different, pre-merger setups of these previous models share a common feature; that is, there are one-to-one relationships between upstream and downstream firms (three pairs of upstream and downstream firms in Lommerud et al. and two pairs in others).

When the upstream industry produces an intermediate product, each upstream firm produces an input exclusively for one downstream producer. When the upstream industry produces a final product, each downstream retailer is an exclusive distributor of one upstream firm's product. The price of each upstream firm's product is determined by its bargaining with the paired downstream firm in Horn and Wolinsky (1988), Milliou and Pertrakis (2007), and Symeonidis (2010), whereas the price is set by each upstream firm in Ziss (1995) and Lommerud et al. (2005). Another common feature is that the final products are differentiated across producers in all these models.² Horizontal mergers in the downstream industry affect not only downstream prices but also upstream prices. Regarding welfare consequences of horizontal mergers, Ziss (1995) shows that a merger between downstream retailers can increase output and hence pro-competitive. Also, Symeonidis (2010) shows that a merger between downstream producers may raise consumer surplus and overall welfare when competition is in quantity.

²The importance of countervailing power (Galbraith, 1952) has been analyzed in models that consist of one manufacturer and a number of retailers (von Ungern-Sternberg, 1996; Dobson and Waterson, 1997; Chen, 2003). In von Ungern-Sternberg and Dobson and Waterson, unit retail prices are determined by manufacturer-retailer bargaining and symmetric retailers compete among themselves. Chen took a different approach by considering a model consisting of one manufacturer, one dominant retailer and n fringe retailers. The contract between the manufacturer and a retailer takes the form of two-part tariffs in Chen's model.

Bilateral oligopoly models are applicable to real-world situations in which products are highly differentiated. For example, final good producers often procure a highly specialized input for one or a few upstream suppliers, and bilateral oligopoly models are suitable to analyze such a situation. At the same time, however, downstream producers often purchase relatively homogeneous inputs from a number of upstream suppliers through markets (“market interface”) as pointed out by Inderst (2010), and successive oligopoly models are applicable to analyze a vertical oligopoly with market interface.

Our paper is the first, to the best of our knowledge, to study horizontal mergers under a successive oligopoly model with a general demand function and to show that horizontal mergers can improve welfare. We discover two new mechanisms, production reallocation effect and rationalization effect, through which horizontal mergers affect welfare. We identify necessary and sufficient conditions for horizontal mergers to improve welfare. Our model does not specify the number of upstream firms (N) and the number of downstream firms (M). This enables us to conduct comparative statics exercises with respect to N or M when N and M are fixed in the model, and to endogenize N by allowing free entry in the upstream industry. Previous bilateral oligopoly models mentioned above, in contrast, use specific functional forms and analyze specific cases in which N ($= M$) is small (2 or 3).

The downstream firms produce a homogeneous product in our model, which is an appropriate assumption under successive oligopoly models. This assumption enables clean comparison between our results and previous findings in the horizontal merger literature without vertical relationships, because firms produce a homogeneous product in most of these previous models.³ In contrast, aforementioned bilateral oligopoly models assume that downstream firms produce differentiated products. This is a reasonable assumption because highly specialized inputs supplied under one-to-one relationships typically lead to differentiated final products. Introducing product differentiation in our model would not change the qualitative nature of our results. Production reallocation effect and rationalization effect, the two key channels of welfare improvement, do not depend on whether products are homogenous or differentiated.

³This is true in Salant et al. (1983), Perry and Porter (1985), Farrell and Shapiro (1990), and McAfee and Williams (1992), whereas Deneckere and Davidson (1985) consider a differentiated oligopoly.

3 Successive oligopoly with asymmetric firms

We consider an industry with two sectors of production, upstream and downstream. In the upstream sector, a homogeneous intermediate product is produced by N upstream firms. Each upstream firm, $k(= 1, 2, \dots, N)$, produces at constant marginal cost c_k . Without loss of generality, assume that $c_1 \leq c_2 \leq \dots \leq c_N$ where at least one inequality is strict. In the downstream sector, the intermediate products are transformed into homogeneous final product with constant marginal cost, which is normalized to zero. Production of one unit of the final product requires one unit of the intermediate product. Initially, there are $M > 1$ downstream firms. A downstream merger is simply modeled as a reduction in M . Following Farrell and Shapiro (1990) we focus on welfare effect of mergers, implicitly assuming that mergers, if proposed, are profitable. The downstream firms face a thrice continuously differentiable and strictly decreasing inverse demand function $P(Q)$, where $Q \geq 0$ denotes the aggregate output in the downstream sector. To ensure that a positive but finite quantity is produced in equilibrium, we assume $P_0 \equiv \lim_{Q \rightarrow 0} P(Q) > \max_{k \in N} c_k > \lim_{Q \rightarrow \infty} P(Q) \equiv P_\infty$.

The firms produce and compete in two stages. In Stage 1, the upstream firms compete in quantity (Cournot) in supplying intermediate goods. In Stage 2, the downstream firms also compete in quantity to supply final products. The input price r is determined at the market-clearing level, which equates the demand of downstream firms to the total amount of the intermediate product supplied by the upstream firms. Note that the downstream firms have no oligopsony power over the upstream sector. This assumption is in line with the previous literature on vertical oligopolies—see, for example, Greenhut and Ohta (1979), Salinger (1988), Ghosh and Morita (2007), and Peitz and Reisinger (2013)—where downstream firms take the input price as given when they make a production decision.⁴

We consider the subgame perfect Nash equilibrium (SPNE) in pure strategies of the game. As is well known, the following assumption guarantees the existence and uniqueness of the Cournot-Nash equilibrium in the downstream competition (see, for instance, Vives, 2001).

Assumption 1 $(M + 1)P'(Q) + QP''(Q) < 0$ for all $Q > 0$ and $M \geq 1$.

⁴The downstream firms' price-taking behavior can be rationalized by assuming that the upstream sector supplies to a large number of downstream sectors. Then, even if some downstream sectors have only a few firms with significant market power, the total number of downstream firms is still large, and hence a quantity change of each downstream firm has a negligible effect on input price. See Reisinger and Schnitzer (2012) for an alternative modeling with differentiated products in which both upstream and downstream markets are modeled as Salop-circles.

The game is solved by using backward induction. In Stage 2, each downstream firm $i (= 1, 2, \dots, M)$ chooses its output, $q_i (\geq 0)$, to maximize its profit:

$$(P(q_i + \sum_{j \neq i}^M q_j) - r)q_i,$$

taking other downstream firms' output and input price as given. Under Assumption 1, there exists a unique interior solution to this maximization problem that solves the first-order condition:

$$P(q_i + \sum_{j \neq i}^M q_j) - r + P'(q_i + \sum_{j \neq i}^M q_j)q_i = 0, \quad (1)$$

where $i = 1, 2, \dots, M$. If $r \in (0, P_0)$, equation (1) yields the sole candidate for the sub-game equilibrium in Stage 2, $q_1 = q_2 = \dots = q_M \equiv q$. If $r \in [P_0, \infty)$, each firm i 's equilibrium decision is to choose $q_i = 0$. Assume $r \in (0, P_0)$. Adding together the first-order conditions for $i = 1, 2, \dots, M$ and rearranging yields:

$$r = P(Mq) + \frac{P'(Mq)Mq}{M}. \quad (2)$$

This condition implicitly defines q as a function of r . Then, the total output can be written as $Q(r) = Mq$ for $r \in (0, P_0)$.

We next consider the Stage 1 game in which N upstream firms compete in supplying inputs. Let x_k denote the output of an upstream firm $k (= 1, 2, \dots, N)$ and let $X = \sum_k^N x_k$. The one-to-one transformation between inputs and final products implies $X = Mq$. Thus, equation (2) becomes:

$$r = P(X) + \frac{P'(X)X}{M} \equiv g(X, M).$$

The inverse demand faced by upstream firms is then equal to P_0 if $X = 0$, $g(X, M)$ if $X \in (0, Q_0)$, and 0 if $X \geq Q_0 \equiv \lim_{r \rightarrow 0} Q(r)$. It is easy to verify that $g_X(X, M) = \frac{\partial g(X, M)}{\partial X} < 0$ for any $X > 0$.

Given the upstream inverse demand function, an upstream firm k 's profit is:

$$\left(g(x_k + \sum_{l \neq k}^N x_l, M) - c_k \right) x_k.$$

Each upstream firm k chooses its output, x_k , to maximize its profit, taking other upstream firms' outputs as given. The next assumption, which is the counterpart of Assumption 1 in the upstream sector, ensures the existence and uniqueness of the solution to the upstream firms' profit-maximization problem.

Assumption 2 $(N + 1)g_X(X, M) + Xg_{XX}(X, M) < 0$ for all $X > 0$, $M \geq 1$, and $N \geq 1$.

Solving the first-order conditions:

$$g\left(x_k + \sum_{l \neq k}^N x_l, M\right) - c_k + g_X\left(x_k + \sum_{l \neq k}^N x_l, M\right) x_k = 0,$$

yields:

$$x_k^* = -\frac{g(X^*, M) - c_k}{g_X(X^*, M)X^*}, \quad (3)$$

where X^* satisfies the following condition:

$$Ng(X^*, M) - \sum_{k=1}^N c_k + g_X(X^*, M)X^* = 0. \quad (4)$$

Here is the summary description of the equilibrium outcomes. In equilibrium, X^* , given by (4), is the aggregate amount of the intermediate input produced in the upstream sector. An upstream firm k ($= 1, 2, \dots, N$) produces x_k^* units of the intermediate input, where x_k^* satisfies (3). Given the one-to-one relationship between the intermediate input and the final good, the aggregate amount of the final good produced in equilibrium is $Q^* = X^*$. Each downstream firm produces $q^* = \frac{Q^*}{M} = \frac{X^*}{M}$ units of the final good. The prices of the final good and the intermediate input are given by $P^* \equiv P(X^*)$ and $r^* \equiv g(X^*, M)$ respectively.

Intermediate inputs, and consequently the input price, are usually absent in the standard analyses of horizontal mergers. Almost exclusively, these analyses focus on single-stage oligopolies producing final goods. We depart from the standard practice by explicitly incorporating an imperfectly competitive upstream sector and allowing for endogenous determination of the input price, r^* . A key component of our welfare results is the reallocation of output shares among the upstream firms following a merger-induced change in r^* .

Input Price: To understand how downstream mergers affect r^* , rewrite (4) as:

$$r^* \left(1 - \frac{1}{N e_u^*} \right) = \frac{\sum_{k=1}^N c_k}{N}, \quad (5)$$

where $r^* = g(X^*, M)$ is the equilibrium input price and $e_u^* = -\frac{r^*}{X^* g_X(X^*, M)}$ is the elasticity of the input demand function evaluated at (r^*, X^*) . Equation (5) captures the familiar negative relationship between price and elasticity: the higher the elasticity of input demand, the lower the input price.

The elasticity term e_u involves the first-order derivative of $g(\cdot)$, or, equivalently, the second-order derivative of $P(\cdot)$. Consequently, a change in e_u involves the second-order derivative of $g(\cdot)$, or, equivalently, the third-order derivative, P''' . To cut through the complication arising from higher order derivatives, we define two notions related to the slopes of the demand functions. Let

$$\epsilon_d = \frac{Q P''(Q)}{P'(Q)}$$

denote the elasticity of slope of the inverse demand function, $P(Q)$. Similarly, let

$$\epsilon_u = \frac{X g_{XX}(X, M)}{g_X(X, M)}$$

denote the elasticity of the slope of the inverse demand faced by the upstream firms, $g(X, M)$. Proposition 1 expresses the necessary and sufficient condition for the reduction in r^* in terms of the elasticity of slopes defined above.

Proposition 1 *A downstream merger reduces (increases) the input price if and only if:*

$$\epsilon_u - \epsilon_d > (<) 0 \quad (6)$$

or, equivalently, $\frac{d\epsilon_d}{dQ} > (<) 0$ where ϵ_u and ϵ_d are evaluated at $X = X^$.*

The condition given in Proposition 1 is general, convenient to check, and depends solely on a property of the demand function: whether ϵ_d increases, decreases, or remains unchanged with a change in Q . For an illustrative example, consider the inverse demand function given by:

$$P(Q) = (1 - Q)^b,$$

where $b > 0$. We have that:

$$\epsilon_d = \frac{QP''(Q)}{P'(Q)} = (1 - b) \frac{Q}{1 - Q}.$$

The demand function is linear when $b = 1$. In this case, $\epsilon_d (= 0)$ is constant and hence, according to Proposition 1, the input price does not change with a merger in the downstream sector. However, when $b < (>)1$, ϵ_d is increasing (decreasing) in Q implying that a merger leads to a lower (higher) input price.

While the demand functions used in imperfect competition models allow some flexibility in the elasticity of demand, few are flexible in the curvature properties of demand. Fabinger and Weyl (2012) and Weyl and Fabinger (2013) argue that while they buy convenience, unnecessarily restrictive assumptions on curvature might lead to biased conclusions.⁵ They show that demand curvature plays a central role in determining the rate of cost-pass through in models of imperfect competition. In the context of price discrimination, Aguirre, Cowan, and Vickers (2011) show that the differences in the curvature of demand between the weak and the strong markets is important for understanding how third-degree price discrimination affects welfare. Cowan (2007) illustrates the role of demand curvature in the context of third-degree price discrimination where the demand function in the weak and the strong markets differ by an additive scalar.⁶

Our finding is in a similar spirit in the sense that the curvature properties of upstream and downstream demand are crucial in understanding the effect of a downstream merger on the input price. It is tempting to focus on a class of demand functions with a particular curvature property, say, constant elasticity of slope. This is convenient, easily tractable, and the class includes popular demand functions such as linear, semi-log, and constant elasticity. However, the input price does not vary with downstream mergers for this class of demand functions. The invariance goes away once we allow for more general demand functions, in particular the ones with decreasing and increasing elasticities of slope.⁷

The input price can go down with a downstream merger. It is then important to

⁵For example, if demand functions are assumed to be logconcave, equilibrium price can never increase more than the rise in cost in a standard monopoly setup.

⁶When third-degree price discrimination is applied to two markets, a weak (strong) market is referred to the market in which the discriminatory price is below (above) the non-discriminatory one.

⁷If upstream firms have increasing marginal costs then the input price can go down even for the class of demand functions with constant elasticities of slope. Consider the class of demand functions with constant elasticities of slope where $\epsilon_u = \epsilon_d = \epsilon$ (say). Instead of $c_k x_k$, suppose an upstream firm k 's cost function is give by $c_k x_k + \frac{dx_k^2}{2}$ where $d > 0$. We find that $\frac{dx^*}{dM} =$

investigate whether or not the reduced input price offsets the anti-competitive effect of higher market concentration caused by the downstream merger. We find that downstream mergers increase welfare under a range of parameterizations, whereas they decrease consumer surplus, despite lower input prices, as long as Assumptions 1 and 2 are satisfied.

Consumer Surplus: Consumers buy Q^* units of the final good in equilibrium. Since one unit of the final good requires one unit of the intermediate input, $Q^* = X^*$, and consumer surplus could be expressed as:

$$CS = \int_0^{X^*} P(y)dy - P(X^*)X^*,$$

where X^* is given implicitly by (4). Differentiating (4) and rearranging, we get

$$\frac{dX^*}{dM} = \frac{X^*(N + 1 + \epsilon_d)}{M(M + 1 + \epsilon_d)(N + 1 + \epsilon_u)}. \quad (7)$$

Analyzing the right-hand side of (7) and noting that a reduction in X^* is necessary and sufficient for a reduction in CS in an oligopoly with a homogenous final good, we obtain the following result:

Proposition 2 *For all demand functions satisfying Assumptions 1 and 2, a horizontal merger in the downstream sector always reduces consumer surplus (CS).*

To understand Proposition 2, rewrite (4) in terms of prices and elasticities:

$$P^* = \frac{r^*}{1 - \frac{1}{Me_d}},$$

where $e_d = -\frac{P}{Q P'(Q)}$ is the elasticity of demand for the final good evaluated at $Q = Q^* = X^*$ and $P = P^* = P(X^*)$. The final good's price, P^* , is the input price, r^* , times the mark-up $\frac{1}{1 - 1/Me_d}$. Mergers increase the market power of the downstream firms and raise the mark-up. This puts an upward pressure on P^* . However, for demand functions satisfying $\frac{de_d}{dQ} > 0$, the input price r^* goes down with merger which puts downward pressure on P^* . Assumption 1, which is effectively $\epsilon_d > -2$, puts an implicit upper bound on the degree of convexity of demand functions, which in turn limits the downward pressure on P^* . We find that the downward pressure on P^* caused by a reduction in r^* is always outweighed by the upward pressure on

$$\frac{dX^*}{M((M+1+\epsilon)(N+1+\epsilon) - \frac{Md}{P'(X^*)})} > 0.$$

P^* caused by higher mark-up. Thus, for the class of demand functions satisfying Assumptions 1 and 2, price never decreases or equivalently consumer surplus never increases following a merger.⁸

Welfare: In our successive oligopoly framework with asymmetric upstream firms, welfare is given by:

$$W = \int_0^{X^*} P(y)dy - \sum_{k=1}^M c_k x_k^*, \quad (8)$$

where $\int_0^{X^*} P(y)dy$ is the gross surplus and $\sum_{k=1}^M c_k x_k^*$ is the aggregate production costs. Define $s_k^* \equiv \frac{x_k^*}{X^*}$ as the output share of the k -th upstream firm and express (8) as:

$$W = \int_0^{X^*} P(y)dy - \left(\sum_{k=1}^M c_k s_k^* \right) X^*. \quad (9)$$

where $\sum_{k=1}^M c_k s_k^*$ is the average cost in the upstream industry of producing X^* . Differentiating (9) with respect to M we have that:

$$\frac{dW}{dM} = \left(P^* - \sum_{k=1}^M c_k s_k^* \right) \frac{dX^*}{dM} - X^* \frac{d}{dM} \left(\sum_k c_k s_k^* \right).$$

While $\frac{dX^*}{dM} > 0$, $\frac{dW}{dM} < 0$ can still hold if $\frac{d}{dM} \left(\sum_k c_k s_k^* \right) > 0$. Even if aggregate output decreases, welfare can still increase with a merger if production efficiency improves, i.e., average production cost, $\sum_k c_k s_k^*$, decreases.

Let $s_i = \frac{x_i^*}{X^*}$ and $s_j = \frac{x_j^*}{X^*}$ denote the share of total intermediate input produced by firms i and j respectively, where $c_i < c_j$. Using (3), we can express:

$$\frac{s_i^*}{s_j^*} = \frac{r^* - c_i}{r^* - c_j} = 1 + \frac{c_j - c_i}{r^* - c_j}.$$

We interpret that $(r^* - c_i)/(r^* - c_j)$ captures the cost-efficient firm i 's competitive advantage over firm j in terms of price-cost margin. Then, firm i 's competitive

⁸Consumer surplus can improve with a merger if we relax Assumptions 1 and 2, which are typically used as sufficient conditions for existence and uniqueness of a Cournot equilibrium. Assumptions 1 and 2 respectively imply $\epsilon_d > -2$ and $\epsilon_u > -2$. We found the possibility of consumer surplus improvement with the following parameterization: $N = 1$, $M > 3$, $P(Q) = a + Q^{-b}$ with $b > 1$. Here, both Assumptions 1 and 2 are violated since $\epsilon_u = \epsilon_d = -(b+1) < -2$.

advantage increases as r^* decreases. This implies that the market shares in the upstream sector shift towards more efficient upstream firms if and only if r^* decreases. We call this effect as the production reallocation effect of an input-price reduction. The relationship between a reduction in r^* and increased dispersion of upstream market shares is best reflected in the Herfindahl Index (H) of the upstream sector given by:

$$H = \sum_{k=1}^N (s_k^*)^2 = \sum_{k=1}^N \left(\frac{x_k^*}{X^*}\right)^2 = \frac{\sum_{k=1}^N (r^* - c_k)^2}{N^2(r^* - \mu)^2} = \frac{1}{N} + \frac{\sigma^2}{N(r^* - \mu)^2},$$

where $\mu = \frac{\sum_{k=1}^N c_k}{N}$ and $\sigma^2 = \frac{\sum_{k=1}^N (c_k - \mu)^2}{N}$ denote the mean and the variance respectively of the unit costs c_1, c_2, \dots, c_k . Observe that H increases as r^* decreases. By increasing the market shares of the relatively more efficient firms, a decrease in r^* lowers average production costs $\sum_{k=1}^M c_k s_k^*$.

The above discussion suggests that a reduction in r^* , or, equivalently:

$$\epsilon_u - \epsilon_d > 0,$$

is a necessary condition for welfare improvement. The necessary condition can be made tighter by considering a slightly different decomposition of $\frac{dW}{dM}$:

$$\frac{dW}{dM} = \sum_{k=1}^N (P^* - c_k) \frac{dx_k^*}{dM}. \quad (10)$$

Even though $\sum_{k=1}^N \frac{dx_k^*}{dM} = \frac{dX^*}{dM} > 0$ (by Proposition 2), $\frac{dW}{dM} < 0$ implies that $\frac{dx_k^*}{dM} < 0$ must hold for some k . At least one upstream firm's output must increase for welfare to improve with downstream mergers. Recall that, N upstream firms are labeled such that $c_k < c_{k+1}$ where $k = \{1, 2, \dots, N - 1\}$. We have that:

$$\frac{dW}{dM} < 0 \Rightarrow \frac{dr^*}{dM} > 0 \Rightarrow \frac{d\frac{s_k^*}{s_{k+1}^*}}{dM} < 0,$$

which implies that if x_k^* increases for a set of upstream firms, that set must include the most efficient firm ($k = 1$). We have that:

$$\frac{dx_k^*}{dM} = \frac{X^*((Ns_k^* - 1)(\epsilon_u - \epsilon_d) - s_k^*(N + 1 + \epsilon_d))}{M(M + 1 + \epsilon_d)(N + 1 + \epsilon_u)}, \quad (11)$$

which is strictly negative for $k = 1$ if and only if the following holds:

$$\epsilon_u - \epsilon_d > 1 + \frac{\left(\frac{1}{s_1^*} + 1 + \epsilon_d\right)}{N - \frac{1}{s_1^*}}. \quad (12)$$

Since firm 1 is the most efficient one among N firms and there are at least two active firms, $\frac{1}{N} < s_1^* < 1$, which in turn implies that (a) $N - \frac{1}{s_1^*} > 0$ and (b) $\frac{1}{s_1^*} + 1 + \epsilon_d > 2 + \epsilon_d > 0$. Thus, the right-hand side of (12) is greater than unity. Starting from parameterizations that satisfy $\epsilon_u - \epsilon_d > 0$, we have narrowed down our search for welfare-improving mergers to a subset of those parameterizations, namely, the ones that satisfy:

$$\epsilon_u - \epsilon_d > 1. \quad (13)$$

While stronger than $\epsilon_u - \epsilon_d > 0$, (13) is still not sufficient for $\frac{dW}{dM} < 0$. Substituting the expressions of $\frac{dx_k^*}{dM}$ from (11) in (10) and analyzing the resultant expression gives us the necessary and sufficient condition for welfare-improving mergers in terms of market structure, the demand curvatures, and concentration in the upstream sector captured by the Herfindahl Index.

Proposition 3 *When the upstream firms have asymmetric unit costs, a downstream merger improves welfare if and only if the following condition holds:*

$$\epsilon_u - \epsilon_d > 1 + \frac{\frac{1}{H}\left(1 + \frac{N+1+\epsilon_d}{M+1+\epsilon_d}\right) + 1 + \epsilon_d}{N - \frac{1}{H}}, \quad (14)$$

where $H = \sum_{k=1}^N (s_k^*)^2$ is the Herfindahl Index corresponding to the upstream sector.

Observe that the right-hand side of (14) is strictly decreasing in H . Loosely speaking, this implies that the higher concentration in the upstream sector makes it more likely that a downstream merger increases welfare. The statement is loose in the sense that ϵ_u , ϵ_d , and H often changes simultaneously (due to change in parameter values) which makes it difficult to isolate the impact of a change in H .

In the case of mean-preserving spread of unit costs, however, only H increases while ϵ_u and ϵ_d remain unchanged. This is immediate from rearranging (4), which gives:

$$X^* = -\frac{N(g(X^*, M) - \mu)}{g_X(X^*, M)},$$

and Herfindahl Index:

$$H = \frac{1}{N} + \frac{\sigma^2}{N(r^* - \mu)^2}.$$

Keeping μ the same, suppose we increase σ . As long as μ remains the same, X^* does not change. Since ϵ_u and ϵ_d depend on X^* only, they do not change either. As σ increases, H increases as well.

Thus, conditional on $\epsilon_u - \epsilon_d > 1$ being satisfied, a mean-preserving spread of unit costs raises H , which increases the right-hand side of (14) and makes welfare improvement more likely. A reduction in the input price increases cost-efficient firms' competitive advantages over cost-inefficient firms, and increases market shares of cost-efficient firms. Under a mean-preserving spread of unit costs, an input-price reduction more drastically increases cost-efficient firms' competitive advantages and their market shares, making it more likely for downstream mergers to improve welfare by reducing input price.

We conclude this section with a concrete example of welfare improving mergers.

Example 1 *The inverse demand function is $P(Q) = (1 - Q)^b$ with $b > 0$ (Malueg, 1992). This demand function is convex for $b > 1$, linear for $b = 1$, and concave for $0 < b < 1$. Let $N = 6$, $b = 0.05$, $c_1 = 0.1$, and $c_k = 0.8$ for $k \neq 1$. The table below presents the equilibrium values of individual output, x_1 and x_k for $k \neq 1$, total output, X^* , input price, r^* , Herfindahl index, H^* , and welfare, W^* .*

M	ϵ_d	ϵ_u	x_1^*	x_k^*	X^*	r^*	H^*	W^*
4	6.380	10.71	0.734	0.0273	0.870	0.827	0.716	0.658
3	5.481	9.394	0.742	0.022	0.852	0.821	0.762	0.662
2	4.357	7.666	0.755	0.013	0.821	0.812	0.848	0.668

Clearly, following downstream mergers, the production shifts to firm 1, the input price decreases, and welfare increases.

4 Free entry in the upstream sector

We have shown that downstream mergers can improve welfare when upstream firms have asymmetric costs. Key to the possibility of welfare improvement is *reallocation* in the upstream sector: downstream mergers can lower the input price, which in turn reallocates upstream production towards more efficient upstream firms and improves welfare. In this section, we focus on an alternative channel of welfare improvement, namely *rationalization* of the upstream sector. A downstream merger can lower the input price, which in turn leads to fewer but bigger upstream firms. In the presence

of fixed costs, bigger firms imply lower average costs, which in turn creates the possibility of welfare improvement. Although the two channels, i.e., rationalization and reallocation, work differently, the necessary condition for welfare improvement is the same for both: a reduction in the input price.

As before, assume that there are $M(\geq 1)$ downstream firms producing a homogeneous final good facing inverse demand $P(Q)$. The demand specification as well as the downstream production remain the same as in Section 3. Modeling changes are only in the upstream sector. Assume that a large number of identical upstream firms exist, each of which must decide whether to enter the upstream sector by incurring a setup cost of $K > 0$. Each upstream firm has a constant marginal production cost, $c > 0$. Notice that in order to focus on the rationalization effect of downstream mergers, we rule out the reallocation effect by assuming that all upstream entrants have the same constant marginal cost. There is free entry in the upstream sector and entry is assumed to take place after a downstream merger (if any) so the number of active upstream firms can differ depending on whether the merger takes place at a prior stage.⁹

Let \hat{N} and \hat{X} respectively denote the number of upstream firms and aggregate output in the free-entry equilibrium. Upon entry, each upstream firm produces $\frac{\hat{X}}{\hat{N}} \equiv \hat{x}$. Free entry in the upstream sector implies that the post-entry profit of each upstream firm exactly offsets the fixed cost of entry, where we ignore the integer constraint, as is standard in the literature. Thus:

$$\frac{(g(\hat{X}, M) - c)\hat{X}}{\hat{N}} = K. \quad (15)$$

Summing up the first-order conditions of the upstream firms' profit-maximization problem, we get:

$$\hat{N}g(\hat{X}, M) - \hat{N}c + \hat{X}g_X(\hat{X}, M) = 0. \quad (16)$$

⁹Alternatively, we could assume that each upstream firm's cost function $C(x_i) = cx_i + K$ if $x_i > 0$ and zero otherwise, in which case even if the entry cost is zero, the number of active upstream firms would differ depending on the merger decision. We assume K to be suitably low such that at least one upstream firm enters and produces a strictly positive amount of output. Furthermore, following the standard practice in this literature, we treat the number of upstream firms, N , as a continuous variable.

Differentiating equations (15) and (16) and rearranging, we find that:

$$\frac{d\hat{N}}{dM} = \left(\frac{\hat{N}}{M}\right) \left(\frac{1}{2\hat{N} + \epsilon_u}\right) \left[\frac{\hat{N} + 1 + \epsilon_d + \hat{N}(\epsilon_u - \epsilon_d)}{M + 1 + \epsilon_d}\right], \quad (17)$$

$$\frac{d\hat{X}}{dM} = \frac{\partial\hat{X}}{\partial M} + \frac{\partial\hat{X}}{\partial\hat{N}} \frac{d\hat{N}}{dM} = \frac{\hat{X}(2\hat{N} + 1 + \epsilon_d)}{M(2N + \epsilon_u)(M + 1 + \epsilon_d)}. \quad (18)$$

Analyzing (17) and (18) gives the following result:

Proposition 4 *For all demand functions satisfying Assumptions 1 and 2, a downstream merger reduces aggregate output (\hat{Q}) and consumer surplus (CS).*

From Section 3, we know that, when the number of upstream firms is fixed, downstream merger reduces output. Proposition 4 tells us that the qualitative nature of the result remains unchanged under free entry of upstream firms.

While consumer surplus decreases, welfare might improve with a downstream merger if production efficiency improves, or, equivalently, if the average cost goes down. In the presence of free entry in the upstream sector, we can write welfare (W) as the gross benefit less the sum of production costs and entry costs:

$$\begin{aligned} W &= \int_0^{\hat{X}} P(y)dy - c\hat{X} - \hat{N}K \\ &= \int_0^{\hat{X}} P(y)dy - \hat{X} \left(c + \frac{K}{\hat{x}} \right) \\ &= \int_0^{\hat{X}} P(y)dy - \hat{r}\hat{X}. \end{aligned} \quad (19)$$

The second equality restates welfare by expressing costs as output, \hat{X} , times average cost, $\frac{c\hat{X} + \hat{N}K}{\hat{X}} (= c + \frac{K}{\hat{x}})$. The third equality follows from rearranging the zero-profit condition in the upstream sector, $(\hat{r} - c)\hat{x} - K = 0$, as $\hat{r} = c\frac{K}{\hat{x}}$, i.e., average cost must equal the input price in the free-entry equilibrium. It is then immediate that, with a downstream merger, the average cost goes down if and only the input price goes down.

Differentiating (19) with respect to M gives:

$$\frac{dW}{dM} = (\hat{P} - \hat{r}) \left(\frac{d\hat{X}}{dM} \right) - \hat{X} \frac{d\hat{r}}{dM}.$$

Since $\frac{d\hat{X}}{dM} < 0$, $\frac{dW}{dM} < 0$ can hold only if $\frac{d\hat{r}}{dM} > 0$. Thus, as in Section 3, a neces-

sary condition for welfare improvement is that the input price goes down with a downstream merger. In the Appendix we show that:

$$\frac{d\hat{r}}{dM} = \frac{-P'(\hat{X})\hat{X}(\epsilon_u - \epsilon_d - 1)}{M^2(2\hat{N} + \epsilon_u)}.$$

Since $P'(\hat{X}) < 0$ and $2\hat{N} + \epsilon_u > 0$ (by Assumption 2), we have the following result.

Proposition 5 *In the presence of free entry in the upstream sector, a downstream merger reduces (increases) the input price if and only if:*

$$\epsilon_u - \epsilon_d > (<)1, \tag{20}$$

where ϵ_u and ϵ_d are evaluated at $(X, N) = (\hat{X}, \hat{N})$.

Observe that (20) is stronger than (6)—the condition for input price reduction in Section 3. In other words, a reduction in input price is less likely under free entry. The curvature-related arguments outlined in Section 3 apply here as well. Under free entry, there is an additional effect on the input price arising from the change in upstream market structure. A downstream merger typically leads to fewer upstream firms, which in turn puts an upward pressure on the input price. As a result, a reduction in input price becomes less likely under free entry. While a reduction in \hat{r} is necessary for welfare improvement, it is not sufficient. Proposition 6 below states the necessary and sufficient condition for welfare improvement.

Proposition 6 *In the presence of free entry in the downstream sector, downstream mergers improve welfare if and only if:*

$$\epsilon_u - \epsilon_d > 1 + \frac{2N + 1 + \epsilon_d}{M + 2 + \epsilon_d}, \tag{21}$$

where both ϵ_u and ϵ_d are evaluated at $(X, N) = (\hat{X}, \hat{N})$.

The right-hand side of (21) is strictly greater than unity since $2N + 1 + \epsilon_d > 0$ and $M + 2 + \epsilon_d > 0$. Recall that $\epsilon_u - \epsilon_d - 1 > 0$ is required for a reduction in r^* , which in turn prompts a reduction in average costs. For welfare gains from lower average costs to outweigh the welfare loss from lower aggregate output, a more stringent condition is needed, namely, $\epsilon_u - \epsilon_d - 1$ needs to be greater than a strictly positive threshold. Below, we present a concrete example where (21) is satisfied.

Example 2 Consider again the inverse demand function $P(Q) = (1 - Q)^b$ and the following parameterization: $M = 5$, $c = 0.01$, $k = 0.3$, and $b = 0.1$. The table below suggests that a horizontal merger among downstream firms can lead to lower total output, lower input price, and fewer upstream firms, but higher welfare.

M	\hat{Q}	ϵ_u	ϵ_d	\hat{r}	\hat{N}	\hat{W}
5	0.874	13.22	6.260	0.700	2.010	0.205
4	0.861	11.76	5.569	0.6942	1.963	0.208
3	0.841	10.02	4.748	0.686	1.894	0.212
2	0.805	7.863	3.724	0.673	1.781	0.217

In our successive oligopoly framework with upstream free entry, a downstream merger improves welfare only if the input price \hat{r} , or, equivalently, the average cost, $c + \frac{K}{(\hat{X}/\hat{N})}$, goes down. A downstream merger decreases aggregate output \hat{X} . Then, average cost decreases if and only if the equilibrium number of upstream firms \hat{N} decreases as well. How can a reduction in \hat{N} improve welfare? It is possible if the number of upstream firms is socially excessive. Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) show that the free-entry number of firms in a homogenous products Cournot oligopoly is socially excessive.¹⁰ In a vertical oligopoly framework such as ours, the free-entry number of upstream firms can be socially insufficient or excessive (see, for example, Ghosh and Morita, 2007). Excessive entry is necessary for welfare improvement of downstream mergers in our framework. We find that downstream mergers can help mitigate excessive entry in the upstream sector when they reduce the input price (r) and lower the upstream average cost $c + \frac{K}{(\hat{X}/\hat{N})}$. For a range of parameterizations, we find that the welfare gain from reduction in the upstream average cost outweighs the welfare loss from the standard anticompetitive effect of a merger.

5 Upstream mergers

Thus far, we have focused on welfare implications of downstream mergers. In this section, we study welfare implications of upstream mergers. Overall, our findings in this section are parallel to our finding in the previous two sections. In the presence of cost asymmetry in the downstream sector, an upstream merger can improve welfare

¹⁰Mankiw and Whinston (1986) have shown that, if an entrant causes incumbent firms to reduce output, entry is more desirable to the entrant than it is to society. There is therefore a tendency toward excessive entry in homogeneous product markets.

by reallocating output towards more efficient downstream firms. Under free entry in the downstream sector, an upstream merger can improve welfare by rationalizing the downstream sector. As in Sections 3 and 4, *reallocation* and *rationalization* respectively are key mechanisms underpinning welfare improvement. We find that upstream mergers increase the input price r as well as the final good's price P . However, $P - r$ can increase or decrease. A reduction in $P - r$ is necessary for an upstream merger to improve welfare in both cases. Lower $P - r$, i.e., a smaller price-cost margin in the downstream sector, increases the output shares of relatively efficient downstream firms, which improves production efficiency. When there is free entry, a squeeze in the price-cost margin leads to fewer but bigger downstream firms generating welfare gains from scale economies. We keep the discussion brief, as several steps in the analyses are quite similar to those in Sections 3 and 4.

5.1 Asymmetric downstream firms

Consider the model in Section 3 with the following variation. All upstream firms have the same constant marginal cost $c > 0$. Each downstream firm requires one unit of the intermediate input (r denotes the input price) to produce one unit of the final good. In addition, downstream firm i incurs the per-unit cost a_i to transform one unit of the intermediate input to one unit of the final good.¹¹ Assume $a_i \leq a_{i+1}$, where the strict inequality holds for some $i \leq M - 1$. Downstream firm i 's constant marginal cost of production is then $r + a_i$. Proceeding as in Section 3, it can be shown that the input demand function faced by each upstream firm is:

$$r = P(X) - \mu_a + \frac{XP'(X)}{M} \equiv g(X, M),$$

where $\mu_a = \frac{\sum_{k=1}^M a_k}{M}$. In the overall equilibrium of the successive oligopoly game, each upstream firm produces $x^* = -\frac{g(X^*, M) - c}{g_X(X^*, M)}$, where X^* satisfies:

$$Ng(X^*, M) - Nc + X^*g_X(X^*, M) = 0.$$

It is straightforward to show that:

$$\frac{dX^*}{dN} = \frac{X^*}{N(N + 1 + \epsilon_u)} > 0. \quad (22)$$

¹¹In Section 3, the cost of transformation was normalized to zero; i.e., $a_i = 0$ for all i .

Since X^* decreases with a merger, consumer surplus always decreases with an upstream merger. However, as in Section 3, welfare can still increase if production efficiency improves.

Welfare is given by the gross surplus less aggregate production costs:

$$W = \int_0^{X^*} P(y)dy - cX^* - \left(\sum_{k=1}^M a_k s_k^* \right) X^*,$$

where $s_k^* = \frac{q_k^*}{Q^*}$ is the share of the final good produced by a downstream firm k and $\sum_{k=1}^M s_k^* = 1$. Differentiating W with respect to N we get:

$$\frac{dW}{dN} = (P^* - c) \frac{dX^*}{dN} - X^* \frac{d}{dN} \left(\sum_{k=1}^M a_k s_k^* \right),$$

where $P^* \equiv P(X^*)$ denotes the equilibrium price of the final good.

Let s_i^* and s_j^* denote the share of the final good produced by firms i and j respectively, where $a_i < a_j$. The ratio of market shares is given by:

$$\frac{s_i^*}{s_j^*} = \frac{P^* - r^* - a_i}{P^* - r^* - a_j} = 1 + \frac{a_j - a_i}{P^* - r^* - a_j},$$

where $r^* \equiv g(X^*, M)$ denotes the equilibrium price of the input. The cost-efficient firm i 's competitive advantage over firm j in terms of the price-cost margin, $(P^* - r^* - a_i)/(P^* - r^* - a_j)$, increases as $P^* - r^*$ decreases. This implies that the market shares in the downstream sector shift towards more efficient firms if and only if $P^* - r^*$ decreases. We have that:

$$\begin{aligned} \frac{dP^*}{dN} &= P'(X^*) \frac{dX^*}{dN} = \frac{X^* P'(X^*)}{N(N+1+\epsilon_u)} < 0, \\ \frac{dr^*}{dN} &= g_X(X^*, M) \frac{dX^*}{dN} = \frac{X^* P'(X^*) (1 + \frac{1+\epsilon_d}{M})}{N(N+1+\epsilon_u)} < 0 \end{aligned}$$

and thus:

$$\frac{d(P^* - r^*)}{dN} > 0 \Leftrightarrow \frac{X^* P'(X^*) (1 + \epsilon_d)}{MN(N+1+\epsilon_u)} < 0 \Leftrightarrow \epsilon_d > -1.$$

That is, an upstream merger decreases $P^* - r^*$ if and only if $\epsilon_d > -1$.

An upstream merger increases the input price r^* and P^* . If $\epsilon_d > -1$, i.e., the inverse demand function is strictly logconcave, P^* does not increase as much as r^*

and hence $P^* - r^*$ decreases. Since $a_j - a_i > 0$, a decrease in $P^* - r^*$ leads to higher $\frac{s_i^*}{s_j^*}$. As the market shares of relatively more efficient firms increases, $\sum_k^M a_k s_k^*$ declines and production efficiency improves. Thus, upstream mergers improve production efficiency in the downstream sector if and only if the inverse demand function is strictly logconcave. However, strict logconcavity of the inverse demand function is not sufficient to guarantee welfare improvement.

Proposition 7 *When the downstream firms have asymmetric marginal costs of production, an upstream merger improves welfare if and only if the following condition holds:*

$$\epsilon_d \left(H_d - \frac{1}{M} \right) > \frac{M + N + 1 + \epsilon_d}{MN}, \quad (23)$$

where $H_d = \sum_{k=1}^N (s_k)^2$ is the Herfindahl index for the downstream sector.

Since $M + N + 1 + \epsilon_d > 0$ (by Assumptions 1 and 2) and $H_d - \frac{1}{M} > 0$, (23) holds only if the demand function is strictly concave (i.e., $\epsilon_d > 0$). Conditional on the strict concavity condition being satisfied, a higher degree of asymmetry makes welfare improvement more likely. To see why, express Herfindahl Index of the downstream sector as:

$$H_d = \frac{1}{N} + \frac{\sigma_a^2}{N(P^* - r^* - \mu_a)^2},$$

where $\mu_a = \frac{\sum_{k=1}^M a_k}{M}$ and $\sigma^2 = \frac{\sum_{k=1}^M (a_k - \mu_a)^2}{N}$ denote the mean and the variance of the unit costs a_1, a_2, \dots, a_k . Keeping μ_a the same, suppose we increase the degree of downstream cost asymmetry by increasing σ_a . As long as μ_a remains the same, the input demand function $g(X, M) = P(X) - \mu_a + \frac{XP'(X)}{M}$ does not change. Consequently, X^* , and ϵ_d , which only depends on X^* , remain unchanged. As σ_a increases, only H_d increases which makes (23) more likely to hold.^{12, 13}

¹²Notice that (i) strict concavity of the inverse demand function (i.e., $\epsilon_d > 0$) is necessary for welfare improvement, and (ii) welfare improvement can occur for demand functions even with a constant elasticity of slope. Neither (i) nor (ii) were true for downstream mergers.

¹³It is easy to construct examples of welfare improving upstream mergers. Rearrange (23) as follows $\epsilon_d \left(H_d - \frac{1}{M} - \frac{1}{MN} \right) > \frac{1}{M} + \frac{1}{N} + \frac{1}{MN}$. Consider a demand function of the form $P = 1 - Q^b$ where the constant elasticity of slope is $\epsilon_d = \epsilon_u = b - 1$. Fix $M > 2$ and $N > 2$. It is always possible to choose b and degree of cost asymmetry high enough such that (23) holds.

5.2 Free entry in the downstream sector

Consider once again the setup described above. Impose symmetry in the downstream sector, i.e., $a_i = a$ for all $i = 1, 2, \dots, M$, which implies that the input demand function faced by each upstream firm is:

$$r = P(X) - a + \frac{XP'(X)}{M} \equiv g(X, M).$$

The number of downstream firms, M , is determined endogenously as there is free entry in the downstream sector. Let \hat{M} and \hat{Q} respectively denote the number of downstream firms and aggregate output in equilibrium. Upon entry, each downstream firm produces $\hat{q} = \frac{\hat{Q}}{\hat{M}}$. Free entry in the downstream sector implies that the post-entry profit of each downstream firm exactly offsets the fixed cost of entry. Ignoring the integer constraint, we can write the free entry condition as:

$$\frac{(\hat{P} - a - \hat{r})\hat{Q}}{\hat{M}} = K,$$

where \hat{P} and \hat{r} denote the prices of the final good and the intermediate input respectively. Using $\hat{Q} = \hat{X}$ and $\hat{r} = g(\hat{X}, \hat{M}) = \hat{P} - a + \frac{\hat{X}P'(\hat{X})}{\hat{M}}$ in the above equation, we can rewrite the free-entry condition as:

$$-\frac{\hat{P}'(\hat{X})\hat{X}^2}{\hat{M}^2} = K. \quad (24)$$

Equation (24) together with the sum of the first-order conditions (in the upstream sector), i.e.,

$$Ng(\hat{X}, \hat{M}) + \hat{X}g_X(\hat{X}, \hat{M}) = Nc, \quad (25)$$

determines \hat{X} and \hat{M} .

Hereafter, we focus on parameterizations for which upstream mergers lead to rationalization of the downstream sector (i.e., a reduction in \hat{M}). These parameterizations include linear demand, constant elasticity demand and in fact all demand functions with weakly increasing elasticity of slope ($\frac{d\epsilon_d}{dQ} \geq 0$). For all such parameterizations, aggregate outputs and consumer surplus decrease with an upstream merger. However, welfare can still increase with an upstream merger.

Welfare in this case is defined as the gross benefit less production costs and entry costs:

$$W = \int_0^{\hat{X}} P(y)dy - (a + c)\hat{X} - \hat{M}K.$$

Since downstream profits are zero, i.e., $\hat{P}\hat{X} - (a + \hat{r})\hat{X} - \hat{M}K = 0$, welfare could be expressed as:

$$W = \underbrace{\int_0^{\hat{X}} P(y)dy}_{\text{consumer surplus}} - \hat{P}\hat{X} + \underbrace{(\hat{r} - c)\hat{X}}_{\text{upstream profits}},$$

which upon differentiation gives:

$$\frac{dW}{dN} = (\hat{r} - c)\frac{d\hat{X}}{dN} - \hat{X}\frac{d(\hat{P} - \hat{r})}{dN}.$$

Even if $\frac{d\hat{X}}{dN} > 0$, $\frac{dW}{dN} < 0$ can hold if $\frac{d(\hat{P} - \hat{r})}{dN} > 0$. In the Appendix, we show that:

$$\frac{d(\hat{P} - \hat{r})}{dN} = -\frac{\epsilon_d P'(\hat{X})}{2M} \frac{d\hat{X}}{dN}. \quad (26)$$

Using (26), we can express $\frac{dW}{dN}$ as:

$$\frac{dW}{dN} = \frac{\hat{X}P'(\hat{X})(N-2)}{2\hat{M}N} \left(\epsilon_d - \frac{2(\hat{M}+1)}{N-2} \right) \frac{d\hat{X}}{dN}. \quad (27)$$

The welfare result stated below follows from analyzing (27).

Proposition 8 *In the presence of free entry in the downstream sector, an upstream merger improves welfare if and only if:*

$$\epsilon_d > \frac{2(\hat{M}+1)}{N-2}, \quad (28)$$

where ϵ_d is evaluated at $(X, M) = (\hat{X}, \hat{M})$.

To better understand the welfare result, rearrange the zero-profit condition in the downstream sector as:

$$\hat{P} - \hat{r} = a + \frac{K}{\hat{q}}.$$

If $\frac{d\hat{X}}{dN} > 0$ and $\epsilon_d > 0$, $\hat{P} - \hat{r}$ decreases with an upstream merger (see equation (26)) and \hat{q} increases. Thus, if demand function is strictly concave, downstream firms that are active after the merger enjoy greater economies of scale. Welfare gains from greater economies of scale offset welfare loss from lower aggregate outputs when demand function is sufficiently concave, in particular when $\epsilon_d > \frac{2(\hat{M}+1)}{N-2}$.¹⁴

¹⁴It is easy to construct examples where (28) holds. Consider the inverse demand function

6 Summary and conclusion

Final-goods producers often procure intermediate products from upstream firms and sell their products through downstream retailers. It is therefore important to study welfare effects of horizontal mergers under models that incorporate vertical relationships between industries. It is well known that horizontal mergers under Cournot oligopoly models (without vertical relationships) can improve welfare in the presence of production reshuffling and synergy or learning associated with mergers. In our study of downstream mergers, we rule out these effects by assuming that downstream firms are symmetric, so that we can focus on the new effects that arise from vertical relationships.

Analyzing mergers of symmetric downstream firms in a successive oligopoly model under a general demand function we have found that mergers can increase welfare if they decrease equilibrium input prices. We have identified the necessary and sufficient condition for reduction in input prices. We have explored two channels through which a reduction in input prices (induced by the merger) can lead to higher welfare. First, in the presence of cost asymmetry in the upstream sector, a lower input price increases the competitive advantage of cost-efficient upstream firms, thereby reallocating some input production from cost-inefficient firms to cost-efficient ones. Second, in the presence of fixed entry costs in the upstream sector, a lower input price makes upstream entry less attractive, thereby rationalizing the upstream sector. Both of these two effects lower average costs in the upstream sector and work in the direction of increasing welfare. For each scenario, we have identified the necessary and sufficient condition for downstream mergers to improve welfare. Also, we have shown that the qualitative nature of our results remain mostly unchanged for upstream mergers.

Although we have ruled out production reshuffling and synergy or learning effects associated with horizontal mergers, they play important roles in the assessment of their welfare effects. Along with these effects, the two new effects that we have identified, production reallocation and rationalization effects, will together help us more accurately assess welfare effects of horizontal mergers in which vertical relationships are important and can be approximated by successive oligopoly models.

$P = 1 - Q^b$ where $\epsilon_d = b - 1$. Fix $N > 3$. It is always possible to choose b and K suitably large such that (28) holds.

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Appendix

Proof of Proposition 1

Totally differentiating equation (4) we get

$$[(N + 1)g_X(X^*, M) + X^*g_{XX}(X^*, M)] dX^* + (Ng_M(X^*, M) + X^*g_{XM}(X^*, M)) dM = 0,$$

where

$$\begin{aligned} g(X^*, M) &= P(X^*) + \frac{X^*P'(X^*)}{M} \\ g_M(X^*, M) &= -\frac{X^*P'(X^*)}{M^2}, \\ g_X(X^*, M) &= \frac{P'(X^*)(M + 1 + \epsilon_d)}{M}, \\ g_{XM}(X^*, M) &= -\frac{P'(X^*)(1 + \epsilon_d)}{M^2} \end{aligned}$$

and $\epsilon_d = \frac{X^*P''(X^*)}{P'(X^*)}$. Substituting these derivatives above and rearranging yields

$$\frac{dX^*}{dM} = \left(\frac{X^*}{M} \right) \left[\frac{N + 1 + \epsilon_d}{(M + 1 + \epsilon_d)(N + 1 + \epsilon_u)} \right].$$

where $\epsilon_u = \frac{X^*g_{XX}(X^*, M)}{g_X(X^*, M)}$.

Since $r^* \equiv g(X^*, M)$ we have that

$$\frac{dr^*}{dM} = g_X(X^*, M) \frac{dX^*}{dM} + g_M(X^*, M).$$

Substituting the expressions for $g_X(X^*, M)$, $g_M(X^*, M)$ and $\frac{dX^*}{dM}$ in the right-hand side of the above equation and simplifying we get:

$$\frac{dr^*}{dM} = \left(\frac{-X^*P'(X^*)}{M^2} \right) \left(\frac{\epsilon_u - \epsilon_d}{N + 1 + \epsilon_u} \right).$$

Since $P'(X^*) < 0$ and $N + 1 + \epsilon_u > 0$ (Assumption 2) it follows that

$$\frac{dr^*}{dM} > (=, <)0 \Leftrightarrow \epsilon_u - \epsilon_d > (=, <)0.$$

The rest of the proof is devoted to establishing that

$$\text{sign}(\epsilon_u - \epsilon_d) = \text{sign} \frac{d\epsilon_d}{dQ}$$

where all expressions are evaluated at $Q(\equiv X) = X^*$. Substituting $g_X(X^*, M) = \frac{(M+1)P'(X^*) + X^*P''(X^*)}{M}$ and $g_{XX}(X^*, M) = \frac{(M+2)P'(X^*) + X^*P'''(X^*)}{M}$ in the expression for $\epsilon_u = \frac{X^*g_{XX}(X^*, M)}{g_X(X^*, M)}$ and simplifying we get

$$\epsilon_u = \frac{X^*P''(X^*)(M+2+\alpha)}{P'(X^*)(M+1+\epsilon_d)} = \epsilon_d + \frac{\epsilon_d(1+\alpha-\epsilon_d)}{M+1+\epsilon_d},$$

where $\alpha = \frac{X^*P'''(X^*)}{P''(X^*)}$. Since $M+1+\epsilon_d > 0$ it follows that

$$\text{sign}(\epsilon_u - \epsilon_d) = \text{sign}(\epsilon_d(1+\alpha-\epsilon_d)).$$

Differentiating $\epsilon_d \equiv \frac{QP''(Q)}{P'(Q)}$ with respect to Q and evaluating at $Q = X^*$ we get

$$\frac{d\epsilon_d}{dQ} = \frac{P'(X^*)(P''(X^*) + X^*P'''(X^*)) - X^*(P''(X^*))^2}{(P'(X^*))^2}$$

which upon simplification gives

$$\frac{d\epsilon_d}{dQ} = \frac{P''(X^*)(1+\alpha-\epsilon_d)}{P'(X^*)} = \frac{\epsilon_d(1+\alpha-\epsilon_d)}{X^*}.$$

The result then follows from observing that

$$\text{sign} \frac{d\epsilon_d}{dQ} = \text{sign}(\epsilon_d(1+\alpha-\epsilon_d)) = \text{sign}(\epsilon_u - \epsilon_d)$$

where all expressions are evaluated at $Q(\equiv X) = X^*$.

Q.E.D.

Proof of Proposition 2

Assumption 1 and Assumption 2 respectively imply that $M+1+\epsilon_d > 0$ and $N+1+\epsilon_u > 0$. Since $M+1+\epsilon_d > 0$ holds for all $M \geq 1$, it holds for $M = N$ where $N \geq 1$. Thus $N+1+\epsilon_d > 0$ and consequently $\frac{dX^*}{dM} > 0$. Furthermore, since $P'(X^*) < 0$ we have that $\frac{dCS}{dM} = -P'(X^*)X^*\frac{dX^*}{dM} > 0$. *Q.E.D.*

Proof of Proposition 3

Expand (10) as

$$\frac{dW}{dM} = \sum_{k=1}^N (P^* - c_k) \frac{dx_k^*}{dM} = (P^* - r^*) \frac{dX^*}{dM} + \sum_{k=1}^N (r^* - c_k) \frac{dx_k^*}{dM}. \quad (29)$$

Rearranging (1) and (3) we have that

$$\begin{aligned} P^* - r^* &= -P'(X^*)q^* = -\frac{X^*P'(X^*)}{M}, \\ r^* - c_k &= g_X(X^*, M)x_k^*. \end{aligned}$$

Differentiating (3) with respect to M and rearranging we have that

$$\frac{dx_k^*}{dM} = \frac{X^*}{M(M+1+\epsilon_d)} \left[1 + s_k^*(1+\epsilon_d) - \frac{(1+s_k^*\epsilon_u)(N+1+\epsilon_d)}{N+1+\epsilon_u} \right],$$

Substituting these expressions and $\frac{dX^*}{dM}$ from (7) in (29) and simplifying we get that

$$\frac{dW}{dM} = - \left(\frac{P'X^{*2}}{M(N+1+\epsilon_u)} \right) \left\{ \frac{N+1+\epsilon_d}{M+1+\epsilon_d} + (\epsilon_u - \epsilon_d) + H[(N+1+\epsilon_u)(1+\epsilon_d) - \epsilon_u(N+1+\epsilon_d)] \right\}.$$

Since $N+1+\epsilon_u > 0$ (by Assumption 2), it follows that:

$$\begin{aligned} \frac{dW}{dM} < 0 &\Leftrightarrow \frac{N+1+\epsilon_d}{M+1+\epsilon_d} + (\epsilon_u - \epsilon_d) + H[(N+1+\epsilon_u)(1+\epsilon_d) - \epsilon_u(N+1+\epsilon_d)] < 0, \\ &\Leftrightarrow 1 + \frac{N+1+\epsilon_d}{M+1+\epsilon_d} + H(1+\epsilon_d) - (NH-1)(\epsilon_u - \epsilon_d - 1) < 0, \\ &\Leftrightarrow \epsilon_u - \epsilon_d > 1 + \frac{\frac{1}{H}(1 + \frac{N+1+\epsilon_d}{M+1+\epsilon_d}) + 1 + \epsilon_d}{N - \frac{1}{H}}. \end{aligned}$$

Q.E.D.

Proof of Proposition 4

Substituting (16) into (15) and simplifying we get

$$-g_X(\hat{X}, M)X^2 = \hat{N}^2K$$

which upon total differentiation gives:

$$- \left(g_{XX}(\hat{X}, M)X^2 + 2g_X(\hat{X}, M)X \right) d\hat{X} - g_{XM}(\hat{X}, M)\hat{X}^2 = 2\hat{N}Kd\hat{N}. \quad (30)$$

Using $g_{XM}(\hat{X}, M) = -\frac{P'(\hat{X})(1+\epsilon_d)}{M^2}$, $\epsilon_d = \frac{\hat{X}P''(\hat{X})}{P'(\hat{X})}$ and $\epsilon_u = \frac{\hat{X}g_{XX}(\hat{X}, M)}{g_X(\hat{X}, M)}$ we can write (30) as

$$(2 + \epsilon_u) \left(\frac{\partial \hat{X}}{\partial M} + \frac{\partial \hat{X}}{\partial \hat{N}} \frac{d\hat{N}}{dM} \right) - \frac{\hat{X}(1 + \epsilon_d)}{M(M + 1 + \epsilon_d)} = \frac{2\hat{X}}{\hat{N}} \frac{d\hat{N}}{dM}. \quad (31)$$

We have that

$$\begin{aligned} \frac{\partial \hat{X}}{\partial M} &= \frac{\hat{X}(\hat{N} + 1 + \epsilon_d)}{M(M + 1 + \epsilon_d)(\hat{N} + 1 + \epsilon_u)}. \\ \frac{\partial \hat{X}}{\partial \hat{N}} &= \frac{\hat{X}}{\hat{N}(\hat{N} + 1 + \epsilon_u)}. \end{aligned}$$

Substituting these expressions in (31) and rearranging yields equation (17) of the text:

$$\frac{d\hat{N}}{dM} = \left(\frac{\hat{N}}{M} \right) \left(\frac{1}{2\hat{N} + \epsilon_u} \right) \left[\frac{\hat{N} + 1 + \epsilon_d + \hat{N}(\epsilon_u - \epsilon_d)}{M + 1 + \epsilon_d} \right],$$

Using (17) and the expressions for $\frac{\partial \hat{X}}{\partial M}$ and $\frac{\partial \hat{X}}{\partial \hat{N}}$ from above, we get:

$$\frac{d\hat{X}}{dM} = \frac{\partial \hat{X}}{\partial M} + \frac{\partial \hat{X}}{\partial \hat{N}} \frac{d\hat{N}}{dM} = \frac{\hat{X}(2\hat{N} + 1 + \epsilon_d)}{M(2\hat{N} + \epsilon_u)(M + 1 + \epsilon_d)}. \quad (32)$$

Assumption 1 implies that $M + 1 + \epsilon_d > 0$ as well as $2\hat{N} + 1 + \epsilon_d > 2 + \epsilon_d > 0$. Assumption 2 implies that $2\hat{N} + \epsilon_u \geq \hat{N} + 1 + \epsilon_u > 0$. Thus $\frac{d\hat{X}}{dM} > 0$ which in turn proves Proposition 4.

Q.E.D.

Proof of Proposition 5

Differentiating $\hat{r} \equiv g(\hat{X}, M)$ totally we get

$$\frac{d\hat{r}}{dM} = g_X(\hat{X}, M) \frac{d\hat{X}}{dM} + g_M(\hat{X}, M).$$

Using the expression for $\frac{d\hat{X}}{dM}$ from (17), $g_X(\hat{X}, M) = \frac{P'(\hat{X})(M+1+\epsilon_d)}{M}$ and $g_M(\hat{X}, M) =$

$-\frac{\hat{X}P'(\hat{X})}{M^2}$ we get

$$\frac{d\hat{r}}{dM} = \left(\frac{P'(\hat{X})(M+1+\epsilon_d)}{M} \right) \left(\frac{\hat{X}(2\hat{N}+1+\epsilon_d)}{M(2\hat{N}+\epsilon_u)(M+1+\epsilon_d)} \right) - \frac{\hat{X}P'(\hat{X})}{M^2}$$

which upon simplification gives

$$\frac{d\hat{r}}{dM} = \frac{-P'(\hat{X})\hat{X}(\epsilon_u - \epsilon_d - 1)}{M^2(2\hat{N} + \epsilon_u)}. \quad (33)$$

Proposition 5 immediately follows from the expression of $\frac{d\hat{r}}{dM}$.

Q.E.D.

Proof of Proposition 6

Differentiating W with respect to M yields

$$\frac{dW}{dM} = (P - r) \frac{d\hat{X}}{dM} - \hat{X} \frac{d\hat{r}}{dM}.$$

$$\begin{aligned} \frac{dW}{dM} &= -\frac{\hat{X}^2 P'}{M^2} \left\{ \left[\frac{(\hat{N} + \epsilon_u)(\epsilon_u - \epsilon_d) - (\hat{N} + 1 + \epsilon_d)}{(\hat{N} + 1 + \epsilon_u)(2\hat{N} + \epsilon_u)} \right] (M + 2 + \epsilon_d) - 1 \right\} \\ &= -\frac{\hat{X}^2 P'}{M^2} \left[\left(\frac{\epsilon_u - \epsilon_d - 1}{2\hat{N} + \epsilon_u} \right) (M + 2 + \epsilon_d) - 1 \right] \end{aligned} \quad (34)$$

Since $-\hat{X}^2 P'/M^2 > 0$, $dW/dM < 0$ is equivalent to the following condition

$$\frac{\epsilon_u - \epsilon_d - 1}{2\hat{N} + \epsilon_u} > \frac{1}{M + 2 + \epsilon_d}. \quad (35)$$

The result then follows.

Q.E.D.

Proof of Proposition 7

From the first-order condition of the profit maximization problem in the downstream sector we get

$$q_k^* = -\frac{P^* - r^* - a_k}{P'(X^*)},$$

where $k = 1, 2, \dots, M$ and X^* satisfies the following

$$Ng(X^*, M) - Nc + X^*g_X(X^*, M) = 0.$$

and $P^* \equiv P(X^*)$ and $r \equiv g(X^*, M)$. We have that

$$\frac{dq_k^*}{dN} = -\frac{P'(X^*)\frac{d(P^*-r^*)}{dN} - P''(X^*)(P^* - r^* - a_k)\frac{dX^*}{dN}}{(P'(X^*))^2}.$$

Substituting the expressions for $\frac{d(P^*-r^*)}{dN}$ and $\frac{dX^*}{dN}$ from (22) in the above equation and simplifying subsequently we get:

$$\frac{dq_k^*}{dN} = -\frac{X^*}{N(N+1+\epsilon_u)}\left(\frac{1+\epsilon_d}{M} - s_k\epsilon_d\right). \quad (36)$$

We have that

$$\begin{aligned} \frac{dW}{dN} &= \sum_{k=1}^M (P^* - a_k - c) \frac{dq_k^*}{dN}, \\ &= \sum_{k=1}^M (P^* - r^* - a_k) \frac{dq_k^*}{dN} + (r^* - c) \frac{dX^*}{dN}. \end{aligned} \quad (37)$$

Substituting $P^* - r^* - a_k = -P'(X^*)q_k^*$, $r^* - c = \frac{-g_X(X^*, M)X^*}{N}$, $\frac{dq_k^*}{dN}$ from (36) and $\frac{dX^*}{dN}$ from (22) in the above equation and simplifying we get

$$\frac{dW}{dN} = -\frac{P'(X^*)X^{*2}}{N(N+1+\epsilon_u)}\left(\frac{1+\epsilon_d}{M} + \frac{M+1+\epsilon_d}{MN} - H_d\epsilon_d\right).$$

Given $P'(X^*) < 0$,

$$\frac{dW}{dN} < 0 \Leftrightarrow \epsilon_d\left(H_d - \frac{1}{M}\right) > \frac{M+N+1+\epsilon_d}{MN},$$

where $H_d = \sum_{k=1}^M (s_k)^2$ is the Herfindahl index for the downstream sector.

Q.E.D.

Proof of Proposition 8

We have that

$$\frac{dW}{dN} = (\hat{r} - c) \frac{d\hat{X}}{dN} - \hat{X} \frac{d(\hat{P} - \hat{r})}{dN}.$$

First we derive $\frac{d(\hat{P}-\hat{r})}{dN}$. When $a_i = a$ for all $i = 1, 2, \dots, M$, the input demand function is

$$r = P(X) - a + \frac{XP'(X)}{M} \equiv g(X, M).$$

Rearranging the above equation and evaluating at the free entry equilibrium values, we get:

$$\hat{P} - \hat{r} = a - \frac{\hat{X}P'(\hat{X})}{\hat{M}}.$$

Differentiating $\hat{P} - \hat{r}$ with respect to N gives:

$$\frac{d(\hat{P} - \hat{r})}{dN} = \frac{\hat{X}P'(\hat{X})}{\hat{M}^2} \frac{d\hat{M}}{dN} - \frac{P'(\hat{X})(1 + \epsilon_d)}{M} \frac{d\hat{X}}{dN}. \quad (38)$$

Differentiating (24) and rearranging we get:

$$\frac{d\hat{X}}{dN} = \frac{2\hat{X}}{(2 + \epsilon_d)\hat{M}} \frac{d\hat{M}}{dN}. \quad (39)$$

Using (39), we can rewrite (38) as:

$$\frac{d(\hat{P} - \hat{r})}{dN} = -\frac{\epsilon_d P'(\hat{X})}{2M} \frac{d\hat{X}}{dN}. \quad (40)$$

Using (40) and $\hat{r} - c = -\frac{g_X(\hat{X}, \hat{M})\hat{X}}{\hat{N}} = -\frac{P'(\hat{X})(\hat{M}+1+\epsilon_d)}{\hat{M}\hat{N}}$ we get

$$\begin{aligned} \frac{dW}{dN} &= (\hat{r} - c) \frac{d\hat{X}}{dN} - \hat{X} \frac{d(\hat{P} - \hat{r})}{dN}, \\ &= \frac{\hat{X}P'(\hat{X})}{M} \left(\frac{\epsilon_d}{2} - \frac{\hat{M} + 1 + \epsilon_d}{N} \right) \frac{d\hat{X}}{dN}, \\ &= \frac{\hat{X}P'(\hat{X})(N-2)}{2\hat{M}N} \left(\epsilon_d - \frac{2(\hat{M} + 1)}{N-2} \right) \frac{d\hat{X}}{dN}. \end{aligned} \quad (41)$$

Recall we focus on parameterizations where $\frac{d\hat{M}}{dN} > 0$.¹⁵ Since $\epsilon_d > -2$ it follows

¹⁵Totally differentiating (24) and (25) and rearranging we get: $\frac{d\hat{M}}{dN} = \frac{(\hat{r}-c)(2+\epsilon_d)\hat{M}}{2\Delta\hat{X}}$, where $\Delta = -\frac{P'(\hat{X})[(\hat{M}+1+\epsilon_d)(N+1+\epsilon_u)-(2+\epsilon_d)(N+1+\epsilon_d)]}{2M}$. Observe that $\Delta > 0 \Leftrightarrow \frac{d\hat{M}}{dN} > 0$. Note that $\epsilon_u - \epsilon_d \geq 0 \Rightarrow \Delta > 0$. From the proof of Proposition 1 we know that $\text{sign} \frac{d\epsilon_d}{dQ} = \text{sign}(\epsilon_u - \epsilon_d)$. Thus, $\frac{d\epsilon_d}{dQ} \geq 0 \Rightarrow \epsilon_u - \epsilon_d \geq 0 \Rightarrow \Delta > 0 \Rightarrow \frac{d\hat{M}}{dN} > 0$. In other words, $\frac{d\hat{M}}{dN} > 0$ holds for all demand functions with increasing elasticity of slope.

from (39) that $\frac{d\hat{X}}{dN} > 0$. Since $P'(\hat{X}) < 0$ and $\frac{d\hat{X}}{dN} > 0$, it is immediate from (41) that

$$\frac{dW}{dN} < 0 \Leftrightarrow \epsilon_d > \frac{2(\hat{M} + 1)}{N - 2}.$$

Q.E.D.