Risk, ambiguity and sovereign rating

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Abstract

Decisions of investing in sovereign assets involve both risk and ambiguity. Ambiguity arises from unknown elements characterizing the value of a generic sovereign. In presence of ambiguity, ambiguity-averse investors are prone to pay for obtaining summary information such as ratings which reduces ambiguity. Ambiguity-neutral and ambiguity-averse investors, then, make decisions on the basis of different informative sources. By presenting a simple model of sovereign rating under ambiguity, three facts occurring in today's financial markets are explained. Sovereign ratings influence decisions of investment of ambiguity-sensitive individuals. Rating-dependent regulations create distortions in financial markets by institutionalising specific summary signals. Providing ratings may be a profitable activity. Some final suggestions propose future areas of theoretical and empirical research.

Keywords: risk, ambiguity, ambiguity aversion, sovereign rating, value of information.

JEL classification: D80, D81, G11, G14.

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1. Introduction

Were they alive today, John Maynard Keynes and Frank Knight seeing the power of some alphabetical letters and few mathematical signs to influence the general behaviour regarding the creditworthiness of a particular country, would be at least disappointed. Not only sovereign credit ratings and rating agencies have reached a prominent position in international financial markets, but rating announcements have been able to dominate most of front-page newspapers and persuade policy makers, especially during the present economic crisis.

Although the limits of sovereign ratings have been highlighted in several occasions, this increased popularity can be ascribed to the progressive institutionalization of sovereign rating and rating agencies pursued by financial regulators at international level since mid-1970s. In particular, specific acts adopted by the SEC for regulating the role of *Nationally Recognized Statistical Rating Organizations* (NRSRO) in the US system and some rules for financial institutions contained in the *Basel II Accord* have contributed to the worldwide affirmation of sovereign rating in financial markets.

But, a question naturally arises: is the only institutional argument able to fully explain the ascendance of rating activities in the last years? Or, this featuring aspect of today's financial markets requests to look for additional answers? The present paper takes the latter question as relevant on the basis of the following intuition: credit rating agencies provide summary information to investors on the probability of country risk of default; in presence of both risk and uncertainty (ambiguity) rating grades can assume the status of valuable information for ambiguity-averse decision makers. This becomes particularly important in periods of greater ambiguity in the system as in the case of financial crisis when sovereign rating news may assume the role of simple, credible and conventional information for investors feeling lost.

In a continuous time horizon, individuals choose to invest in the sovereign debt issued by a particular country given the available set of information. In general, the *ex ante* value of a sovereign is expressed in the market with certainty, by means of diverse informative signals about fundamentals and other aspects related to the creditworthiness of a country. These unambiguous attributes mostly determine the interest rate at which sovereign assets are traded. When considering sovereign debt assets, however, the occurrence of unknown elements influencing their values is far from being a rare event. For instance, the *ex ante* probability of a national default is partly uncertain due to several institutional reasons such as the peculiarity of enforcement procedures and the specific lender-borrower relationship (Eaton et al., 1986; Roubini, 2001). In other words, the event of a national default is partly uncertain in the Knightian sense (Keynes, 1921; Knight, 1921).

Ambiguity or uncertainty occurs when a decision-maker is unable to fully attribute measurable probabilities to a generic event for whatsoever reasons. In such a situation, as extensively pointed out in the well-known Ellsberg's paradox (Ellsberg, 1961), there is a clear distinction between risk and uncertainty, which needs to be addressed. While the presence of risk implies looking at subjective and objective numerical probabilities in order to set up a decision problem; Knightian's uncertainty requires adding further informative elements for modeling an environment where there are unmeasurable or unknown probabilities.

To analyze the relevance of sovereign rating activities in presence of both risk and ambiguity in financial markets, the flexible theoretical framework recently presented in Caskey (2009) results very helpful. It is based upon the smooth ambiguity preferences approach analyzed in Klibanoff et al. (2005), with the introduction of an articulated informative structure. Ambiguity-averse investors are compared to Savage decision-makers in terms of preference for different informative sources, namely disaggregate and aggregate, in order to unveil the importance of certain kind of information for reducing investors' exposure

towards ambiguity. In this context, both investors make their decision on the basis of the riskiness of a generic sovereign (i.e. the conditional variance), but only ambiguity-averse investors are also influenced by the presence of ambiguity.

By considering sovereign rating as a valuable information in the process of decision-making under ambiguity, this paper provides some intuitive explanations for three features occurring in today's financial markets. First, rating activities are relevant for ambiguity-averse investors and their importance increases with higher ambiguity aversion, though they are biased information. Second, the progressive institutionalization of rating activities by means of rating-dependent regulations can be related to the need of widening the spectrum of financial investors as a consequence of the financial deregulation trend started in late Seventies. Third, the preference for summary rating grades rather than more precise disaggregate information is able to create mispricing effects of sovereigns and divergent gains between investors-type.

The paper is organised as follows. Section 2 briefly illustrates sovereign rating activities. The theoretical framework is discussed in Section 3. Section 4 investigates three facts regarding sovereign ratings in financial markets. A final section summarizes and proposes some suggestions for future research.

2. Sovereign rating

Sovereign credit ratings are condensed assessment by credit rating agencies of a government's ability and willingness to repay its public debt both in principal and in interests on time. They are considered forward-looking qualitative measures of the probability of default of a country. In general, the default risk is defined as probability of default by Standard & Poor's (S&P), expected loss by Moody's or both by Fitch. Credit rating agencies

assign to all financial commitments of a sovereign State a alphabetical letter (frequently with a number or +/-) in order to classify its overall creditworthiness.

The procedures adopted by prominent rating agencies derive from a mix of quantitative forecasts and qualitative opinions, and there are no substantial differences between issued categories (corporate bond, sovereign, structured financial instruments). A rating assessment can be solicited (formally requested by the issuer) or unsolicited (voluntarily started by the rating agency). Under particular circumstances, rating agencies re-assess upward (upgrading) or downward (downgrading) their judgments on the creditworthiness of a debtor.

As extensively discussed by Barry Eichengreen (2009), in every crisis rating agencies are targets of criticism for several reasons. Looking at the aftermath of the Asian financial crisis or at the more recent financial turmoil in Europe and the United States, rating activities *lato sensu* have been criticized for their inability of forecasting out of sample, operating within an oligopolistic market structure, showing many conflicts of interest, lacking of transparency and providing not well-defined ancillary services in combination with ratings.

From an economic point of view, rating agencies 'offer financial markets an estimate of the probability that borrowers will not fulfill the obligations specified in their debt issues' (Ferri et al., 1999). Thus, rating agencies act as informational intermediaries in the assessment of credit risk, contributing to reduce asymmetries and information costs between investors and issuers in the financial markets. In addition, ratings can serve as monitoring technology in principal-agent problems between individual investors and aggregate financial institutions such as banks and pension funds, and reduce collective action problems between dispersed investors and debtors.

Sovereign credit ratings have various implications for the economic and financial system. They contribute to determine country's borrowing costs in international capital markets in two ways: defining the threshold between investment-grade and speculative-grade,

and amplifying particular financial phenomena such as cliff effects and herd behavior. In this direction, many empirical studies have found robust relations between sovereign ratings and variation in government yield spreads (Afonso et al., 2011).

The attribution of a national rating may influence ratings of both private (banks, companies) and public actors (regional and local governments) within a country, creating a sort of internal spillover effects with evident distortions in the financing costs of related subjects. The downgrade of Portugal operated by Moody's in summer 2011, for instance, was followed by the downgrade of some Portuguese financial institutions. Similar consequences had the downgrade of Italy, announced by S&P and Moody's between September and October 2011, and directly affecting some Italian banks, companies and local governments.

Sovereign rating activities are able to influence neighbouring countries or contiguous regions, sustaining the propagation of external contagion effects. Since the seminal paper of Cantor and Packer (1996) for the United States, diverse empirical contributions have confirmed this pattern for different areas. Arezki et al. (2011), for instance, have provided empirical evidence of cross-country spillover effects in the Eurozone, during the years 2007-2010. Bissoondoyal-Bheenick (2012) explained the external impact of sovereign rating announcements in emerging markets in the presence of financial and commercial links among countries.

3. A simple model of sovereign rating under ambiguity

3.1. Setting the scene

This section describes the main aspects of the theoretical framework hereafter adopted for subsequently investigating some facts related to sovereign rating activities occurring in today's financial markets. It is mainly based upon the model presented in Caskey (2009) as 'ambiguity in fundamentals'. In a financial market where there are α Savage and $(1-\alpha)$

ambiguity-sensitive investors, sovereigns assets are traded together with a riskless asset with a rate of return normalized to zero. Individual preferences towards ambiguity are represented by smooth ambiguity functions as in Klibanoff et al. (2005) and Gollier (2011)[1]. As a result, ambiguity and ambiguity preferences can be separated, with the latter captured by a non-decreasing transformation function.

For simplicity, we assume that all investors share the same constant absolute risk tolerance utility:

$$U(x) = -e^{-\frac{x}{\rho}}, \qquad (1)$$

with the parameter $\rho > 0$ specifying the degree of constant absolute risk tolerance. Moreover, preferences for ambiguity are described by a constant relative ambiguity aversion form as follows:

$$\varphi(E_F[U]) \; = \; -(-E_F[U])^a, \qquad (2)$$

where the function $\varphi(E_F[U])$ denotes a concave transformation of the utility function, which is able to capture ambiguity in this decision-making problem. This representation derives from the smooth ambiguity preferences approach here adopted[2]. The value of the parameter a captures the ambiguity profile: for $a = 1$ we have ambiguity-neutrality, or alternatively Savage investors; while $a > 1$ is the case of ambiguity aversion.

The *ex ante* value of a generic sovereign asset can be described as the sum of two independent normal random variables $y = y_1 + y_2$ with mean $E[y_i] = \mu_{yi}$ and variance $var[y_i] = \sigma_{yi}^2$. The unambiguous component y_2 has mean $E[y_2] = \mu_{y2}$ and variance $var[y_2] = \sigma_{y2}^2$; while y_1 represents the ambiguous element having unknown mean $E[y_1] = \mu_{y1} + v$. From the investors' subjective prior belief for the ambiguous part v it is assumed

[1] It is outside the boundaries of the present paper discussing the assumptions, the implications and the shortcomings of this particular preference representation (for recent discussions, see Hey et al., 2010 and Maccheroni et al., 2013).

[2] It shall be noted that specifying ambiguity aversion in this way can be equivalent to increasing risk aversion while leaving the elasticity of intertemporal substitution unchanged. I thank John Y. Campbell to have highlighted this point. However, this critical feature holds when the support of the prior is a singleton or when we do not recognize the presence of different sources of risk aversion (for a more detailed discussion, see Maccheroni et al., 2013).

that v is normally distributed with unknown mean μ_v and unknown variance σ_v^2. Therefore, the variance of y_1 can be obtained as the sum of independent normal random variables, namely $\sigma_{y1}^2 = \sigma_{z1}^2 + \sigma_v^2$, with σ_{z1}^2 denoting the variance of the random variable z_1.

Investors receive unambiguous information about the overall value of the sovereign asset, which is partially ambiguous. Ambiguity is driven by the unknown element v for which investors only assume normality of its distribution. Regarding sovereign assets, this situation can occur when investors are not able to fully disentangle the uncertain component of a generic sovereign (e.g. some aspects of the probability of national default) given their information.

At this point some comments are worth mentioning. The simple way of modeling ambiguity presented in this context makes it possible to unveiling how the ambiguous part v influences the value of the generic sovereign. Indeed, it affects both the expected value and the variance of the unknown component y_1. For example, when the *ex post* realization of v is negative the overall value of the sovereign will be reduced as in the case of an adverse shock. This can occur with an expected realization of v restricted to assume only negative values (μ_v^-). In addition, the degree of ambiguity incorporated in this decision-making process can be ascribed to the variance of the ambiguous element v. The more variable is the ambiguous component v, the higher is the impact of ambiguity on the sovereign asset value. This implies, then, that in a situation of higher variance $\sigma_v^2 > \sigma_{v\prime}^2$, investors face a more ambiguous environment $\sigma_{y1}^2 > \sigma_{y1}^2$.

At time t_0 investors observe a composite information set $\mathcal{I} \in \{J, (j, r)\}$ with J denoting a disaggregate vector of information, j representing a summary informative report obtained as a linear aggregation of J and r being the interest rate paid by a generic sovereign[3]. More specifically, we have:

[3] In this model the informative content of interest rate is secondary, therefore without loss of generality it can be ignored.

$$J \equiv \begin{bmatrix} j_1 \\ j_2 \end{bmatrix} = \begin{bmatrix} y_1 + e_1 \\ y_2 + e_2 \end{bmatrix} \quad (3)$$

where j_1 and j_2 represent the informative content of the overall (ambiguous and unambiguous) value of the sovereign asset and $e_1 \sim N(0, \sigma_{e1}^2)$ and $e_2 \sim N(0, \sigma_{e2}^2)$ are unambiguous informative errors.

This informative structure can be thought as a situation where investors acquire market signals about sovereigns from each component (ambiguous and unambiguous) characterizing the value of a generic sovereign plus a white noise. For example, the individual decision of investing in a specific sovereign can be influenced by several informative elements such as the characteristics of both the economic (e.g. debt to GDP ratio or current account) and political (e.g. stability or instability of the government) environment. However, ambiguity in fundamentals implies that the true value of the sovereign asset is partially unknown *a priori*.

Alternatively, investors may observe a summary signal j which is a linear function of the vector in (3):

$$j = \gamma \frac{\sigma_{y1}^2}{\sigma_{j1}^2} j_1 + \frac{\sigma_{y2}^2}{\sigma_{j2}^2} j_2 \quad (4)$$

where the formulation in (4) derives from the relation between a generic summary signal j $(\gamma \neq 1)$ and a benchmark case $(\gamma = 1)$ where j is a sufficient statistics for J (i.e. j is informationally equivalent to J).

From (4) it can be observed that the informative content of the summary signal j depends on the individual signals j_1 and j_2, and in particular on the relative variance of y_i on j_i. The parameter γ captures the presence of the ambiguous signal j_1 in the summary information j. For $\gamma > 1$ the summary signal j reflects more ambiguity in fundamentals than the benchmark sufficient statistics; while for $0 \leq \gamma < 1$ the opposite is true, namely that the summary signal j include less ambiguous information than the benchmark case. In other words, γ denotes the ambiguity content of the summary signal.

In this case, the summary signal j can be thought as being the rating grade associated to a given sovereign asset in financial markets. Put it differently, sovereign ratings provide *ex ante* condensed information on the true value of a sovereign asset, which is obtained by a combination of both known and unknown elements. Moreover, a specialized information provider such as rating agencies can choose to set different values of the parameter γ (i.e. the degree of ambiguous information in the summary signal) on the basis of the investor willingness to pay for receive this information.

3.2. Investment decisions

To complete the description of this decision-making process we need to specify the maximization problem of each investor (Savage and ambiguity-sensitive) and derive both optimal quantities and equilibrium prices. Given the utility function and the form of ambiguity preferences previously discussed, investors choose the optimal quantity of sovereign assets which maximizes:

Ambiguity-neutral: $$\max_q q(E[y|\mathcal{J}] - r) - \frac{1}{2\rho} q^2 var(y|\mathcal{J}) \quad (5a)$$

Ambiguity-sensitive: $$\max_q q(E[y|\mathcal{J}] - r) - \frac{1}{2\rho} q^2 var(y|\mathcal{J})[1 + (a-1)G(y, v|\mathcal{J})]. \quad (5b)$$

Expressions (5a) and (5b) are similar to those presented in Caskey (2009), to which the interested reader can refer to a formal derivation. For our purposes, however, three aspects are worth commenting. As usual, optimal demands of both Savage and ambiguity-sensitive investors positively depend on the conditional expected utility of the sovereign asset value given the information set, namely $E[y|\mathcal{J}]$. Another common aspect in the financial literature is the influence of the risk component given the conditional variance $var(y|\mathcal{J})$.

The main difference between (5a) and (5b), then, is represented by the ambiguous element $(a-1)G(y, v|\mathcal{J})$ with:

$$G(y, v|\mathcal{J}) = 1 - \frac{var(y|\mathcal{J},v)}{var(y|\mathcal{J})} = corr(y, v|\mathcal{J})^2, \quad (6)$$

denoting the degree of ambiguity experienced by ambiguity-sensitive investors. Note that $var(y|\mathcal{J}, v) \leq var(y|\mathcal{J})$, and the value of the expression in (6) is bounded between 0 and 1.

When $var(y|\mathcal{J}, v) = var(y|\mathcal{J})$, the ambiguous term v does not affect the conditional variance of the value of the sovereign asset given the information set $var(y|\mathcal{J})$, implying that $G(y, v|\mathcal{J}) = 0$. The opposite will be true the greater is the difference between $var(y|\mathcal{J}, v)$ and $var(y|\mathcal{J})$, having $G(y, v|\mathcal{J}) \to 1$. Clearly, the unknown variance of the ambiguous element σ_v^2 becomes crucial for the ambiguity level. In presence of ambiguity-neutral investors ($a = 1$) the expression (5b) boils down to the canonical Savage investment decision problem. Whereas, ambiguity-averse decision-makers ($a > 1$) add a negative term into their maximization problem.

For simplicity, we assume that investors belief *ex ante* that the supply of sovereign assets is $x \sim N(\mu_x, \sigma_x^2)$ and x is independent of both the *ex post* sovereign asset value y and the vector of informative signals J. Moreover, we assume that the population of both Savage and ambiguity-sensitive investors observe homogenous information[4], namely they view J or alternatively j. Before deriving equilibrium quantities and prices, we need to specify an additional function for each investor, defined as the posterior precision given the different information sets.

These functions are:

$$g(y|\mathcal{J}) \equiv \frac{1}{var(y|\mathcal{J})} \quad \text{and} \quad g_a(y|\mathcal{J}) \equiv \frac{1}{var(y|\mathcal{J})[1+(a-1)G(y,v|\mathcal{J})]}, \quad (7)$$

where $g(y|\mathcal{J})$ is the posterior precision for ambiguity-neutral investors and $g_a(y|\mathcal{J})$ is the posterior precision for ambiguity-sensitive investors. Note that, in presence of high conditional variance of the sovereign asset value given the available information sets, both

[4] In concrete, investors manage different information sets, resulting in heterogeneous information in the system. The heterogeneous case adds some complications, without affecting the main message of this framework.

type of investors experience lower posterior precision. In addition, the posterior precision of ambiguity-averse investors suffers from the additional effect arising from both the ambiguity level and their ambiguity preferences.

For a given information set $\mathcal{I} \in \{J, j\}$ the equilibrium in this market is obtained by solving the following expression for the optimal interest rate:

$$\alpha q_S(\mathcal{I}, r^*) + (1-\alpha) q_A(\mathcal{I}, r^*) = x, \tag{8}$$

where $q_S(\mathcal{I}, r^*)$ is the demand function for Savage investors calculated from the maximization of the objective function in (5a) and $q_A(\mathcal{I}, r^*)$ is the demand function for ambiguity-averse investors from (5b). In both cases, demand is a function of the interest rate and the available information.

From (8) and the investor maximization problem we can obtain:

$$r^* = E[y|\mathcal{I}] - \frac{1}{\rho \bar{g}} x, \quad q_S^*(\mathcal{I}, x) = \frac{g(y|\mathcal{I})}{\bar{g}} x, \tag{9}$$

$$q_A^*(\mathcal{I}, x) = \frac{g_a(y|\mathcal{I})}{\bar{g}} x = \frac{1}{var(y|\mathcal{I})[1+(a-1)G(y,v|\mathcal{I})]} q_S^*(\mathcal{I}, x),$$

with r^*, $q_S^*(\mathcal{I}, x)$ and $q_A^*(\mathcal{I}, x)$ be the optimal interest rate, the optimal quantity of sovereigns held by Savage investors and the optimal quantity of sovereigns held by ambiguity-averse investors, respectively. The function $\bar{g}$ can be defined as the average posterior precision in the market given the homogenous information set $\mathcal{I} \in \{J, j\}$, namely:

$$\bar{g} = \alpha g(y, \mathcal{I}) + (1-\alpha) g_a(y, \mathcal{I}). \tag{10}$$

As usual, the equilibrium interest rate is negatively related to the supply quantity and it is positively affected by the risk tolerance parameter ρ. In addition, we can observe a positive relation between the interest rate and the conditional expected value of the sovereign asset. The fact that the average precision in the market $\bar{g}$ contributes to increase the equilibrium interest rate can be interpreted in the sense that, in a better informative environment (i.e. with

a lower conditional variance), the interest rate is able to absorb its informative role with more precision.

With regard to optimal sovereign assets holdings, two aspects are worth noting. First, in both cases (Savage and ambiguity-averse) the higher is the specific posterior precision relative to the average precision in the market, the greater will be the optimal amount of sovereigns held by each investor. Second, ambiguity-averse investors have lower equilibrium quantities of sovereigns than Savage investors given their sensitivity to ambiguity. Moreover, both an higher ambiguity aversion ($a \uparrow$) and an higher ambiguity level in the system $G(y, v|\mathcal{J})$ contribute to further reduce the optimal quantity held by ambiguity-averse investors.

3.3. The value of information

Once equilibrium characteristics have been determined, what we need is to investigate the role of the different informative sources in this financial market. In other words, we are interested in disentangling the diverse effects of the vector of information J and the summary signal j in terms of ambiguity. The starting point is the relation in (6), which can be rewritten in the following form:

$$G(y, v|J) = \left(1 - \frac{\sigma_{y1}^2}{\sigma_{j1}^2}\right)^2 \frac{\sigma_{j1}^2}{var(j_1|v)} \frac{\sigma_v^2}{var(y|J)} \tag{11a}$$

$$G(y, v|j) = \left(1 - \gamma \frac{cov(y,j)}{var(j)} \frac{\sigma_{y1}^2}{\sigma_{j1}^2}\right)^2 \frac{var(j)}{var(j|v)} \frac{\sigma_v^2}{var(y|j)}. \tag{11b}$$

Expressions (11a) and (11b) describe the impact of the alternative informative sources J and j on ambiguity, perceived by ambiguity-averse investors. They have been obtained by rearranging the formulation in (6) and separating it for each component of the information

set[5]. The choice of what kind of informative environment, J or j, is preferred (i.e. reduce ambiguity) by ambiguity-averse investors is the crucial point of this model. In particular, it can be affirmed that ambiguity-averse investors are better off when they view the summary signal j rather than the disaggregate vector of information J. This result has been presented and proved in Section 2 in Caskey (2009). The intuition behind this outcome is quite simple.

When the summary signal j is considered information about ambiguity j_1 is processed together with information concerning the unambiguous element j_2. Indeed, by remembering the structure of j described in (4), we can note that it is an aggregation of both ambiguous and unambiguous individual signals or, alternatively, a synthesis of information on the known and unknown value of a given sovereign. Therefore, the joint informative content incorporated by the summary signal is able to reduce the perceived ambiguity. More formally, $var(j)/var(j|v) < \sigma_{j1}^2/var(j_1|v)$.

In today's financial markets this feature can be translated as follows. The value of a given sovereign is characterized by known and partially unknown attributes. Moreover, ambiguity-averse investors are sensitive to the degree of ambiguity present in the system and they are willing to reduce their exposure to uncertainty. In this setting, sovereign rating activities provide summary information about the value of a generic sovereign, capturing both ambiguous and unambiguous aspects. For this reason, they become attractive only for non-Savage decision-makers.

When the summary signal j is a sufficient statistics for J ($\gamma = 1$), it can be quite easily showed that the only difference between (11a) and (11b) is represented by the second component in the right-hand side of both equations. In general, however, the restriction $\gamma \neq 1$ shall hold, given that even a very precise summary signal is not able to completely eliminate

[5] The first term of the right-hand side of both (11a) and (11b) represents the square of 1-coefficient on v in $E[y|J]$ and $E[y|j]$ respectively; the second term is the contribution of the ambiguous element v to uncertainty about the disaggregate signal j_1 and the summary report j respectively; the third terms denotes the impact of posterior precision.

the intrinsic ambiguity attached to the value of a sovereign asset. Indeed, the unknown element of a sovereign will always be present.

As a consequence, it is important to study the relation between the ambiguous content of the summary report described by the parameter γ and the variation of the overall degree of ambiguity. From (11b) we can see that the higher is the ambiguous content of the summary report ($\gamma > 1$), the lower is the ambiguity faced by ambiguity-averse investors. In other words, ambiguity-averse investors are interested in receive more precise information about the unknown part of a sovereign, which they view as uncertain.

But, this story provides three further insights. First, from (9) it can be observed that a lower level of ambiguity resulting from a greater ambiguous content of the summary signal ($\gamma \uparrow$) determines higher equilibrium quantities $q_A^*(\mathcal{J}, x)$ for ambiguity-averse investors: a reduction in the perceived ambiguity, then, is reflected by the creation of new trade opportunities for ambiguity-averse investors. Second, when the summary signal is able to process ambiguous information in a more effective way, the average precision in the system $\bar{g}$ increases. This relation can be observed by combining the expressions in (7) and (9). Moreover, in a market with a greater number of ambiguity-averse investors ($\alpha \downarrow$) this effect is magnified. Third, the increased average precision in the market contributes to rise the equilibrium interest rate in (9) for a given supply of sovereigns. This effect is due to the variation in the informative environment caused by a better process capability registered by the summary signal, which is preferred by ambiguity-averse investors.

4. Three facts on sovereign rating

4.1 Rating for whom

Answering to some questions about the relevance of rating grades and rating agencies in influencing sovereign positions, in January 2012 French economist Jean-Paul Fitoussi

declared to 'simply ignore them because they are worthless stuffs'[6]. Why the reality is quite different, namely that sovereign grades invade worldwide front-page newspapers? How meaningless things may become seemingly new oracles?

An intuitive answer relies upon the theoretical framework heretofore discussed. Recognizing the presence of some unknown elements incorporated in the value of a generic sovereign asset, and given a market populated by both Savage and ambiguity-averse investors, rating grades can be thought as summary information about sovereigns. For ambiguity-averse investors this summary information becomes a crucial aspect in order to reduce their exposure towards ambiguity. More precisely, when ambiguity-averse investors deal with uncertainty about sovereign fundamentals, they may prefer to view a quick and dirty signal relying on aggregate information, rather than more punctual disaggregate informative sources. This situation occurs given that an aggregate signal is a composition of both known and unknown characteristics, implying a superior perceived precision.

In addition, an increase in the level of ambiguity aversion ($a' > a$) makes more attractive the summary signal j than the disaggregate information vector J. Therefore, more ambiguity-averse decision makers are more willing to looking for sovereign ratings even in presence of more detailed information. The relative relevance of sovereign rating activities is also positively associated to the number of ambiguity-averse investors in the system $(1 - \alpha)$. As a limiting case, if we assume that there are no Savage decision-makers in the market, all the remaining population of investors is formed by ambiguity-averse subjects, which are prone to receive summary information.

This discussion is able to shed light on some peculiar aspects observable in financial markets. Steiner and Heinke (2001), for instance, provide evidences on the greater importance of sovereign rating news at international level rather than within the borders of the country

[6] This quotation has been reported by an interview of Jean-Paul Fitoussi to the Italian newspaper *'Il Messaggero'* (January, 14 – 2012).

directly interested. This situation can occur due to the fact that home and foreign investors may show different degrees of ambiguity-aversion[7]. In particular, domestic decision-makers can be less ambiguous-averse towards sovereign assets issued by their home country, as a consequence of a lower level of feeling of incompetence or explicit home bias behaviour.

Another related argument is the evidence showing how less professional financial investors are deeper influenced by sovereign rating announcements than more professional ones (Baker and Mansi, 2003). It is not so unrealistic to assume that the former are more ambiguity-averse than the latter. In this case, non-professional investors associate a major importance to the summary information provided by sovereign ratings than other informative sources.

Finally, rating activities may be valuable for rating agencies. Acting as information providers, rating agencies are able to match the demand of summary signals arising from ambiguity-averse investors. Remembering the decision-maker maximization problem described in (5b), it emerges the individual willingness to pay for reduce ambiguity. And, the demand of rating activities is positively related to an higher degree of ambiguity in the system. Probably, it is not a case if the three prominent rating agencies have increased their market value during the present financial turmoil.

4.2. The effects of rating-dependent regulations

As extensively documented in Partnoy (1999 and 2009) and Mathis et al. (2009), since mid-1970s sovereign rating activities have been progressively incorporated in international financial regulations, though mostly of the historical rating grades provided by the prominent rating agencies were only partially correct. For example, during the East Asian financial crisis occurred in the second half of 1990s, rating downgrades had been operated with some months

[7] Alternatively, we can think at a situation in which the proportion of ambiguity-averse investors is higher in the foreign population than in the domestic one.

of delay with respect to the effective economic situation of most of Asian countries (Ferri et al, 1999).

A question, then, naturally arises: why international regulators have been so willing to introduce sovereign ratings in most of the rules defining the functioning of today's financial markets? And, why several public and private institutions have sponsored the adoption of specific rating grades? A simple rational answer can be linked to our previous discussions.

The institutionalization of sovereign rating activities can be interpreted as the creation of an official ranking between summary signals. Other things being equal, the explicit recognition of some rating agencies has the effect of restricting the available information sets $\mathcal{J}' \in \{J, (j', r)\}$ to investors; where now j' denotes the summary signals of recognized rating agencies. It is worth noting that the set $\mathcal{J}'$ is a subset of $\mathcal{J}$, restricting the spectrum of all possible rating grades or summary signals.

The force of law attained by some rating activities reinforce the preference for particular summary signals when compared to the disaggregate information vector, creating a sort of *de jure* bias. More precisely, considering sovereign ratings like formal rules of the financial system implies as if the difference between processing the ambiguous information in (11a) and (11b) is amplified by a sort of external constraint. This structure can be imagined through the introduction of a distorting parameter $0 < c \leq 1$ in the relation (11b):

$$G(y, v|j) = \left(1 - \gamma \frac{cov(y,j)}{var(j)} \frac{\sigma_{y1}^2}{\sigma_{j1}^2}\right)^2 \frac{var(j)}{var(j|v)} \frac{\sigma_v^2}{var(y|j)} \times \mathrm{c} \qquad (12)$$

with a deeper institutionalization of sovereign ratings reflected by a lower level of the parameter c. When rating-dependent regulations are absent (c = 1) expressions (11b) and (12) are equivalent.

In presence of a downward distortion of the relation in (11b), other things being equal, the equilibrium sovereign holdings of ambiguity-averse investors increase $q_A^*(\mathcal{J}, x) \uparrow$ through the channel of a greater perceived precision $g_a(y|\mathcal{J})$. In other words, the introduction of

rating-dependent regulations offers new trade opportunities for ambiguity-averse investors, which are now more confident about their exposure towards ambiguity, where this confidence is bolstered by official regulations.

At this point, it should not be so difficult to imagine why rating-dependent rules have flowed since mid-Seventies. At that time, financial deregulations started and novel international financial opportunities were created. Making this new financial environment more attractive for ambiguity-averse investors, then, requested the institutionalization of some informative sources able to reduce the perceived ambiguity. By setting new rules characterized by the incorporation of rating grades financial regulators reached the aim of widening the population of potential financial investors.

4.3. Good, bad and profitable news

One more fact needs to be explored in order to complete our tale: how diverse rating grades can differently affect the market value of a given sovereign asset and the profits of each investor-type. In this direction, the existing literature has emphasized the quite relevant role played by rating announcements in influencing sovereign yield spreads and, in particular, the higher sensitivity of markets during periods of higher volatility (Afonso et al., 2011; Arezki et al., 2011).

To further investigate this aspect it is convenient to rewrite the equilibrium interest rate making a distinction between investors' types as follows:

$$r^* = \beta_J E[y|J] + (1-\beta_J)E[y|j] - \frac{1}{\rho\bar{g}}x - \beta_x(x-\mu_x) \qquad (13)$$

where $\beta_J \in [0,1]$ denotes the share of investors which are able to view the disaggregate information J, and $1-\beta_J$ is the share of investors observing the summary report j. For simplicity, we have ignored possible supply mismatching, namely we have assumed that

$x = \mu_x$. Moreover, it can be observed that the optimal interest rate does not depend on the value of the parameter γ.

The first aspect to worth mentioning is the mispricing effect introduced by the choice of viewing summary information pursued by ambiguity-averse investors when there are also Savage decision-makers. The explanation of this effect is quite simple. Market news (good or bad) are captured by the overall informative vector J, given that the summary report j is a linear aggregation of J. Therefore, market news are described by variations in the term $E[y|J]$: where $E[y|J] > E[y]$ represents good news and $E[y|J] < E[y]$ bad ones.

In the benchmark case (i.e. when the summary signal is a sufficient statistics of disaggregate information, $j = E[y|J]$), the impact of good or bad announcements is fully reflected in terms of interest rate. However, by remembering that ambiguity-averse investors prefer to view the distorted summary signal ($\gamma \neq 1$) rather than the disaggregate vector of information, variations in $E[y|J]$ are partly reduced by the component $E[y|j]$.

The idea behind this aspect is linked to the presence of two types of investors willing to pay for a generic sovereign asset. Each investor prefer different informative sources, with ambiguity-averse investors showing preferences for (distorted) summary signals such as rating grades. But, while rating grades contribute to partly reduce the perceived ambiguity, they have lower informative content than (undistorted) disaggregate signals. More formally, $var(E[y|J]) > \text{var}(E[y|j])$ when $\gamma \neq 1$.

In addition, this framework allows to suggest an intuitive explanation behind possible different gains experienced by Savage and ambiguity-averse investors in the market for sovereigns. Moreover, it permits to motivate the presence of rational incentives for Savage investors to provide summary information for ambiguity-averse decision makers. In this sense, it can be probably interpreted the increasing formal and informal sponsorship of rating activities operated by several public and private institutional investors in the last thirty years.

The starting point is now the introduction of the possibility for each investor of viewing different information sources at the same time (i.e. heterogeneous information case, using Caskey's terminology). As a result, both investors-type may rely upon the disaggregate informative vector J as well as the summary signal j. In this case, it can be imagined a population where some Savage investors $\theta_S \in [0,1]$ observe the disaggregate vector J; and, this information source is also viewed by a certain proportion $\theta_A \in [0,1]$ of ambiguity-averse decision-makers.

Therefore, the average precision in the market now becomes:

$$\bar{g} = \alpha\theta_S g(y|J) + \alpha(1-\theta_S)g(y|j,r) + (1-\alpha)\theta_A g_a(y|J) + (1-\alpha)(1-\theta_A)g_a(y|j,r) \quad (14)$$

where the first two terms in the right-hand side of (14) refer to Savage investors and the last two to ambiguity-averse investors. From our previous discussion on the value of the summary information in presence of ambiguity, and remembering that Savage investors prefer to observe the most precise available information $\theta_S \to 1$ (a result due to Blackwell, 1953), we can expect that Savage investors prefer to view the disaggregate vector J, while ambiguity-averse decision makers show preferences for the summary signal j.

The *ex ante* profits for each investor can be written as follows:

Ambiguity-neutral: $$\rho\, g(y|J)E[(E[y|J]-r)\,(y-r)] \quad (15a)$$

Ambiguity-sensitive: $$\rho\, g_a(y|j,r)E[(E[y|j,r]-r)\,(y-r)] \quad (15b)$$

Expressions (15a) and (15b) have been obtained by combining the individual maximization problem described in (5a) and (5b), together with equilibrium optimal quantities. A comparison between (15a) and (15b) is necessary in order to analyze possible divergent gains arising from diverse decision-makers. For simplicity, we ignore differences in investors' behavior concerning short or long term investment positions.

As a first step, we start to remark that $g_a(y|j,r) \leq g(y|J)$, given that $var(y|j,r) \geq var(y|J)$. Then, differences in profits between Savage and ambiguity-averse investors are determined by comparing the expected value of the terms inside the brackets. This implies that we shall consider the joint character of the following relations:

$$E[(E[y|J]-r)(y-r)] = \overbrace{\left(1-\beta_j\right)^2[var(E[y|J]) - var(E[y|j])]}^{\text{I}} + \left(\frac{1}{\rho\bar{g}}+\beta_x\right)\sigma_x^2 + \left(\frac{1}{\rho\bar{g}}\mu_x\right)^2 \quad (16a)$$

$$E[(E[y|j,r]-r)(y-r)] = \overbrace{-\left(1-\beta_j\right)\left(1-\frac{cov(y,r|j)}{var(r|j)}\right)\beta_j[var(E[y|J]) - var(E[y|j])]}^{\text{II}} + \frac{1-\tau}{\rho\bar{g}}\left(\frac{1}{\rho\bar{g}}+\beta_x\right)\sigma_x^2 + \left(\frac{1}{\rho\bar{g}}\mu_x\right)^2 \quad (16b)$$

with $\tau \in [0,1]$ denoting a composite function, which captures the influence of the disaggregate information and the interest rate for ambiguity-averse investors[8]. Equations (16a) and (16b) are similar to equations (25a) and (25b) presented in Caskey (2009) to which the interested reader can refer to for a more detailed discussion.

For our purposes we can concentrate on the first component of the above expressions remarked as I and II, ignoring the last two terms which represent the equivalent of gains from variations in sovereigns' value. Due to fact that $var(E[y|J]) \geq var(E[y|j])$, the above element I in (16a) results positive. On the contrary, noting that $\left(\frac{cov(y,r|j)}{var(r|j)}\right) < 1$, the component II in (16b) has a negative sign. Thus, Savage investors gain from viewing the (unbiased) disaggregate informative vector *J*, while ambiguity-averse investors suffer from preferring the (biased) aggregate report *j*, which is able to reduce their exposure towards ambiguity. Both investor-type, however, gain from variations in sovereigns' value: given the positive sign of the last two terms in both equations.

[8] More specifically, $\tau \equiv \frac{var(y|j)-var(y|J)}{var(v|j)-var(v|J)+\left(\frac{1/\rho}{\alpha\bar{g}(y|J)+(1-\alpha)\theta_A g_a(y|J)}\right)^2\sigma_x^2}$.

The main implication of this simple story can be linked to some regularities occurring in today's financial markets in a quite direct way. On the one hand, investors making their decisions on the basis of an undistorted set of available information regarding a generic sovereign exploit positive results. On the other hand, ambiguity-averse investors may experience losses given that they prefer distorted summary signals about sovereigns for counterbalancing the presence of ambiguity.

5. Conclusions

While banks have traditionally shown reluctance to assign formal ratings to countries, credit rating agencies have built part of their conspicuous businesses on this activity. By attaining the force of law in the form of rating-dependent regulations in international financial markets, sovereign ratings activities have been institutionalized into the informative framework available to investors, improving their credibility and reliability. This public investiture, furthermore, has been amplified by the proclivity of both media and policy-makers at international level.

This paper has tried to shed light on some regularities observable in today's financial markets by using a simple model of sovereign rating under ambiguity. To summarize, the following main results have been achieved: summary informative signals about sovereign assets, such as credit ratings, contribute to influence investment decisions of ambiguity-sensitive individuals. Rating-dependent regulations set at international level create a sort of *de jure* bias in favour of particular summary signals. When both Savage and ambiguity-averse investors are active, the latter may suffer from losses due to differences in the sources of information and the relevance of ambiguity in the system.

Some direct policy implications derive from this contribution. First, sovereign ratings need to be distinguished from other kinds of credit ratings such as corporate bonds or

structured financial instruments. Second, international financial regulations shall progressively reduce their overreliance on specific credit ratings in order limit the occurrence of distortions in the markets. Third, the reduction of the influence of prominent rating agencies' and the potential rise of systemic confidence (Welfens, 2008) can be pursued by making more transparent public information on countries' economic profiles periodically released by institutions like the European Commission, the IMF and the OECD.

These and other policy prescriptions have been progressively discussed at European level in recent times. In particular, since the informal EU ECOFIN in April 2002, European policymakers have adopted several regulations with the aim of supervising and monitoring credit rating agencies and their activities. More precisely, specific rules have been defined for sovereign ratings, like the *ex-ante* communication of a calendar with the dates for publication of sovereign ratings and the explicit prohibition of ratings announcements of a given group of countries made without providing individual country reports. The European Union, moreover, is currently involved in removing rating-dependent rules from its regulatory framework and limiting the overreliance of private actors on credit ratings. This paper has provided theoretical rationales for these policies.

Finally, some possible avenues for future research can be suggested. From a theoretical point of view, it would be worth promising to model ambiguity in a different way and try to find out similar (or different) results. A more articulated informative structure could provide further insights about the multifaceted role of sovereign ratings or other summary information inside the present financial decision-making environment. On the empirical side, it would be interesting to setting up an experiment, if it is possible, able to capture the main features of this model. In addition, other intriguing practical questions are linked to the detection of discordant profits when there are diverse financial portfolios with both Savage and ambiguity-

averse investors, or the comparison of different profits during periods of high uncertainty such as in the middle of a financial crisis. These and other questions are left for future research

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