Public Subsidy on Education, Welfare, and Wage Inequality in a Small Open Developed Economy: A Two-period Analysis

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29 November 2014
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(November 29, 2014)

ABSTRACT: As per the conventional wisdom there should be provision for public assistance for skill acquirement for achieving higher economic growth and improving relative wage inequality in the future. However, empirical observations on certain small OECD countries over the period 2000-2011 tell somewhat a different story. The present paper develops a simple two-sector, specific factor general equilibrium framework with endogenous skill formation and provision for public subsidy on education which shows that these atypical empirical observations are theoretically plausible and can possibly be used in examining the data empirically to uncover the reality. The analysis questions the desirability of the policy on the ground that it may not only aggravate inequality in both present and future periods but also does not necessarily improve intertemporal social welfare.

Keywords: Skill formation, Education Subsidy, Intertemporal Social Welfare, Wage Inequality, General Equilibrium.

JEL Classification: J31, I24, I28, D58.

* The author is thankful to Dr. Anindya Biswas, Spring Hill College, USA for providing him with the necessary empirical support. The usual disclaimer, however, applies.
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1. Introduction and motivation

The aspect of skill formation is extremely crucial in all countries irrespective of whether developed or developing.\(^1\) It promotes human capital formation through positive externalities which is conducive to economic growth and prosperity of an economy as well as has an egalitarian effect because it is likely to lower the difference in wages between the two groups of worker differentiated with respect to their skills. The conventional wisdom is that skill acquisition, given the size of the workforce, lowers the endowment of unskilled labour in the short run and raises the supply of skilled labour in the future. In the current period, given other things, national income falls and relative wages should move in favour of unskilled labour as the supply of this type of labour decreases although the endowment of skilled labour does not change. In the future unskilled workers going for skill acquisition in period 1 become skilled and therefore, the endowment of skilled labour goes up while that of unskilled labour does not change. National income must increase and the wage inequality again should improve because of increased supply of skilled labour. If these arguments are valid it logically follows that there should be a provision for public assistance for skill acquirement. In the post-reform regime, the need for this type of public assistance has become extremely critical in view of the empirical evidences of strong symmetrical wage movements against unskilled labour in different countries across the globe.\(^2,\(^3\)

\(^1\) Different facets of skill formation have been discussed in works like Autor (2014), Becker (1964), Brown et al. (2001), Crouch et al. (1999), Heckman and Krueger (2003), and Vanhuysse (2007).


\(^3\) In this connection, it is important to mention that Beyer et al.(1999), while studying the consequence of trade liberalization on wages in Chile, have found that an increase in the proportion of labour force with higher education exerted downward pressure to reduce the
In this connection, it is extremely important to present some empirical observations on certain small OECD countries over the period 2000-2011. Here earnings difference can possibly be considered as a proxy for wage inequality. On the other hand, from initial calculations carried out on available data for the period 2005-2010, we find that the simple correlation coefficients between GDP growth (annual %) and aggregate public spending on education (% of GDP) are negative for some small OECD economies.

In view of these atypical findings, there is urgent necessity to build up a simple theoretical framework, which can show that these observations are theoretically plausible and can readily be used in examining the data empirically to unearth the truth. This is exactly what has been done in the present paper. We have developed a two sector-three factor general equilibrium model with endogenous skill formation and provision for public subsidy designed at encouraging acquisition of expertise on the part of the unskilled workers for the purpose of our analysis. It is a two-period model where the number of people going for skill acquisition and hence the supply of unskilled labour in period 1 is endogenously determined from the intertemporal utility maximizing behaviour of the unskilled working families where the population size including skilled and unskilled workers remains the same over time. The question of skill acquisition on the part of skilled workers does not arise. Each unskilled working family in its maximization exercise takes the wage rates as datum. Although both the wages in period 2 change the family cannot foresee them as their decision for skill formation is taken in period 1. In the given circumstances, the consequences of the public subsidy provided in period 1 on the relative wage inequality in both the periods are examined. The effect of the policy on the intertemporal national income has also been scrutinized. We find that the subsidy aggravates the inequality in the current period if the high-skill sector is capital-intensive skilled-unskilled wage inequality. Therefore, as per their findings education had some equalizing effects on the wage gap in that country which fortifies the necessity of providing public incentives to skill acquisition.

We have basically incorporated endogenous skill formation in an otherwise Jones’ (1971) model.
and that the policy does not necessarily improve the inequality in the future period.\textsuperscript{5} Furthermore, there is no guarantee that the sum of discounted national incomes of the two periods increases.

The work deserves some attention because of two reasons. First, it provides a tractable structure that is capable of theoretically explaining the plausibility of the peculiar empirical findings as mentioned earlier. Second, the results of the analysis are interesting and can at least question the desirability of public assistance for skill formation especially when it may fail to ensure the desired effects on national welfare and wage inequality among the different sections of the working class. Nevertheless, these results and the appropriateness of the analytical structure are to be empirically examined in details. However, performing a rigorous econometric analysis of the available data of the countries, taking ‘terms-of-trade (TOT)’, ‘population size’ and ‘openness’ as control variables, is beyond the scope of the preset work. It is, therefore, left to future research in this area.\textsuperscript{6}

\textsuperscript{5} That the difference in distributive shares of capital between high-skill and low-skill sectors play a crucial role in determining the final outcome on the relative wage inequality resulting from any exogenous shock has been pointed out in many theoretical works like Chaudhuri (2004), Marjit and Kar (2005), Chaudhuri and Yabuuchi (2007), Beladi et al. (2008) and Chaudhuri (2008). However, Chaudhuri (2008) has found a special case where these distributive shares do not matter. Although there are a couple of papers in the literature like Kar and Beladi (2004) and Yabuuchi and Chaudhuri (2009) that have introduced skill formation in terms of a separate education sector in a static general equilibrium framework and have studied its impact on the relative wage inequality we do not come across any theoretical work that has analyzed the consequence of the skill-promoting subsidy policy in a dynamic set-up with endogenous skill formation and has simultaneously demonstrated the possibility of its failure in improving both wage inequality and social welfare.

\textsuperscript{6} The theoretical structure that we have developed here relies heavily on certain assumptions like given population size, TOT etc. In reality, however, the TOT index figures and population sizes do change over time and the degrees of openness of different countries to international trade vary overtime.
2. The Model and Analysis

We consider a small open economy with two sectors: low-skill (sector 1) and high-skill (sector 2). Sector 1 produces the export commodity \( X_1 \) with the help of unskilled labour \( L \) and capital \( K \) while sector 2 produces the import good \( X_2 \) by means of skilled labour \( S \) and capital. Markets are competitive and all the standard assumptions including CRS hold. While capital is perfectly mobile intersectorally unskilled labour and skilled labour are specific to sectors 1 and 2, respectively. The endowments of both skilled labour and capital are given exogenously while the aggregate supply of unskilled labour in the economy is endogenously determined. Commodity prices, \( P_i \)s are given by the small open economy assumption. Finally, commodity 1 is taken to be the numeraire.

The economy is endowed with \( L \) number of unskilled families each possessing 1 unit of labour. The supply of unskilled labour by each family is endogenously determined from its intertemporal utility maximizing behavior in period 1. Let \( W \) and \( W_s \) denote the existing unskilled and skilled wages, respectively. As \( W_s > W \) in period 1, a fraction of family labour, \( l_s \) goes for skill formation while the rest, \( (1-l_s) \) works in the unskilled labour market at the wage, \( W \). The family receives a government assistance (subsidy) designed at encouraging skill formation at the rate, \( s \) for the part of family labour that goes for skill formation. There are no direct costs of acquiring skills.\(^7\) The family consumes its entire income in each period. Hence, the question of savings or borrowing at the time of their decision-making (period 1) does not arise. The indirect (opportunity) cost of skill formation is \( (W - s) \). The endogenously determined aggregate supply function of unskilled labour in the economy in general form is given as follows.\(^8\)

\(^7\) One can introduce direct costs of acquiring skills which would not affect the basic results of the paper. For further details see the concluding section.

\(^8\) It has been derived in Appendix 1.
\[ L = L(W,W_s,s) \text{ with } \frac{\partial L}{\partial W} > 0; \quad \frac{\partial L}{\partial W_S} , \frac{\partial L}{\partial s} < 0. \] (1)

Besides, \( E_W = (\frac{\partial L}{\partial W} \frac{W}{L}) > 0 \); \( E_{W_S} = (\frac{\partial L}{\partial W_S} \frac{W_S}{L}) < 0 \); and, \( E_s = (\frac{\partial L}{\partial s} \frac{s}{L}) < 0 \) respectively, denote the elasticities of \( L(.) \) with respect to \( W, W_s \) and \( s \), respectively. We present the properties of the \( L(.) \) function in terms of the following proposition.\(^9\)

**Proposition 1:** Given other determinants, the aggregate supply of unskilled labour in the economy is an increasing function of unskilled wage and decreasing in both skilled wage and rate of subsidy on skill formation provided in period 1.

The reasons for the signs of the partial derivatives are obvious. An increase (a decrease) in unskilled wage, \( W \) (rate of subsidy, \( s \)) raises the opportunity cost of acquiring skills, \( (W - s) \) and hence leads to higher supply of unskilled labour by each family and vice versa. On the other hand, an increase in skilled wage, \( W_s \) raises the present discounted return from skill formation which in turn lowers the supply unskilled labour. Note that the number of unskilled working families, \( L \) is exogenously given and does not change over time.

The general equilibrium structure of the model consists of the following set of equations.

\[ W a_{L1} L_1 + r a_{K1} K_1 = 1 \] (2)

\[ W_s a_{S2} S_2 + r a_{K2} K_2 = P_2 \] (3)

\[ a_{L1} X_1 = L(W,W_s,s) \] (4)

\[ a_{S2} X_2 = S \] (5)

\[ a_{K1} X_1 + a_{K2} X_2 = K \] (6)

\(^9\) This result holds in period 2 as well. See proposition 4.
where \( a_{ji} \) denotes the per unit requirement of the \( j \)th input in the \( i \)th sector with \( j = L, S, K \) and \( i = 1, 2 \) while \( r \) is the return to capital. Besides, we define \( \theta_j = \left( \frac{w_j a_{ji}}{P_i} \right) \) as the distributive share of the \( j \)th input in the \( i \)th sector which would be used subsequently in stating the results.

Equations (2) and (3) are the two zero-profit conditions for the low-skill and high-skill sectors, respectively. Equations (4) – (6) are the full-employment conditions for unskilled labour, skilled labour and capital.

Using equations (7) and (8), equation (9) can be rewritten as follows.

\[
\frac{a K_1}{a L_1} L \left( W, W_s, s \right) + \frac{a K_2}{a S} S = K
\]

(6.1)

This is an indecomposable production system. Factor prices, \( W, W_s \) and \( r \) are determined simultaneously from equations (2), (3) and (6.1) as functions of the system parameters including \( s \). Then \( X_1 \) and \( X_2 \) are obtained from equations (4) and (5), respectively as \( a_{ji} \)s are now known.

As \( W_s > W \), the absolute wage gap is \( (W_s - W) \) and the relative wage inequality is given by \( (\hat{W}_s - \hat{W}) \) where Here ‘\( \wedge \)’ implies proportional change e.g. \( \hat{W}_s = \left( \frac{dW_s}{W_s} \right) \). The skilled–unskilled wage gap improves (worsens) in absolute terms if the gap between \( W_s \) and \( W \) falls (rises). On the other hand, the wage inequality improves (deteriorates) both in absolute and relative terms if \( (\hat{W}_s - \hat{W}) < (>)0 \).
2.1. Results on wage inequality

Differentiating equations (1) – (3) and (6.1), using (1) and simplifying the following two propositions can be ascertained.

**Proposition 2:** An increase in the subsidy designed to encourage skill formation lowers the supply of unskilled labour in the economy if

\[
\frac{E_{W}}{E_{W_s}} \geq \left( \frac{\theta_{l1} \theta_{K,2}}{\theta_{K1} \theta_{S2}} \right)^{10}.
\]

**Proposition 3:** The relative wage inequality worsens (in period 1) due to the subsidy policy if

\[
\frac{E_{W}}{E_{W_s}} > \left( \frac{\theta_{l1} \theta_{K,2}}{\theta_{K1} \theta_{S2}} \right).
\]

On the contrary, it improves if

\[
\frac{E_{W}}{E_{W_s}} < \left( \frac{\theta_{l1} \theta_{K,2}}{\theta_{K1} \theta_{S2}} \right).
\]

Propositions 2 and 3 can intuitively be explained in the following fashion. If the subsidy rate, \(s\), increases there would be a direct negative effect on \(L(.)\). As the supply of unskilled labour initially falls, the unskilled wage, \(W\), rises and the low-skill sector (sector 1) contracts releasing capital to the high-skill sector (sector 2). Consequently, sector 2 expands and demands more skilled labour that raises the skilled wage, \(W_s\). So, both \(W\) and \(W_s\) increase which produce two opposite induced (indirect) effects on \(L(.)\). The supply of unskilled labour rises as \(W\) rises while it falls as \(W_s\) increases. Therefore, there are three different effects on \(L(.)\). Our analysis shows that the sum of the two negative effects dominates over the positive effect of an increase in \(W\) and the net effect would be a fall in the aggregate supply of unskilled labour in the economy under the sufficient condition that

\[
\frac{E_{W}}{E_{W_s}} \geq \left( \frac{\theta_{l1} \theta_{K,2}}{\theta_{K1} \theta_{S2}} \right).
\]

We have already noted that both \(W\) and \(W_s\) have increased and sector 1 (sector 2) has contracted (expanded). The contracting sector 1 releases capital to sector 2. However, as in our model capital is fully utilized, the released capital by sector 1 would be absorbed by sector 2 only if its rate of return, \(r\), falls. Thus, \(r\) plummets which is also clear from the two

\[10\] This result also holds in period 2. See proposition 4 in this context.
zero-profit conditions (equations 2 and 3). The fall in \( r \) leads to saving on capital cost in both the sectors. So, the proportions of increase in \( W \) and \( W_s \) depend on the extent of saving on capital costs in the two sectors which in turn depends on their distributive shares of capital, \( \theta_{ki} \). If \( \theta_{k2} > (<) \theta_{k1} \) i.e. if \( \theta_{l1} \theta_{k2} > (<) \theta_{k1} \theta_{s2} \), the saving on capital cost would be higher in sector 2 (sector 1) vis-à-vis that in sector 1 (sector 2) and the relative wage inequality worsens (improves).

**Period 2**

We consider the same broad production system in period 2 also. It has already been assumed that the aggregate workforce (skilled plus unskilled) i.e. \((L + S)\) does not change over time although their composition changes due to skill formation. In period 2 unskilled labours that went for skills acquisition (and not working in the unskilled labour market) in period 1 becomes skilled labour. So, the endowment of skilled labour increases from \( S \) (in period 1) to \((\bar{S} = S + L - L,(.)\) (in period 2) while that of unskilled labour remains the same, \( L,(.) \). As in our indecomposable production structure all factor prices depend on factor endowments apart from commodity prices, \( W, W_s \) and \( r \) would take different values in equilibrium and so would be the values of \( a_{ji} \)s, and \( \theta_{ji} \)s in period 2. In order to avoid complications in the use of notations we leave them unchanged.

Equations (5) and (6.1) would now have to be replaced by the following.

\[
a_{s2} X_2 = \bar{S} \quad (5.1)
\]

\[
\frac{a K_1}{a L_1} \left( W, W_s + s \right) + \frac{a K_2}{a S_2} \bar{S} = K
\]

\[
\bar{S} = S + (L - L,(.))
\]

where,

Differentiating equations (1) – (3) and (6.2), using (1) and simplifying the following two propositions follow.
**Proposition 4:** Propositions 1 and 2 continue to hold even in period 2.

**Proposition 5:** In period 2 the skilled-unskilled wage inequality improves under all situations except when \( \theta_{L1} \theta_{K2} < \theta_{K1} \theta_{S2} \) and the \( \frac{W_S}{W} \) ratio is sufficiently high.

Although the intuitive explanations of proposition 4 remain the same as before those of the remaining one would be somewhat different.\(^{11}\) These are as follows. First, let us see what happens to the wage inequality in period 2 resulting from skill formation in period 1. Given the rate of skills subsidy, \( s \), (or in the absence of any subsidy) the skilled wage, \( W_S \), falls as the supply of skilled labour, \( \overline{S} \), rises following the joining of skill-acquired unskilled labour in the army of skilled workers in period 2. This raises the return to capital, \( r \) (equation (3)) which in turn lowers the unskilled wage, \( W \) (equation (2)). Hence, in complete contrast to our findings in period 1, we here find that both the wages fall. The working class irrespective of whether skilled or unskilled becomes worse-off. In each of the two sectors the cost on capital has increased. What happens to the relative wage inequality now depends on the difference in intersectoral distributive shares of capital i.e. \( \theta_s \). The higher the value of \( \theta_s \) the higher would be the fall in the wage rates. Thus, given the rate of skills acquiring subsidy, \( s \) (\( \geq 0 \)) the wage inequality rises due to skill formation if \( \theta_{K1} > \theta_{K2} \). Let us now consider the situation where \( s > 0 \) and \( s \) rises. \( L \) falls in both the periods while \( \overline{S} \) rises even further. Sector 1 contracts while sector 2 expands as unskilled labour and skilled labour are specific to those sectors, respectively. Consequently, the expanding sector 2 demands more capital while the contracting sector 1 releases capital. If \( \frac{aK2}{aS2} > \frac{aK1}{aL1} \), the net effect would be an increase in demand for capital that raises its return, \( r \). Both \( W \) and \( W_S \) decrease.

When \( \theta_{L1} \theta_{K2} > (<) \theta_{K1} \theta_{S2} \), i.e. when \( \theta_{K2} > (<) \theta_{K1} \) the proportionate fall in \( W_S \) would be greater (less) than that in \( W \). Hence the wage inequality improves (worsens). However, if

\(^{11}\) See verbal explanations provided for propositions 1 and 2.
\( \frac{a_{K2}}{a_{S2}} < \frac{a_{K1}}{a_{L1}} \), the net effect would be a fall in the demand for capital that leads to a decrease in \( r \).\(^{12}\) Both the wage rates increase. In this case, the wage inequality improves due to the subsidy if \( \theta_{L1} \theta_{K2} < \theta_{K1} \theta_{S2} \).

2.2. Education subsidy and national welfare

We assume that the subsidy for skill formation is financed by non-distortionary lump-sum taxes on factor incomes. The national income at world prices in the \( t \)th period is then given by

\[
Y^t = X_1^t + P_2 X_2^t
\]  

for \( t = 1, 2 \).

We here consider the discounted sum of national incomes of the two periods as the measure of social welfare in this small open economy which is written as follows.

\[
Y = Y^1 + \delta Y^2
\]

where \( \delta (< 1) \) is the time discount factor.

Differentiating equations (7), (8) and the production functions the following proposition can be easily established.\(^{13}\)

\(^{12}\) The net demand for capital falls in period 2 in the case where \( \frac{W}{W} > 1 \) is very low and close to unity. But, in the opposite case we always have \( \frac{a_{K2}}{a_{S2}} > \frac{a_{K1}}{a_{L1}} \) since \( W_s > W \). So, the demand for capital cannot fall in this situation.

\(^{13}\) This has been proved in Appendix 3.
Proposition 6: The subsidy to encourage skill formation does not necessarily improve intertemporal social welfare.

Let us now intuitively explain proposition 6 as follows. We have already explained why the supply of unskilled labour, $L(\cdot)$, which remains the same in both periods, is a decreasing function of the rate of subsidy, $s$ if \[
\frac{E_{w_1}}{E_{w_1}} \geq \left(\frac{\theta_{11} \theta_{32}}{\theta_{21} \theta_{32}}\right) \geq 1.
\] If $s$ rises some amount of unskilled labour goes out of the production activities in period 1 while the endowments of other factors of production remain the same. Quite naturally, the national income in period 1 decreases. In period 2 also that amount of unskilled labour is lost but exactly the same amount of skilled labour is added to the economy’s resources. As the skilled wage, $W_s$ is greater than the unskilled wage, $W$, national income in period 2 unambiguously rises. However, what happens to the intertemporal national income, $Y$ must, therefore, depend on the two wages and the discount factor. If the sum of discounted unskilled wages in the two periods, $(W^1 + \delta W^2)$ is greater (lower) than the discounted skilled wage, $\delta W^2_s$ intertemporal welfare worsens (improves). This clearly depends on the technological and trade-related parameters and the discount factor.

5. Concluding remarks

This paper has developed a two-sector, specific factor general equilibrium model with endogenous skill formation and education subsidy assisting skill acquisition with an eye to examine the theoretical plausibility of some recent empirical evidences on certain small OECD countries that suggest that public assistance for skill formation may not lead to higher economic growth and lower wage inequality in the future. There are two types of labour, skilled and unskilled, where unskilled workers have the opportunity to go for skills acquisition, become skilled and earn a higher wage in future. The aggregate supply of
unskilled labour in the economy is determined from the intertemporal utility maximizing-behaviour of the unskilled working families in the current period (period 1). The unskilled workers after acquiring skills join the army of skilled labour in future thereby increase the endowment of skilled labour. The aggregate size of the workforce is assumed to remain unchanged over time. A public subsidy designed to promote skill formation lowers the supply of unskilled labour in both the periods but raises that of skilled labour in future. The analysis has found that an increase in the public subsidy raises the skilled-unskilled wage inequality in the present period if the high-skill sector is capital-intensive (in the Jones-Neary sense) and may accentuate inequality even in period 2 under certain parametric restrictions. In period 1 both the wages increase while the return to capital falls; thereby leading to a redistribution of income in favour of the working class although the income inequality may become more uneven among the two groups of the working population differentiated according to skills. However, in period 2 both the wages fall while the return to capital increases except in a special case that leads to a redistribution of income against the working class although their inter-group inequality is most likely to diminish. On the other hand, we have also demonstrated that the policy does not necessarily improve intertemporal social welfare. Hence, the policy may fail to deliver the goods from the perspectives of both welfare and distribution.

Our analysis may be criticized on the ground that some important issues like labour market imperfection especially those of unskilled labour, unemployment problem, efficiency wage considerations, and collective bargaining have not been captured. Furthermore, we do not take into account savings and direct costs associated with skill acquisition and its financing problem. Hence, the role of capital market imperfection has not been taken care of. The absence of any non-traded skill-intensive sector like services is also a limitation of the analysis. If some of these features are taken into consideration the results of the model may hold subject to one or two additional sufficient conditions. In defense, we can modestly argue that these are some of the salient features of the developing countries and that our structure is more appropriate for a small developed country rather than a developing one.

14 See Jones and Neary (1984).
Finally, it should be clearly mentioned that although the analytical framework that we have developed can theoretically explain the plausibility of the recent empirical observations on certain small OECD countries which are contrary to the common wisdom a rigorous econometric analysis of the data has to be carried out in future in order to judge the aptness of our theoretical structure. This has been left to future empirical research because it is beyond the scope of this paper. However, despite simplicity and abstraction the present theoretical analysis deserves some attention because it questions the desirability of providing public subsidy to assist skill formation not only from the perspective of improving social welfare but also from the viewpoint of lowering the relative wage inequality among the different sections of the working population.

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