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Positional Preferences, Endogenous Growth, and Optimal Income- and Consumption Taxation¹

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Abstract. This paper analyzes the impact of positional preferences, exhibiting conspicuous consumption and conspicuous wealth, on optimal consumption- and income taxes, for an endogenous growth model with public capital. Positional preferences raise the endogenous growth rate if the elasticity of intertemporal substitution is larger than one. Even if labor supply is exogenous, the consumption externalities introduce distortions so long as preferences are *wealth-dependent*, and with or without the presence of conspicuous wealth. Consequently, optimal consumption- and income taxes differ from zero. Numerical simulations present the effects of fiscal policy on the balanced growth path and transitional dynamics.

Keywords and Phrases: Conspicuous consumption, conspicuous wealth, endogenous growth, public capital, optimal consumption tax

JEL Classification Numbers: D62, D91, E21, H21, O41

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1. Introduction

This paper analyzes the impact of positional preferences, exhibiting conspicuous consumption and conspicuous wealth, on optimal consumption- and income-taxes, in the context of an endogenous growth model.

Positional preferences refer to the situation in which consumption and wealth of an individual have a direct effect on the utility of other individuals.² Such preferences were already studied by ancient philosophers,³ and more recently by political philosophers and classical economists such as Adam Smith or Thorstein Veblen.⁴ Meanwhile, a large body of literature has established significant empirical evidence for positional preferences.⁵

Within a growth set-up, our consumers' utility thus depends not only on the level of their own consumption but also on how their consumption *compares* to some standard, which is referred to as the *reference level*. There are two specific versions of this: the consumer's reference level could be his own past consumption, or it could be the consumption of others, *a la* Duesenberry (1949). These two versions are referred to as the *inward-* and *outward-looking* versions, respectively, by Carroll et al. (1997). The inward version is adopted by Carroll et al. (2000), Monteiro et al. (2013) following Ryder and Heal (1973), etc., while the outward version is considered by Liu and Turnovsky (2005), Turnovsky and Monteiro (2007), Wendner (2011), among others. Finally, both types of consumption reference levels are

² Different terms for *positional preferences* have been used in the literature, with slightly differing meanings. They include status preferences, status consumption, conspicuous consumption, conspicuous wealth, relative consumption, relative wealth, keeping up/catching up with the Joneses preferences, jealousy/envy, external habits, or simply consumption externality. In this article, we use these terms synonymously, though we focus on the relative wealth aspect.

³ That social distinction or status is an important motivation of human (and non-human) behavior. was already shown by Darwin (1871), who emphasized *sexual* selection besides *natural* selection: "To spread across the population, genes of sexual species not only need to survive in their natural and social environment, but also need to be or appear a more attractive mating partner than their same sex competitors." (Truyts, 2010, p.137). Clearly, Darwin was not the first to think about social distinction. Philosophers have thought about the issue of social distinction more than 2400 years ago. Plato, in his *The Republic* (Book II) wrote: "Since … appearance tyrannizes over truth and is lord of happiness, to appearance I must devote myself." This passage astoundingly resembles Darwin's argument on sexual selection.

⁴ Adam Smith (1759) devoted a section to conspicuous consumption in his *Theory of Moral Sentiments*. "The poor man's son … when he begins to look around him, admires the condition of the rich. He finds the cottage of his father too small … It appears in his fancy like the life of some superior rank of beings, and, in order to arrive at it, he devotes himself for ever to the pursuit of wealth and greatness." (Smith, 1759, p. 181). Likewise, in his Theory of the Leisure Class, Veblen (1899) emphasized the quest for status – via conspicuous consumption and wealth – as an important component of the pursuit of self-interest. Veblen, thus, argues: "Conspicuous consumption of valuable goods is a means of reputability to the gentleman of leisure." (Veblen, 1899, p. 64).

⁵ The literature emphasizes three approaches: survey experimental studies (Johansson-Stenman et al. 2002, 2006, Solnik and Hemenway 1998, 2005), econometric studies (Ferrer-i-Carbonell 2005, Luttmer 2005), and revealed preference studies (Glazer and Conrad 1996). An excellent review is provided by Truyts (2010).

considered by Carroll et al. (1997), Alvarez-Cuadrado et al. (2004), and Alonso-Carrera et al. (2005). In this article, we adopt the second of the two specifications, as we are more interested in the externality aspects of consumer behavior (and, to a lesser extent, producer behavior), and issues about how far tax/subsidy policies in a decentralized economy can replicate the social optimum, and it is clear that only in the 'outward' case is the reference level an externality (as such an agent ignores the effect that his own consumption has on utility via the average consumption).

Importantly, in our paper, wealth in the form of capital is an argument in agents' welfare function, as in Zou (1994, 1995), Corneo and Jeanne (1997), Futagami and Shibata (1998), Nakamoto (2009), among others, which creates a 'wealth externality' effect.⁶ We show in our paper that higher positional preferences via conspicuous consumption and conspicuous wealth have a direct and positive impact on the endogenous growth rate, providing the intertemporal elasticity of substitution is less than 1.⁷ Moreover, we demonstrate that if wealth is present in the consumer's utility function.⁸ then the consumption externality does have a distortionary effect, even if labor supply is exogenous. For plausible parameter values, the decentralized economy here achieves a lower growth rate compared to what could be attained via a notional central planner,⁹ and consequently we characterize the optimal fiscal policy that would enable the first-best to be attained, as in Liu and Turnovsky (2005). Given that here the private return on capital falls below its socially optimal return, a positive tax on consumption helps offset this deviation. As positional concerns for consumption rise, both optimal growth and welfare rise, which necessitates an even lower income tax rate and a higher consumption tax rate for the decentralized economy: this, together with the higher complementary public spending, raises the growth rate and also improves welfare.

⁶ The initial idea is due to Zou (1994), who argues that the incentive for accumulating capital lies not only in maximizing long-run consumption, but also to increase wealth, which in itself adds to agents' utility. Zou's model, in turn, is based on 'the theory of the spirit of capitalism' by Weber (1958), and the mathematical model of Kurz (1968). By adding a 'cultural' dimension to existing models, his set-up is able to embody all the contributions of both traditional and new growth theories.

⁷ In Futagami and Shibata (1998), if all consumers are identical, the long-run balanced growth rate is positively related to the degree of status preference (but this may not hold with heterogeneous agents).

⁸ Cole et al. (1992) consider status preferences affecting capital accumulation, but do not include status directly in the utility function.

⁹ Likewise, in Liu and Turnovsky (2005), Section 6, where endogenous growth – but not via public capital – is considered, a positive production externality leads to the decentralized growth rate falling short of the socially optimal rate (with inelastic labour). Here, consumption externalities affect the magnitude of the distortion caused by the production externality. By contrast, within an endogenous growth set-up, Corneo and Jeanne (1997) show that the quest for status may result in a competitive economy growing too fast relative to the social optimum if the marginal status utility of relative wealth exceeds a certain threshold.

It is important to compare and contrast our findings with those of other studies on the topic in the context of a key result that emerges from much of the related literature: in the absence of a labour-leisure choice, a consumption externality does not have any impact on the steady state equilibrium of a decentralized economy in a neoclassical growth model (see, for example, Rauscher (1997), Fisher and Hof (2000), Liu and Turnovsky (2005)).¹⁰ If, however, as in Turnovsky and Monteiro (2007), there is also a positive (negative) production externality, then the steady-state equilibrium capital stock and output are below (above) their respective optimal levels, while the equilibrium output-capital ratio is too high (low), so a consumption externality does introduce distortions in the presence of a production externality in a growing economy.¹¹ In Carroll et al. (1997), who consider a simple Rebelo-type AK technology, the more individuals care about how consumption compares to the reference level, and the less they care about the absolute level of consumption, the higher will be the growth rate of consumption in the steady state. The introduction of consumption externalities leads to the economy approaching its balanced growth equilibrium along a transitional path, this sluggishness in adjustment being caused by the consumption externality (i.e., in spite of the AK technology). However, as demonstrated by Alvarez-Cuadrado et al. (2004), the transitional adjustment paths may exhibit non-monotonic behaviour if the production function is neoclassical rather than AK-type. This is because then the transitional dynamics are governed by two opposing forces: one generated by preferences (the status effect) and the other by technology (diminishing returns to capital). It is important to note that in our framework, a consumption externality of the type considered above does not have a distortionary effect on the economy, regardless of the presence of the production externality; however, the presence of wealth or a wealth externality alters the situation.

In models with consumption externalities but with elastic labour supply, the decentralized economy diverges from the social optimum in the long-run, as in Dupor and Liu (2003), Liu and Turnovsky (2005), and Turnovsky and Monteiro (2007). In the first of these papers, an increase in aggregate consumption may raise the marginal utility of individual consumption relative to leisure when others consume more. At the same time, higher per capita consumption (holding individual consumption fixed) can trigger jealousy (admiration) so that individual utility falls (rises). In Liu and Turnovsky (2005), if labour supply is elastic, a

¹⁰ With exogenous technical change, the consumption externality - by affecting the elasticity of marginal utility of consumption - does impact on the equilibrium (see Wendner, 2011). For a similar result, but with the reference level comprising current and past consumption, see Alvarez-Cuadrado et al. (2004).

¹¹ The same is true in Liu and Turnovsky (2005) when a production externality is introduced.

negative (positive) consumption externality leads to over- (under-) consumption and over-(under-) supply of capital and labour, relative to the optimum, in the steady state. With endogenous labour supply, in Turnovsky and Monteiro (2007), the consumption externality affects the steady state even in the absence of any production externality. This is because it affects the marginal valuation of consumption, which in turn changes the optimal utility value of the marginal product of labour. Thus, consumption distortion results in distortion in the labour-leisure trade-off and therefore creates production inefficiency. In our paper, labour supply is inelastic, and there is a production externality, but the decentralised equilibrium differs from the social optimum mainly because agents derive utility from their own wealth relative to a wealth reference level (in addition to a consumption externality), necessitating the use of corrective income and consumption taxes mentioned earlier. In this respect, our paper comes closest perhaps to Nakamoto (2009), where also labour supply is inelastic. However, despite this, in both set-ups the distortionary effect of consumption externalities persists in the long-run because of wealth preferences. A key difference between Nakamoto (2009) and our paper is that ours is an endogenous growth model where output is produced by public (in addition to private) capital, while he considers a neoclassical growth model.

As is clear from the discussion above, an important aspect of papers examining externality issues in consumption (and production) is to study optimal fiscal policy, i.e., to devise appropriate tax/subsidy policies that enable the decentralized economy to replicate the social optimum. In Alonso-Carrera et al. (2005), a consumption externality makes the decentralized equilibrium allocation inefficient, which can be corrected by either a consumption tax or an income tax. If consumers' willingness to shift current consumption to the future is suboptimally low (high), then optimal fiscal policy consists of either a decreasing (an increasing) sequence of consumption taxes or a subsidy (tax) on income/output. On the other hand, in Nakamoto (2009), the reason for the decentralized outcome to differ from the first-best is due to wealth preference: when households feel jealousy (admiration) about others' consumption, the long-run levels of consumption and the capital stock are lower (higher) than the social optimum, calling for a positive (negative) consumption tax and a negative (positive) income tax. In our case, if wealth is an argument in the utility function, and providing the desire to raise consumption is different from the desire to increase saving (wealth), the optimal consumption and income tax rates differ from zero even if government spending is chosen optimally. In the case where the private return on capital falls below its socially optimal return, a positive tax on consumption helps offset this deviation; consequently, a larger weight on consumption relative to the average results in even larger divergence between private and social returns, and therefore calls for an even larger value of the consumption tax rate to compensate.

The value added of our paper arises also from the five fiscal policy experiments that we conduct in the penultimate section, and their effects on the economy along the balanced growth as well as transition paths. Three of those experiments involve an increase in public capital spending financed by lump-sum, income or consumption taxes, while the remaining two are about an income and consumption tax increase, respectively (without a corresponding spending increase). Our results indicate that public spending positively affects both growth and welfare in the steady state, and does so quite strongly, and so demonstrates that the production externality clearly dominates the consumption externalities in this regard. The latter is reflected also in the 'decisive' way in which some of the key variables adjust along the transition path in response to the first three fiscal shocks. In addition, it can be observed that for all the policy experiments considered, the transitional paths of all the important variables are monotonic.

The rest of the paper is organized as follows: Section 2 develops the model, emphasizing the preference structure and characterizing the steady state. In Section 3 we derive the social optimum, identify the fiscal policies that would enable the decentralized economy to replicate the first-best scenario, and link this with growth and welfare. In Section 4, the growth and welfare effects of five fiscal policy shocks are studied, both along the balanced growth path and in transiting from one steady state to another. Finally, Section 5 concludes the paper.

2. The Model

We consider a dynamic general equilibrium model of a closed economy that allows for fully endogenous growth. Time is considered to be continuous. The source of endogeneity of growth is a public good, public capital K_g , that serves as an input to production. There is a large number of households and firms, the respective number of which we normalize to unity. Households are homogeneous and exhibit positional preferences. They derive utility not only from own consumption but also from own consumption *relative to some consumption reference level*, and from own wealth *relative to some wealth reference level*.

2.1 Preferences

The representative household has preferences for consumption, relative consumption, and relative wealth. Relative consumption is given by individual consumption relative to some consumption reference level, $\overline{C}: C/\overline{C}$. As households are homogeneous in our framework, we consider the economy's average consumption level as a natural choice for a household's consumption reference level.¹² By the same token, relative wealth is given by individual wealth relative to the average wealth in the economy, $\overline{K}: K/\overline{K}$. That is, conspicuous consumption (CC, in the following) is captured by a relative consumption term, and conspicuous wealth (CW, in the following) is captured by a relative wealth term in the instantaneous utility function. Both \overline{C} and \overline{K} are considered exogenous by individual households.

The instantaneous utility function is given by:

$$u\left(C,K,\frac{C}{\overline{C}},\frac{K}{\overline{C}}\right) = C^{1-\eta_c}\left(\frac{C}{\overline{C}}\right)^{\eta_c}\left(K^{1-\eta_k}\left(\frac{K}{\overline{K}}\right)^{\eta_k}\right)^{\xi} = C\overline{C}^{-\eta_c}K^{\xi}\overline{K}^{-\eta_k\xi}, \ 0 \le \eta_i < 1, \ i \in \left\{c,k\right\}, \ \xi \ge 0$$
(1)

where parameters η_i represent marginal degrees of positionality (Johansson-Stenman et al. 2002). A marginal degree of positionality reflects the share of marginal utility of individual consumption (or wealth) that is due to the fact that own consumption (or wealth) raises the ratio C/\overline{C} or K/\overline{K} ceteris paribus. There is robust empirical evidence that $\eta_i > 0$ with estimates found in the range of [0.2, 0.8] (cf. Johansson-Stenman et al. 2002, Solnik and Hemenway 1998, 2005, Wendner and Goulder 2008). Parameter ξ indexes the strength of CW. If $\xi = 0$, the household does not exhibit positional preferences with respect to wealth. However, if $\xi > 0$, the household's preferences exhibit CW in addition to CC.

The intertemporal utility function is given by:

¹² In a model with heterogeneous households, a household's consumption reference level may be specified quite more generally (cf. Eckerstorfer and Wendner 2013).

$$U\left(C,K,\frac{C}{\overline{C}},\frac{K}{\overline{K}}\right) = \int_0^\infty \frac{1}{\gamma} u\left(C,K,\frac{C}{\overline{C}},\frac{K}{\overline{K}}\right)^\gamma e^{-\beta t} dt = \int_0^\infty \frac{1}{\gamma} \left[C\overline{C}^{-\eta_c}K^{\xi}\overline{K}^{-\eta_k\xi}\right]^\gamma e^{-\beta t} dt, -\infty < \gamma \le 1$$
(2)

The household has a constant rate of time preference $\beta > 0$ and an instantaneous CRRA utility function with absolute elasticity of marginal utility of consumption equal to $(1-\gamma)$. Facing given market prices, reference levels \overline{C} and \overline{K} , and equipped with perfect foresight the household chooses a plan $\{C(t)\}_{t=0}^{\infty}$ so as to

$$\max U\left(C, K, \frac{C}{\overline{C}}, \frac{K}{\overline{K}}\right)$$

s.t.
$$\dot{K} = (1 - \tau_{y})Y - (1 + \tau_{c})C - T - \delta_{k}K,$$

$$\lim_{t \to \infty} K_{t}e^{-\int_{0}^{t} r_{s}ds} \ge 0.$$
(3)

The first constraint in (3) is the household's flow budget constraint, where, τ_y and τ_c are respectively the income- and consumption tax rate, and *T* denotes lump sum taxes. In our framework, the labor-leisure decision is exogenous. Under standard assumptions (in the absence of CC and CW), the *optimal* consumption tax is nil. Below, we are interested in the mechanisms affecting the optimal consumption tax rate in the presence of CC and CW.

The second constraint in (3) is the No-Ponzi-Game condition. In equilibrium, the transversality condition requires the No-Ponzi-Game condition to hold with strict equality.

2.2 Technology

A homogeneous output, Y, is produced by private and public capital using a CES technology:

$$Y = A \left[\alpha K^{-\rho} + (1 - \alpha) K_g^{-\rho} \right]^{-1/\rho}, \ 0 < \alpha < 1, \ -1 < \rho < \infty,$$
(4)

where K denotes private capital. The elasticity of substitution between private capital and the public good is given by $1/(1+\rho)$. To ensure positivity of growth rates along the BGP (see below), we assume

$$\alpha A - \delta_k > \beta . \tag{A.1}$$

The assumption roughly implies that the rate of interest strictly exceeds the rate of time preference.

The public good evolves according to:

$$\dot{K}_{g} = G - \delta_{g} K_{g}, \quad G = gY, \quad 0 < \delta_{g} < 1, \tag{5}$$

where G represents the flow of public expenditures for public capital and δ_g is the rate of depreciation of public capital. The flow of public expenditures is a fixed share g of output.

Let C denote aggregate consumption. As we consider a closed economy, the aggregate resource constraint is given by:

$$\dot{K} = Y - C - G - \delta_{\nu} K, \tag{6}$$

where δ_k is the rate of depreciation of private capital.

2.3 Macroeconomic Dynamics and the Steady State

Let the current-value Hamiltonian be given by:

$$H = \frac{1}{\gamma} \left[C \overline{C}^{-\eta_c} K^{\xi} \overline{K}^{-\eta_k \xi} \right]^{\gamma} + \lambda \left[(1 - \tau_y) Y - (1 + \tau_c) C - T - \delta_k K \right], \tag{7}$$

where *C* is a control variable, *K* is a state variable, and λ is a costate variable. An interior solution satisfies the following necessary first-order conditions.

$$\frac{\partial H}{\partial C} = C^{\gamma-1} \overline{C}^{-\eta_c \gamma} K^{\xi \gamma} \overline{K}^{-\eta_k \xi \gamma} - \lambda (1+\tau_c) = 0$$
(8)

$$\frac{\partial H}{\partial K} = \xi C^{\gamma} \overline{C}^{-\eta_{c}\gamma} K^{\xi\gamma-1} \overline{K}^{-\eta_{k}\xi\gamma} + \lambda \left[(1-\tau_{y}) Y_{K} - \delta_{k} \right] = \beta \lambda - \dot{\lambda} , \qquad (9)$$

where Y_K is the partial derivative of Y with respect to K.

Ex post, as households are homogeneous, $\overline{C} = C$ and $\overline{K} = K$. The first-order conditions then imply:

$$C^{\gamma(1-\eta_c)-1}K^{\xi\gamma(1-\eta_k)} = \lambda(1+\tau_c) , \qquad (10)$$

$$\xi C^{\gamma(1-\eta_c)} K^{\xi\gamma(1-\eta_k)-1} + \lambda \left[(1-\tau_y) Y_K - \delta_k \right] = \beta \lambda - \dot{\lambda} .$$
⁽¹¹⁾

Next, we define $c \equiv C / K$, $y \equiv Y / K$, $z \equiv K_g / K$. Considering (10) in (11) yields

$$\xi c(1+\tau_c) + (1-\tau_y)Y_K - \delta_k = \beta - \frac{\dot{\lambda}}{\lambda} .$$
⁽¹²⁾

Differentiating gives

$$(\gamma(1-\eta_c)-1)\frac{\dot{C}}{C} + (\xi\gamma(1-\eta_k))((1-g)y-c-\delta_k) = \frac{\dot{\lambda}}{\lambda}, \qquad (13)$$

where we took (6) into account. (13) in (12) and considering $\dot{c} / c = \dot{C} / C - \dot{K} / K$ yields:

$$\frac{\dot{c}}{c} = \frac{(1-\eta_k)\xi\gamma((1-g)y-c-\delta_k)+\xi(1+\tau_c)c+(1-\tau_y)Y_K-\beta-\delta_k}{1-(1-\eta_c)\gamma} - (1-g)y+c+\delta_k, \quad (14)$$

where (4) implies that $y = A \left[\alpha + (1 - \alpha) z^{-\rho} \right]^{-1/\rho}$, and $Y_{\kappa} = \alpha A^{-\rho} y^{1+\rho}$. Finally, from (5) and (6), it follows:

$$\frac{\dot{z}}{z} = g \frac{y}{z} - \delta_g - (1 - g)y + c + \delta_k \,. \tag{15}$$

Differential equations (14) and (15) represent the model's two-dimensional dynamic system in the dynamic variables c and z.

The economy will, in a steady state, follow a balanced growth path (BGP). Along the BGP $\dot{c} = \dot{z} = 0$, and *C*, *K*, K_g and *Y* grow at the same constant endogenous growth rate.

In the following, we employ the following parameter restriction, which ensures positivity of (c,z) along the BGP.

$$Ag(1-\alpha)^{-1/\rho} > \delta_g \ge Ag.$$
(A.2)

Proposition 1. (Existence and Stability)

(1) Assume (A.1) and (A.2). Then, a non-trivial steady state (c,z) exists and is unique. The steady state is associated with a BGP along which C, K, K_g and Y grow with the constant growth rate $\Gamma = g y/z - \delta_g$.

(2) The unique steady state is a saddle point and is saddle-point stable.

Proof. See the Appendix.

Parameter restrictions (A.1) and (A.2) are sufficient, not necessary, for a steady state to exist. In fact, as shown in the Appendix, 0 < z < 1 along the BGP. Assumption (A.1) requires the rate of interest to exceed the pure rate of time preference at z = 1. Assumption (A.2) requires the rate of growth of public capital investment to be strictly positive at z = 0 (left hand inequality) and negative at z = 1 (right hand inequality).

In the Appendix it is shown that the Jacobian matrix associated with the dynamic system, evaluated in the steady state, has one eigenvalue with negative real part and one positive eigenvalue. There is one predetermined variable, z, and one jump variable, c. Thus, the saddle point is saddle path stable.

Ceteris paribus, the endogenous growth rate, $\Gamma = g y / z - \delta_g$, rises in g, due to the production externality. The following proposition shows how positional preferences impact on the endogenous growth rate.

Proposition 2. (Positional Preferences and Endogenous Growth)

Assume (A.1) and (A.2). Then, positional preferences ($\eta_c > 0$ or $\eta_k > 0$) impact on the endogenous balanced-growth growth rate, Γ , independently of the presence of a production externality. Specifically,

$$\frac{\partial \Gamma}{\partial \eta_i} \geq 0 \Leftrightarrow \gamma \leq 0, \ i \in \{c, k\}$$

Proof. See the Appendix.

We consider the case $\gamma < 0$ (the intertemporal elasticity of substitution is less than one) to be the main case. For this case, positional preferences raise the endogenous growth rate, irrespective of whether or not individual households exhibit a concern for relative wealth or for relative consumption. Intuitively, consider a rise in η_c (a parallel argument can be given for a rise in η_k). From (14), if $\gamma < 0$, it is seen that ceteris paribus a rise in η_c leads an individual household to raise her steady state consumption growth rate (in the pursue to outshine the others). A higher steady state consumption growth rate is attained by higher savings initially, as of the increase in η_c . As higher saving raises a household's capital oneby-one, it increases output and consumption by less than one-by-one. As a consequence, both the consumption-to-capital ratio, c, as well as the public capital-to capital ratio, z, decline. As $\Gamma = g y / z - \delta_g$, and (y/z) declines in z, the endogenous BGP-growth rate increases as of a rise in η_c .

In order to derive optimal consumption- and income tax rates under CC and CW we will now consider the socially optimal allocation. From this allocation, by comparing with the market economy's allocation, we derive optimal consumption- and income tax rates for the BGP below.

3. The Social Optimum

We adopt the primal approach to derive the socially optimal allocation. In contrast to private households, the government takes into account both externalities, CC and CW. The current value Hamiltonian of the government's problem is given by:

$$H = \frac{1}{\gamma} \left[C^{1-\eta_c} K^{\xi(1-\eta_k)} \right]^{\gamma} + \lambda \left[(1-g)Y - C - \delta_k K \right] + \mu \left[gY - \delta_g K_g \right], \tag{16}$$

where C is a costate variable, and K, K_g are state variables. An interior solution satisfies the following necessary first-order conditions:

$$\frac{\partial H}{\partial C} = (1 - \eta_c) C^{\gamma(1 - \eta_c) - 1} K^{\xi \gamma(1 - \eta_k)} - \lambda = 0 , \qquad (17)$$

$$\frac{\partial H}{\partial K} = \xi (1 - \eta_k) C^{(1 - \eta_c)\gamma} K^{\xi \gamma (1 - \eta_k) - 1} + \lambda [(1 - g)Y_K - \delta_k] + \mu g Y_K = \beta \lambda - \dot{\lambda} , \qquad (18)$$

$$\frac{\partial H}{\partial K_g} = \lambda (1-g) Y_{K_g} + \mu (g Y_{K_g} - \delta_g) = \beta \mu - \dot{\mu} .$$
⁽¹⁹⁾

Let $q \equiv \mu / \lambda$. Then from (19) it directly follows that

$$\frac{\dot{q}}{q} + \frac{1}{q}(1 - g - qg)Y_{K_g} - \delta_g = \beta - \frac{\dot{\lambda}}{\lambda} , \qquad (20)$$

where $Y_{K_g} = (1 - \alpha) A^{-\rho} (y/z)^{1+\rho}$. First-order condition (17) in (18) yields:

$$\frac{\xi(1-\eta_k)}{1-\eta_c}c + (1-g+qg)Y_K - \delta_k = \beta - \frac{\lambda}{\lambda}.$$
(21)

Combining (20) with (21) yields a differential equation in q, where both partial derivatives of Y are functions of z (only):

$$\frac{\dot{q}}{q} = \frac{\xi(1-\eta_k)}{1-\eta_c}c + (1-g+qg)Y_K - \delta_k - \frac{1}{q}(1-g-qg)Y_{K_g} + \delta_g \quad .$$
⁽²²⁾

As in the section above,

$$\frac{\dot{z}}{z} = g \frac{y}{z} - \delta_g - (1 - g)y + c + \delta_k \quad .$$
⁽²³⁾

Finally, the dynamic equation for *c* is found by differentiating (17) with respect to time and taking (21) into account for $\dot{\lambda}/\lambda$:

$$\frac{\dot{c}}{c} = \frac{\xi\gamma(1-\eta_k)[(1-g)y-c-\delta_k] + \xi(1-\eta_k)/(1-\eta_c)c + (1-g+qg)Y_K - (\beta+\delta_k)}{1-\gamma(1-\eta_c)}$$
(24)
-(1-g)y+c+\delta_k.

For a given government expenditure share for public investment, the three-dimensional dynamical system of the economy is given by the differential equations (22) - (24) in the dynamic variables (c,q,z).

However, if the government follows its optimal policy, $\partial H / \partial g = 0$, implying q = 1 and $\dot{q} = 0$. In this case, the dynamical system becomes two-dimensional (as for the market economy). Again, the economy will, in a steady state, follow a BGP. Along the BGP $\dot{c} = \dot{z} = 0$, and C, K, K_g and Y grow at the same constant endogenous growth rate. In a parallel way as presented for Proposition 1, one can establish existence of a unique, nontrivial, saddle point stable steady state. We are now ready to consider the optimal taxation results.

3.1 Optimal Taxation

Given that income and consumption taxes impact the economy in very different ways, what tax and expenditure rates in the decentralized economy will replicate the social planner's optimum? Let these choices be represented by the vector $(\hat{g}, \hat{\tau}_y, \hat{\tau}_c)$. Then, by definition, this vector is a description of optimal fiscal policy in the decentralized economy. To determine these optimal choices, we will compare the equilibrium outcome in the decentralized and centrally planned economies. Since our focus is on the two distortionary tax rates, we will assume that g is set optimally at \hat{g} , given by the solution to (22) – (24), and is appropriately financed by some combination of non-distortionary lump-sum taxes. Given \hat{g} , a comparison of (14) and (24) yields the following long-run optimal relationship between the income and consumption tax rates:

$$\tau_{y} = \frac{\left[\tau_{c} + (\eta_{k} - \eta_{c})/(1 - \eta_{c})\right]\xi c}{\alpha A^{-\rho} y^{1+\rho}} .$$
(25)

From (25), we see that only *one* tax rate can be chosen independently to attain the first-best equilibrium. This implies that the government has a choice in the 'mix' between the income and consumption tax rates: if one is set arbitrarily, the other automatically adjusts to satisfy (25) to replicate the first-best allocation. But what kind of a policy 'mix' should the government choose? Even if one individual tax instrument is at its non-optimal level, (25) suggests that the government can still adjust the other appropriately to attain the social optimum.

To see this flexibility in designing optimal fiscal policy, note that, in (25), the income and consumption tax rates are positively related. A useful benchmark, then, is to derive the tax on income, say, $\hat{\tau}_y$, when $\tau_c = 0$. Given this benchmark rate, we can evaluate the role of the consumption-based tax when the actual income tax rate differs from its benchmark rate, $\hat{\tau}_y$. Likewise, we can evaluate the role of the income-based tax when the actual consumption tax rate differs from its benchmark rate, $\hat{\tau}_c$. When consumption (income-) taxes are absent, that is, $\tau_c = 0$ (that is $\tau_y = 0$), the optimal taxes on income and consumption are given by

$$\hat{\tau}_{y} = \frac{\left[(\eta_{k} - \eta_{c})/(1 - \eta_{c})\right]\xi c}{\alpha A^{-\rho} y^{1+\rho}},$$

$$\hat{\tau}_{c} = \frac{\eta_{c} - \eta_{k}}{1 - \eta_{c}}, \ \xi > 0.$$
(26)

Now suppose that the actual income tax rate is different from its benchmark rate derived in (26). The government has a choice to use the consumption tax to correct for this deviation, yet attain the first-best optimum without altering the income tax rate. To see this, subtract (26) from (25):

$$\tau_{c} = \frac{\alpha A^{-\rho} y^{1+\rho}}{\xi c} (\tau_{y} - \hat{\tau}_{y}), \quad \xi > 0 .$$
⁽²⁷⁾

Therefore, when $\tau_y > \hat{\tau}_y$, the government must introduce a positive consumption tax to attain the first-best equilibrium. On the other hand, if $\tau_y < \hat{\tau}_y$, a consumption subsidy ($\tau_c < 0$) is the appropriate corrective fiscal instrument. These results are summarized by

Proposition 3. (Optimal Taxation).

Assume (A1) and (A2). Then, the first-best allocation is attained through a mix of income- and consumption taxation as given by

$$\tau_{y} = \frac{\left[\tau_{c} + (\eta_{k} - \eta_{c})/(1 - \eta_{c})\right]\xi c}{\alpha A^{-\rho} y^{1+\rho}}.$$

The income tax rate differs from zero only if wealth is an argument in the household utility function.

Proof. Follows immediately from comparing (24) with (14).

In general $(\xi > 0)$, income- or consumption taxes are needed to correct for the distortions caused by the concern for relative wealth and relative consumption. As numerical simulations show (see below), the optimal tax rates may become quite large.

Corollary 1.

If wealth is not an argument of the household utility function, and labor supply is inelastic, then the consumption externality by itself does not cause any distortionary effect in spite of the presence of a production externality. This holds true, even if g is not chosen optimally.

A related argument in a framework in which the engine of growth stems from private capital accumulation is provided by Liu and Turnovsky (2005). We extend this argument to a framework in which the public capital stock serves as an engine of growth. Liu and Turnovsky (2005, p.1121) show that their consumption externality alone does not introduce a distortion. However, in the presence of an additional production externality, the consumption externality exacerbates or reduces the distortion created by the production externality. In contrast, in our framework, as long as $\xi = 0$, the consumption externality does not have a distortionary effect, regardless of the presence of the production externality, and regardless of whether or not g is optimally chosen.¹³

¹³ If g is not optimally chosen, though, then the optimal income tax rate becomes $\tau_y = g(1-q)$.

Proposition 4. (Distortionary effects of the consumption externalities)

If wealth is an argument of the household utility function $(\xi > 0)$, and labor supply is inelastic, then the consumption externality by itself does cause a distortionary effect, even in the absence of a production externality.

In the presence of wealth in the household utility function, the Keynes-Ramsey rule does not hold anymore. This is because the derivative of the Hamiltonian with respect to the capital stock contains a term, the marginal utility of wealth, that itself depends on the consumption externality. Individuals do not internalize this externality, whereas the government does so. As a consequence, the so modified Keynes-Ramsey rule requires the government to choose a capital stock that is affected by the strength of the consumption externality. Notice that this result also holds if preferences exhibit no concern for *relative* wealth ($\xi > 0$, $\eta_k = 0$). The Keynes-Ramsey rule requires to be even more strongly modified if η_k is strictly positive *in addition* to $\xi > 0$. That is, the marginal benefit from consuming an additional unit of capital today rises not only by a preference for *relative* wealth in addition to wealth per se. In a different framework, in which there are no externalities in production and therefore growth is not endogenous and in which households do not have a concern for relative wealth, Nakamoto (2009) points out a parallel argument.

Corollary 2.

If g is chosen optimally, the optimal tax rates $\hat{\tau}_{y}$ and $\hat{\tau}_{c}$ differ from zero if and only if (i) $\xi > 0$ and (ii) $\eta_{k} \neq \eta_{c}$.

The corollary unravels three important characteristics of the optimal tax program derived above. As long as $\xi = 0$ (wealth does not enter the utility function), the concern for relative consumption is non-distortionary. Hence, the optimal tax rates equal zero. However, if $\xi > 0$, the optimal tax rates differ from zero if and only if $\eta_k \neq \eta_c$. In this case, the opposing forces of the consumption- and wealth externalities, which cause an increase in consumption and a corresponding reduction in saving (wealth), do not cancel out. Intuitively, if $\eta_k = \eta_c$, the desire to raise consumption is exactly matched by the desire to increase saving (wealth). In this case, the consumption externalities do not lead to a change in household behavior relative to the social optimum. Therefore, even if $\xi > 0$, the social planner does not need a tax instrument to correct for any distortion.

To get a flavor of how the optimal values of the income-and consumption taxes are affected by the key behavioral parameters of the model (η_c, η_k, ξ) , we calculate $\hat{\tau}_y$ and $\hat{\tau}_c$ based on benchmark parameter values commonly employed in the literature. Preference parameters are assigned the following values: $\beta = 0.04$, $\gamma = -1.5$. The latter parameter gives rise to an intertemporal elasticity of substitution equal to $1/(1-\gamma) = 0.4$, as suggested by Guvenen Technology (2006). the parameters are assigned following values: $A = 0.6, \alpha = 0.8, \rho = 1, \delta_g = \delta_k = 0.08$. First, following common practice, we use the total factor productivity, A, as a scale parameter to help us obtain plausible values for the growth rate, and a value of 0.6 looks reasonable. The value of α (which is the elasticity of private capital) is set at 0.8, which is plausible if private capital is meant to include human capital, as in Romer (1986). This also implies that the elasticity of public capital is 0.2, which is consistent with the empirical evidence provided by Gramlich (1994). There is not much empirical evidence on the elasticity of substitution between private and public capital (Lynde and Richmond, 1993, provides an exception); $\rho=1$, which corresponds to this elasticity being equal to $1/(1+\rho) = 0.5$, is one of the values for this parameter chosen by Chatterjee and Ghosh (2011). Finally, the depreciation rates for the private and public capital stocks are each set at 8% in line with Chatterjee and Ghosh (2011).¹⁴

Based on these benchmark values, we focus on the impact of different values of the key parameters (η_c, η_k, ξ) on the optimal tax rates $(\hat{\tau}_c, \hat{\tau}_y)$ as well as on the optimal level of government spending, g.

In Table 1, we focus on the following preferred benchmark values for the key behavioral parameters: $(\eta_c, \eta_k, \xi) = (0.3, 0.25, 0.5)$. Empirical evidence supports the chosen values of the strength of positional concerns. Compiling several empirical studies, Wendner and Goulder (2008) find that η_1 and η_2 are found to fall into the range $\eta_i \in [0.2, 0.4]$. Other studies find empirical evidence for even larger values of η_i (cf. Johansson-Stenman et al. 2002, Solnik and Hemenway 1998, 2005). Newer empirical studies corroborate this evidence

¹⁴ See also Baxter and King (1993), where the value for the rate of depreciation of the capital stock in the US is chosen at 10%.

(Alvarez-Cuadrado et al. 2012, Dynan and Ravina 2007). Panel C shows that a rise in ξ has a minor impact on ($\hat{\tau}_c, g$) and only slightly raises $\hat{\tau}_v$.

Panel A.	$\eta_c = 0$	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0.3$	$\eta_c = 0.4$	$\eta_c = 0.5$
$\eta_k = 0, \xi = 0.5$						
Optimal g	0.1528	0.1572	0.1622	0.1679	0.1745	0.1821
$\hat{\tau}_{c} (\tau_{y} = 0)$	0.0000	0.1111	0.2500	0.4286	0.6667	1.0000
$\hat{\tau}_{y}(\tau_{c}=0)$	0.0000	-0.0416	-0.0896	-0.1458	-0.2124	-0.2927
Panel B.	$\eta_k = 0$	$\eta_k = 0.1$	$\eta_k = 0.2$	$\eta_k = 0.3$	$\eta_k = 0.4$	$\eta_k = 0.5$
$\eta_c = 0, \xi = 0.5$						
Optimal g	0.1528	0.1540	0.1552	0.1564	0.1577	0.1589
$\hat{\tau}_{c} (\tau_{y} = 0)$	0.0000	-0.1000	-0.2000	-0.3000	-0.4000	-0.5000
$\hat{\tau}_{y}(\tau_{c}=0)$	0.0000	0.0386	0.0770	0.1152	0.1532	0.1910
Panel C.	$\xi = 0$	$\xi = 0.1$	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$
$\eta_c=0.3, \eta_k=0.25$						
Optimal g	0.1841	0.1817	0.1792	0.1768	0.1743	0.1719
$\hat{\tau}_{c} (\tau_{y} = 0)$	0.0000	0.0714	0.0714	0.0714	0.0714	0.0714
$\hat{\tau}_{y} (\tau_{c}=0)$	0.0000	-0.0048	-0.0096	-0.0144	-0.0193	-0.0241

Table 1. The optimal levels of $(g, \hat{\tau}_y, \hat{\tau}_c)$ when respectively η_c , η_k and ξ are gradually increased

Note. (c,z,g) are simultaneously derived employing the benchmark parameter values. $\hat{\tau}_c$ is implicitly given by (25) for $\tau_y = 0$.

A more comprehensive sensitivity analysis with respect to ξ reveals the following robust patterns. The optimal consumption tax rate is not affected by ξ , as seen in (26). The impact of the consumption externalities on the optimal income tax becomes stronger with ξ . The impact of ξ on optimal g is small irrespective of (η_c, η_k) .¹⁵

¹⁵ These results are not reported but are available from the authors upon request.

3.2 Growth and Welfare along the Balanced Growth Path

The endogenous growth rate along the BGP, Γ (decentralized) together with (c,z) is derived from (14) - (15) in the decentralized framework. Without loss of generality, we consider the baseline income- and consumption tax rates to be zero. We assume that g = 0.05.¹⁶ The endogenous growth rate for the social optimum Γ (optimal) together with (c,z,g) is derived from (22) - (24).

In the Appendix, we show that for both, the decentralized as well as the centralized framework, the steady state welfare expression is given by:

$$W_{0} = \frac{c^{\gamma(1-\eta_{c})}}{\gamma \left\{ \beta - \gamma \Gamma \left[(1-\eta_{c}) + \xi(1-\eta_{k}) \right] \right\}}$$
(28)

Welfare expression (28) is an implicit function of c and $\Gamma(z)$. Both variables, (c,z), generally differ between the decentralized and centralized economies. Consequently, so do growth rates and welfare.

A rise in η_i impacts upon both, the growth rate Γ and c. Unfortunately, as seen in (28), the effects on welfare W_0 are ambiguous. For example, if $\gamma < 0$, a rise in η_c raises both the numerator and the denominator. The sign of the steady state welfare effect then depends on the respective changes in c and Γ .

To gain more insight, we employ numerical simulations (Table 2). Specifically, we gradually raise respectively η_c and η_k and calculate the associated (decentralized and optimal) growth rates and welfare levels.

For the social optimum, as η_c rises, g rises, but (c, z, y) fall, and growth rises. The fall in c is greater than the fall in y (and there is higher g as well). Looking at it the other way, the rise in g and fall in z more than compensate for the fall in y. Optimal growth and optimal welfare, both rise, following the rise in η_c . Optimal fiscal policy in this case calls for a lower $\hat{\tau}_y$

¹⁶ The pre-shock value for g is set at 5% also in Chatterjee and Ghosh (2011).

(which is negative) and a higher $\hat{\tau}_c$ as the value of η_c is raised. So higher η_c results in a lower income tax rate (higher income subsidy in our case) and a higher subsidy on private saving (a higher consumption tax rate), which together with the higher complementary public spending, raises the growth rate, and also improves welfare (because of the growth effect, despite the lower consumption-to-capital ratio).

Panel A.	$\eta_c = 0$	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0.3$	$\eta_c = 0.4$	$\eta_c = 0.5$
$\eta_k = 0, \xi = 0.5$						
Γ (decentralized)	0.0166	0.0168	0.0171	0.0174	0.0177	0.0180
Γ (optimal)	0.0899	0.0959	0.1028	0.1108	0.1201	0.1311
W (decentralized)	-138.34	-108.92	-85.68	-67.33	-52.87	-41.48
W (optimal)	-24.946	-21.269	-18.089	-15.335	-12.946	-10.866
Panel B.	$\eta_k = 0$	$\eta_k = 0.1$	$\eta_k = 0.2$	$\eta_k = 0.3$	$\eta_k = 0.4$	$\eta_k = 0.5$
$\eta_c = 0, \xi = 0.5$						
Γ (decentralized)	0.0166	0.0167	0.0168	0.0170	0.0171	0.0172
Γ (optimal)	0.0899	0.0896	0.0894	0.0891	0.0887	0.0883
W (decentralized)	-138.34	-141.09	-143.95	-146.96	-150.11	-153.42
W (optimal)	-24.946	-25.415	-25.913	-26.443	-27.009	-27.614

Table 2. Growth rates (Γ) and welfare (W) along the BGP when η_c and η_k are gradually increased

Note. $\tau_c = \tau_y = 0$, g = 0.05.

Also, for the social optimum, as η_k rises, *g*, *c*, *z*, *y* rise, and growth falls. The rise in *c* is greater than the rise in *y* (and there is higher *g* as well). Looking at it the other way, although *g* and *y* rise, *z* rises more, which leads to growth falling. Optimal growth and optimal welfare, both fall, following the rise in η_k . (This contrasts with optimal growth and optimal welfare, both rising, following a rise in η_c .) Optimal fiscal policy in this case calls for a higher $\hat{\tau}_y$ and a lower $\hat{\tau}_c$ (which is negative) as the value of η_k is raised. So higher η_k results in a higher income tax rate and higher tax on private saving due to a lower consumption tax rate (higher consumption subsidy in our case), which together lead to lower growth, despite the higher public spending. Welfare is also lower (due to the growth effect, despite the higher consumption-to-capital ratio).

At first sight it might seem surprising that in the decentralized economy a rise in η_c is associated with a higher endogenous growth rate. On closer inspection it is possible that households postpone current for future consumption, which boosts saving and provides an impact to the growth rate. This is in fact what happens as is clear from the expression given by (15) where y is only a function of z, and given that g is fixed, the growth rate can also be expressed in terms of $z = K_g / K$ only: $\Gamma = Ag[(1-\alpha) + \alpha z^{\rho}]^{-1/\rho} - \delta_g$. A decline in z raises the growth rate by increasing the marginal productivity of capital (due to the complementarity between public and private capital). As households rise saving (K), z in fact declines.

While the effects of the consumption externalities on welfare and growth in the social optimum roughly correspond to those in the decentralized economy, Table 2 displays one important difference. Households in the decentralized economy have a tendency to overaccumulate capital corresponding to higher values of η_k (due to their concern for relative wealth). The central planner, in an effort to correct this externality, picks a growth rate that reduces the rate of capital accumulation.

3.3 Link between Optimal Taxation, Growth and Welfare

In Table 1, Panel A, the decentralized income tax rate (which is =0) is above the rate $\hat{\tau}_y$ (which is <0): so the private return on capital falls below its socially optimal return. In this case, a positive tax on consumption helps offset this deviation by raising the private return to capital relative to consumption. Consequently, higher η_c , which implies that $\hat{\tau}_y$ becomes even lower (a larger value in absolute terms) results in even larger divergence between private and social returns, and therefore calls for an even larger value of the consumption tax rate to compensate. This in turn implies a higher growth rate, which has a positive effect on welfare.

In Table 1, Panel B, note that the decentralized income tax rate (=0) is below $\hat{\tau}_y$: so the private return on capital exceeds its social return and a consumption subsidy corrects this deviation by lowering the private return on capital relative to consumption. Consequently, higher η_k (which implies that the benchmark rate becomes even higher (a larger positive value) results in even larger divergence between private and social returns, and therefore calls for an even larger value of the consumption subsidy to compensate. This also implies that the growth rate falls with η_k , which has a negative effect on welfare.

4. Fiscal Policy Experiments

The panels of Table 3 report the long-run impact of five fiscal policy shocks, PS1 – PS5, on equilibrium growth rates and welfare levels in the decentralized economy. PS1 – PS3 pertain to an increase in *g* from 5% to 8% of GDP. For PS1 the increase in *g* is financed by an increase in lump-sum taxes (with $\tau_c = \tau_y = 0$). PS2 considers a simultaneous increase in *g* and an increase in the income tax rate, τ_y , from zero to 3%. Likewise PS3 considers a simultaneous increase in *g* and an increase in the consumption tax rate, τ_c , from zero to 3%. The last two policy shocks relate to the replacement of the lump-sum tax as a means of financing the benchmark rate of government spending (g = 0.05) by introducing an income tax, $\tau_y = 0.03$ (PS4) and a consumption tax, $\tau_c = 0.03$ (PS5). In our discussion below, we will focus on growth and welfare implications respectively of a gradual increase in η_c , keeping $\eta_k = 0$ (Panel A in Table 3), and a gradual increase in η_k , keeping $\eta_c = 0$ (Panel B in Table 3).

4.1 Growth and Welfare Effects along the Balanced Growth Path

We report the growth- and welfare effects of the five fiscal policy experiments in the table below. In the first two rows of both panels A and B we report the initial (pre-policy) values of the growth rate and welfare. In the policy experiments, we report percentage deviations from those values.

The effects on growth and welfare for PS1 to PS3 are driven by the strong effect of public spending, irrespective of whether or not part of the additional government spending is financed by income- or consumption taxation. As the table shows, the gradual increase in η_c or η_k does not have a pronounced influence on the magnitude of the growth- and welfare effects. Clearly, the production externality dominates the consumption externalities.

Panel A.	$\eta_c = 0$	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0.3$	$\eta_c = 0.4$	$\eta_c = 0.5$
$\eta_k = 0, \xi = 0.5$						
Pre-policy initial Γ	0.0166	0.0168	0.0171	0.0174	0.0177	0.0180
initial W	-138.34	-108.92	-85.68	-67.33	-52.87	-41.48
PS1 % change in Γ	+178.2	+179.0	+179.9	+180.8	+181.8	+182.8
% change in W	+62.91	+60.71	+58.39	+55.94	+53.34	+50.60
PS2 % change in Γ	+174.4	+175.3	+176.2	+177.1	+178.1	+179.2
% change in W	+63.21	+60.98	+58.63	+56.13	+53.49	+50.69
PS3 % change in Γ	+180.4	+181.2	+182.1	+182.9	+183.9	+184.8
% change in W	+62.72	+60.54	+58.24	+55.82	+53.25	+50.54
PS4 % change in Γ	-1.80	-1.78	-1.77	-1.76	-1.74	-1.72
% change in W	+0.75	+0.64	+0.54	+0.43	+0.31	+0.19
PS5 % change in Γ	+1.46	+1.45	+1.44	+1.42	+1.41	+1.40
% change in W	-0.63	-0.54	-0.45	-0.36	-0.27	-0.17
Panel B.	$\eta_k = 0$	$\eta_k = 0.1$	$\eta_k = 0.2$	$\eta_k = 0.3$	$\eta_k = 0.4$	$\eta_k = 0.5$
Panel B. $\eta_c = 0, \xi = 0.5$	$\eta_k = 0$	$\eta_k = 0.1$	$\eta_k = 0.2$	$\eta_k = 0.3$	$\eta_k = 0.4$	$\eta_k = 0.5$
Panel B. $η_c = 0, \xi = 0.5$ Pre-policy initial Γ	$\eta_k = 0$ 0.0166	$\eta_k = 0.1$ 0.0167	$\eta_k = 0.2$ 0.0168	$\eta_k = 0.3$ 0.0170	$\eta_k = 0.4$ 0.0171	$\eta_k = 0.5$ 0.0172
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial W	$\eta_k = 0$ 0.0166 -138.34	$\eta_k = 0.1$ 0.0167 -141.09	$\eta_k = 0.2$ 0.0168 -143.95	$\eta_k = 0.3$ 0.0170 -146.96	$\eta_k = 0.4$ 0.0171 -150.11	$\eta_k = 0.5$ 0.0172 -153.42
Panel B.	$\eta_k = 0$ 0.0166 -138.34 +178.2	$\eta_k = 0.1$ 0.0167 -141.09 +178.6	$\eta_k = 0.2$ 0.0168 -143.95 +179.0	$\eta_k = 0.3$ 0.0170 -146.96 +179.5	$\eta_k = 0.4$ 0.0171 -150.11 +179.9	$\eta_k = 0.5$ 0.0172 -153.42 +180.3
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial WPS1 % change in Γ % change in W	$\eta_k = 0$ 0.0166 -138.34 +178.2 +62.91	$\eta_k = 0.1$ 0.0167 -141.09 +178.6 +62.51	$\eta_k = 0.2$ 0.0168 -143.95 +179.0 +62.10	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial WPS1 % change in Γ % change in WPS2 % change in Γ	$\eta_k = 0$ 0.0166 -138.34 +178.2 +62.91 +174.4	$\eta_k = 0.1$ 0.0167 -141.09 +178.6 +62.51 +174.8	$\eta_k = 0.2$ 0.0168 -143.95 +179.0 +62.10 +175.3	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66 +175.7	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19 +176.2	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70 +176.6
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial WPS1 % change in Γ % change in WPS2 % change in Γ % change in W	$\eta_k = 0$ 0.0166 -138.34 +178.2 +62.91 +174.4 +63.21	$\eta_k = 0.1$ 0.0167 -141.09 +178.6 +62.51 +174.8 +62.83	$\eta_k = 0.2$ 0.0168 -143.95 +179.0 +62.10 +175.3 +62.43	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66 +175.7 +62.01	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19 +176.2 +61.56	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70 +176.6 +61.08
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial WPS1 % change in Γ % change in WPS2 % change in Γ % change in W PS3 % change in Γ	$\eta_k = 0$ 0.0166 -138.34 +178.2 +62.91 +174.4 +63.21 +180.4	$\eta_{k} = 0.1$ 0.0167 -141.09 +178.6 +62.51 +174.8 +62.83 +180.8	$\eta_k = 0.2$ 0.0168 -143.95 +179.0 +62.10 +175.3 +62.43 +181.2	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66 +175.7 +62.01 +181.6	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19 +176.2 +61.56 +182.1	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70 +176.6 +61.08 +182.5
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial WPS1 % change in Γ % change in WPS2 % change in Γ % change in W PS3 % change in Γ % change in W	$\eta_k = 0$ 0.0166 -138.34 +178.2 +62.91 +174.4 +63.21 +180.4 +62.72	$\eta_{k} = 0.1$ 0.0167 -141.09 +178.6 +62.51 +174.8 +62.83 +180.8 +62.32	$\eta_k = 0.2$ 0.0168 -143.95 +179.0 +62.10 +175.3 +62.43 +181.2 +61.90	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66 +175.7 +62.01 +181.6 +61.45	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19 +176.2 +61.56 +182.1 +60.97	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70 +176.6 +61.08 +182.5 +60.46
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial W PS1 % change in Γ % change in W PS2 % change in Γ % change in W PS3 % change in Γ % change in W PS3 % change in Γ % change in W PS4 % change in Γ	$\eta_{k} = 0$ 0.0166 -138.34 +178.2 +62.91 +174.4 +63.21 +180.4 +62.72 -1.80	$\eta_{k} = 0.1$ 0.0167 -141.09 +178.6 +62.51 +174.8 +62.83 +180.8 +62.32 -1.79	$\eta_k = 0.2$ 0.0168 -143.95 +179.0 +62.10 +175.3 +62.43 +181.2 +61.90 -1.78	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66 +175.7 +62.01 +181.6 +61.45 -1.78	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19 +176.2 +61.56 +182.1 +60.97 -1.77	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70 +176.6 +61.08 +182.5 +60.46 -1.76
Panel B. $\eta_c = 0, \xi = 0.5$ Pre-policy initial Γ initial WPS1 % change in Γ % change in WPS2 % change in Γ % change in W PS3 % change in Γ % change in W PS4 % change in Γ % change in W	$\eta_{k} = 0$ 0.0166 -138.34 +178.2 +62.91 +174.4 +63.21 +180.4 +62.72 -1.80 +0.75	$\eta_{k} = 0.1$ 0.0167 -141.09 +178.6 +62.51 +174.8 +62.83 +180.8 +62.32 -1.79 +0.78	$\eta_{k} = 0.2$ 0.0168 -143.95 +179.0 +62.10 +175.3 +62.43 +181.2 +61.90 -1.78 +0.81	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66 +175.7 +62.01 +181.6 +61.45 -1.78 +0.84	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19 +176.2 +61.56 +182.1 +60.97 -1.77 +0.87	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70 +176.6 +61.08 +182.5 +60.46 -1.76 +0.90
Panel B. $η_c = 0, \xi = 0.5$ Pre-policy initial Γ initial W PS1 % change in Γ % change in W PS2 % change in Γ % change in W PS3 % change in Γ % change in W PS4 % change in Γ % change in W PS4 % change in Γ % change in W PS5 % change in Γ	$\eta_k = 0$ 0.0166 -138.34 +178.2 +62.91 +174.4 +63.21 +180.4 +62.72 -1.80 +0.75 +1.46	$\eta_k = 0.1$ 0.0167 -141.09 +178.6 +62.51 +174.8 +62.83 +180.8 +62.32 -1.79 +0.78 +1.45	$\eta_k = 0.2$ 0.0168 -143.95 +179.0 +62.10 +175.3 +62.43 +181.2 +61.90 -1.78 +0.81 +1.45	$\eta_k = 0.3$ 0.0170 -146.96 +179.5 +61.66 +175.7 +62.01 +181.6 +61.45 -1.78 +0.84 +1.44	$\eta_k = 0.4$ 0.0171 -150.11 +179.9 +61.19 +176.2 +61.56 +182.1 +60.97 -1.77 +0.87 +1.44	$\eta_k = 0.5$ 0.0172 -153.42 +180.3 +60.70 +176.6 +61.08 +182.5 +60.46 -1.76 +0.90 +1.43

Table 3. Growth and welfare effects along the BGP for different fiscal policy experiments

The rise in g strongly positively affects both growth and welfare. The positive impact on growth is evident from the fact that endogenous growth is generated by public spending, which complements private spending. The positive impact on welfare stems from the fact that

the pre-policy level of government spending is well below the optimal level. E.g., the optimal level of *g* equals 0.1528 (see Table 1, Panel A).

For PS4 and PS5 the impact on growth and welfare is rather small, which can immediately be attributed to the fact that in these two policy experiments the production externality from an increase in government spending – which was present for PS1 to PS 3 - is absent.

For PS4 The effect on the growth rate is negative, which is intuitive. A higher income tax rate reduces the private rate of return on capital, and there is no complementary increase in public capital spending. Regarding the welfare effect, a rise in the income tax rate to finance the fixed amount of government spending implies a lowering of lump-sum taxes, which can lead to a rise in consumption. Our numerical results show that in this case not only is there a rise in the consumption-capital ratio but this rise also outweighs the negative growth effect resulting from a rise in income taxes leading to a rise in welfare.

For PS5 the effect on the growth rate is positive, which is intuitive. A higher consumption tax rate discourages consumption and raises the relative return on private capital, and thereby encourages saving and boosts growth. Regarding the welfare effect, a rise in the consumption tax directly affects consumption, and tends to reduce it. On the other hand, higher consumption taxes to finance the fixed amount of government spending imply a lowering of lump-sum taxes, which can lead to a rise in consumption. Our numerical results show that in this case, not only is there a fall in the consumption-capital ratio but this fall also outweighs the positive growth effect resulting from a rise in consumption taxes, and this leads to an overall decrease in welfare.

4.2 Transitional Dynamics

Finally, we consider the transitional dynamics of the five policy shocks. Specifically, we consider the transitional paths of (c,z) as well as those of the growth rates (g_K, g_{Kg}, g_C) . To solve numerically for the transitional paths, we employ the Mathematica implementation of the *Relaxation Algorithm* (Trimborn et al., 2008).

Figures 1 and 2 contain grids of graphs displaying the transitional effects of PS1 to PS5 on c and z (Figure 1) as well as on the growth rates of C, K, and K_g (Figure 2). Both figures show the results for $\eta_c = \eta_k = 0$.¹⁷



Figure 1. Transitional dynamics of (z_t, c_t) under the policy shocks PS1 to PS5

Note: In these simulations, $\eta_i = 0$. For different values of η_i , the transitional dynamics are similar.

Along the transitional paths, we observe three robust patterns.¹⁸ First, for all investigated policy shocks, the transitional paths of (c, z, g_K, g_{Kg}, g_C) are monotone. As the dynamic system of the decentralized economy is characterized by one differential equation of state variable z and by one differential equation of jump variable c, we essentially expect transitional paths of these variables to be monotone.

Second, for PS1 to PS3, both c and z change "strongly" along the transitional path, while for PS4 and PS5 the policy impact on the transitional paths of these variables is small. This behavior becomes clear when considering the steady state effects of the policy shocks on c and z. As discussed above, the steady state effects of PS4 and PS5 are minor. Also, as the

¹⁷ Figures for the transitional paths for $(\eta_c, \eta_k) = (0.5, 0); (\eta_c, \eta_k) = (0, 0.5)$ are available from the authors upon request. These figures, though, are similar to the ones presented here.

¹⁸ These patterns also occur for all other parameter constellations we simulated. In particular, these patterns also hold true for different values of η_{i} .

transitional paths are monotone, we conclude that the effects of PS4 and PS5 on the transitional paths of c and z must be minor as well.

Third, focusing on a tax reform with *g* being unchanged, the transitional dynamics (and steady state) effects of PS4 are opposite to those of PS5.

PS4. Consider a rise in τ_y with (g, τ_c) constant. Initially, the net-return on savings declines, and individuals respond with an upwards jump in consumption (thereby *c*). Initially, *z*, being a state variable, does not change. The lowering in savings lowers *K*. Both (K, K_g) still grow at a positive rate. But $g_{Kg} > g_K$ as the former is directly proportional to output, while the latter is reduced by a rising *c* (cf. (5) and (6)). Consequently *z* starts to increase. The rise in *z*, lowers the rate of interest. Subsequently individuals lower the growth rate of consumption, as seen in the modified Keynes-Ramsey rule (14) due to a still lower net return on savings. As a consequence, private capital starts to accumulate at a rate *higher* than initially, just after the introduction of the policy shock, towards its new steady state level. In the BGP, though g_K is still below its pre-policy level, as discussed in the previous subsection.

PS5. Consider a rise in τ_c with (g, τ_y) constant. Initially, the net-return on saving increases as income is taxed while consumption is not taxed. That is, initially, individuals respond with a downward jump in consumption (thereby *c*). The initial rise in savings increases the growth rate of private capital (initial upwards jump). As the growth rate of public capital is initially fixed (cf. (5)), $g_{Kg} < g_K$, and z starts to decline. Thus, the rate of interest rises, and so does the growth rate of consumption, due to (14). As a consequence, the growth rate of private capital declines towards its new steady state level. Both, along the transitional path and in the new BGP, g_K is above its pre-policy level. As a consequence, Y increases and the growth rate of public capital rises towards its new BGP-level. In the post-policy BGP, the endogenous growth rate is higher than in the pre-policy BGP (see previous subsection).

The transitional effects of the policy shocks on the growth rates (as discussed above) are shown in Figure 2. Along the transitional paths (K, K_g, C) grow at *differing* rates.



Figure 2. Transitional dynamics of (gK_t, gK_{gt}, g_{Ct}) under the policy shocks PS1 to PS5

Note: In these simulations, $\eta = 0$. For different values of η , the transitional dynamics are similar.

5. Conclusions

This paper contributes to the literature on positional preferences by introducing conspicuous wealth in the agent's utility function, in addition to conspicuous consumption. And it does so within an endogenous growth set-up where the engine of growth is public capital. Production externalities have been captured extensively in much of the growth literature, but the same cannot be said about consumption externalities. And even when the latter have been considered, the reference level has mostly been conspicuous consumption rather than wealth. Our paper attempts to plug this gap, given that one objective in foregoing current consumption and accumulating capital, which increases wealth, is that this in itself adds to agents' utility. Also, in the process of enhancing wealth, individual wealth relative to the average is considered as an argument in the utility function.

In the paper we demonstrate that if wealth is present in the consumer's utility function, then – despite labor supply being inelastic – the consumption externality does have a distortionary effect, irrespective of the production externality. This modifies the previous results from some endogenous growth models where, with inelastic labor supply, such distortionary effects are obtained only with production externalities. Interestingly, in our framework, if wealth is not present in the consumer's utility function, this distortion disappears. In some sense, this result resembles those in models with conspicuous consumption (but not wealth), where there are no distortions; however, such models are typically neoclassical rather than endogenous growth models. While the effects of consumption externalities on growth and welfare in the decentralized economy broadly correspond to those in the social optimum, the effect of wealth externalities is to cause over-accumulation of capital by households in the decentralized economy. Here the social planner, in an effort to correct this externality, picks a growth rate that reduces the rate of capital accumulation to optimal levels. We also conduct some fiscal policy experiments where our results demonstrate that where an increase in public spending occurs, this positively and strongly affects both growth and welfare in the steady state and along the transition path: here the production externality clearly dominates the consumption externalities.

We have performed our analysis in the context of a closed economy, following much of the literature. Our paper could be extended to an open economy context – either a small open economy that has to take the world interest rate as given, or a large economy where economic policies would determine the domestic interest rate – where consumption and wealth externalities could be generated not only at home but also abroad. This would add an interesting new dimension to the growth and welfare analysis that we have conducted thus far, and make our analysis richer. To our knowledge, there have not yet been many studies that proceed in this direction: Fisher and Hof (2005) provides an attempt.

Also, the standard growth models typically consider a constant rate of time preference, but recently a "preference-driven theory of economic growth" has been proposed by Strulik (2012), among others, where the rate of impatience varies negatively with wealth, i.e., as wealth increases, individuals tend to become more patient. Given that in our existing set-up, the inclusion of wealth and conspicuous wealth in the utility function makes a significant difference to the workings of the baseline model (where positional preferences are defined

with respect to consumption alone), the introduction of wealth-driven time preference will surely introduce another interesting element in the determination of growth and welfare. We have made some progress in both these directions, but they would obviously be the subject of other papers and beyond the scope of the current one.

Appendix

Proof of Proposition 1. Part (1). From differential equations (14) and (15), the steady state values (c,z) cannot be explicitly derived. However, at $\dot{c} = \dot{z} = 0$, both differential equations can be analytically solved for c as a function of z. Let cc(z) denote this solution associated with (14) and cz(z) denote the solution associated with (15). Furthermore, let $\Delta(z) \equiv cc(z) - cz(z)$. Obviously, at a steady state $\Delta(z) = 0$. We first note that

$$\Delta'(z) = -\frac{Bl\left(B2 + z(1-\alpha)\left(A^{\rho}(1-g)\xi + \alpha\left(A\left(z^{-\rho}(1-\alpha) + \alpha\right)^{-1/\rho}\right)^{\rho}(1+\rho)\right)\right)}{z^{2}\left(1 + (-1+z^{\rho})\alpha\right)\left(1 + \xi + \gamma\left(-1+\eta_{c} + (-1+\eta_{k})\xi\right)\right)} < 0,$$

$$Bl \equiv A^{1-\rho}\left(z^{-\rho}(1-\alpha) + \alpha\right)^{-1/\rho} > 0,$$

$$B2 \equiv A^{\rho}g \, z^{\rho}\alpha\left(1 + \xi + \gamma\left(-1+\eta_{c} + (-1+\eta_{k})\xi\right)\right) > 0.$$
(29)

That is, the slope of $\Delta(z)$ is *strictly* negative. As a consequence, a steady state, if it exists, is unique. We now argue that $\Delta(0) > 0$ and $\Delta(1) < 0$. Then, by the Intermediate value theorem (and by strict monotonicity), there exists a unique, strictly positive $z \in (0,1)$ for which $\Delta(z) = 0$. Furthermore,

$$\Delta(0) = \underbrace{\left(1 + \xi - \gamma \left(1 - \eta_c + (1 - \eta_k)\xi\right)\right)}_{\geq 0} \underbrace{\left(Ag - \delta g \left(1 - \alpha\right)^{\frac{1}{p}}\right)}_{>0 \text{ by}(A.2)} + \underbrace{\left(1 - \alpha\right)^{\frac{1}{p}} \left(\beta + \delta_k \left(1 + \xi\right)\right)}_{>0} > 0,$$

$$\Delta(1) = -\underbrace{\left(\alpha A - \delta_k - \beta\right)}_{>0 \text{ by}(A.1)} - \underbrace{\xi \left(A \left(1 - g\right) - \delta_k\right)}_{\geq 0} - \underbrace{\left(1 + \xi + \gamma \left(-1 + \eta_c + (-1 + \eta_k)\xi\right)\right)}_{\geq 0} \underbrace{\left(\delta_g - Ag\right)}_{\geq 0 \text{ by}(A.2)} < 0$$

As can easily be seen, (A.1) and (A2) imply $\Delta(0) > 0$ and $\Delta(1) < 0$.

Part (2). The determinant of the Jacobian matrix, evaluated at the steady state, is unambiguously negative:

$$-\frac{B3\left(B2+z\left(1-\alpha\right)\left(A^{\rho}\left(1-g\right)\xi+\alpha\left(A\left(z^{-\rho}\left(1-\alpha\right)+\alpha\right)^{-1/\rho}\right)^{\rho}\left(1+\rho\right)\right)\right)}{z\left(1+\left(-1+z^{\rho}\right)\alpha\right)\left(1-\gamma\left(1-\eta_{c}\right)\right)}<0,$$

$$B3\equiv A^{1-\rho}c\left(z^{-\rho}\left(1-\alpha\right)+\alpha\right)^{-1/\rho}>0.$$

Therefore one eigenvalue is positive and the other eigenvalue of the dynamic system is negative. As we have one predetermined variable, z, and one jump variable, c, the steady state is a saddle point and saddle path stable.

Proof of Proposition 2. As

$$\Gamma = g y / z - \delta_g = g A \left[\alpha z^{\rho} + (1 - \alpha) \right]^{-1/\rho} - \delta_g,$$

the endogenous growth rate *negatively* depends on z: $\Gamma(z)$. In what follows, we graphically analyze the impact of a rise of η_i on the steady state value of z. Specifically, we consider the cc(z) - and cz(z) -loci (as defined in the proof of Proposition 1) in (z,c)-plane.

$$cz(z) = (1-g)y - g(y/z) + (\delta_g - \delta_k),$$

$$cz(0) = -\left[Ag(1-\alpha)^{-1/\rho} - \delta_g\right] - \delta_k < 0,$$

$$cz(1) = \left[A(1-g) - \delta_k\right] - (Ag - \delta_g) > 0.$$

cz(0) is strictly negative (cz(1) is strictly positive) by Assumption (A.2). As y is increasing in z, and (y/z) is decreasing in z, the slope of the cz(z)-locus is strictly positive in (z,c)plane. Notice that the cz(z)-locus is independent of the preference parameters η_i . The cc(z)-locus is given by

$$cc(z;\eta_{c},\eta_{k}) = \underbrace{\left[(1-g)y - \delta_{k}\right]}_{\geq 0} E_{1}(\eta_{c},\eta_{k}) + \underbrace{\left[(1-\tau_{y})Y_{K} - \delta_{k} - \beta\right]}_{>0 \text{ by}(A.1)} E_{2}(\eta_{c},\eta_{k})$$
$$E_{1}(\eta_{c},\eta_{k}) = \frac{\xi\gamma(1-\eta_{k}) - 1 + \gamma(1-\eta_{c})}{\xi\gamma(1-\eta_{k}) - 1 + \gamma(1-\eta_{c}) - \xi(1+\tau_{c})},$$
$$E_{2}(\eta_{c},\eta_{k}) = \frac{1}{\xi\gamma(1-\eta_{k}) - 1 + \gamma(1-\eta_{c}) - \xi(1+\tau_{c})},$$

where the auxiliary terms E_1 and E_2 depend only on parameters. For z = 0, $cc(0;\eta_c,\eta_k) > cz(0)$, as shown in the proof of Proposition 1. That is, at the unique steady state, the cc(z)-locus crosses the cz(z)-locus from above. In other words, at the unique steady state, the (positive or negative) slope of the cc(z)-locus is lower than the (positive) slope of the cz(z)-locus in (z,c)-plane.

For any given z, a rise in η_i lowers (raises) E_i , j = 1,2 if $\gamma < 0$ (if $\gamma > 0$). That is,

$$\operatorname{sgn}\frac{\partial E_{j}}{\partial \eta_{i}} = \operatorname{sgn}\gamma, \ i \in \{c,k\}, \ j \in \{1,2\}.$$

If $\gamma < 0$, which is overwhelmingly suggested by empirical evidence, a rise in η_i makes the cc(z)-locus shift downwards. As a consequence, the steady state value of z decreases. As $\Gamma(z)$, the endogenous growth rate increases as of a rise in η_i .

Welfare. Regardless of whether we consider a decentralized economy or a centralized framework, along a BGP (where *c* is constant and *K* grows at the constant rate Γ), welfare is given by:

$$\begin{split} W_{0} &= \int_{0}^{\infty} \frac{1}{\gamma} \Big[C^{1-\eta_{c}} K^{\xi(1-\eta_{k})} \Big]^{\gamma} e^{-\beta t} dt = \frac{1}{\gamma} \int_{0}^{\infty} C^{(1-\eta_{c})\gamma} K^{\gamma\xi(1-\eta_{k})} e^{-\beta t} dt = \frac{1}{\gamma} \int_{0}^{\infty} c^{(1-\eta_{c})\gamma} K^{(1-\eta_{c})\gamma+\gamma\xi(1-\eta_{k})} e^{-\beta t} dt \\ &= \frac{c^{(1-\eta_{c})\gamma}}{\gamma} \int_{0}^{\infty} K^{(1-\eta_{c})\gamma+\gamma\xi(1-\eta_{k})} e^{-\beta t} dt = \frac{c^{(1-\eta_{c})\gamma}}{\gamma} \int_{0}^{\infty} K_{0}^{(1-\eta_{c})\gamma+\gamma\xi(1-\eta_{k})} e^{\Big[(1-\eta_{c})\gamma+\gamma\xi(1-\eta_{k})\Big]\Gamma t} e^{-\beta t} dt \\ &= \frac{K_{0}^{(1-\eta_{c})\gamma+\gamma\xi(1-\eta_{k})} c^{(1-\eta_{c})\gamma}}{\gamma} \int_{0}^{\infty} e^{-\Big[\beta-\gamma\Gamma\big((1-\eta_{c})+\xi(1-\eta_{k})\big)\Big]t} dt = \frac{c^{(1-\eta_{c})\gamma}}{\gamma} \int_{0}^{\infty} e^{-\Big[\beta-\gamma\Gamma\big((1-\eta_{c})+\xi(1-\eta_{k})\big)\Big]t} dt, \end{split}$$

where the last line follows from the initial condition $K_0 = 1$. This expression is defined only if $\left[\beta - \gamma \Gamma \left((1 - \eta_c) + \xi(1 - \eta_k)\right)\right] > 0$. Noting (10), this inequality is equivalent with the transversality condition $\lim_{t\to\infty} \lambda_t K_t e^{-\beta t}$. As the transversality condition is required to hold at a solution to the optimization problem, we find the welfare expression

$$W_0 = \frac{c^{(1-\eta_c)\gamma}}{\gamma \left[\beta - \gamma \Gamma \left((1-\eta_c) + \xi(1-\eta_k)\right)\right]}$$

where c and $\Gamma(z)$ are implicitly given.

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