



Munich Personal RePEc Archive

**Willingness to overpay for insurance and  
for consumer credit: search and risk  
behavior under price dispersion**

Malakhov, Sergey

Pierre-Mendès France University

28 October 2014

Online at <https://mpra.ub.uni-muenchen.de/60358/>  
MPRA Paper No. 60358, posted 03 Dec 2014 07:30 UTC

Sergey Malakhov  
Ph.D., Applied Economics,  
Pierre-Mendès-France University,  
Grenoble, France

**Willingness to overpay for insurance and for consumer credit:  
search and risk behavior under price dispersion.**

**Abstract**

When income growth under price dispersion reduces the time of search and raises prices of purchases, the increase in purchase price can be presented as the increase in the willingness to pay for insurance or the willingness to pay for consumer credit. The optimal consumer decision represents the trade-off between the propensity to search for beneficial insurance or consumer credit, and marginal savings on insurance policy or consumer credit. Under price dispersion the indirect utility function takes the form of cubic parabola, where the risk aversion behavior ends at the saddle point of the comprehensive insurance or the complete consumer credit. The comparative static analysis of the saddle point of the utility function discovers the ambiguity of the departure from risk-neutrality. This ambiguity can produce the ordinary risk seeking behavior as well as mathematical catastrophes of Veblen-effect's imprudence and over prudence of family altruism. The comeback to risk aversion is also ambiguous and it results either in increasing or in decreasing relative risk aversion. The paper argues that the decreasing relative risk aversion comes to the optimum quantity of money.

**Keywords** : consumer search, risk, insurance, credit, optimum quantity of money, Veblen effect, family altruism, mathematical catastrophe

JEL Classification: D11, D81.

**Introduction to indirect utility function of satisficing optimal decision**

The analysis of the consumption-leisure choice  $U=U(Q,H)$  with respect to the wage rate  $w$  and to the purchase price reduction and marginal savings got from the search, or to the value  $\partial P/\partial S$ , can be presented as the static photograph of a step in the dynamic satisficing decision procedure. The **satisficing** consumer decision procedure ignores unacceptable high prices  $P_S$ ; it starts at the reservation level of labor income  $wL_0$  and finishes at the purchase price level  $P_P=wL < wL_0$ , where the satisficing procedure results in **optimal decision** because it equalizes marginal costs of search with its marginal benefit and that equality provides the maximization of the utility function (Malakhov 2014). The use of the truly **relative price**, i.e., purchase price  $P_P$  with regard to the time of search  $S$  or to **the given place of purchase**, gives new economic explanations for some anomalies of behavior like endowment effect, sunk costs sensitivity, little pre-purchase

search of big ticket items, and, finally, Veblen effect and money illusion. From the point of view of the problem  $\max U(Q,H)$  subject to  $w/\partial P/\partial S|_{const}=Q/\partial L/\partial S$ , where the value  $\partial P/\partial S_{const}$  represents the given place of purchase and the value  $\partial L/\partial S$  represents the **propensity to search**, i.e., propensity to substitute labor  $L$  for search  $S$ , the constraint is created by the core equality of marginal values of search derived from the satisficing decision procedure:

$$w \frac{\partial L}{\partial S} = Q \frac{\partial P}{\partial S} \quad (1)$$

The equilibrium price  $P_e$  becomes equal to the sum of consumers' labor costs  $wL$  and transaction cost  $wS$ , or  $P_e=w(L+S)$ :

$$\frac{\partial U / \partial H}{\partial U / \partial Q} = - \frac{w}{\partial P / \partial S} \partial^2 L / \partial S \partial H = - \frac{w}{T \partial P / \partial S} = \frac{w}{w(L+S)} = \frac{w}{P_e} \quad (2)$$

where the value  $T=1/\partial^2 L/\partial S \partial H$  represent the time horizon until the similar purchase, or the **commodity lifecycle**.

As we can see, the Equation (2) specifies the paradox formulated by P.Diamond that when search costs are positive the equilibrium price is becomes equal to the monopoly price (Diamond 1971). Moreover, the Equation (2) gives another view on home production where G.Becker's model is still the dominant vector of analysis. Indeed, if we consider the household activity to be a specific form of search, the equilibrium price for the final product or the **willingness to accept** will be equal to the sum of purchase price of inputs  $P_P$ , i.e., of labor costs  $wL$ , and transformation costs  $wS$ .

Although the original values of the model  $\partial P/\partial S$  and  $\partial L/\partial S$  look unusual, their modeling tries not to forget the testament of A.Marshall, who told that “*when a great many symbols have to be used, they become very laborious to any one but the writer himself*” (Marshall 1920[1890], p.12). Sometimes such relative values are indispensable, especially when the original G.Stigler's assumption of the diminishing marginal efficiency of search (Stigler 1961) is used ( $\partial P/\partial S < 0$ ;  $\partial^2 P/\partial S^2 > 0$ ), or when the behavior of the propensity to search is derived ( $\partial L/\partial S < 0$ ;  $\partial^2 L/\partial S^2 < 0$ ) (Malakhov 2014). However, the understanding of these relative values can be simplified by the graphical illustration of the interrelation between static ( $Q_{variable}; \partial P/\partial S_{const}$ ) **implicit optimal decision** and dynamic ( $Q_{const}; \partial P/\partial S_{variable}$ ) **explicit satisficing decision** (Fig.1):

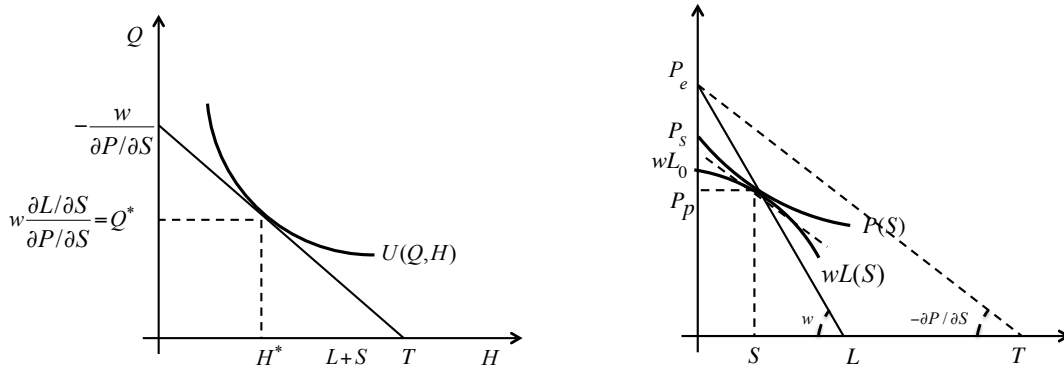


Fig.1. Implicit optimal decision and explicit satisficing decision

The presentation of relative values in absolute terms,  $|\partial P/\partial S|$  and  $|\partial L/\partial S|$  simplifies their mathematical treatment without logical losses. This tactic facilitates the comparative static analysis and we can easily derive marginal utilities of money income and money expenditures with respect to optimal values of consumption and leisure (Malakhov 2013):

$$MU_w = \lambda; \quad (3.1)$$

$$MU_{|\partial P/\partial S|} = -\lambda \frac{w}{|\partial P/\partial S|} \quad (3.2)$$

The analysis of the second order cross partial derivatives, i.e, the change in the marginal utility of received money income with the change of the place of purchase, or  $\partial MU_w / \partial |\partial P/\partial S|$ , and the change in the marginal utility (disutility) of the habitual place of purchase with the change in money income, or  $\partial MU_{|\partial P/\partial S|} / \partial w$ , results in the equation that demonstrates the behavior of the marginal utility of money under the optimal consumption-leisure choice:

$$e_{\lambda, |\partial P/\partial S|} + e_{\lambda, w} = e_{|\partial P/\partial S|, w} - 1 \quad (4)$$

Under the assumption of the diminishing efficiency of search the elasticity of price reduction  $e_{|\partial P/\partial S|, w}$  illustrates both the increase in the **willingness to overpay** and the decrease in time of search after the increase in the wage rate ( $|\partial P_i/\partial S_i| > |\partial P_j/\partial S_j| \rightarrow P_i > P_j; S_i < S_j$ ). Hence, it is always positive. When the value of the elasticity of price reduction  $e_{|\partial P/\partial S|, w}$  is equal to one, we have

$$e_{\lambda, |\partial P/\partial S|} + e_{\lambda, w} = 0 \quad (5)$$

The Equation (4) also enlightened the way for the comparative static analysis of the indirect utility function where subsequent satisficing decisions optimize consumption-leisure trade-offs with respect to changes in both parts of the constraint. The increase in the wage rate moves consumers from low-price stores to high-price stores. Indeed, the Equation (4) shows us that the indirect utility function depends on two variables in the following manner:

$$v(w, |\partial P/\partial S|) = v(w, |\partial P/\partial S|(w)) \quad (6)$$

The total derivative of this utility function gives us the following:

$$dv(w, |\partial P / \partial S| (w)) = dw \left( \frac{\partial v}{\partial w} \Big|_{|\partial P / \partial S| \text{ const}} + \frac{\partial v}{\partial |\partial P / \partial S|} \frac{\partial |\partial P / \partial S|}{\partial w} \right); \quad (7)$$

$$\frac{dv}{dw} = \lambda - \lambda \frac{w}{|\partial P / \partial S|} \frac{\partial |\partial P / \partial S|}{\partial w} = \lambda (1 - e_{|\partial P / \partial S|, w})$$

We see that when the price reduction is unit elastic ( $e_{|\partial P / \partial S|, w} = 1$ ), the Equation (5) takes place and the utility stays constant, or  $dv/dw = 0$ . And the following choice of the purchase price which is accompanied by a greater price reduction ( $e_{|\partial P / \partial S|, w} > 1$ ) decreases the utility of consumption-leisure choice. The consumption growth is followed by the disproportionately important reduction in leisure time.

### Willingness to overpay as insurance premium

Usually, guarantees and insurance contracts increase both prices of purchases and price dispersion and we can await that guarantees and insurance contracts raise the equilibrium price reduction  $|\partial P / \partial S|$  that equalizes marginal costs of search with its marginal benefit.

We can assume that *the increase in the wage rate results not in the simple increase in the purchase price with respect to the increased income but in the increase in the insurance premium, accompanied by the increase in price reduction*. The consumer details his insurance policy and increases the insurance premium with every increase in the wage rate. Our assumption is really *illustrative* because here the consumer behaves like a homeowner who raises progressively the fence with any subsequent increase in income. And more insurance policy is detailed, the more efficient is the search, i.e., the greater is the absolute value of the equilibrium price reduction.

The appearance of the saddle point in the utility function gives an answer to the question what the consumer should do in order to avoid the decrease in utility. Obviously, he should decrease relative price reduction i.e., to be... not more modest, but less ambitious with regard to purchase prices after the following increase in the wage rate. We see that the decrease in the willingness to overpay is really possible. The only way to increase both consumption and real balances is not to reduce *absolute* overpayments (the value  $\partial |\partial P / \partial S| / \partial w$  is always positive) but to reduce *relative* overpayments, or to make them less income elastic, i.e.,  $e_{|\partial P / \partial S|, w} = 0,9; 0,8; 0,7...$  etc., other words, to accept *incomplete insurance and guarantees* for items to be bought.

However, this change represents the change in the model of behavior – from risk aversion to risk seeking. Indeed, the prospect theory tells us that facing the inevitable loss, here the decrease in utility, the consumer should take risk (Kahneman and Tversky 1979). Hence, the utility function changes its shape and becomes close to the cubic parabola (Fig.2).

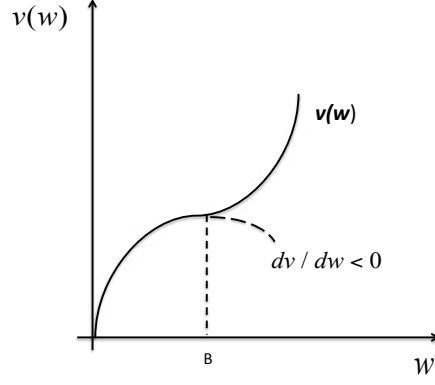


Fig.2. Utility function under price dispersion

### Unwillingness to overpay for insurance as driver of risk behavior

When we determine the second derivative of the utility function, we should keep in mind the marginal utility of money income  $\lambda$  as well as the **unwillingness to overpay** ( $1 - e_{|\partial P/\partial S|,w}$ ) also represent functions of two variables. We can omit labor-intensive intermediate calculations and present the second derivative directly in its total form and in its elasticity form:

$$\frac{d^2v}{dw^2} = \frac{d\lambda}{dw} (1 - e_{|\partial P/\partial S|,w}) + \lambda \frac{d(1 - e_{|\partial P/\partial S|,w})}{dw} \quad (8)$$

$$\frac{d^2v}{dw^2} = \frac{\lambda}{w} (1 - e_{|\partial P/\partial S|,w}) (e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w} + e_{(1 - e_{|\partial P/\partial S|,w}),w}) \quad (9)$$

The form of the total second derivative is very useful for the step-by-step analysis of changes in the model of behavior. The elasticity form, although its use is limited by critical points, is helpful in the derivation of the relative measure of risk aversion and in following optional high-order derivations of measures of prudence, which are omitted from the present analysis and left for analysts who are not afraid to work with relative values of the model. Thus, the relative Arrow-Pratt measure takes the following form:

$$\eta = -(e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w} + e_{(1 - e_{|\partial P/\partial S|,w}),w}) \quad (10)$$

Although we get here the second order elasticity, it is rather simple to understand it. We can denote the value ( $1 - e_{|\partial P/\partial S|,w}$ ) as the **unwillingness to overpay** and consider its elasticity with respect to the wage rate. When the increase in wage rate decreases the unwillingness to overpay, the second derivative  $d^2v/dw^2$  is strictly negative. Moreover, while the unwillingness to overpay is decreasing ( $e_{(1 - e_{|\partial P/\partial S|,w}),w} < 0$ ), the absolute value of its elasticity  $e_{(1 - e_{|\partial P/\partial S|,w}),w}$  is increasing. And with the increase in absolute value of the elasticity of the unwillingness to overpay the relative risk aversion is **increasing**, i.e., the share of risky assets, i.e., unsecured consumption, is decreasing. Of course, it certainly happens because the subsequent growth in the wage rate and

in the equilibrium value of price reduction always results in the increase in real balances, which follow the optimal consumption path of the indirect utility function. It means that the total elasticity of the marginal utility of money is negative, or  $(e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w}) < 0$ . The last assumption can be verified by the following transformation with the help of the Equation (4):

$$e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w} = e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w} + e_{\lambda,|\partial P/\partial S|} - e_{\lambda,|\partial P/\partial S|} = (e_{|\partial P/\partial S|,w} - 1)(1 + e_{\lambda,|\partial P/\partial S|}) \quad (11)$$

The price reduction elasticity of the marginal utility of money is positive, or  $e_{\lambda,|\partial P/\partial S|} > 0$ , because it simply states the growth in the marginal utility of money with increase in price of purchase. Hence, the Equation (11) shows us that, when  $e_{|\partial P/\partial S|,w} < 1$ , any increase in wage rate raises real balances and decreases the marginal utility of money because the total elasticity of the marginal utility of money is negative, or  $(e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w}) < 0$ .

The behavior of the utility function at this stage is described by the following expressions:

$$1 - e_{|\partial P/\partial S|,w} > 0; \lambda > 0; d\lambda/dw < 0; de_{(1-e_{|\partial P/\partial S|,w}),w}/dw < 0 \rightarrow d^2v/dw^2 < 0 \quad (12)$$

Here the relative risk aversion is increasing because the consumer raises the overpayments or, in the case of insurance, makes the latter more and more detailed. The homeowner begins with insurance for the house and he details it with furniture and paintings. Once there is no object to be insured except the coffer with cash. And the consumer insures it by the following increase in the wage rate and he spends on the coffer's insurance the total increase in income. This action means that neither consumption nor cash kept in the coffer are changed. The insurance policy becomes full or comprehensive. The elasticity of price reduction becomes equal to one ( $e_{|\partial P/\partial S|,w} = 1$ ), the unwillingness to overpay becomes equal to zero ( $e_{(1-e_{|\partial P/\partial S|,w}),w} = 0$ ), and, according to the Equation (5), the increasing marginal utility of money expenditures completely offsets the decreasing marginal utility of money income:

$$e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w} = e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} = 0 \quad (13)$$

This stationary point B also represents the decision node (Fig.2). If the consumer decides to re-insure his comprehensive insurance ( $e_{|\partial P/\partial S|,w} > 1$ ) for the given level of consumption, he will decrease his real balances. The utility function will go down ( $dv/dw < 0$ ). Thus, the only way to increase both consumption and real balances is to accept incomplete insurance and guarantees for items to be bought.

This decision results in the increase in the unwillingness to overpay  $e_{(1-e_{|\partial P/\partial S|,w}),w}$ . However, when the increase in the wage rate raises the unwillingness to overpay, the second derivative  $d^2v/dw^2$  becomes positive. The consumer begins to seek risk:

$$1 - e_{|\partial P/\partial S|,w} > 0; \lambda > 0; d\lambda/dw < 0; de_{(1-e_{|\partial P/\partial S|,w}),w}/dw > 0; d^2v/dw^2 > 0 \quad (14)$$

It happens because at the beginning of risk-seeking the positive ( $e_{(1-e_{|\partial P/\partial S|,w}),w} > 0$ ) elasticity of the unwillingness to overpay outweighs the total negative elasticity of the marginal utility of money, or

$$(e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w}) + e_{(1-e|\partial P/\partial S|,w),w} > 0.$$

Here we need some comments on the relationship between real balances and overpayments. The risk-seeking behavior means that the increase in consumption is not well secured. However, the insurance is provided not only by insurance policy but also by real balances, which could represent the *precautionary savings*. The risk-seeking model of behavior means that the total of precautionary savings and insurance policy is insufficient for the optimal level of consumption. It happens because here the relative increase in real balances is followed by the relative decrease in overpayments. Real balances as the tool of protection of consumption, i.e., of wealth, begin to *substitute* overpayments.

Here we come to the question whether precautionary savings and insurance are substitutes or complements. In spite of some analytical solutions of this problem (Ehrlich and Becker (1972)), this question is still open in the general economic analysis. Moreover, when this issue is studied, the attention is usually paid to health and social insurance (Hubbard, Skinner and Zeldes (1995), Guariglia and Rossi (2004)). Here we can only assume the substitutability between money balances and overpayments. The only reason for this assumption is the response of relative overpayments to the continuous decrease in the value of  $\lambda$ , i.e., in the marginal utility of increasing real balances. The economic sense of the decrease in the relative overpayments with respect to the decrease in the marginal utility of money, i.e., in the “price” of money, presumes the substitutability. In addition, the increase in relative overpayments with respect to the decrease in the marginal utility of money presumes that when the consumer is risk-averse, real balances and overpayments becomes complements from the standpoint of the protection of wealth. In any way, the rather harmonic assumption that precautionary savings and insurance are complements in the risk-aversion model and they are substitutes in the risk-seeking model needs, and we are going to see it, more profound analysis.

The comeback from risk seeking to risk aversion is ambiguous. While the positive elasticity of the unwillingness to overpay  $e_{(1-e|\partial P/\partial S|,w),w}$  is decreasing, once it certainly matches the total negative elasticity of the marginal utility of money:

$$(e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w}) + e_{(1-e|\partial P/\partial S|,w),w} = 0 \quad (14)$$

The analysis of the second derivative of the utility function discovers two possible outcomes from the risk neutrality. While the total elasticity of the marginal utility of money is always negative ( $e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w} < 0$ ), the model of behavior depends here on the decision whether to continue to decrease relative overpayments and to increase the unwillingness to overpay ( $e_{(1-e|\partial P/\partial S|,w),w} > 0$ ), or to increase relative overpayments and to decrease the unwillingness to overpay ( $e_{(1-e|\partial P/\partial S|,w),w} < 0$ ). The continuous increase in real balances with the negative total elasticity of the marginal utility of money ( $e_{\lambda,w} + e_{\lambda,|\partial P/\partial S|} e_{|\partial P/\partial S|,w} < 0$ ) provides the negative second derivative



$d^2v/dw^2 < 0$  for both outcomes. However, the increase in the unwillingness to overpay, i.e., in the unwillingness to detail insurance policy, results in the “steeper” *sortie* from the risk neutrality. We can verify this fact without laborious calculations of high-order derivatives but with simple back-on-the envelope sketch. The increase in the unwillingness to overpay ( $e_{(1-e|\partial P/\partial S|,w),w} > 0$ ) simply states the fact that the consumer relies more on precautionary savings than on insurance and he increases the share of risky assets, i.e., the share of uninsured commodities or, more precisely, the *share of commodities with incomplete insurance and guarantees*. Hence, his relative risk aversion becomes decreasing. On the other hand, if he chooses the extension of insurance policy or the decrease in the unwillingness to overpay, he increases his risk aversion. The option to decrease the unwillingness to overpay and to detail insurance policies ( $e_{(1-e|\partial P/\partial S|,w),w} < 0$ ) results in the flat transformation of the utility curve. And with the increasing relative risk aversion the consumer comes again to the next saddle point with the unit elasticity of the price reduction  $e_{|\partial P/\partial S|,w} = 1$  that represents the next decision node (Fig.3):

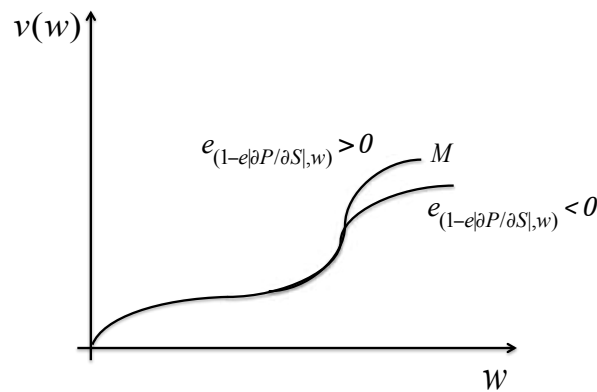


Fig.3. Decreasing vs. increasing relative risk aversion

The path of the decreasing relative risk aversion is more intriguing. There, the consumer can continue to decrease relative overpayments until the moment when the value of price reduction  $|\partial P/\partial S|$  becomes definitely constant. At this moment the elasticity of the unwillingness to pay  $e_{(1-e|\partial P/\partial S|,w),w}$  as well as the elasticity of price reduction  $e_{|\partial P/\partial S|,w}$  becomes equal to zero, and the derivatives of the utility function gets its “true” values, or  $dv/dw = \lambda$  and  $d^2v/dw^2 = d\lambda/dw$ , i.e., the marginal utility of income becomes unit elastic. Evidently, the marginal utility of money  $\lambda$  is equal here to the opportunity costs of holding cash. However, while the value of price reduction  $|\partial P/\partial S|$  doesn’t affect here the marginal utility of money in dynamics because its elasticity is equal to zero, it doesn’t disappear at all and continues to bother the consumer by its constant value. Here this residual constant  $|\partial P/\partial S|$  value can represent the prolongation of the insurance policy for the coffer, leaving all other wealth unsecured.

The insurance for the coffer simply substitutes the costs of illiquidity in the model of the precautionary demand for cash (Whalen 1966, p.316). Thus, the “true” value of money is decreased by the costs of guarding the cash. This assumption corresponds to M.Friedman’s reasoning on the optimum quantity of money:

*“The amount held will, at the margin, reduce utility – because of concern about the safety of the cash, perhaps, or because of pecuniary costs of storing and guarding the cash.” (Friedman 2005 [1969], p.18).*

Indeed, if the consumer follows this path once he could come to the point M of the optimum quantity of money. The volume of precautionary saving with respect to consumption becomes so important that it protects the wealth against any disaster. However, if the marginal utility of the optimum quantity of money equals to zero, the consumer doesn’t need to insure it.

These considerations raises the question why the consumer cannot change the manner of risk aversion and get the “true” value of money at low levels of income, i.e., why the shift from the increasing to the decreasing risk aversion cannot take place at low values of relative overpayments  $e_{|\partial P/\partial S|,w} \ll 1$ . Moreover, it seems that in this case the consumer could avoid saddle points and he could reproduce the exact contour of the Friedman-Savage’s utility function (Friedman and Savage 1948). However, in this case high values of the marginal utility of real balances of low-income levels could hardly be offset by the marginal decrease in the unwillingness to overpay and the consumer will come to the saddle point where he will meet “catastrophic” consequences of both imprudence and over prudence.

### **Economic and mathematical catastrophes: Veblen effect and family altruism**

When G.Becker issued his famous rationalization of family altruism, he stressed the importance of the role of security:

*Therefore, altruism helps families insure their members against disasters and other consequences of uncertainty: each member of an altruistic family is partly insured because all other members are induced to bear some of the burden through changes in contributions from the altruist (Becker 1981, pp.3-4).*

Hence, the family altruism can be introduced in our model as an additional insurance. There are two possible outcomes for this extra insurance from the saddle point.

We can reproduce the decrease in the individual utility function of the head of the family when relative overpayments really become disproportionate to his individual security, or  $e_{|\partial P/\partial S|,w} > 1$ . The extra insurance is provided by the decrease in real balances ( $\partial \lambda / \partial w > 0$ ). However, the following set of equations demonstrates that the decrease in utility ( $\partial v / \partial w < 0$ ) is accompanied

there not by the risk-seeking behavior but by risk-aversion ( $\partial^2 v/\partial w^2 < 0$ ). The utility function takes the form of parabola:

$$1 - e_{|\partial P/\partial S|,w} > 0; \lambda > 0; d\lambda/dw > 0; de_{(1-e_{|\partial P/\partial S|,w}),w}/dw < 0 \rightarrow d^2 v/dw^2 < 0 \quad (15)$$

Here we could wait for the moment when money balances become equal to zero and the family changes her model of behavior. Unfortunately, in the absence of budget constraints the family could borrow. In this case the marginal utility of money income  $\lambda$  becomes negative. However, when the marginal utility of money income  $\lambda$  becomes negative the head of the family can **increase** his utility if he continues to increase overpayments ( $\lambda < 0$ ;  $(1 - e_{|\partial P/\partial S|,w}) < 0$ ;  $dv/dw > 0$ ).

Here the head of the family reproduces the Veblen effect. The previous analysis discovered the correspondence between negative marginal utility of money and the extra overpayments (Malakhov 2013).<sup>1</sup> This is the first “pitfall” the stationary point B prepares for imprudent consumers. Moreover, from the individual point of view the Veblen-effect-like leaving of the saddle point looks more positive than the increase in the unwillingness to overpay. This way can provide more utility until the moment when real balances will be exhausted or the borrowing will be closed and the comeback either to risk aversion or to risk-seeking behavior will take place (Fig.4). In addition, only here we can definitely talk about **maximizing** behavior. Indeed, if the aspiration level motivates the consumer to get from the search more than from the labor, i.e., to get marginal savings on purchase greater than the wage rate, the consumer immediately follows the Veblen effect (Malakhov 2013):

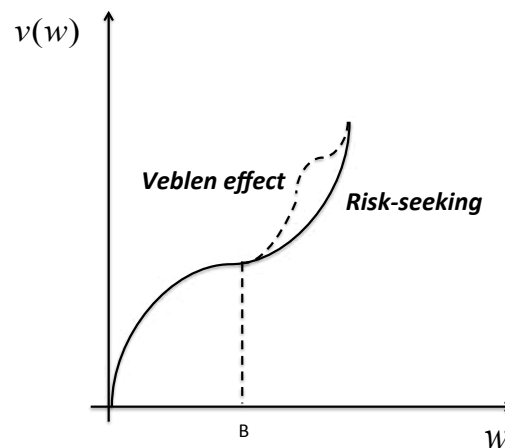


Fig.4. The option of Veblen effect in risk-seeking behavior

The equilibrium at the saddle point B is unstable. The consumer can take either maximizing or satisficing decision. The maximizing decision results in the Veblen effect and the satisficing decision produces the ordinary risk seeking behavior. However, the maximizing decision is the decision to purchase a “bad” item with negative marginal utility due to the value  $\lambda < 0$ . The rules

<sup>1</sup> If the attribute of the negative marginal utility of money represents a subjective value, the Veblen effect decision could be estimated from the external satisficing point of view as the decision that decreases the utility.

of the optimization of consumption-leisure stop working, the constraint line takes the north-east direction, and the increase in utility happens only due to an important increase in leisure time that increases the purchase price and compensates the consumption of “bad” item. It really happens when imprudent young family considers holidays on the seaside or in mountains to be vital and parents agree to sponsor vacations for grandchildren. *Hélas*, in the search model of behavior even skiing might become “bad”.

The occurrence of Veblen effect with regard to the previous reasoning on the optimum quantity of money tells us that Veblen effect can take place at rather modest levels of income where consumption is far from satiation. However, although this scenario can take place, it does not seem well compatible with the description of the individual utility function within the family. There is another possibility to present family altruism. We can pretend the head of the family to be more “economic man” and to separate altruism from the individual utility function. If we take the factor of **giving** as the share of the individual wage rate, we get the following utility function  $v^g(w)=v(w)-gw$ . However, there we automatically get the other “pitfall” or the mathematical “fold”-type catastrophe due to the existence of the saddle point B and to its unstable equilibrium in the original utility function (Fig.5):

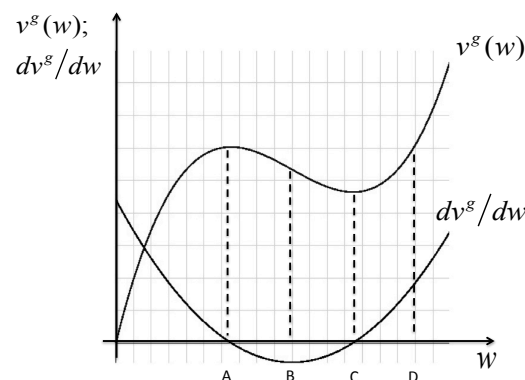


Fig.5. "Fold" catastrophe of family altruism

In this case the decrease in the utility function starts at point A when the consumer, the head of the family, is still risk averse and he continues to make protection of his wealth by the increasing real balances and by increasing overpayments. The continuous increase in overpayments discovers the unwillingness of the head of the family to economize. Here, the behavior looks like “pure” altruism. However, once the head of the family changes the model of his behavior and he begins to make risky decisions. It happens at point B when he passes the saddle point of the original utility function with the unit elastic price reduction ( $e_{|P/P|,w}=1$ ). The following increasing unwillingness to overpay gives an idea that the nature of his altruism has been changed. The head of the family becomes more “pragmatic”. Although his altruism does not exhausted, his purchase decisions become more prudent. They begin to look like investments.

The investments in family reach its peak at point C. Finally, the head of the family begins to feel again the increase in his utility function and at point D he no longer suffers from his altruism, or he finally gets returns on investments:

*“Altruistic parents might not have more children than selfish parents, but they invest more in the human capital or quality of children because the utility of altruistic parents is raised by investment returns that accrue to their children.” (Becker 1981, p.12).*

Indeed, the movement of the utility curve from point A to point D reminds the parental behavior from the birth of a child till the go-out of a young man from the nest. At the beginning parents do not economize on purchases for babies. They are trying to buy everything of high quality and with guarantees. Once, at point B, these purchases take the form of investments, which even in prudent manner lead to point C in the bottom due to their importance. However, the earlier decision at point B to reduce relative overpayments continues to work and finally it pulls out the head of the family from the “pitfall”.<sup>2</sup>

### **Interest rate and willingness to overpay for consumer credit**

The common question addressed to the model presented here why it doesn't follow the original G.Stigler's presentation of the equality of marginal values of search with respect to the interest rate. Indeed, the core equation of the model could be presented in that manner:

$$i \times w \frac{\partial L}{\partial S} = Q \frac{\partial P}{\partial S} \quad (16)$$

However, even G.Stigler agreed, that interest rate made *“expected reduction in price...be smaller than the smallest unit of currency”* (Stigler 1961, p.219). While the dynamics of the satisficing decision procedure is short, the model assumes that consumers usually ignore interest rate during the search. If the satisficing consumer doesn't calculate marginal values of search, why he should compute decimals of interest rate and of probabilities?

However, the methodological concern about interest rate can be gratified if we envisage the risk of delay of consumption, i.e., the risk of unexpected rise in prices, and explain overpayments as ***payments for consumer credit***. Other words, interest rate increases price dispersion as well as marginal savings on purchase. The greater is an item under consumer credit, the greater are the marginal savings on this purchase. In this case the comprehensive insurance is transformed into

---

<sup>2</sup> When G.Becker cited King Lear's Fool in order to illustrate the Rotten Kid Theorem by the parental willingness to delay contributions until last stage of life he did not take into account the possibility of saddle points in the parental utility function. We have seen that if the consumer continues to increase overpayments without change in the model of behavior at the saddle point his utility goes down infinitely. Once upon a time King Lear simply missed that point. And from the literature point of view it would be better here to remember d'Artagnan-father, who contributed to his son only *quinze écus*, his horse, and some parental advices.

the comprehensive consumer credit and the extra comprehensive insurance ( $e_{|\partial P/\partial S|,w} > 1$ ) is transformed into the refinancing of existing debt.

When the consumer buys an item against **coming** increase in the wage rate it means that the value  $e_{|\partial P/\partial S|,w} = 1$  also is coming. This consideration with respect to consumer credit tells us that saddle point with its **risk neutrality** is more common economic phenomenon than it was seen from the point of view of insurance. People hold cash for everyday expenses where the cash represents the residual of interest payments. And at  $e_{|\partial P/\partial S|,w} = 1$  level the total increase in income is going to finance the debt. Neither consumption nor real balances are changed. After that, if the consumer wants to buy another big-ticket item he should either refinance current debt or search this item more intensively in order to decrease relative overpayments, i.e., to find more beneficial credit for the new purchase. The first way decreases the utility and the second way increases risk of unexpected rise in prices during the search.

We remember that while the positive elasticity of the unwillingness to overpay  $e_{(1-e|\partial P/\partial S|,w),w}$  is decreasing, once it certainly matches the total negative elasticity of the marginal utility of money and the second derivative of the utility function becomes equal to zero, or  $d^2v/dw^2 = 0$ . The following increase in the wage rate again gives a chance to expand consumer credit by the increase in relative overpayments. Facing price uncertainty, the consumer chooses this way of the increasing relative risk aversion. But we already know that real balances at this moment can also protect consumption. If the consumer chooses the decreasing relative risk aversion path, once overpayments become definitely constant. Here, the constant  $|\partial P/\partial S|$  value of consumer credit could mean that products are delivered every day by a boy from the neighboring grocery store and once a month the consumer signs a check to the grocer like he renews the insurance policy for the coffer every year.

***The constant  $|\partial P/\partial S|$  value and the constant place of purchase mean that the consumer is satiated by items that could be bought in other places, i.e., by items that could produce another marginal savings on purchase.***

In addition, the consumer also can get the optimum quantity of money but he should decrease for that liquidity costs to the zero level, for example, to give to the grocer a right to debit his current account. With that the consumer reproduces the optimal precautionary model of money holdings – credit is not used, liquidity costs are zero, and the marginal utility of money also equals to zero (Fenestra 1986, p. 283).

However, this theoretical assumption is really illustrative. There are more realistic paths and both of them are well known to us because they represent “catastrophic” solutions. Coming to very low values of the marginal utility of money, either the consumer buys an extraordinary item

and, therefore, increases the time of leisure to consume it or he starts the practice of charity that might take a form of the sponsorship for venture investments.

### **Conclusion**

The analysis of consumer behavior presented in this paper discovers the methodological power of relative values, which are produced by the process of search. The consumer's search for beneficial price reduction can be interpreted as the search for reduction in insurance or in interest payments.

The motivation to reduce time of search and to increase quality in consumption after the increase in the wage rate inevitably leads a consumer to the saddle point of the utility function. And the equilibrium in the saddle point is unstable. The consumer can follow maximizing path where he produces the Veblen effect or he can follow common satisficing path where he should take risk. However, even the satisficing path comes to the economic catastrophe of the decrease in utility if consumer takes into account the factor of giving or family altruism.

The model also provides a graphical difference between increasing and decreasing relative risk aversion. The increasing relative risk aversion path could come to the new saddle point of comprehensive insurance or complete consumer credit and the decreasing relative risk aversion could come to the optimum quantity of money.

In addition, this approach can revive the discussion on the optimum quantity of money with an interesting argument. Indeed, when overpayments become constant they could represent not direct interest payments but some fixed expenditures the consumer pays to the government to finance the interest payments on money (Bewley 1983, Mehrling 1995).

The question of the lump-sum taxation leads to the understanding that the model presented here could be useful in the analysis of the optimal taxation. If we substitute in the individual utility function the factor of giving by income tax we also get the "fold"-type catastrophe. However, if one tries to go further and to explain overpayments by VAT or excise tax, the coming trade-off between income taxes and overpayments should be examined with prudence.

### **References**

1. Becker G.S. (1981) 'Altruism in the Family and Selfishness in the Market Place', *Economica*, New Series, 48 (189), 1-15.
2. Bewley, T. (1983). 'A Difficulty with the Optimum Quantity of Money', *Econometrica*, 51, 5, 1485-1504.

3. Diamond, P. (1971). 'A Model of Price Adjustment', *Journal of Economic Theory*, 3, 156-168.
4. Ehrlich, I., Becker, G.S. (1972). 'Market Insurance, Self-Insurance and Self-Protection' *Journal of Political Economy*. 80(4), 623-648.
5. Fenestra, R. (1986). "Functional equivalence between Liquidity Costs and the Utility of Money", *Journal of Monetary Economics*, 17, 271-291.
6. Friedman, M., Savage, L.J. (1948). 'The Utility Analysis of Choices Involving Risk', *Journal of Political Economy*, 56, 279-304.
7. Friedman, M. (2005 [1969]) 'The optimum quantity of money' Transaction Publishers.
8. Guariglia, A., Rossi, M. (2004). 'Private medical insurance and saving: evidence from the British Household Panel Survey' *Journal of Health Economics*. 23, 761-783.
9. Hubbard, R.G., Skinner, J., Zelders, P. (1995) 'Precautionary Saving and Social Insurance' *Journal of Political Economy*. 103(2), 360-99.
10. Kahneman, D., Tversky, A. (1979) 'Prospect Theory: An Analysis of Decision under Risk', *Econometrica*, XLVII (1979), 263—291.
11. Mehrling, P. (1995). 'A Note on the Optimum Quantity of Money', *Journal of Mathematical Economics*, 24, 249-258.
12. Malakhov, S. (2013). 'Money Flexibility and Optimal Consumption-Leisure Choice' *Theoretical and Practical Research in Economic Fields*, IV (1), 77-88. ([http://www.asers.eu/asers\\_files/tpref/TPREF%20Volume%20IV%20Issue%201\\_7\\_%20Summer%202013\\_last.pdf](http://www.asers.eu/asers_files/tpref/TPREF%20Volume%20IV%20Issue%201_7_%20Summer%202013_last.pdf))
13. Malakhov, S. (2014) 'Satisficing Decision Procedure and Optimal Consumption-Leisure Choice.' *International Journal of Social Science Research*, 2 (2), 138-151(<http://dx.doi.org/10.5296/ijssr.v2i2.6158>)



14. Marshall, A. (1920 [1890] 'Principles of Economics. An Introductory Volume.' London: Macmillan and Co., Ltd.
15. Stigler, J. (1961). 'The Economics of Information.' *Journal of Political Economy*, 69(3), 213-225.
16. Whalen, E. (1966). 'A Rationalization for the Precautionary Demand for Cash', *Quarterly Journal of Economics*, 80 (2), 314-324.