A detailed analysis of fulfilling and delinquency of payments on loan

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Abstract. A new model for predicting the future expected cash flows from a loan is developed. It is based on a detailed analysis of the events of fulfilling, delinquency and default of each individual payment on the loan. The proposed model has significantly less uncertainty compared with the Markov chain model with the same detailing. The model is expected to have greater predictive power in comparison to the traditional models, and its usage will allow reducing the interest rate on the loan. The results of estimation of the probabilities of payments over time and the future expected cash flows from the loan with monthly equal principal repayment are given.

Keywords: loan, payment, delinquency, default, cash flow, present value, interest rate, credit spread, credit risk, liquidity risk, Markov chain, soft collection

JEL Classifications: G21, G12

1. INTRODUCTION

To effectively manage both the credit and liquidity risks a bank should carefully assess the future expected cash flows from loans. The traditional models often overestimate the credit and liquidity risks. So, the task of improving the approach to appraising the future expected cash flows from the loans is actual.

2. AN ANALYSIS OF THE LITERATURE

The traditional approach to estimating the future expected cash flows from loans assumes that each individual payment on the loan has only two states being paid or default. The default is presumed to occur immediately after the event of failure of individual payment on loan, neglecting its overdue term (see, for example, Bohn and Stein, 2009; Jorion, 2003; Resti and Sironi, 2007).

However, the "current" (i.e. without delinquency) state of the loan does not pass into the "default" state immediately. Indeed, the loan state should consistently pass from "current" state through "30 days past due" and "90 days past due" states to the "default" one (Grimshaw and Alexander). In addition, the traditional approach ignores the fact that the delinquent payment could be paid by the borrower. But such payments have particular importance as a bank begins to work with the borrowers on early stages of delinquency at soft-collection phase.

Thus, the traditional approach ignores the migration of a loan through the terms of delinquency. As a result, the future expected cash flows from the loan and its present value are turned out to be underestimated. Meanwhile, the risky interest rate on the loan is overvalued.

To overcome these shortcomings Grimshaw and Alexander have developed the Markov chain model to predict the outstanding balances of subprime mortgages having different terms of delinquency. Bidyuk and Torovets (2010) have investigated a similar model for evaluation of delinquency of retail loan portfolio.

The matrix of transition probabilities in the Markov chain model shows the probabilistic changes of delinquency states of the loan for the one month (Table 1 and Fig. 1). That is, the matrix elements are the transition probabilities that the loan changes the one delinquency state on another for the one month. Due to the transition matrix the bank could control the change of the credit quality (Barkman, 1977).

Table 1. The matrix of probabilities of transition from one delinquency state to another* (Grimshaw and Alexander)

		Delinquency state as of current month				
		Current	1M	2M	3M	>3M
Delinquency state as of previous month	Current	p_{11}	p_{12}	0	0	0
	1M	p_{21}	p_{22}	p_{23}	0	0
	2M	p_{31}	p_{32}	p_{33}	p_{34}	0
	3M	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}
	>3M	0	0	0	0	1

**Note. In Table 1 the denotation of “1M” means “1 month past due”, etc.*

The advantage of the Markov chain model (Grimshaw and Alexander, Bidyuk and Torovets, 2010) is that it emphasizes the importance of consideration of loan migration. It gives a clear insight that there is a given sequence of changes of delinquency terms.

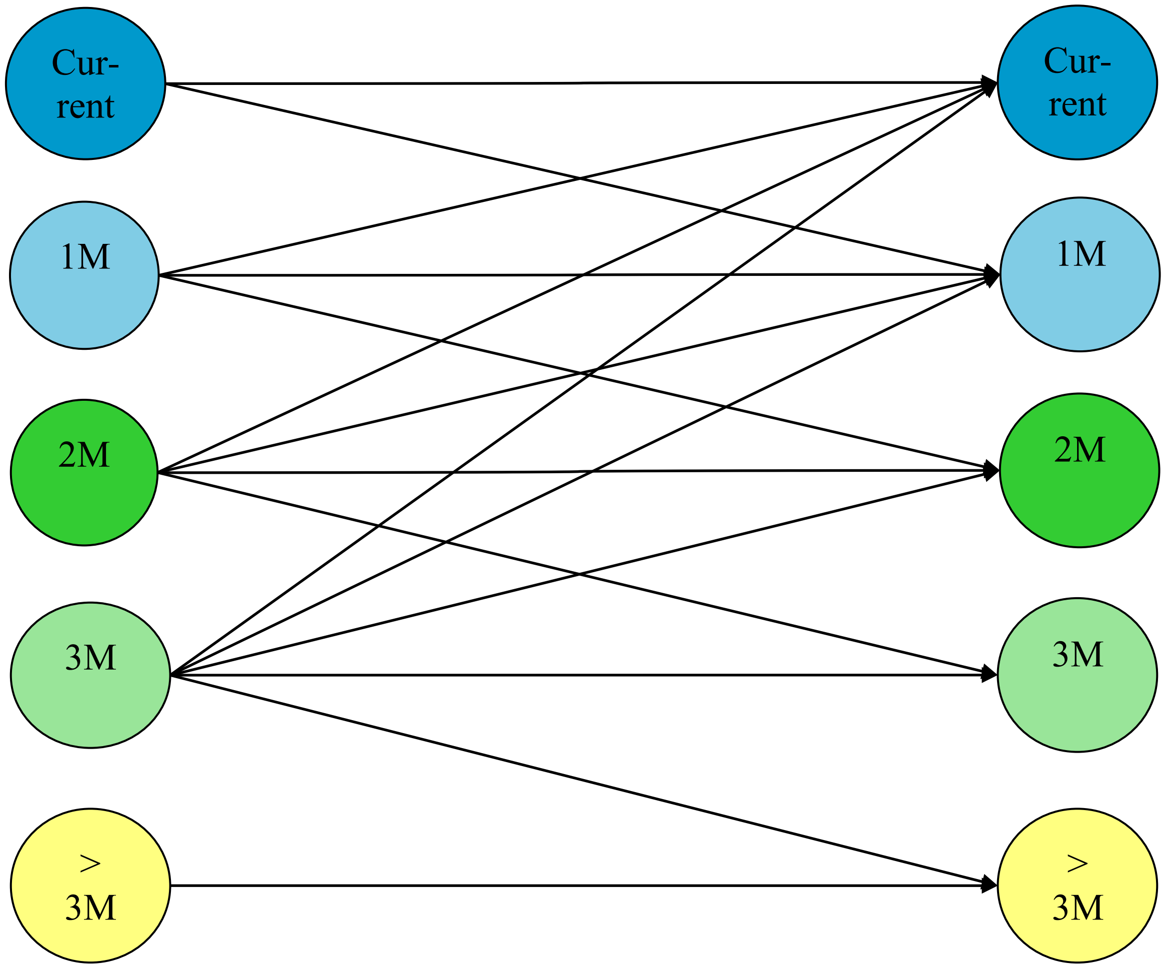

Fig. 1. The possible paths of transition from one delinquency state to another*

**Note. On Fig.1 the denotation of "1M" means "1 month past due", etc.*

However, this approach contains unobservable transitions (Bidyuk and Torovets, 2010). For example, the observed transition from the "current" to the "90 days past due" one actually contains the unobservable transitions through successive "30 days past due" and "60 days past due" states. As a result, this approach may capture the statistical relationships that do not exist in practice (Voloshyn, 2008). In other words, the Markov chain model does not take into account all the cause and effect relationships between payments. As a result, it replaces them by statistical relationships.

To overcome this disadvantage, it is necessary to use the long series of historical data (Bidyuk and Torovets, 2010). However, the delinquency states of loan may rapidly vary due to the dynamic changes in the macroeconomic situation. As a result, the historical data quickly become obsolete too.

The transition matrix has relatively many independent transition probabilities. In other words, the problem of determining the transition probabilities has a relatively large statistical uncertainty. For example, for the loans with the five delinquency states the number of independent transition probabilities is equal to 10. *Note that to calculate this number it was taken into account that the sum of the probabilities for each row of the matrix must be equal to one.*

In addition, to evaluate the transition probabilities it requires to use the complex mathematical algorithms such as Bayesian estimation, the method of least squares, etc. (Grimshaw and Alexander, Bidyuk and Torovets, 2010). That in turn needs keeping the non-core for bank specialists in mathematics.

These problems are generated on author's point of view that the Markov chain model is utilized to analyze the loan balances that are in various delinquency states, but not to analyze the events of execution and delinquency of each individual payment according to contractual payment schedule for the loan.

The purpose of the paper is to reduce the statistical uncertainty of the problem of the forecasting the future expected cash flows from the loan by a detailed analysis of the events of execution, delinquency and default of each individual payment according to contractual payment schedule for the loan.

3. A MODEL OF FULFILLING AND DELINQUENCY OF EACH INDIVIUAL PAYMENT ON THE LOAN

For further exposition of the material let's utilize the commonly used definition of default: a borrower incurs default if it will not fulfill the contract payment during 90 days. In other words, the bank does not expect to execute only those payments that are overdue for more than three months. The rest delinquent payments could be paid.

The subject of study is the cash flows generated by the loan, but not cash flows from the sale of collateral. Thus, let's consider the cash flows from the **unsecured** loan with monthly installments, and track execution and delinquency of each individual contract payment on loan belonging the one generation. Note that the borrower can (with some probability) to execute the next installment only when it fulfilled the previous one.

It is proposed to distinguish the three states of the payment execution and the three states of its delinquency (total six states):

$A_1(t)$ is the paid in time (in t-th month) current payment under condition that the previous payment was made. This previous payment could be executed in the previous (t-1)-th months (i.e. in time) or in t-th month if it had one month overdue;

$B_1(t)$ is the delinquent for one month current payment under condition that the previous payment was made;

$A_2(t)$ is the repaid in t-th month current payment which had one month overdue under condition that the previous payment was earlier made or the delinquent for two months previous payment was paid in t-th month;

$B_2(t)$ is the delinquent for two months current payment under condition that the previous payment was made;

$A_3(t)$ is the repaid in t-th month current payment that was delinquent for two months under condition that the previous payment was earlier made;

$B_3(t)$ is the delinquent for three months current payment, i.e. credit default.

Note that number of payment states depends on the loan schedule and the delinquency term after which the default is recognized.

Thus, all payments can be arranged in three pairs of payments with opposing states being "paid" and "delinquent", namely: $A_1(t)$ and $B_1(t)$; $A_2(t)$ and $B_2(t)$, $A_3(t)$ and $B_3(t)$.

Note that unlike the transition matrix (Table 1) the all payments (besides $A_1(t)$) cannot be leaved in the initial state. They can be executed and move “up”, reducing the delinquency, or be delinquent and move “down”, increasing the delinquency.

Using these states let’s track execution and delinquency of each individual payment on the loan. The scheme of payments and arrears and their corresponding transition probabilities are presented on Fig. 2 and in Table 2.

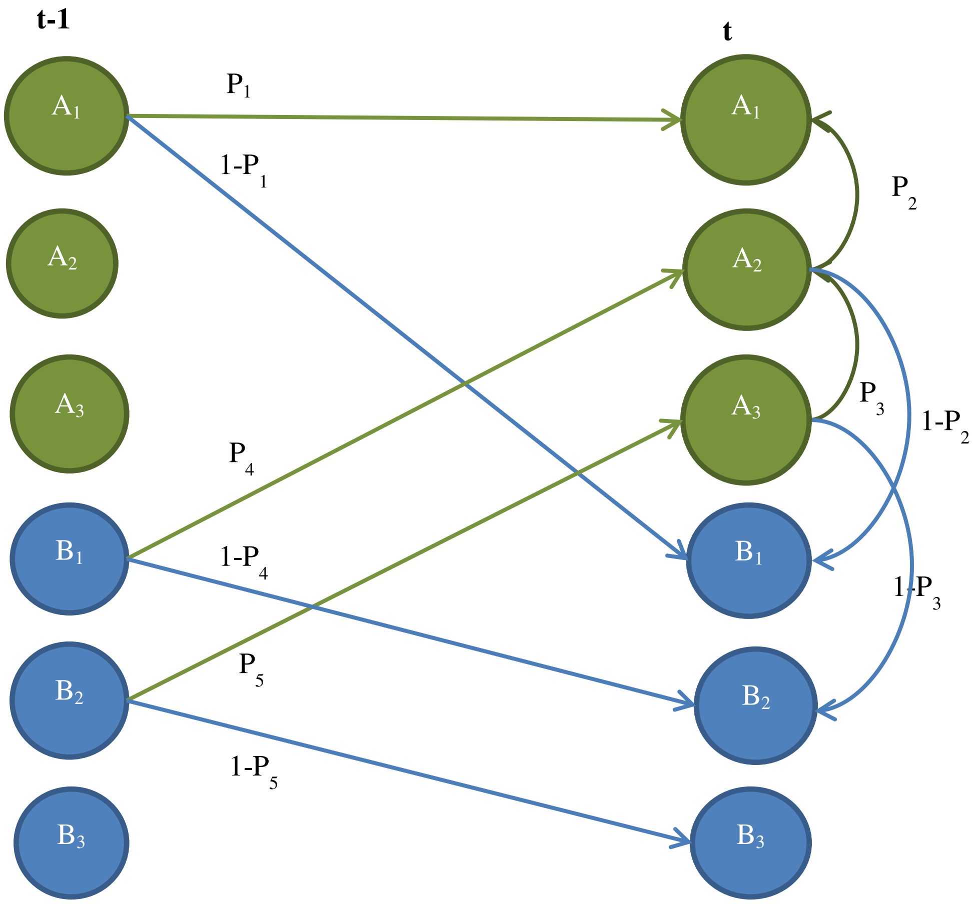

Fig. 2. Scheme of execution and delinquency of payments on one monthly step

Table 2. Unconditional probabilities of execution and delinquency of payments depending on its states

Payment state		Paid			Delinquent		
		$A_1(t)$	$A_2(t)$	$A_3(t)$	$B_1(t)$	$B_2(t)$	$B_3(t)$
Paid	$A_1(t)$	P_1			$1- P_1$		
	$A_2(t)$	P_2			$1- P_2$		
	$A_3(t)$		P_3			$1- P_3$	
Delinquent	$B_1(t)$		P_4			$1- P_4$	
	$B_2(t)$			P_5			$1- P_5$
	$B_3(t)$						1

Taking into account the scheme on Fig. 2 and the unconditional probabilities in Table 2 let's write the equations for determining the probabilities of execution and delinquency of payments at a certain time t:

$A_3(t) = B_2(t-1) \times P_5,$ (1)

$B_3(t) = B_2(t-1) \times (1-P_5).$ (2)

$A_2(t) = A_3(t) \times P_3 + B_1(t-1) \times P_4,$ (3)

$B_2(t) = A_3(t) \times (1-P_3) + B_1(t-1) \times (1-P_4),$ (4)

$A_1(t) = A_1(t-1) \times P_1 + A_2(t) \times P_2,$ (5)

$B_1(t) = A_1(t-1) \times (1-P_1) + A_2(t) \times (1-P_2),$ (6)

where $A_1(t)$, $A_2(t)$, $A_3(t)$ are the probabilities of execution of payment in the current (in time), the second and the third month from the scheduled payment data, respectively;

$B_1(t)$, $B_2(t)$, $B_3(t)$ are the probabilities of delinquency of payment on one, two and three months, respectively.

Note that the equations (1-6) are well-arranged helping to simplify calculations.

Thus, the proposed model is based on the detailed analysis of existing cause and effect relationships between payments. As a result, the cash flows become more organized, and the number of unconditional probabilities (Table 2) is significantly reduced (from 10 to 5 probabilities). Decreasing the statistical uncertainty of the problem of forecasting the future expected cash flows from loans is expected to increase the predictive power of the proposed model (1-6).

4. EXAMPLE OF CALCULATION

Let's consider the task of predicting the future expected cash flows from a loan belonging one generation. Let a bank issue the unsecured (LGD = 100%) fixed rate loan for a period of 12 months with the initial amount of \$1,200 and the equal monthly principal payment that is equal to \$100 per month. Assume that default of the loan stops accruing the interests on this loan, and all probabilities of the payment execution are the same and equal to 0.9, i.e.:

$$P = P_1, P_2, P_3, P_4, P_5 = 0.9. \qquad (7)$$

Using the model (1-6) let's calculate the probabilities of execution (Table 3), delinquency and default (Table 4) of payments.

Table 3. Probabilities of fulfilling of payments over time

Payment state	Time *t*, month							
	0	1	2	3	4	5	6	7
$A_1(t)$	1,000	0,900	0,891	0,889	0,888	0,887	0,886	0,885
$A_2(t)$			0,090	0,097	0,098	0,098	0,098	0,097
$A_3(t)$				0,009	0,010	0,010	0,010	0,010
Cumulative probability of payment, Y(t)	1,000	0,900	0,981	0,996	0,996	0,995	0,994	0,993

Table 3 (continued). Probabilities of fulfilling of payments over time

Payment state	Time *t*, month						
	8	9	10	11	12	13	14
$A_1(t)$	0,884	0,884	0,883	0,882	0,881		
$A_2(t)$	0,097	0,097	0,097	0,097	0,097	0,097	
$A_3(t)$	0,010	0,010	0,010	0,010	0,010	0,010	0,010
Cumulative probability of payment, Y(t)	0,992	0,990	0,989	0,988	0,987	0,106	0,010

Table 4. Probabilities of delinquency and default of payments over time

Delinquency state	Time *t*, month							
	0	1	2	3	4	5	6	7
$B_1(t)$		0,100	0,099	0,099	0,099	0,099	0,098	0,098
$B_2(t)$			0,010	0,011	0,011	0,011	0,011	0,011
$B_3(t)$				0,001	0,001	0,001	0,001	0,001
Cumulative probability of default, Z(t)				0,001	0,002	0,003	0,004	0,005

Table 4 (continued). Probabilities of delinquency and default of payments over time

Delinquency state	Time *t*, month						
	8	9	10	11	12	13	14
$B_1(t)$	0,098	0,098	0,098	0,098	0,098		
$B_2(t)$	0,011	0,011	0,011	0,011	0,011	0,011	
$B_3(t)$	0,001	0,001	0,001	0,001	0,001	0,001	0,001
Cumulative probability of default, Z(t)	0,006	0,007	0,009	0,010	0,011	0,012	0,013

Using the probabilities from Tables 3 and 4 let's calculate the cash flows from the loan and the risky interest rates on the one. For this purpose it is applied the following formulas.

The actual cash flow of principal is equal to $pCF(t) = pCF_{contr} \times Y(t)$,
where Y(t) is the cumulative probability of payment;
pCF_{contr} = \$100 per month is the cash flow of principal according to contract payment schedule.

The cash flow of defaulted principal is equal to $dCF(t) = pCF_{contr} \times Z(t)$,
where Z(t) is the cumulative probability of default.

The outstanding balance of loan is equal to $B(t) = B(t-1) – pCF(t)$,
where B(t), B(t-1) are the outstanding balances of the loan at times t and t-1, respectively;
B(0) = \$1200.

The balance of working loan at the time t is equal to $WB(t) = B(t) – dCF(t)$.

The interest on working loan is equal to $iCF(t) = WB(t) \times R/12$.

The total cash flow is the sum of cash flows of principal and interest on the working loan: $CF(t) = pCF(t) + iCF(t)$.

The discounted total cash flow is equal to $CFD(t) = CF(t) \times D(t)$,
where $D(t) = 1 / (1 + R_D/12)^t$ is the discount factor, R_D = 14,00% is the discount rate.

The results of evaluation of the future expected cash flows from the loan are presented in Table 5.

In order to the present value of cash flows from loan would be equal to the initial value of the loan, i.e.:

$$PV = \sum CF(t) \times D(t) = \$1200,$$

the annual interest rate should be equal to R = 15.19%. Accordingly, the credit spread is equal to 15.19% - 14.00% = 1.19%.

It should be noted the following. At the 12-th month (at the end of contract) the default on the loan is not recognized because the delinquency of the last payment has not yet exceeded three months. Therefore, the time horizon in Tables 3-5 was expanded from 12 to 14 months. The borrower uses the loan during these two months (13 and 14 months) and therefore it should pay interest on loan.

Note that the cumulative probability of payment Y(t) in the Table 3 is the survival probability at the time t. At the end of the loan contract term the survival probability is equal to Y(12) = 0,987.

If it is utilized the traditional model which does not take into account repayment of arrears the survival probability of payments at time t = 12 will be equal only to Y(12) = $P^{12} = 0{,}9^{12} = 0{,}282$. Of course, such a low survival probability causes dramatic overvaluation of credit and liquidity risks of the bank which in turn leads to a significant overestimation of the interest rate on the loan to cover credit risk.

Table 5. Cash flows and outstanding balances of the loan in USD.

Time, month	Contract cash flows	Actual cash flows	Defaulted cash flows	Outstanding loan balances	Working loan balances	Interest from working loans	General cash flows	Discounted cash flows
1	100,00	90,00	0,00	1 200,00	1 200,00	25,15	115,15	112,89
2	100,00	98,10	0,00	1 110,00	1 110,00	23,26	121,36	116,65
3	100,00	99,56	0,10	1 011,90	1 011,80	21,20	120,76	113,80
4	100,00	99,58	0,21	912,34	912,03	19,11	118,69	109,65
5	100,00	99,48	0,32	812,77	812,14	17,02	116,50	105,52
6	100,00	99,37	0,43	713,29	712,24	14,93	114,30	101,49
7	100,00	99,26	0,53	613,92	612,33	12,83	112,10	97,59
8	100,00	99,15	0,64	514,65	512,43	10,74	109,89	93,79
9	100,00	99,05	0,75	415,50	412,52	8,65	107,69	90,11
10	100,00	98,94	0,86	316,45	312,62	6,55	105,49	86,54
11	100,00	98,83	0,97	217,51	212,71	4,46	103,29	83,07
12	100,00	98,72	1,07	118,68	112,81	2,36	101,09	79,71
13		10,65	1,18	19,96	12,90	0,27	10,92	8,44
14		0,97	1,29	9,31	0,97	0,02	0,99	0,75
	Total repaid:	**1 191,66**	**Defaulted loans:**	**8,34**	**0,00**		**Present value:**	**1 200,00**

It should be noted that the proposed model is easily extended to loan with more complex payment schedule (for example, with unequal installments); to the case when the default is considered as more long delinquency of payment on loan, for example, more than 180 days past due.

In addition, this model can be used to evaluate not only the credit risk and liquidity risk of bank, but also to estimate the effectiveness of the pretrial soft debt collection.

5. SUMMARY

The proposed model of events of execution, delinquency and default of each individual payment on a loan, based on consideration of cause and effect relationships between payments in different states, enables reducing the statistical uncertainty of forecasting the future expected cash flows from the loan. Thus, if it is distinguished the five delinquency states of the loan that the number of independent probabilities is reduced from 10 (for the Markov chain model) to 5 (for the proposed model), that is in 2 times! This model is expected to have greater predictive power in comparison to the traditional models, and its usage will allow reducing the interest rate on the loan.

The future studies will be directed on examining the adequacy of the proposed model to actual data on loans; on comparison with results obtained by conventional models; and on developing models that take into account the dependence of the probability of payments on the borrower's characteristics, macroeconomic factors and the strategies of soft debt collection.

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