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Abstract

This paper replicates [Cheung and Lai, 1995], who use response surface analysis to obtain approximate finite-sample critical values adjusted for lag order and sample size for the augmented Dickey-Fuller test. We obtain results that are quite close to their results. We provide the Ox source code. We also provide a Windows application with a graphical user interface, which makes obtaining custom critical values quite simple.

Keywords. Finite-sample critical value; Monte Carlo; Response surface.

1 Introduction

The augmented Dickey-Fuller (ADF) test is the most used unit root test in econometrics. [Dickey and Fuller, 1979] derived the asymptotic distribution of the ADF test and showed that it is independent of the lag order k. [MacKinnon, 1991] used response surface analysis to obtain the approximate finite-sample critical values for any sample size for $k = 1$. Also using response surface analysis, [Cheung and Lai, 1995] extends the results of [MacKinnon, 1991] for $k > 1$. Their study has important implications for econometric practice since the test results can be affected by the sample size and the lag order. In this paper, we replicate their study.

2 Replication

The augmented Dickey-Fuller test involves the following auxiliary regression

$$\triangle x_t = \mu + \gamma t + \alpha x_{t-1} + \sum_{j=1}^{k-1} \beta_j \triangle x_{t-j} + u_t \tag{1}$$

where x_t is the time series to be tested for unit root, $\triangle$ is the difference operator, t is the time trend, and u_t is a white-noise error term. The test is based on the t ratio of the α coefficient. Note that $k \geq 1$ and for $k = 1$, the test does not include

Table 1: Response Surface Estimation of Critical Values

Coefficients	No constant or trend			Constant, no trend			Constant and trend		
& statistics	10%	5%	1%	10%	5%	1%	10%	5%	1%
τ_0	-1.613	-1.934	-2.562	-2.563	-2.858	-3.424	-3.128	-3.416	-3.982
s.e.	0.001	0.001	0.003	0.002	0.003	0.005	0.002	0.003	0.005
τ_1	-0.325	-1.036	-2.455	-1.688	-2.802	-5.487	-2.462	-3.525	-5.727
s.e.	^{f}0.158	0.179	0.316	0.272	0.380	0.702	0.386	0.484	0.864
τ_2	-2.359	-9.018	-33.470	-8.965	-19.754	-64.657	-22.873	-47.512	-130.728
s.e.	^{n}2.577	2.889	4.950	^{t}5.059	7.031	12.962	7.452	9.167	16.787
ϕ_1	0.312	0.325	0.340	0.663	0.736	0.802	0.973	1.085	1.434
s.e.	0.034	0.039	0.062	0.040	0.053	0.092	0.057	0.069	0.137
ϕ_2	-0.470	-0.552	-0.986	-0.642	-1.146	-1.863	-0.931	-1.607	-3.810
s.e.	0.110	0.128	0.183	0.112	0.162	0.299	0.163	0.188	0.493
R^2	0.587	0.761	0.918	0.853	0.902	0.960	0.875	0.928	0.968
$\hat{\sigma}_\epsilon$	0.010	0.012	0.021	0.014	0.017	0.029	0.020	0.024	0.039
Mean $\lvert\hat{\epsilon}\rvert$	0.008	0.009	0.017	0.010	0.013	0.023	0.015	0.017	0.028
Max $\lvert\hat{\epsilon}\rvert$	0.034	0.039	0.059	0.048	0.073	0.115	0.072	0.087	0.144
*Mean $\lvert\hat{\epsilon}\rvert$	0.007	0.008	0.015	0.007	0.008	0.015	0.007	0.008	0.015
*Max $\lvert\hat{\epsilon}\rvert$	0.028	0.031	0.053	0.028	0.031	0.053	0.028	0.031	0.053

Significance is indicated by ^{f} for the 5% level, and by ^{t} for the 10% level.^{n} indicates "not significant".
$|\hat{\epsilon}|$ indicates the absolute value of the residuals. * indicates computed from residuals for $T \geq 30$.

any augmentation and is simply called the Dickey-Fuller test. The critical values of the test are tabulated for $k = 1$ in most econometrics textbooks, e.g., [Fuller, 1976] and [Hamilton, 1994]. However, [Cheung and Lai, 1995] shows that the critical values are affected by the lag order as well as the sample size. In an extensive Monte Carlo experiment, they obtain the improved critical values which take them into account.

[Cheung and Lai, 1995]'s Monte Carlo experiment is conducted in the following steps:

STEP 1. Generate $I(1)$ series

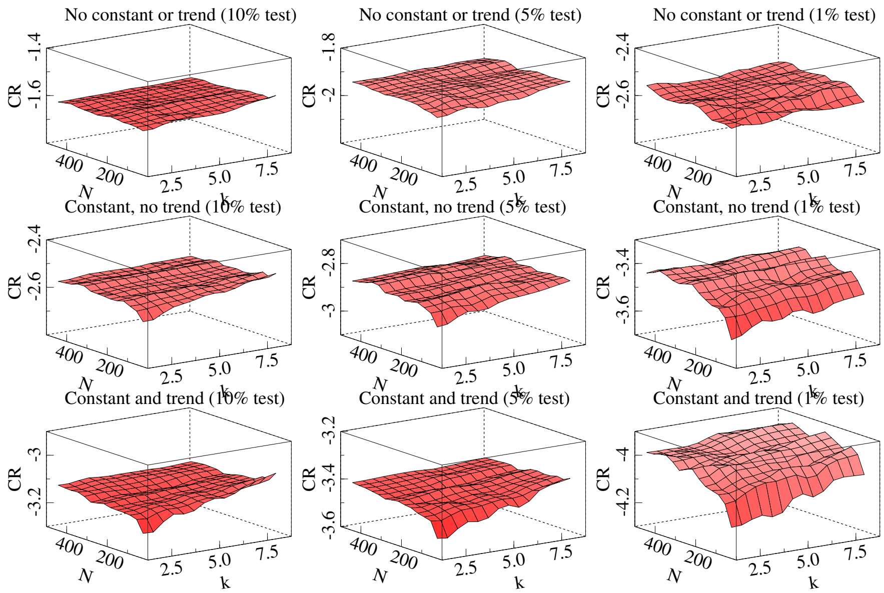
$$x_t = x_{t-1} + e_t \tag{2}$$

where $e_t \sim N(0, 1)$[1]. The initial value x_0 is set to zero. The sample sizes come from the set N = {18, 20, 22, 25, 27, 30, 33, 36, 39, 42, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 150, 200, 300, 350, 400, 500}. For $N \leq 30$, the number of replications is 40,000. For the rest, it is 30,000. For each replication, the first 50 observations are discarded to get rid of the initialization effect.

STEP 2. For each generated sample, conduct the ADF test for the three specifications: i) no constant, no trend, ii) no constant, trend, and iii) constant and trend. The lag orders considered are k = {1, 2, 3, 4, 5, 6, 7, 9}. For $N \leq 25$, $k \leq 5$ is used. For each sample size, lag order, and specification triple, critical values at the 10%, 5%, and 1% levels are calculated.

[1] We used the Ox function `rann` to generate the standard normal errors.

Figure 1: Plots of Monte Carlo Estimated Critical Values



STEP 3. For each lag order and sample size pair, estimate the following response surface equation

$$CR_{N,k} = \tau_0 + \sum_{i=1}^{2} \tau_i \left(\frac{1}{T}\right)^t + \sum_{j=1}^{2} \phi_j \left(\frac{k-1}{T}\right)^j + \epsilon_{N,k} \tag{3}$$

where $CR_{N,k}$ is the finite-sample critical value of the ADF test for the sample size N and the lag order k and $T = N - k$ is the effective number of observations. Notice that τ_0 represents estimated asymptotic critical values. Repeat the estimation for each specification and level pair.

Table 1 shows the replicated response surface regressions for different test specifications and levels. The values under every coefficient are the heteroskedasticity-consistent (HC1) standard errors. All the coefficients are significant at 1% level except the marked three. As can be seen from the table, the critical values and their robust standard errors are quite close to those of [Cheung and Lai, 1995][2]. The signs of the coefficients match perfectly. The estimates of the coefficient τ_2 and their standard errors are the source of the biggest difference. This is not unexpected since it is the coefficient of the inverse of T^2, which can be a very small number, especially for large sample sizes. We also replicate the response surfaces, which are shown in Figure 1[3]. The surfaces confirm

[2] Since the experiment involves generating random numbers, one should not expect exact matches.

[3] In order to make the visual comparison consistent, the critical value axis is set to 0.4 in length in each sub-plot.

[Cheung and Lai, 1995]'s finding that, for a given test size, critical values for the test with no constant and no trend approach their limiting values most rapidly and those for the test with constant and trend most slowly.

3 Conclusion

[Cheung and Lai, 1995] used response surface analysis to obtain approximate finite-sample critical values adjusted for lag order and sample size for the augmented Dickey-Fuller test. In this paper, we have been able to replicate their results reasonably closely. We provide the Ox code to give others an opportunity to double-check our results. The code can be easily modified to be used in new experiments. We also provide a Windows application with a graphical user interface to obtain custom critical values for any sample size and lag order, which should be valued to applied econometricians.

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