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Abstract. This paper studies the aggregate substitution and expansion effects triggered by changes in input prices in a context where firms supply a homogeneous commodity and compete in quantities à la Cournot. We derive a sufficient condition for the existence of a Cournot equilibrium and show that this condition also ensures that the Le Chatelier-Samuelson principle is satisfied in the aggregate at the Cournot equilibrium, although it may not be satisfied at the firm level.

Keywords: Aggregation, returns to scale, market power, markup, own-price elasticity.

JEL Classification: D21, D43.

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1. Introduction

This paper investigates the consequences of input price changes on input demands when the output market is imperfectly competitive. The impact of input price changes on input adjustment is described by the Le Chatelier principle, introduced in economics by Samuelson (1947). This principle states that the sensitivity of input demands with respect to own price variations is smaller when the output level is held constant than when it is adjusted. It is apparently not widely known, however, that Samuelson (1947, p.45-46) showed that the Le Chatelier principle is satisfied whether competition on the output market is perfect or imperfect, provided the production level of competitors is held constant. At the firm level, the Le Chatelier principle attracted the attention of many researchers who derived it by weakening or changing underlying assumptions (see e.g. Eichhorn and Oettli, 1972, Diewert, 1981, and Milgrom and Roberts, 1996). However, these authors did not consider whether the principle is still satisfied when negative externalities between firms affect their behavior. The aim of this paper is to fill this gap in the literature and to extend the Le Chatelier-Samuelson (LCS) principle to the case of Cournot competition with endogenous levels of competitors' output.

For a given level of output, a cost minimizing firm has an incentive to use more intensively the input whose price has decreased and to substitute the cheaper one for the other inputs (the substitution effect). When the firm is less constrained and becomes able to set its output level in order to maximize its profit, it will choose the optimal output level in order to benefit even further from the input price reduction. This adjustment corresponds to an expansion effect. In a competitive output market this expansion effect is always negative, because firms do not consider that the aggregate increase in output induces a drop in the output price. With imperfect competitive output markets à la Cournot, the sign of the expansion effect is ambiguous, because the externality provides incentives to reduce input demand: if all competing firms increase their output level in order to exploit the reduction in input price, the output price must fall, and this reduces each firm's incentives to expand its level of output supply and input demand. Firm level comparative statics are therefore undetermined. Only further restrictions on firm technologies or inverse demand, as discussed by Roy and Sabarwal (2008, 2010), make it possible to obtain well-determined results.

In this paper we show that, despite ambiguous results at the firm level, under Novshek's (1985) type of conditions (which ensure the existence of a Cournot equilibrium), the aggregate expansion effect is negative and the LCS principle is valid in the aggregate Cournot model. The existing literature deriving comparative static results for Cournot oligopolies does not cover this paradox because it investigates comparative statics at the firm level (Dixit, 1986, Hoernig, 2003). We show that aggregation is helpful for resolving the ambiguity at the firm level.

This result can in turn be applied to study the aggregate impact of taxes, subsidies, or, more generally, shocks affecting aggregate demand or firm's cost functions. When firms are heterogenous with respect to their size and technologies, identical and symmetric demand shocks affect them differently: the input demands of smaller firms may shrink while those of bigger firms increase. A related issue has, for instance, been studied by Février and Linnemer (2004) who consider the impact of a cost shock on aggregate profits and welfare. Our paper focuses on input demands, and shows that despite heterogenous reactions at the micro level, the aggregate reaction is well determined.

The next section outlines the microeconomic model and derives the LCS principle at the firm level, when the output market is imperfectly competitive. Section 3 exposes Novshek's (1985) sufficient conditions for the existence of a Cournot equilibrium. Section 4 extends the LCS principle to the case of Cournot competition at the aggregate level, it also describes the aggregate consequences of Cournot competition in terms of input adjustment. Section 5 concludes.

2. Input demands with Cournot competition

The model is developed at the microeconomic level of the production unit. Vector $x \in \mathbb{R}_{+}^{J}$ denotes input quantities and w is the corresponding $J \times 1$ price vector. The production unit's output level is denoted by $y \in \mathbb{R}_{+}$. Under suitable regularity conditions the technology of a cost minimizing production unit is fully described by a twice continuously differentiable cost function c. By definition, $c(w,y) = w^{\intercal} x^{*}(w,y)$, where x^{*} denotes the cost minimizing input vector. The aim of this section is to describe how input demands react to input prices, at the level of the firm.

In an imperfectly competitive product market, the production unit knows the inverse product demand function $p : Y \mapsto p(Y)$ it faces. Let Y_{-h} denote the production level of all competitors to firm h. The profit function π_h is given by

$$\begin{aligned} \pi_h(w, Y_{-h}) &= \max_y \{p(y + Y_{-h})\, y - c_h(w, y)\} && (1) \\ &= p(y_h^o + Y_{-h})\, y_h^o - c_h(w, y_h^o), && (2) \end{aligned}$$

where $y_h^o(w, Y_{-h})$ denotes the optimal solution to (1) and represents the output supply correspondence. One difficulty with y_h^o is that it is not necessarily a function since for some values of (w, Y_{-h}) there might be several profit-maximizing output supplies. In the following we assume that the solution y_h^o is locally unique. At this point we should emphasize that our analysis is purely local, i.e. we consider only small changes in input prices.

Let p' and p'' denote the first and second derivatives of function p. The first order condition for an interior optimum is given by

$$p(y_h^o + Y_{-h}) + p'(y_h^o + Y_{-h})\, y_h^o = \frac{\partial c_h(w, y_h^o)}{\partial y_h}. \tag{3}$$

Output supply changes when the demand function shifts (variation in Y_{-h}) or when the cost parameters w change.

Some authors – reviewed by Appelbaum (1982) and Bresnahan (1989) – consider that this simple framework encompasses a variety of non-competitive pricing behaviors. In this section, we follow the Cournot-Nash conjecture and consider the production level of competitors as fixed while firm h is choosing its optimal production level. Note that Y_{-h} is specific to firm h. A sufficient condition for an interior maximum is that, in addition to (3),

$$a_h^o(w, Y_{-h}) < 0 \tag{4}$$

with

$$a_h^o(w, Y_{-h}) \equiv \left[2p'(y_h^o + Y_{-h}) + p''(y_h^o + Y_{-h})\, y_h^o - \frac{\partial^2 c_h}{\partial y_h^2}(w, y_h^o)\right]^{-1}. \tag{5}$$

Inequality (4) can be fulfilled even in the case of decreasing marginal costs ($\partial^2 c_h / \partial y_h^2 < 0$), provided that the inverse demand function has the adequate shape.

By Hotelling's lemma the input demand functions are given by:

$$x_h^o(w, Y_{-h}) = -\frac{\partial \pi_h}{\partial w}(w, Y_{-h}) = \frac{\partial c_h}{\partial w}(w, y_h^o(w, Y_{-h})) = x^*(w, y_h^o(w, Y_{-h})), \tag{6}$$

where the second equality follows from (3).[1] Thus, just as in the perfect competition case, the constant-output and the unrestricted input demand functions coincide at the optimal output level. Concerning comparative statics, the $J \times J$ matrix of the partial derivatives of the column vector of input demands x_h^o w.r.t. the row vector of input prices $w^\intercal$ can be expressed as:

$$\begin{aligned} \frac{\partial x_h^o}{\partial w^\intercal}(w, Y_{-h}) &= \frac{\partial x_h^*}{\partial w^\intercal}(w, y_h^o) + \frac{\partial x_h^*}{\partial y_h}(w, y_h^o(w, Y_{-h}))\frac{\partial y_h^o}{\partial w^\intercal}(w, Y_{-h}) \\ &= \frac{\partial x_h^*}{\partial w^\intercal}(w, y_h^o) + a^o(w, Y_{-h})\frac{\partial x_h^*}{\partial y_h}(w, y_h^o)\frac{\partial x_h^{*\intercal}}{\partial y_h}(w, y_h^o), \end{aligned} \tag{7}$$

where the second equality follows from the differentiation of (3) with respect to w, yielding:

$$\frac{\partial y_h^o}{\partial w}(w, Y_{-h}) = a_h^o(w, Y_{-h})\frac{\partial x_h^*}{\partial y_h}(w, y_h^o). \tag{8}$$

This allows to obtain the LCS principle in imperfect competition.

Proposition 1. Assuming $a_h^o(w, Y_{-h}) < 0$,
(i) the LCS result is satisfied:

$$\frac{\partial x_{j,h}^o}{\partial w_j}(w, Y_{-h}) \leq \frac{\partial x_{j,h}^*}{\partial w_j}(w, y_h^o) < 0, \tag{9}$$

(ii) an increase in input price w_j decreases the output level iff input demand x_j^* is normal:

$$\partial y_h^o / \partial w_{j,h} < 0 \Leftrightarrow \partial x_{j,h}^* / \partial y_h > 0, \tag{10}$$

(iii) an increase in input price w_j increases the output price $p^o(w, Y_{-h}) \equiv p\left(y_h^o(w, Y_{-h}) + Y_{-h}\right)$ if output demand is decreasing and x_j^* is normal:

$$\left\{p'(Y) < 0 \wedge \partial x_{j,h}^* / \partial y_h > 0\right\} \Rightarrow \partial p_h^o / \partial w_j > 0. \tag{11}$$

Statement *(i)* directly follows from (7), *(ii)* from (8) and *(iii)* from the inverse demand function and (8). This result shows how increases in input prices reduce input demand, which in turn decreases production and creates inflation. Part *(i)* of Propo-

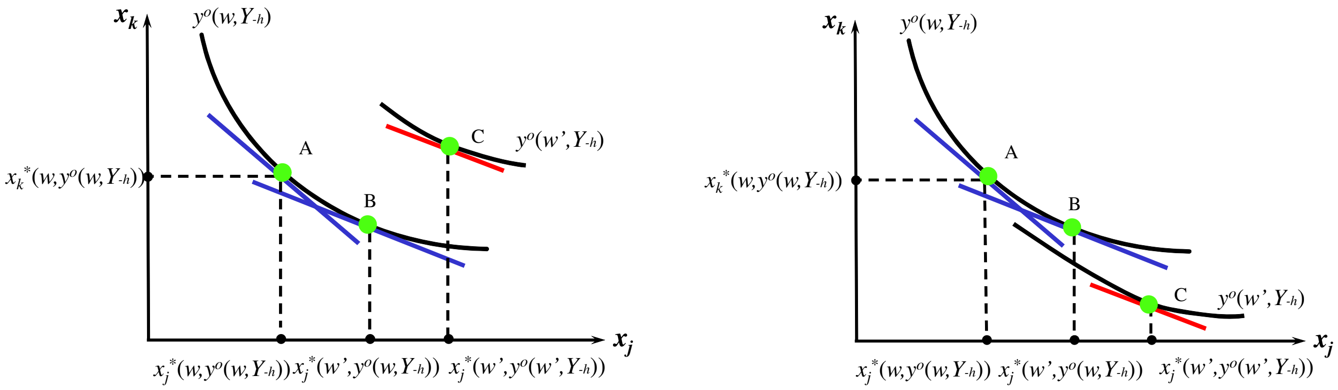
[1] For the convenience of the reader, we summarize Hotelling's lemma in the Appendix.

sition 1 is satisfied without requiring input demand to be normal.[2] The conditions ($a_h^o(w, Y_{-h}) < 0$, $\partial x_j^*/\partial y > 0$ and $p' < 0$) necessary for obtaining statements *(ii)* and *(iii)* of Proposition 1 can be investigated empirically. Note that the comparative static statement of Proposition 1 assumes that Y_{-h} is exogenous.

Samuelson (1947, p.45-46) derived this principle using a revenue function noted $R(x)$, which is compatible with a perfectly competitive output market, when $R(x) = pf(x)$, but also with imperfect competition for $R(x) = p(f(x) + Y_{-h}) f(x)$. A more general formulation of the Le Chatelier principle, yielding Proposition 1*(i)* as a special case, was provided by Eichhorn and Oettli (1972). In comparison to Samuelson's result, the above derivation of the LCS principle has the advantage of relying on the dual: it yields thereby Equation (7) which resembles the Slutsky decomposition in consumer theory.

Figure 1 illustrates the optimal adjustment of output and its implication for the inputs. This figure, presented by Sakai (1973) in the competitive setup, is also valid when production functions are not concave and production units have market power, *as long as Y_{-h} is constant.* The shift from point A to point B along the isoquant corresponding to production level $y^o(w, Y_{-h})$ represents input substitution caused by a decrease in the price of input j from w_j to w_j'. The shift from B to C arises when the production unit chooses the profit-maximizing output level, and depicts the expansion (or scale) effect. For normal inputs, this expansion effect is positive and by (8) it turns out that in this case the production unit increases output to its optimal level $y^o(w', Y_{-h})$. When input j is inferior, the converse applies (see Figure 1b): profit is maximized when the firm decreases output after the decrease of w_j (see 10). Figure 1 illustrates that in both cases the unrestricted move in the j^{th} input demand from $x_j^o(w, Y_{-h})$ to $x_j^o(w', Y_{-h})$ will be larger than the restricted move from $x_j^o(w, Y_{-h})$ to $x_j^*(w', y^o(w, Y_{-h}))$. The LCS principle differs from the Slutsky decomposition because production units maximize profit and not production: in the situation of Figure 1b, profits are maximized by reducing production.

[2] The normality requirement of input demand x_j^* is in fact equivalent to the statement that marginal cost is increasing in w_j.



(a) Input j normal

(b) Input j inferior

Figure 1: Substitution and expansion effects and input adjustment

There are alternative sets of assumptions which yield the conclusions of Proposition 1 (see Milgrom and Roberts, 1996). However, as our objective is to identify both the substitution and scale effects of (7) we rely mainly on duality theory.

In terms of elasticities, (9) becomes

$$\varepsilon\left(x_j^o; w_j\right) \leq \varepsilon\left(x_j^*; w_j\right) < 0, \tag{12}$$

where

$$\varepsilon\left(x_j^o; w_j\right) \equiv \frac{\partial x_j^o\left(w, Y_{-h}\right)}{\partial w_j} \frac{w_j}{x_j^o\left(w, Y_{-h}\right)}.$$

The own-price elasticities of profit-maximizing input demands are smaller than those derived from cost-minimizing input demands. The economic intuition behind this result is that when the output level can be adjusted after a decrease in input price w_j, this change in scale is made in such a way as to fully benefit from the input price reduction, which is achieved by increasing x_j (and y if x_j is normal). Note that input demands are not required to be normal (that is, increasing in the level of output) to obtain the LCS principle.

3. Existence of a Cournot-Nash equilibrium

Changes in input prices affect all firms simultaneously, which in turn affects the inverse output demand function through changes in Y_{-h}. Therefore the result in the former section (derived for constant Y_{-h}) only partly describes the consequences of changes in input prices. We consider an industry that can be relatively well described as a market with competition à la Cournot. Firms produce a similar product and are heterogenous with respect to their cost function, their market power and their market share, measured by y_h/Y. In contrast to the contestable market literature, we do not require that all (potential) firms have access to the same technology.

When products within an industry are perfectly substitutable, all active firms charge or face the same price at equilibrium. The number of incumbent firms H is exogenous. In this setup, some firms make positive profits because they are able to produce more cheaply than others as their technology is more efficient.

In this section we consider strategic interactions between firms and describe their influence on input-demand adjustments. A look at the reaction functions $y_h^o(w, Y_{-h})$, $x_h^o(w, Y_{-h})$ suffices to see that strategic interactions have an important impact on the output and input demand choices. Using (3) and (5), it can be verified that the sign of $\partial y_h^o/\partial Y_{-h}$ is the same as that of $p' + y_h p''$, and for this reason the Cournot game can exhibit a nonmonotone best response, including strategic substitution ($\partial y_h^o/\partial Y_{-h} \leq 0$) and complementarity ($\partial y_h^o/\partial Y_{-h} \geq 0$).

A *Cournot equilibrium* is any H-tuple $y_h^N(w)$ and $x_h^N(w)$ such that, for any active firm, (3) and (4) are satisfied and the product market is cleared. So, at a Cournot equilibrium,

$$y_h^N(w) = y_h^o\left(w, Y_{-h}^N(w)\right), \quad x_h^N(w) = x_h^*\left(w, y_h^N(w)\right) = x_h^o\left(w, Y_{-h}^N(w)\right). \tag{13}$$

If we assume that y_h^o is a continuous function in Y_{-h} for every w, then Brouwer's fixed point theorem can usually be applied to show that a Cournot equilibrium exists. However, in Cournot oligopolistic markets, it is restrictive to assume that y_h^o is a continuous function, because the profit function is not necessarily concave in y_h for all values of (y_h, Y_{-h}, w). Several economists have tried to get around the assumption of concave profits to prove the existence of a Cournot equilibrium.

Novshek (1985) has shown that a H-firm Cournot equilibrium exists provided that a "firm's marginal revenue be everywhere a declining function of the aggregate output of others", that is:

$$p'(y_h + Y_{-h}) + y_h p''(y_h + Y_{-h}) \leq 0. \tag{14}$$

This condition also implies that firms' reaction functions $y_h^o(w, Y_{-h})$ are nonincreasing in Y_{-h}. Inequality (14) is, for instance, satisfied if the (nonincreasing) inverse demand function is linear or concave in y, in which case the existence of a Cournot equilibrium is guaranteed. Since this condition has to be satisfied for any value of y_h and Y_{-h}, it can equivalently be written as

$$p'(Y) + Yp''(Y) \leq 0, \tag{15}$$

for any Y. This inequality depends on aggregate data only and implies that condition (14) is fulfilled for *any* firm.[3]

There are two difficulties with this aggregate condition. On the one hand (15) is sufficient for the existence of a Cournot equilibrium, but not necessary, and is therefore not the weakest possible condition for achieving existence. On the other hand, the fact that (14) has to be satisfied for any value of y_h and Y_{-h}, is very demanding. It must even be satisfied for the case in which one firm produces the total output, which could reasonably be excluded if there is a competition law enforcing an upper bound for the market share, or alternatively, if the firms' cost functions lead them to always choose an output level which is smaller than Y. We therefore first derive a new condition which ensures the existence of a Cournot equilibrium.

Proposition 2. Assume that for any firm, there is a maximal capacity $\overline{y}$, so that no firm chooses $y_h > \overline{y}$. If

$$p'(Y) + p''(Y)\, y_h \leq 0, \tag{16}$$

for any $0 \leq y_h \leq \overline{y}$ then a Cournot equilibrium exists.

Proposition 2 is a reformulation of Novshek's (1985) Theorem 3.[4] The assumption $y_h \leq \overline{y}$ implies that aggregate output is bounded from above by $H\overline{y}$. The proof of Proposition 2 follows from the fact that the output space $[0, \overline{y}]^H$ is a complete lattice, and imposing y_h to be included in the interval $[0, \overline{y}]$ still yields a reaction correspondence that is nonincreasing in Y_{-h} just as in Novshek's case. The first interesting consequence of Proposition 2 is that we can derive a simple and testable aggregate condition, which implies that (16) is satisfied for any firm, and which is weaker than condition (15) obtained by Novshek (in some sense, see footnote 4). Condition (16) is trivially valid if

[3] Amir (1996) provided an alternative sufficient condition ensuring the existence of Cournot's equilibrium. This condition is discussed and empirically investigated by Koebel and Laisney (2012).

[4] The new requirement that firm's choice is bounded above, $y_h \leq \overline{y}$, is weaker than Novshek's condition $\{\exists \overline{Y} : p(\overline{Y}) = 0\}$, but in the absence of a regulatory authority (instead of a maximum capacity, $\overline{y}$ can be interpreted as a firm's maximum output level that a competition commission tolerates in this oligopoly market), the condition $y_h \leq \overline{y}$ puts restrictions on firms' cost functions, in contrast to Novshek's approach.

$p'' \leq 0$ since $p' \leq 0$. If $p'' > 0$, then (16) is implied by the aggregate condition

$$p'(Y) + p''(Y)\,Y\sqrt{\mathbb{H}} \leq 0, \tag{17}$$

for any aggregate and elementary output levels $Y \leq H\overline{y}$ and $y_h \leq \overline{y}$ compatible with the Hirschman-Herfindahl index of concentration $\mathbb{H}$. This is because the highest possible market share $\overline{y}/Y$ satisfies $\overline{y}/Y \leq \sqrt{\mathbb{H}}$, and so $y_h \leq \overline{y} \leq Y\sqrt{\mathbb{H}}$ for any h. Provided the restriction on the distribution of output is valid ($y_h \leq \overline{y}$), condition (17) is weaker than (15), in the sense that for a given value of Y, it is satisfied for a broader set of values for p' and p'' than (15).

4. Aggregate comparative statics

Firm level comparative statics have been studied by Roy and Sabarwal (2008, 2010) who derive conditions ensuring monotone comparative statics at the firm level in games with strategic substitutes. In Section 2 (Proposition 1), we show that the LCS principle is satisfied at the firm level for a given level of the aggregate production of all competitors. At a Cournot equilibrium, however, the total impact of a change in input prices follows from (13):

$$\begin{aligned}
\frac{\partial y_h^N}{\partial w}(w) &= \frac{\partial y_h^o}{\partial w}\left(w, Y_{-h}^N(w)\right) + \frac{\partial y_h^o}{\partial Y_{-h}}\frac{\partial Y_{-h}^N}{\partial w}(w) && (18)\\
\frac{\partial x_h^N}{\partial w}(w) &= \frac{\partial x_h^*}{\partial w}\left(w, y_h^N(w)\right) + \frac{\partial x_h^*}{\partial y_h}\frac{\partial y_h^N}{\partial w}(w) && (19)\\
&= \frac{\partial x_h^*}{\partial w}\left(w, y_h^N(w)\right) + \frac{\partial x_h^*}{\partial y_h}\frac{\partial y_h^o}{\partial w}\left(w, Y_{-h}^N(w)\right) + \frac{\partial x_h^*}{\partial y_h}\frac{\partial y_h^o}{\partial Y_{-h}}\frac{\partial Y_{-h}^N}{\partial w}(w).
\end{aligned}$$

Even if $\partial y_h^o/\partial w < 0$ (see Proposition 1*(ii)*) we cannot be sure that $\partial y_h^N/\partial w \leq 0$ unless the last term in (18), corresponding to a change in firm h's output triggered by the strategic interaction with all other firms, does not outweigh the direct impact of an increase in w. As this last term can be positive or negative, the overall sign of $\partial y_h^N/\partial w$ is undetermined. The same remark applies to (19) since a further and indeterminate "externality-induced input adjustment" is added to the substitution and expansion effects of (7) and this explains why the LCS principle is not necessarily satisfied at the firm level. It will now be interesting to analyze whether the LCS principle holds at the aggregate level of the industry.

4.1 Cournot equilibrium

We show that the LCS principle is satisfied in the aggregate, provided some additional and plausible regularity conditions hold. Let us define the aggregate input demand functions X^* for fixed levels of individual production as:

$$X^*\left(w, \{y_h\}_{h=1}^H\right) \equiv \sum_{h=1}^H x_h^*\left(w, y_h\right).$$

Similarly, let $Y^N\left(w\right)$ and $X^N\left(w\right)$ denote the aggregate Nash equilibrium outcome. We are now able to describe how these aggregate quantities vary with w (see the Appendix for a proof).

Proposition 3. The impact of a change in w on the Cournot equilibrium aggregate quantities is given by:

$$\frac{\partial Y^N}{\partial w}\left(w\right) = b^N\left(w\right)\sum_h^H a_h^N\left(w\right)\frac{\partial x_h^*}{\partial y}\left(w, y_h^N\left(w\right)\right) \tag{20}$$

$$\begin{aligned}\frac{\partial X^N}{\partial w^\top}\left(w\right) &= \frac{\partial X^*}{\partial w^\top}\left(w, \left\{y_h^N\left(w\right)\right\}_{h=1}^H\right) \\ &+\sum_h^H a_h^N\left(w\right)\frac{\partial x_h^*}{\partial y}\left(w, y_h^N\left(w\right)\right)\frac{\partial x_h^{*\top}}{\partial y}\left(w, y_h^N\left(w\right)\right) \\ &+b^N\left(w\right)\sum_h^H a_h^N\left(w\right)\left(p'\left(Y^N\left(w\right)\right)+p''\left(Y^N\left(w\right)\right)y_h^N\left(w\right)\right)\frac{\partial x_h^*}{\partial y}\left(w, y_h^N\left(w\right)\right)\sum_k^H a_k^N\frac{\partial x_k^{*\top}}{\partial y}\left(w, y_k^N\left(w\right)\right),\end{aligned} \tag{21}$$

where

$$a_h^N\left(w\right) \equiv \left[p'\left(Y^N\left(w\right)\right)-\frac{\partial^2 c_h}{\partial y_h^2}\left(w, y_h^N\left(w\right)\right)\right]^{-1} \tag{22}$$

$$b^N\left(w\right) \equiv \left[1+\sum_h^H a_h^N\left(p'\left(Y^N\left(w\right)\right)+p''\left(Y^N\left(w\right)\right)y_h^N\left(w\right)\right)\right]^{-1}. \tag{23}$$

The three matrices involved in the LCS decomposition (21) have an interesting interpretation: the first corresponds to the impact of w on X^N keeping all individual output levels constant; the second matrix represents the impact on X^N due to the adjustment of the individual output levels; and the third matrix describes the consequence of the output price adjustment on X^N (it vanishes if p is constant). At first sight, Proposition 3 looks too intricate to be useful. However, if we adopt an assumption made in many contributions to oligopoly theory:[5]

$$a_h^N(w) < 0, \tag{24}$$

then, together with (14), it turns out that

$$b^N(w) > 0. \tag{25}$$

These inequalities place some structure on (20)-(21) and guarantee that the second matrix on the RHS of (21) is negative semidefinite so that the determinateness of $\partial X^N / \partial w^\top$ depends on the third matrix.

Corollary 1. Assume that inequalities (4), (16), and (24) are satisfied at the Cournot equilibrium, and that either

(i) all firms are in a symmetric Nash equilibrium for any value of w,

(ii) all firms exhibit normal input demand functions x_h^*,[6]

(iii) the output demand function is linear.

Then the own-price elasticity of aggregate input demand is greater (in absolute value) than for fixed levels of outputs:

$$\varepsilon\left(X_j^N; w_j\right) \leq \varepsilon\left(X_j^*; w_j\right) \leq 0. \tag{26}$$

Note that conditions *(i)*-*(iii)* of Corollary 1 are not equivalent (Example 1 below illustrates that *(ii)* and *(iii)* together do not imply *(i)*). Corollary 1 gives three sufficient conditions which ensure that the last matrix of (21) is negative semidefinite. It is important to note that it is actually not necessary for the third matrix on the RHS of (21) to be negative semidefinite for obtaining (26).

As a referee pointed out, condition *(i)* is too specific to be interesting since under symmetry, individual comparative statics are determinate if and only if aggregate comparative statics are determinate. Condition *(ii)* introduces the assumption of normal input demands, and relaxes the restriction on heterogeneity. It is compatible with a wide variety of heterogenous technologies, for instance, any homothetic production function is appropriate. If $y_h = g_h(f_h(x_h))$ where f_h is homogeneous of degree one

[5] See for instance Szidarovszky and Yakowitz (1977), or Vives (1999, p.99).

[6] It can be shown that condition *(ii)* also implies that the elasticity of aggregate output with respect to the input price is negative, $\varepsilon\left(Y^N; w_j\right) \leq 0$.

and g_h is strictly increasing, then the cost and input demand functions take the form $c_h(w, y_h) = d_h(w) g_h^{-1}(y_h)$ and $x^*(w, y) = e_h(w) g_h^{-1}(y_h)$ with $e_h(w) \equiv \partial d_h/\partial w(w)$. Condition *(ii)* guarantees that $\partial y_h^N/\partial w \leq 0$ and $\partial x_h^*/\partial y_h \geq 0$ for any firm, in which case the LCS principle is satisfied at the micro level by (19) and, as a consequence, also in the aggregate. Condition *(ii)* also implies that the elasticity of aggregate output with respect to the input price is negative, $\varepsilon(Y^N; w_j) \leq 0$.

The assumptions underlying this corollary are quite strong, and can be rejected *prima facie*. However, Proposition 3 shows that the LCS principle is more generally valid in the aggregate, without requiring the strong restrictions given in the above corollary. We therefore have good reasons to expect that aggregate input j and output are decreasing when the price of input j increases. This result is not straightforward in an imperfectly competitive context. On the one side, a firm has an incentive to increase its own output and input levels in reaction to decreases in its competitors' output and input levels following an increase in input prices: $\partial y_h^o/\partial Y_{-h} \leq 0$ and $\partial x_h^o/\partial Y_{-h} \leq 0$. On the other side, naive intuition suggests that any ambiguity at the micro level should be inherited at the macro level.

These results show that strategic interaction, more than heterogeneity, hampers determinate comparative statics at the firm level and, to a lesser extent, in the aggregate. Let us consider the case of a monopoly, which annihilates both issues of heterogeneity and strategic interaction. Then, determinate comparative statics hold (by Proposition 1) for this monopoly. If we now aggregate several such monopolies (from disjoint markets), with technologies which can be *arbitrarily* different, then the aggregate input demand for these monopolies still obeys the LCS principle, because there is no strategic interation between them, and adding up nonincreasing functions yields a nonincreasing aggregate function. Heterogeneity is therefore not the source of the problem, but rather the strength of strategic interaction. In the competitive case, Heiner (1982) showed the validity of the LCS principle when the output price adjusts to clear the market, without requiring any restriction on individual technologies (see Section 4.2 below for a discussion).

The following example illustrates why the LCS can be valid in the aggregate without being necessarily satisfied at the firm level.

Example 1. In a standard Cournot duopoly with linear inverse demand $p = a - b(y_1 + y_2)$, and cost function $c_h(w, y_h) = d_h(w)\, y_h$, the reaction functions are given by:

$$y_h^o(w, y_{-h}) = \frac{a - by_{-h} - d_h(w)}{2b},$$

and the Cournot equilibrium is:

$$y_h^N = \frac{a - 2d_h(w) + d_{-h}(w)}{3b}.$$

What happens when w increases? At the firm level the impact on y_h^N is undetermined, but at the aggregate level, the impact is negative, because:

$$Y^N(w) = \frac{2a - d_1(w) - d_2(w)}{3b},$$

decreases when w increases (since d_h is increasing in w). This example is a special case of Corollary 1*(ii)* and *(iii)* as both firms have a technology with constant returns to scale and output demand is linear.

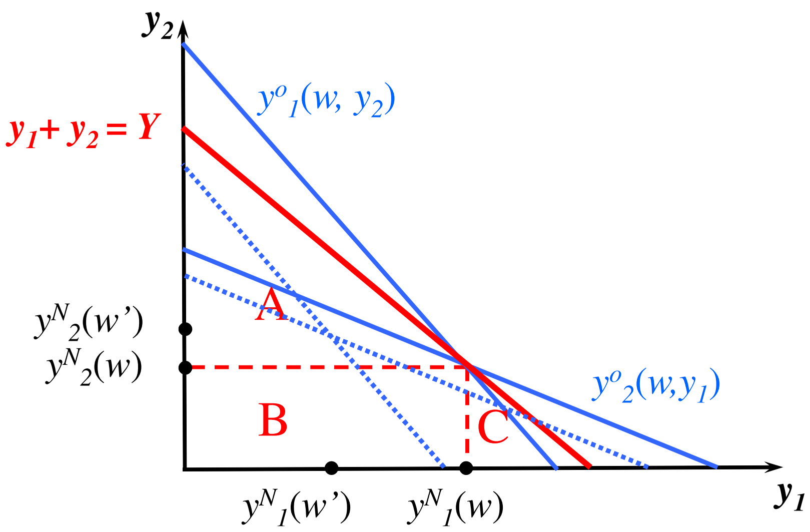


Figure 2: Comparative statics at the firm level and in the aggregate

The reaction curves and Nash equilibria are depicted in Figure 2. This figure also includes the iso-output line $y_1 + y_2 = Y$ going through the aggregate Nash equilibrium Y^N. Any point below this line corresponds to a smaller aggregate output level than Y^N. When w increases to w', the reaction functions are shifted downwards (dotted lines) because $d_h(w)$ increases in w. The new Cournot equilibrium is reached at the intersection of the dotted reaction curves, in one out of the three areas A, B, C. In triangle A the output level of firm 2 increases and the output of firm 1 decreases. In rectangle B the output levels of both firms decrease, and in triangle C the output levels of both firms go in opposite directions. In all three cases, however, the total output level $y_1^N + y_2^N$ decreases after a price increase from w to w'.

Note that the result depicted in Figure 2 does not decisively depend upon the slope of the reaction function as such a figure can also be obtained for both y_h^o increasing in Y_{-h} or when one reaction function is increasing and the other decreasing in Y_{-h}. The important ingredient for obtaining the aggregate comparative statics result is that at least one reaction function is shifted downwards after an increase in w_j, which is ensured if x_j^* is normal (see Proposition 1*(ii)*).[7]

Since in Example 1, micro input demands $x_h^*(w, y_h)$ are proportional to y_h, a similar figure can be drawn in the (x_1, x_2) coordinate plane. Regarding aggregate inputs, we can write (21) as follows:

$$\frac{\partial X_j^N}{\partial w_j}(w) = \frac{\partial X_j^*}{\partial w_j}\left(w, y_1^N(w), y_2^N(w)\right) - \frac{1}{b}\sum_{h=1}^{2}\left(\frac{\partial x_{jh}^*}{\partial y}\left(w, y_h^N(w)\right)\right)^2 - \frac{b}{3}\left(\sum_{h=1}^{2}\frac{1}{b}\frac{\partial x_{jh}^*}{\partial y}\left(w, y_h^N(w)\right)\right)^2.$$

All three terms on the RHS of the equality are negative, and the LCS principle holds.□

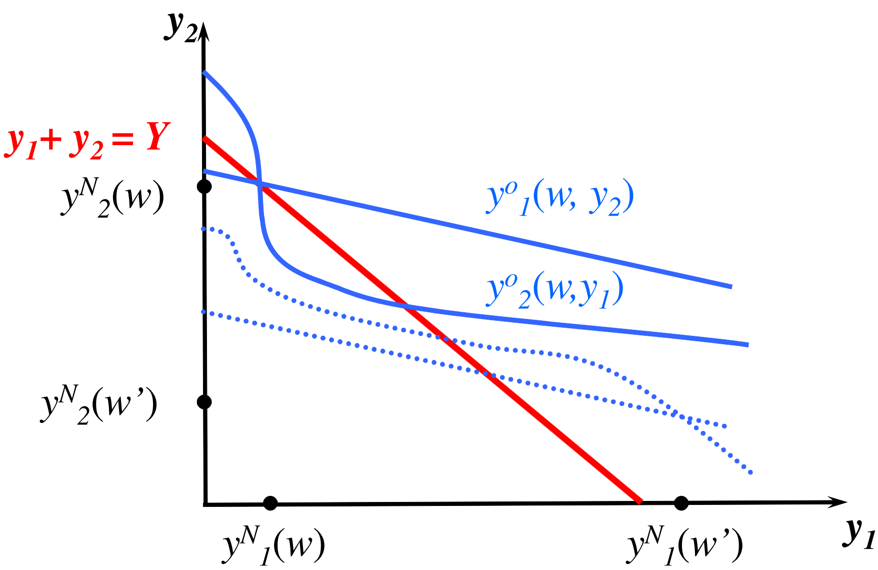


Figure 3. A counter-example to Corollary 1

Aggregate comparative statics, however, becomes tricky when $p''(Y) \neq 0$, and Figure 3 illustrates that the claim of Corollary 1 can be violated in these nonlinear cases. This

[7] The result can still be satisfied if only one of the reaction curves is shifted upwards, but not when all reaction functions shift upwards when w_j increases. This example shows that $\partial Y^N/\partial w_j \leq 0$ is satisfied in more general contexts than those of supermodular or submodular games.

counter-example works because both reaction curves y_1^o and y_2^o cross the iso-aggregate output line $y_1+y_2=Y$ after w increases to w'. In this counterexample the market shares of both firms are drastically shifted by a marginal change in w, which is empirically not often observed.

There are three reasons why inequalities (26) can be violated at a Cournot equilibrium. First, a Cournot equilibrium can exist even if (14) or (17) is violated, in which case $\varepsilon\left(X_j^N;w_j\right)$ may become positive. However, the validity of (17) can be investigated empirically. In the case where (17) cannot be rejected, this provides evidence both for the existence of a Cournot equilibrium and for the validity of the aggregate LCS principle. Nonnormal input demands represent a second source of violation. Third, with multiple equilibria, large shocks on w can shift the economy from one Nash equilibrium to the other. However, as clearly emphasized, the above analysis is only valid locally around the initial equilibrium.

These results and figures help understand why some firms with market power (like recently Deutsche Post in Germany) are pushing trade-unions and the government to *increase (or introduce)* a minimum wage. The resulting increase in labor cost for the lobbying firm can be compensated by an increase in its market share (and even profits) because it hurts the competitors more than the firm itself.

It is of course possible to find weaker sufficient conditions than those given in Corollary 1, at the cost however of being economically less intuitive to interprete. One possibility is to make a plausible assumption on average input demand sensitivity w.r.t. output. For instance, assuming that $\sum_h^H a_h^N\left(p'+p''y_h^N\right)\partial x_h^*/\partial y$ is positive and that $\sum_k^H a_k^N \partial x_h^*/\partial y$ is a negative vector also yields the aggregate LCS principle and this assumption is clearly weaker than Corollary 1*(ii)*.

4.2 Input demand reactivity and degree of competition

In order to better understand the role played by imperfect competition in obtaining the results above, let us compare the Cournot outcome with the benchmark of a market where all firms are in perfect competition. This case was studied by Heiner (1982) and Braulke (1984). We present an alternative derivation of their main result and compare it to ours below.

In perfect competition, the price level $\overline{p}$ is exogenous at the level of production unit h, and the profit maximising output supply $\overline{y}_h(\overline{p}, w)$ satifies

$$\overline{p} = \frac{\partial c_h}{\partial y_h}\left(w, \overline{y}_h(\overline{p}, w)\right), \tag{27}$$

with

$$\overline{a}_h(\overline{p}, w) \equiv \left[-\frac{\partial^2 c_h}{\partial y_h^2}\left(w, \overline{y}_h(\overline{p}, w)\right)\right]^{-1} < 0. \tag{28}$$

The aggregate product supply $\overline{Y}(\overline{p}, w)$ is related to the profit-maximising output supplies (27) by $\overline{Y}(\overline{p}, w) \equiv \sum_{h=1}^{H} \overline{y}_h(\overline{p}, w)$. The competitive output price level $p^c(w)$ is the solution in $\overline{p}$ to the market clearing condition:

$$\overline{p} = p\left(\overline{Y}(\overline{p}, w)\right), \tag{29}$$

where p still represents the inverse demand function. The corresponding microeconomic and aggregate competitive equilibrium output levels are denoted by

$$\begin{aligned} y_h^c(w) &= \overline{y}_h\left(p^c(w), w\right) && (30)\\ Y^c(w) &= \overline{Y}\left(p^c(w), w\right) = \sum_h^H y_h^c(w). && (31) \end{aligned}$$

Similarly, the microeconomic and aggregate equilibrium input quantities are given by

$$\begin{aligned} x_h^c(w) &\equiv x_h^*\left(w, y_h^c(w)\right), && (32)\\ X^c(w) &\equiv \sum_h^H x_h^c(w). && (33) \end{aligned}$$

If we evaluate (27) at the equilibrium price $\overline{p} = p^c(w)$ and differentiate w.r.t. w, we obtain:

$$\frac{\partial y_h^c}{\partial w}(w) = \overline{a}_h\left(p^c(w), w\right)\left(\frac{\partial x_h^*}{\partial y}\left(w, y_h^c(w)\right) - p'\left(Y^c(w)\right)\frac{\partial Y^c}{\partial w^{\intercal}}(w)\right), \tag{34}$$

which can be compared with (39) in imperfect competition.

Whereas at the microeconomic level it is not possible to say how $x_h^c(w)$ or $y_h^c(w)$ vary with w, because the output-price response effect is indeterminate, Heiner (1982) and Braulke (1984) have shown that this effect is well determined in the aggregate. "This reassuring effect [...] represents one of the few cases where an ambiguity at the micro level is resolved at the macro level by aggregation" (Braulke, 1984, p.75). Summing up (34) over all firms yields the aggregate impact of a change of input price on output:

$$\frac{\partial Y^c}{\partial w} = \left(1 + p'\sum_h^H \overline{a}_h\right)^{-1} \sum_h^H \overline{a}_h \frac{\partial x_h^*}{\partial y}.$$

Although the sign of $\partial Y^c/\partial w$ is indeterminate (unless one assumes that input demands are normal), the aggregate impact of input prices on input demands is well determined and given by

$$\begin{aligned}
\frac{\partial X^c}{\partial w^\intercal}(w) &= \sum_h^H \frac{\partial x_h^*}{\partial w^\intercal}(w, y_h^c(w)) + \sum_h^H \frac{\partial x_h^*}{\partial y}(w, y_h^c(w)) \frac{\partial y_h^c}{\partial w^\intercal}(w) \qquad (35) \\
&= \frac{\partial X^*}{\partial w^\intercal}\left(w, \{y_h^c\}_{h=1}^H\right) + \sum_h^H \overline{a}_h \frac{\partial x_h^*}{\partial y}\frac{\partial x_h^{*\intercal}}{\partial y} + \frac{p'}{1+p'\sum_h^H \overline{a}_h}\left[\sum_h^H \overline{a}_h \frac{\partial x_h^*}{\partial y}\right]\left[\sum_h^H \overline{a}_h \frac{\partial x_h^*}{\partial y}\right]^\top.
\end{aligned}$$

The first equality is a consequence of identity (32), the second equality is obtained after substituting $\partial Y^c/\partial w^\intercal$ into the expression of $\partial y_h^c/\partial w^\intercal$. The last term on the RHS corresponds to the adjustement of output price p^c and is negative semidefinite. Altogether we have shown that with perfect competition on the output market:

$$\frac{\partial X^c}{\partial w^\intercal}(w) \ll \frac{\partial X^*}{\partial w^\intercal}\left(w, \{y_h\}_{h=1}^H\right) \ll 0. \qquad (36)$$

These inequalities mean that in the aggregate, output quantity and price adjustment amplify the shock in input prices on X^*, and the reactions in aggregate input quantities then become more important than for constant output and price level. With perfect competition, no restrictions on firms' heterogeneity (technology or production level) is necessary to obtain monotone comparative statics in the aggregate.

How do the non-competitive aggregate input reactions $\partial X^N/\partial w^\top$ compare to the corresponding matrix $\partial X^c/\partial w^\top$ obtained with a perfect competitive output market? Some authors, like Cahuc and Zylberberg (2004, p.186) rely on (7) in order to argue that the expansion effect "diminishes in absolute terms" when market power rises. This claim is true *ceteris paribus*, that is, when the technology is independent from market power, but it is not necessarily satisfied otherwise. As a consequence, their conjecture will not necessarily be satisfied at the aggregate level of an industry, where the link between the degree of competition and the size of the expansion effect becomes an empirical issue. This quantification is the purpose of the companion paper Koebel and Laisney (2013).

In summary, this subsection shows that competitive output markets do not necessarily exhibit more variability than less competitive markets.

5. Conclusion

Output adjustments have important consequences on input demands. This impact, however, is rarely considered in economic contributions, because with imperfectly competitive output markets, increasing returns to scale and externalities disturb the usual representative firm's comparative statics. This paper derives the circumstances under which the LCS principle holds in an aggregate Cournot economy with heterogenous firms.

Whereas at the firm level the impact of changes in the input price on output supply and input demand is ambiguous, aggregation over firms belonging to the same industry has a regularizing effect. We show that when a Cournot equilibrium exists, then aggregate input demand is decreasing in the price of this input under plausible conditions. In the oligopoly case, restricting heterogeneity is a way to canalise the price adjustment effect and enforce the LCS principle in the aggregate.

6. Appendix: Proofs of results

Hotelling's lemma with imperfect competition. We derive Equation (6) using the definition of the profit function,

$$\pi_h (w, Y_{-h}) = p\left(y_h^o (w, Y_{-h}) + Y_{-h}\right) y_h^o (w, Y_{-h}) - c_h\left(w, y_h^o (w, Y_{-h})\right).$$

It implies that the impact of a marginal change in input prices on profit is given by

$$\begin{aligned} \frac{\partial \pi_h}{\partial w} (w, Y_{-h}) &= \left[p'\left(y_h^o (w, Y_{-h}) + Y_{-h}\right) y_h^o (w, Y_{-h}) + p\left(y_h^o (w, Y_{-h}) + Y_{-h}\right)\right] \frac{\partial y_h^o}{\partial w} (w, Y_{-h}) \\ &\quad - \frac{\partial c_h}{\partial w}\left(w, y_h^o (w, Y_{-h})\right) - \frac{\partial c_h}{\partial y}\left(w, y_h^o (w, Y_{-h})\right) \frac{\partial y_h^o}{\partial w}\left(w, Y_{-h} (w, Y_{-h})\right) \\ \Leftrightarrow \quad \frac{\partial \pi_h}{\partial w} (w, Y_{-h}) &= -x_h^o (w, Y_{-h}), \end{aligned}$$

where the last line is a consequence of (3) and Shephard's lemma, which states that cost minimizing input demands x_h^o coincide with the partial derivatives of the cost function w.r.t. w.

Proof of Proposition 3.

Differentiating

$$p\left(Y^N (w)\right) + p'\left(Y^N (w)\right) y_h^N (w) = \frac{\partial c_h\left(w, y_h^N (w)\right)}{\partial y} \tag{37}$$

with respect to w, we obtain (suppressing the arguments in the result)

$$p'\frac{\partial Y^N}{\partial w}+p''\frac{\partial Y^N}{\partial w}y_h^N+p'\frac{\partial y_h^N}{\partial w}=\frac{\partial x_h^*}{\partial y}+\frac{\partial^2 c_h}{\partial y^2}\frac{\partial y_h^N}{\partial w}, \tag{38}$$

and it turns out that

$$\frac{\partial y_h^N}{\partial w}=a_h^N\left(\frac{\partial x_h^*}{\partial y}-\left(p'+p''y_h^N\right)\frac{\partial Y^N}{\partial w}\right). \tag{39}$$

Summing up (39) over all firms yields the impact of a change of input price on aggregate output:

$$\frac{\partial Y^N}{\partial w}=\left(1+\sum_h^H a_h^N\left(p'+p''y_h^N\right)\right)^{-1}\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}=b^N\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}.$$

The impact of input prices on aggregate input demands is given by

$$\begin{aligned}\frac{\partial X^N}{\partial w^\intercal} &= \sum_h^H\frac{\partial x_h^*}{\partial w^\intercal}+\sum_h^H\frac{\partial x_h^*}{\partial y}\frac{\partial y_h^N}{\partial w^\intercal}\\ &= \frac{\partial X^*}{\partial w^\intercal}+\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}\frac{\partial x_h^{*\intercal}}{\partial y}+b^N\left[\sum_h^H a_h^N\left(p'+p''y_h^N\right)\frac{\partial x_h^*}{\partial y}\right]\left[\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}\right]^\top.\end{aligned}$$

□

Proof of Corollary 1.

(i) When firms are in a symmetric Nash equilibrium, $y_h^N=y^N$ for any h, and so the last matrix of (21) can be written as

$$b^N\left[\sum_h^H a_h^N\left(p'+p''y_h^N\right)\frac{\partial x_h^*}{\partial y}\right]\left[\sum_h^H a_h^N\frac{\partial x_h^{*\top}}{\partial y}\right]=b^N\left(p'+p''y^N\right)\left[\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}\right]\left[\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}\right]^\top$$

which is negative semidefinite.

(ii) When input demands are normal, $\partial x_{hi}^*/\partial y>0$ for any w, any firm h and any input j. Then there exist J numbers $k_j<0$ such that

$$\sum_h^H a_h^N\left(p'+p''y_h^N\right)\frac{\partial x_{hj}^*}{\partial y}=k_j\sum_h^H a_h^N\frac{\partial x_{hj}^*}{\partial y}.$$

In fact, this equation defines k_j. Let D_k be a diagonal matrix with $(k_1,\ldots,k_J)^\top$ on the diagonal. So it turns out that

$$b^N\left[\sum_h^H a_h^N\left(p'+p''y_h^N\right)\frac{\partial x_h^*}{\partial y}\right]\left[\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}\right]^\top=b^N D_k\left[\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}\right]\left[\sum_h^H a_h^N\frac{\partial x_h^*}{\partial y}\right]^\top,$$

and all entries on the diagonal of this matrix are negative. However, the matrix itself is not negative semi-definite, as can be checked in the case $J=2$ when the two components of $\partial x_h^*/\partial y$ coincide. With simplified notations, we look at the eigenvalues of the symmetrized matrix $DUU^\top+UU^\top D$ and with $D=\operatorname{diag}(a,b)$ and $U^\top=(u,u)$. These are $u^2\left(a+b-\sqrt{2}\sqrt{a^2+b^2}\right)\leq 0$ and $u^2\left(a+b+\sqrt{2}\sqrt{a^2+b^2}\right)$ which is strictly positive for $a\neq b$.

(iii) The proof is similar to point *(i)* but with $p'' = 0$. □

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